Modeling dynamical fluctuations in heavy-ion collisions

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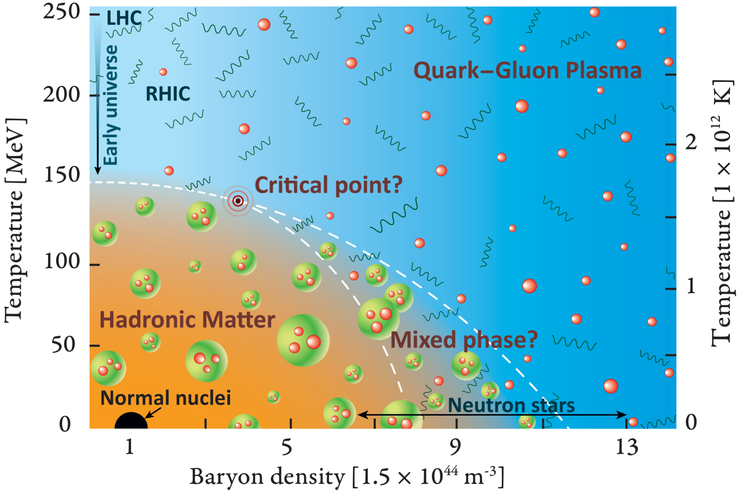
1. Phase transitions in QCD
2. Finding the critical point
3. Finding the first-order phase transition
4. Nonequilibrium Chiral Fluid Dynamics
5. Box calculations
6. Expanding plasma

The QCD phase diagram



Symmetries and order parameters in QCD

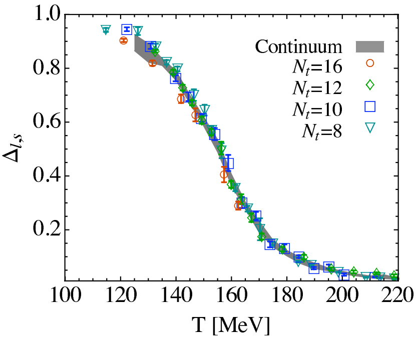
$SU(2)_V \times SU(2)_A$ chiral symmetry

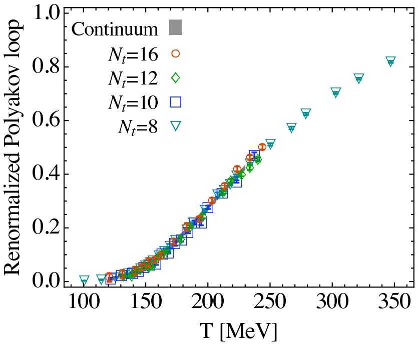
- Explicitly broken by m_q
- Approximate symmetry for small m_q
- Order parameter:
  Chiral condensate $\langle \bar{q}q \rangle$, sigma field σ

Z_{N_c} center symmetry of $SU(N_c)$ gauge group

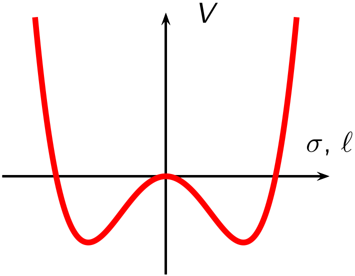
- Only exact in pure gauge theory
- Approximate symmetry for large m_q
- Order parameter for confinement-deconfinement phase transition:
  Polyakov loop $\ell = \frac{1}{N_c} \langle tr_c \mathcal{P} \rangle_\beta$ with $\mathcal{P} = P \exp\left(i g_{QCD} \int_0^\beta \mathrm{d}\tau A_0 \right)$

Symmetries and order parameters in QCD





Phase transitions



First-order phase transition

- Two degenerate minima at T_c
- Phase coexistence

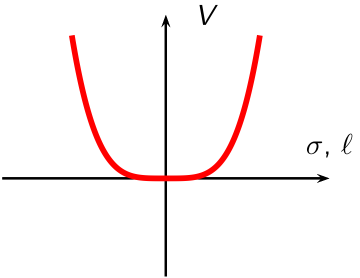
Nucleation

- Supercooled, $\frac{\partial^2 V}{\partial \sigma^2} > 0$
- Large fluctuations
- Bubble formation and growth

Spinodal decomposition

- Unstable, $\frac{\partial^2 V}{\partial \sigma^2} < 0$
- Small fluctuations
- Phase separation uniformly

Phase transitions

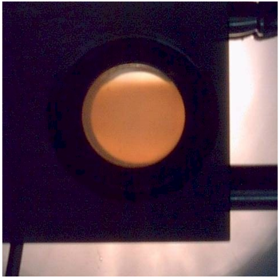


Critical point

- $m_\sigma = 0$
- Divergent correlation length

Critical phenomena

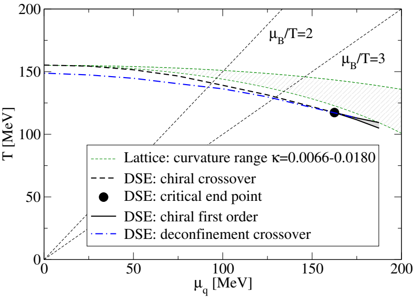
- Divergent susceptibilities
- Critical slowing down
- Long-range fluctuations
- Critical opalescence

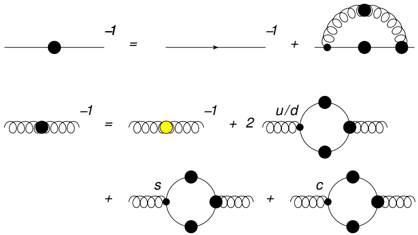


Finding the critical point - theory

1. First principle calculations

- Solve partition function $\mathcal{Z}$ on a lattice (sign problem for finite μ)
- Solve Dyson-Schwinger equations



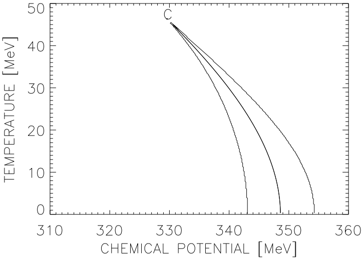


(Fischer, Luecker, Welzbacher, Phys. Rev. D **90** (2014))

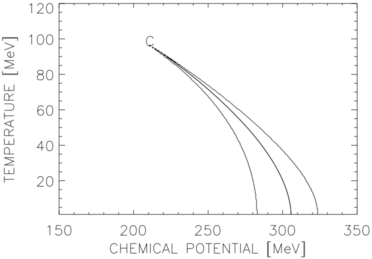
Finding the critical point - theory

2. Effective models

- Respect chiral symmetry (NJL model, Linear sigma model)
- Existence/location of CP not universal!



(Mocsy et al. Rev. C **81** (2010) 045201)

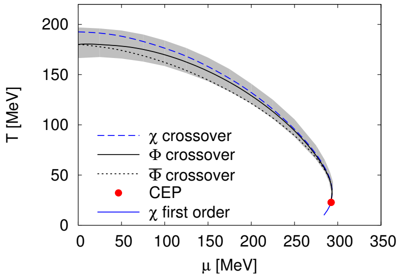


(Mocsy et al. Rev. C **81** (2010) 045201)

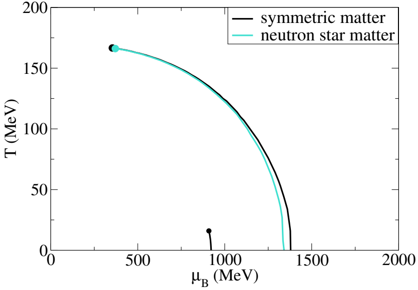
Finding the critical point - theory

2. Effective models

- Extension with Polyakov loop, baryonic degrees of freedom
- Existence/location of CP not universal!



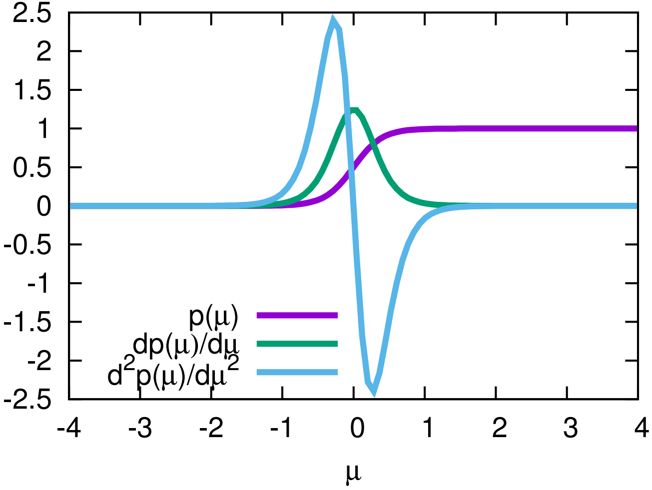
(Herbst, Pawlowski, Schaefer, Phys. Lett. B **696** (2011) 58-67)



(Dexheimer, Schramm, Phys. Rev. C **81** (2010) 045201)

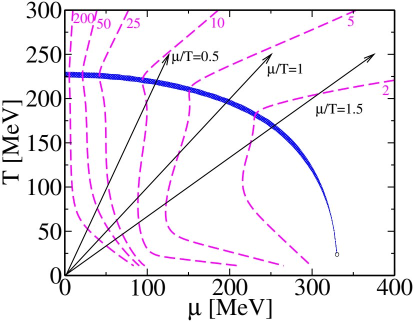
Finding the critical point - theory

3. Susceptibilities

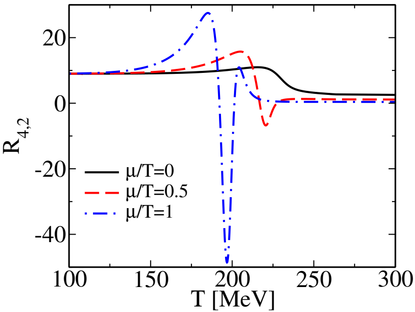


Finding the critical point - theory

3. Susceptibilities



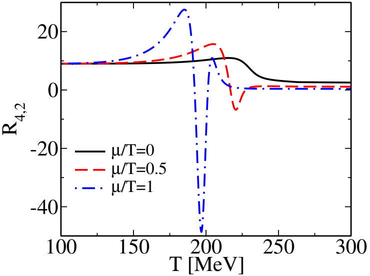
(Skokov, Friman, Redlich, Phys. Rev. C. **83** (2011))



(Skokov, Friman, Redlich, Phys. Rev. C. **83** (2011))

Finding the critical point - theory

4. Susceptibilities and cumulants



(Skokov, Friman, Redlich, Phys. Rev. C. **83** (2011))

- How to measure:

$$c_2 = \frac{\partial^2 (p/T^4)}{\partial(\mu/T)^2} = \frac{1}{VT^3}\langle \delta N^2 \rangle$$

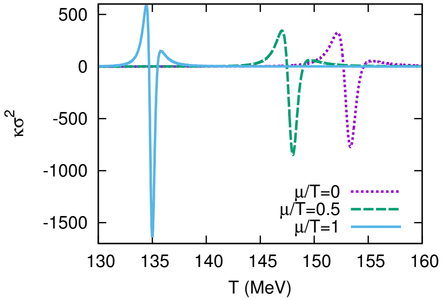
$$c_4 = \frac{\partial^4 (p/T^4)}{\partial(\mu/T)^4} = \frac{1}{VT^3}\left[\langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2\right]$$

$$\kappa\sigma^2 = c_4/c_2 = \frac{\langle \delta N^4 \rangle}{\langle \delta N^2 \rangle} - 3\langle \delta N^2 \rangle$$

- Independent of volume and temperature

Finding the critical point - theory

4. Susceptibilities and cumulants



- Generalized sigma susceptibilities ($\tilde{\sigma} = \sigma/T$):

$$c_2 = \left(\frac{\delta^2\Gamma}{\delta\tilde{\sigma}^2}\right)^{-1}$$

$$c_4 = -\frac{\delta^4\Gamma}{\delta\tilde{\sigma}^4}\left(\frac{\delta^2\Gamma}{\delta\tilde{\sigma}^2}\right)^{-1} + 3\left(\frac{\delta^3\Gamma}{\delta\tilde{\sigma}^3}\right)^2\left(\frac{\delta^4\Gamma}{\delta\tilde{\sigma}^4}\right)^{-5}$$

(CH et al. *in preparation*)

$$\kappa\sigma^2 = c_4/c_2 = \frac{\langle\delta\tilde{\sigma}^4\rangle}{\langle\delta\tilde{\sigma}^2\rangle} - 3\langle\delta\tilde{\sigma}^2\rangle$$

Finding the critical point - experiment

1. Event-by-event fluctuations

Event-by-event fluctuations of multiplicity, mean p_T

$$\langle \Delta n_p \Delta n_k \rangle = v_p^2 \delta_{pk} + \xi^2 \frac{G^2}{T} \frac{v_p^2 v_k^2}{\omega_p \omega_k}$$

(Stephanov, Rajagopal and Shuryak, PRD **60** (1999))

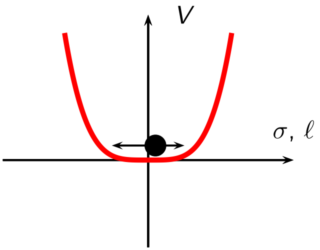
Higher cumulants even more sensitive, e. g. $\kappa_4 \sim \xi^7$

(M. A. Stephanov, Phys. Rev. Lett. **102** (2009))

But:

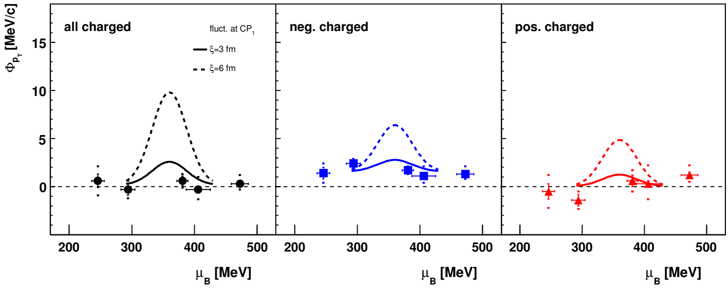
- Finite size effects
- Finite time effects
- Critical slowing down

Will crucially influence the signal



Finding the critical point - experiment (NA49)

2. Event-by-event fluctuations with NA49



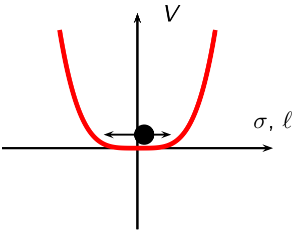
(K. Grebieszkow, NA49 collaboration Nucl. Phys. A **830** (2009))

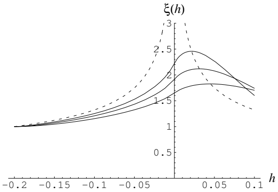
Finding the critical point - experiment

3. Event-by-event fluctuations: caveats

- Finite size effects
- Finite time effects
- Critical slowing down

Will influence potential signals





Phenomenologically

$$\frac{\mathrm{d}}{\mathrm{d}t} m_\sigma(t) = \Gamma(m_\sigma(t)) \left(m_\sigma(t) - \frac{1}{\xi_{\mathrm{eq}}(t)} \right)$$

(Berdnikov, Rajagopal, Phys. Rev. D **61** (2000))

Finding the critical point - experiment

4. Higher order cumulants: beam energy scan (BES) at STAR

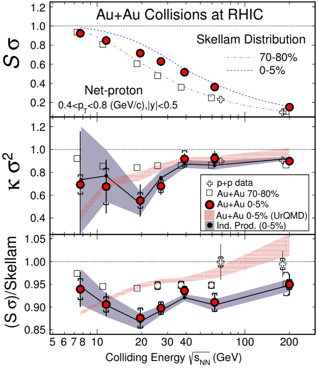
- Higher order cumulants

$$\sigma^2 = \langle \delta N^2 \rangle \sim \xi^2$$

$$S\sigma = \frac{\langle \delta N^3 \rangle}{\langle \delta N^2 \rangle} \sim \xi^{2.5}$$

$$\kappa\sigma^2 = \frac{\langle \delta N^4 \rangle}{\langle \delta N^2 \rangle} - 3\langle \delta N^2 \rangle \sim \xi^5$$

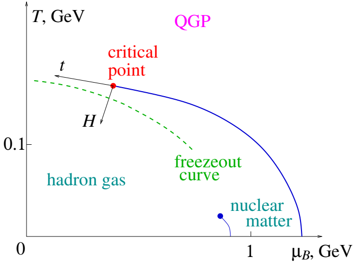
(Stephanov, Phys. Rev. Lett. **102** (2009))



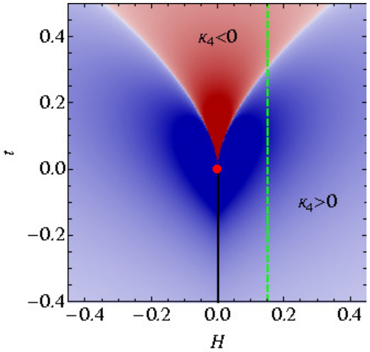
(STAR collaboration, Phys. Rev. Lett. **112** (2014))

Finding the critical point - experiment

5. Higher order cumulants: a negative kurtosis



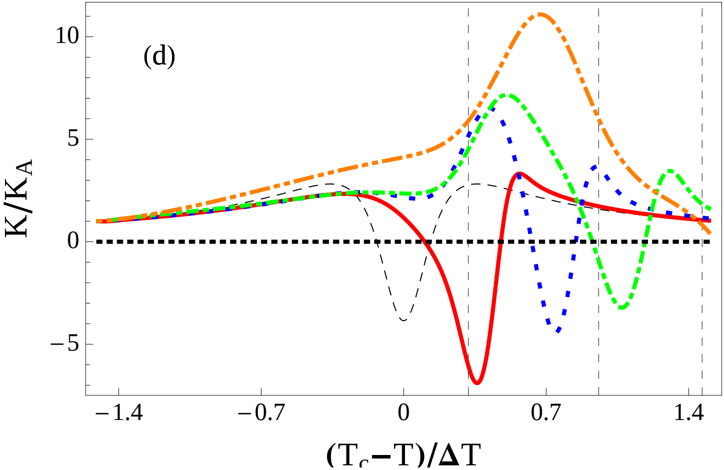
(Stephanov, Phys. Rev. Lett. **107**, (2011))



$$\langle \delta N^4 \rangle = \langle N \rangle + \kappa_4 \left(\frac{gd}{T} \int_p \frac{n_p}{\gamma_p} \right)^4 + \ldots$$

Finding the critical point - experiment

6. Higher order cumulants: a dynamical kurtosis

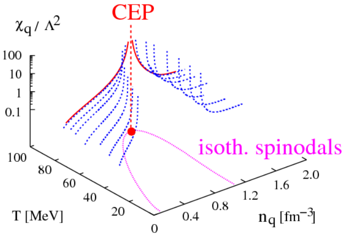


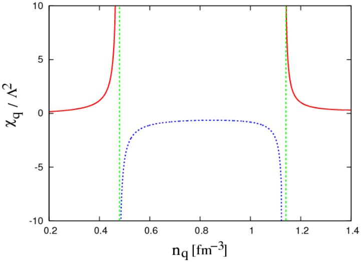
(Mukherjee, Venugopalan, Yin, Phys. Rev. C **92**, (2015))

Finding a first-order phase transition

1. Nonequilibrium enhancement of fluctuations

- Nonequilibrium fluctuations interesting at first-order transition
- Spinodal decomposition
- Amplification of inhomogeneities

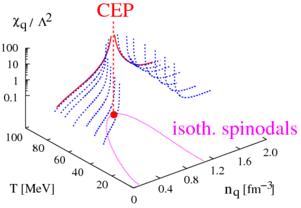




(Sasaki, Friman and Redlich, J. Phys. G **35** (2008))

Finding a first-order phase transition

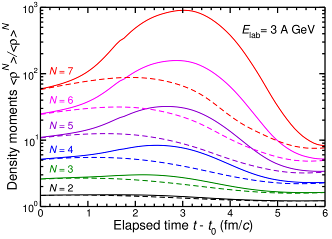
2. Dynamical model



(Sasaki, Friman, Redlich, PRD 77 (2008))

- Allow instable EoS
- Spinodal decomposition
- Non-statistical multiplicity fluctuations

- Formation of metastable phase
- Dynamical fragmentation
- Droplets
- Non-statistical multiplicity fluctuations



(Steinheimer, Randrup, PRL 109 (2012))

NχFD - idea

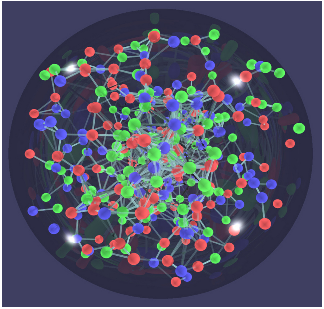
Ingredients for Nonequilibrium Chiral Fluid Dynamics **NχFD** model

- Fluctuations (chiral fields)
- Fluid (quarks)

$$\frac{\partial^2\sigma}{\partial t^2} - \nabla^2\sigma + \eta\frac{\partial\sigma}{\partial t} + \frac{\delta\Omega}{\delta\sigma} = \xi$$
$$\partial_\mu T^{\mu\nu}_{\mathrm{q}} = S^\nu_\sigma \ , \quad \partial_\mu n^\mu = 0$$

- coarse-grained noise, $\xi_{\mathrm{corr}} = 1/m_\sigma$

(Nahrgang, Leupold, CH, Bleicher, Phys. Rev. C 84 (2011))



Based on LσM

$$\mathcal{L} = \bar{q}\,(i\gamma^\mu\partial_\mu - g\sigma) + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma)\ ,$$ possibly extended with ℓ, χ

- **Self-consistent** coupling of order parameter fluctuations to fluid dynamical space-time evolution
- Successfully describes: **critical fluctuations in and out of equilibrium**

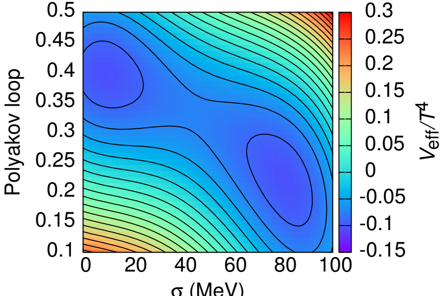
NχFD - thermodynamics

Grand canonical potential at $\mu_B = 0$, $\ell = \bar{\ell}$, mean-field

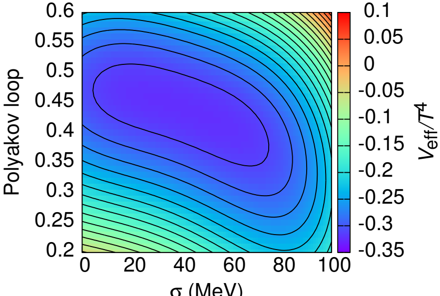
$$\Omega_{\bar{q}q} = -4 N_f T \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \ln\left[1 + 3\ell \mathrm{e}^{-\beta E} + 3\ell \mathrm{e}^{-2\beta E} + \mathrm{e}^{-3\beta E}\right]$$

Effective potential

$$V_{\mathrm{eff}}(\sigma, \ell, T) = U(\sigma) + \mathcal{U}(\ell) + \Omega_{\bar{q}q}(\sigma, \ell, T)$$

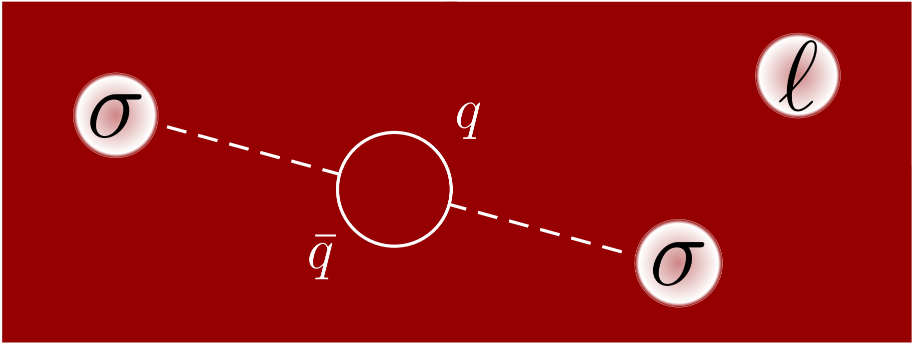


first-order transition



critical point

NχFD - Langevin dynamics



- Damping of chiral field cause by $\sigma \leftrightarrow q\bar{q}$
- Influence functional method

NχFD - Langevin dynamics

Damping kernel

$$D(x) = ig^2 \int_{y_0}^{x_0} \mathrm{d}^4 y \bar{\sigma}(y) \left[S^<(x-y) S^>(y-x) - S^>(x-y) S^<(y-x) \right]$$

Noise kernel

$$\mathcal{N}(x,y) = -\frac{1}{2} g^2 \left[S^<(x-y) S^>(y-x) + S^>(x-y) S^<(y-x) \right]$$

Determine equation of motion

$$-\frac{\delta S_{cl}[\bar{\sigma}]}{\delta \bar{\sigma}} - D = \xi \,,$$

$$\langle \xi(x) \xi(y) \rangle = \mathcal{N}(x,y) \,.$$

NχFD - Langevin dynamics

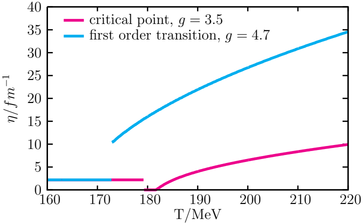
$$\partial_\mu \partial^\mu \sigma + \eta_\sigma \partial_t \sigma + \frac{\partial V_{eff}}{\partial \sigma} = \xi$$

With damping coefficient η_σ for $\mathbf{k} = \mathbf{0}$

$$\eta_\sigma = \frac{12g^2}{\pi} \left[1 - 2n_\mathrm{F}\left(\frac{m_\sigma}{2}\right)\right] \frac{\left(\frac{m_\sigma^2}{4} - m_q^2\right)^{\frac{3}{2}}}{m_\sigma^2}$$

And $\eta_\sigma = 2.2/fm$ for $\sigma \leftrightarrow \pi\pi$ reaction

(T. S. Biro and C. Greiner, Phys. Rev. Lett. **79** (1997))



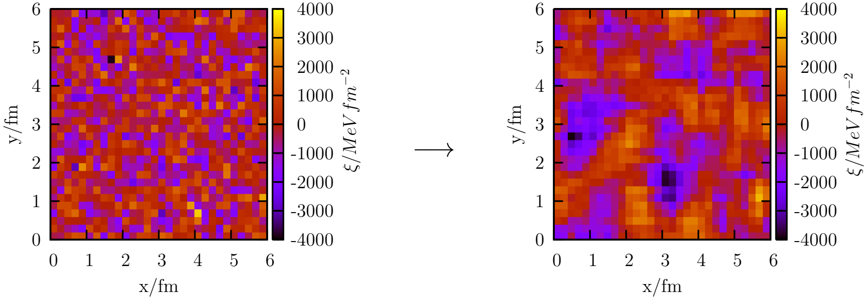
And the dissipation-fluctuation theorem

$$\langle \xi(t)\xi(t') \rangle = \frac{1}{V}\delta(t - t') m_\sigma \eta_\sigma \coth\left(\frac{m_\sigma}{2T}\right)$$

(Nahrgang, Leupold, CH, Bleicher, Phys. Rev. C 84 (2011))

NχFD - coarse-grained noise

- Correlate stochastic noise field over volume of ξ^3:



NχFD - Polyakov loop

Allow for dynamical evolution of the Polyakov loop

$$\mathcal{L} \to \mathcal{L} + \frac{N_c}{g_{QCD}^2} |\partial_\mu \ell|^2 T^2$$

Add a phenomenological damping term $\eta_\ell \sim 1/fm$

$$\frac{2N_c}{g_{QCD}^2} \partial_\mu \partial^\mu \ell T^2 + \eta_\ell \partial_t \ell + \frac{\partial V_{eff}}{\partial \ell} = 0$$

(A. Dumitru and R. D. Pisarski, Nucl. Phys. A **698** (2002))

NχFD - Propagation of the quark fluid

Ideal relativistic hydrodynamics

$$\partial_\mu \left(T_q^{\mu\nu} + T_\sigma^{\mu\nu} + T_\ell^{\mu\nu} \right) = 0$$

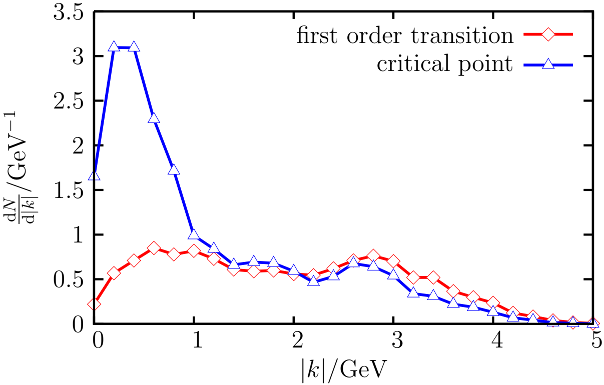
Equation of state $e = e(p)$ from

$$\begin{aligned} e(\sigma, \ell, T) &= T \frac{\partial p(\sigma, \ell, T)}{\partial T} - p(\sigma, \ell, T) \\ p(\sigma, \ell, T) &= -\Omega_{\bar{q}q}(\sigma, \ell, T) \end{aligned}$$

Investigate two scenarios:

- Thermalization in a box
- Hydrodynamic expansion of a hot and dense plasma

Box - Fourier analysis of fluctuations



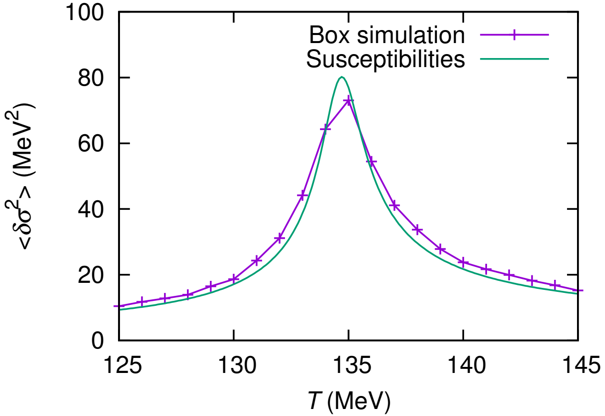
Intensity of sigma fluctuations:

$$N = \int_{\Delta k} \mathrm{d}^3 k \; N_k = \int_{\Delta k} \mathrm{d}^3 k \frac{a_k^\dagger a_k}{(2\pi)^3 2\omega_k} = \int_{\Delta k} \mathrm{d}^3 k \frac{\omega_k^2 |\delta\sigma_k|^2 + |\delta\dot{\sigma}_k|^2}{(2\pi)^3 2\omega_k}$$

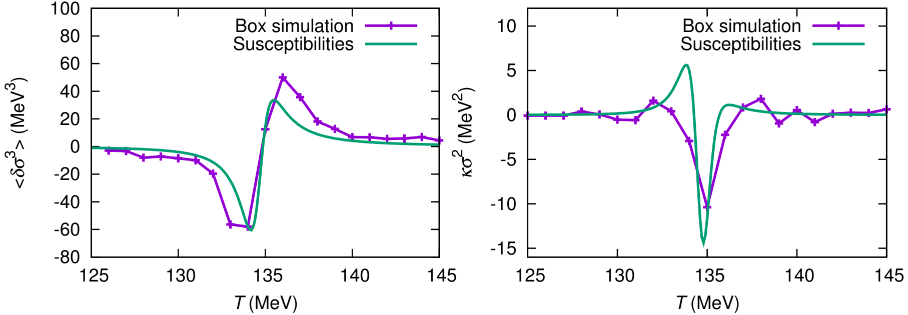
(CH, Nahrgang, Mishustin, Bleicher, Phys. Rev. C 87 (2013))

Box - Fluctuations and susceptibilities

- Isothermal box with periodic boundary conditions

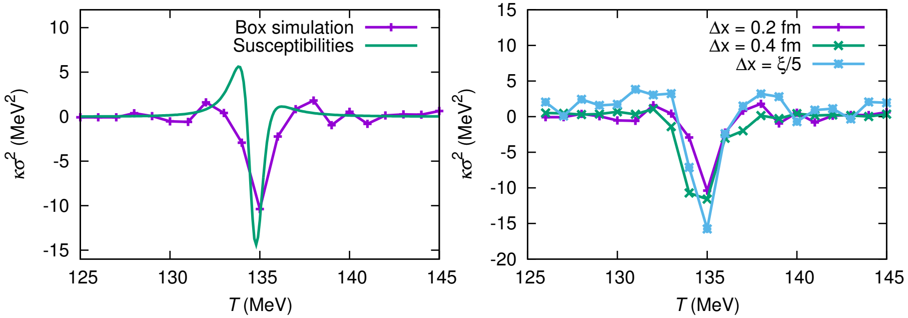


Box - Fluctuations and susceptibilities



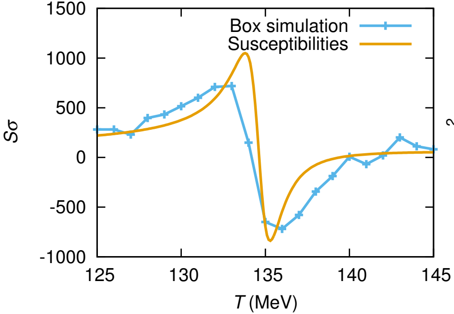
- Moments/Cumulants follow trend of susceptibilities

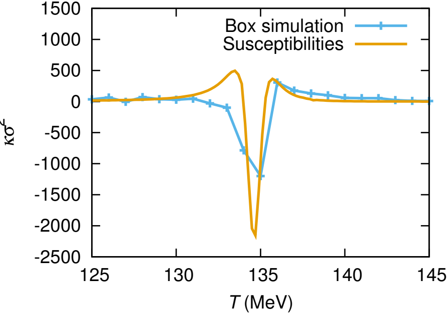
Box - Fluctuations and susceptibilities



- Moments/Cumulants follow trend of susceptibilities
- Stable for several lattice spacings

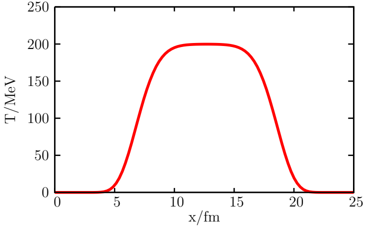
Box - Fluctuations and susceptibilities





- Calculate $n(x)$ locally from T, μ, $\sigma(x)$
- Moments/Cumulants follow trend of susceptibilities

Expanding plasma - Initial conditions



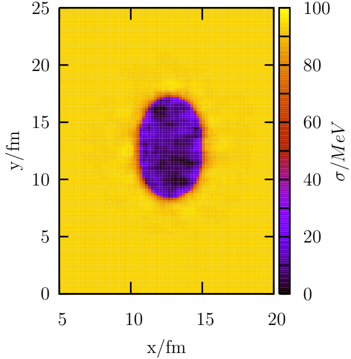
Temperature:
Woods-Saxon distribution

σ-field:

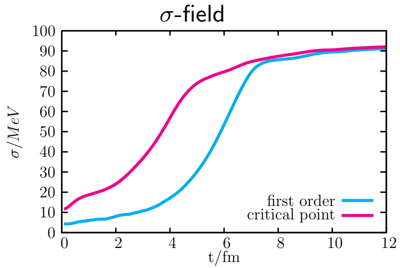
$$\sigma = \sigma_{eq}(T) + \delta\sigma(T)$$

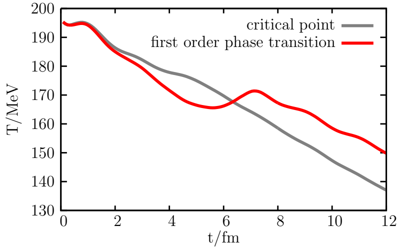
thermal distribution, correlated

$$\begin{aligned} e &= e(\sigma, T, \mu) \\ n &= n(\sigma, T, \mu) \\ p &= p(\sigma, T, \mu) \end{aligned}$$

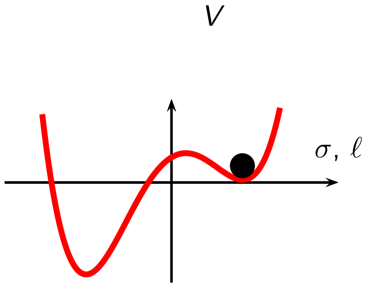


Expanding plasma - Supercooling

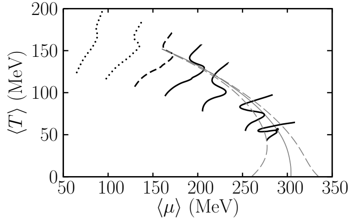


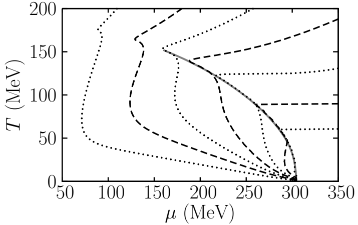


- Averaged sigma field in central volume
- Critical point: intermediate μ
- First-order: large μ



Expanding plasma - Nonequilibrium trajectories



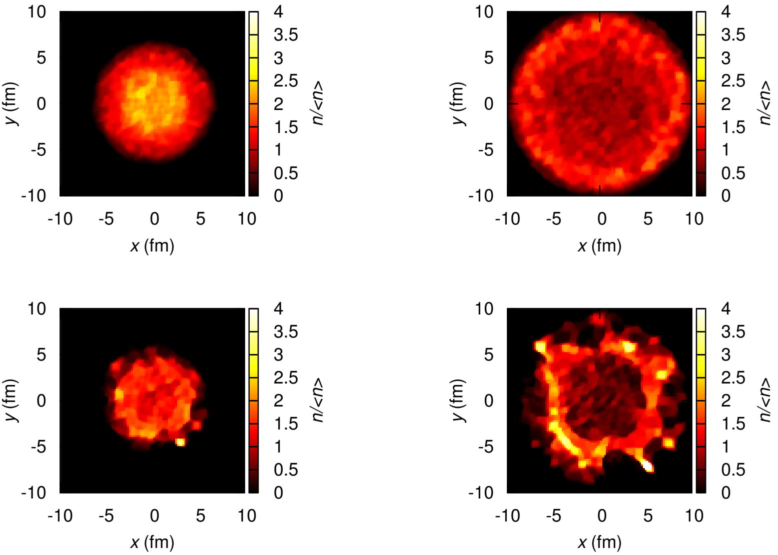


- Trajectories close to isentropes at crossover and CEP
- Trajectories influenced by nonequilibrium effects at first-order transition
- At high densities system remains in spinodal region for long time

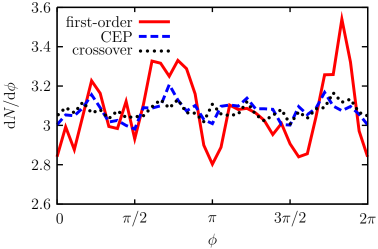
Possibility for domain formation?

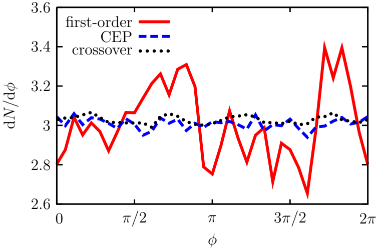
(CH, Nahrgang, Mishustin, Bleicher, Nucl. Phys. A 925 (2014))

Expanding plasma - Droplet formation



Expanding plasma - Droplet formation

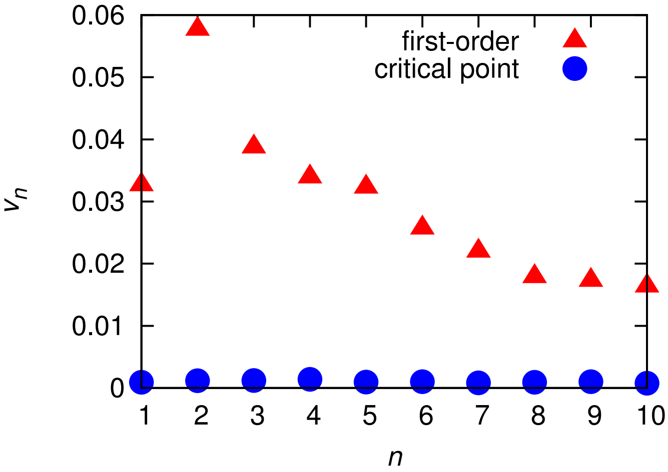




- Fluctuations in azimuthal spatial direction
- Signal for first-order phase transition

(CH, Nahrgang, Mishustin, Bleicher, Nucl. Phys. A 925 (2014))

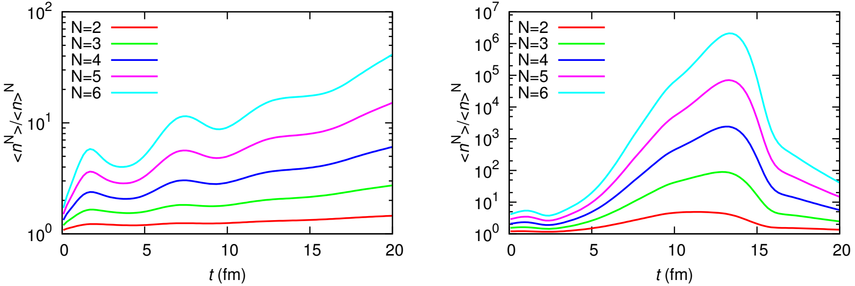
Expanding plasma - Droplet formation



(CH, Nahrgang, Mishustin, Bleicher, Nucl. Phys. A 925 (2014))

Expanding plasma - Droplet formation

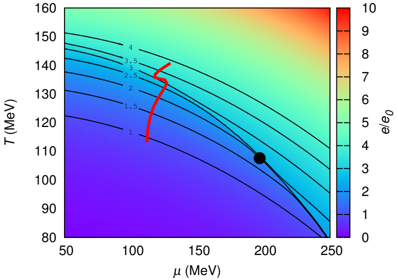
- PQM vs. quark-hadron model
- Equation of state influences stability of inhomogeneities

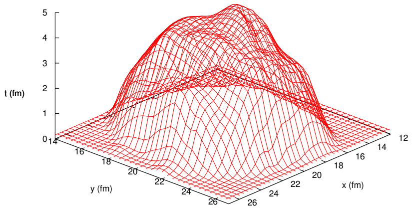


(CH, Nahrgang, NICA white paper, EPJA (2016))

Expanding plasma - Crossover

- Isothermal box with linearly decreasing temperature



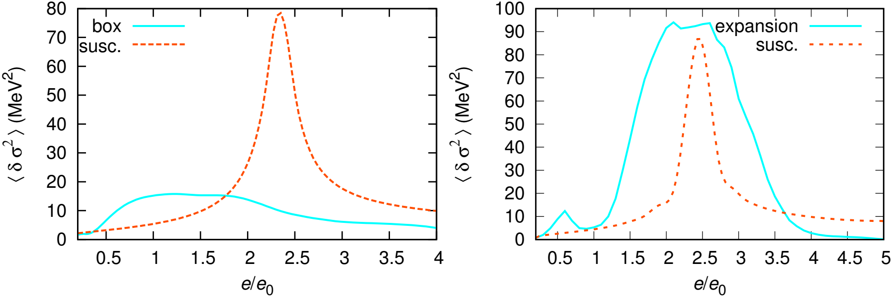


(CH, Nahrgang, Yan, Kobdaj, PRC 93 (2016))

- Study crossover evolution left of CP
- Determine sigma and net-proton cumulants on energy hypersurfaces
- Smooth hypersurfaces at crossover

Expanding plasma - Sigma fluctuations

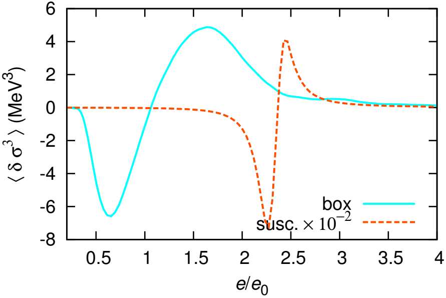
- Isothermal box with linearly decreasing temperature

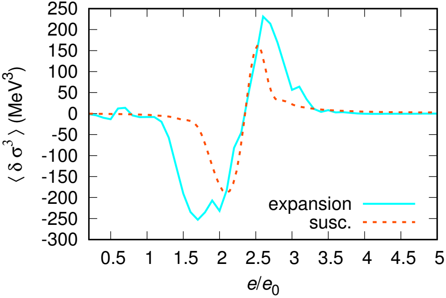


- Box shows clear delay and memory effect
- Critical region widened in inhomogeneous expanding medium

Expanding plasma - Sigma fluctuations

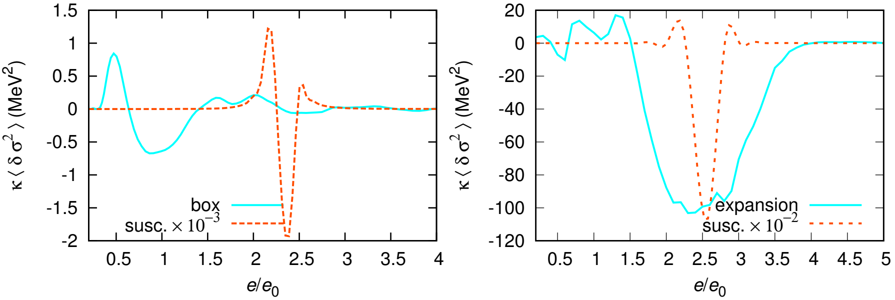
- Isothermal box with linearly decreasing temperature





- Box shows clear delay and memory effect
- Critical region widened in inhomogeneous expanding medium

Expanding plasma - Sigma fluctuations

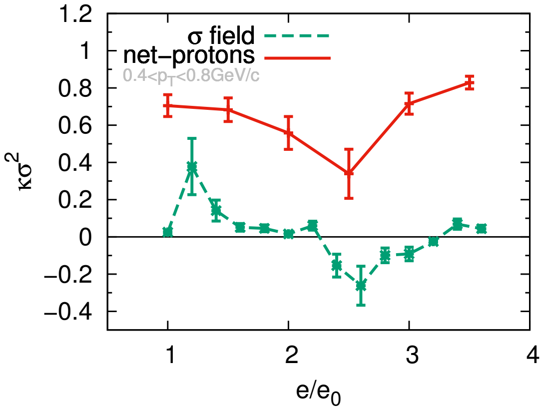


- Box shows clear delay and memory effect
- Critical region widened in inhomogeneous expanding medium

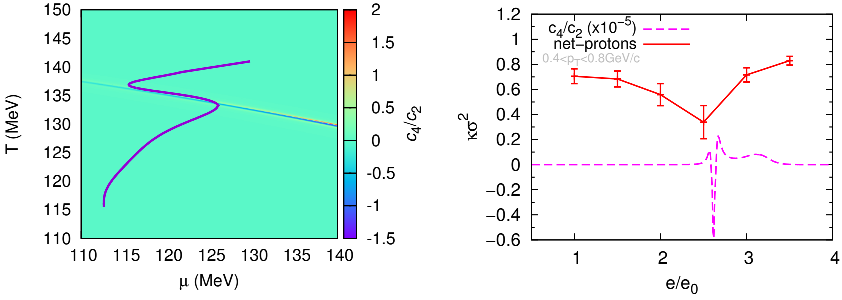
Expanding plasma - Net-proton fluctuations

- Perform Cooper-Frye freezeout
- Total net-baryon number exactly conserved in each event
- Net-proton kurtosis follows kurtosis of sigma field

(CH, Nahrgang, Yan, Kobdaj, PRC 93 (2016))



Expanding plasma - Net-proton fluctuations



(CH, Nahrgang, Yan, Kobdaj, PRC 93 (2016))

- Comparison of net-proton kurtosis to equilibrium fluctuations
- Characteristic dip imprints signal on net-proton kurtosis

Importance of dynamical fluctuations

Two possible evolutions:

- Mean-field, local thermal equilibrium without fluctuations

$$\left.\frac{\partial\Omega(T,\mu;\sigma)}{\partial\sigma}\right|_{\sigma=\sigma_{\mathrm{eq}}} = 0$$
$$p(T,\mu;\sigma) = -\Omega(T,\mu;\sigma)\ , \quad \partial_\mu T^{\mu\nu} = 0$$

- Full nonequilibrium dynamics with damping and stochastic fluctuations

$$\frac{\partial^2\sigma}{\partial t^2} - \nabla^2\sigma + \eta\frac{\partial\sigma}{\partial t} + \frac{\delta\Omega}{\delta\sigma} = \xi$$
$$p(T,\mu;\sigma) = -\Omega_{\bar{\mathrm{q}}\mathrm{q}}(T,\mu;\sigma)\ , \quad \partial_\mu T^{\mu\nu} = S^\nu$$

In both cases quark densities are propagated via

$$\partial_\mu n^\mu = 0$$

Importance of dynamical fluctuations

- Net-proton kurtosis follows kurtosis of sigma field
- In contrast: Mean-field kurtosis remains flat
- In mean-field (hydro/eos): critical fluctuations do not build up

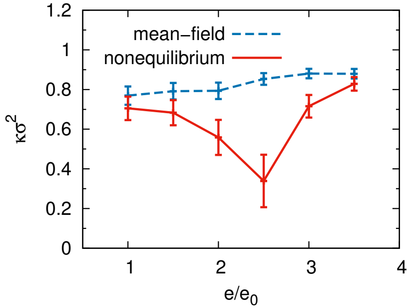
(CH, Nahrgang, Yan, Kobdaj, PRC **93** (2016))

- Mean-field

$$\left.\frac{\partial\Omega}{\partial\sigma}\right|_{\sigma=\langle\sigma\rangle} = 0$$

- Nonequilibrium

$$\frac{\partial^2\sigma}{\partial t^2} - \nabla^2\sigma + \eta\frac{\partial\sigma}{\partial t} + \frac{\delta\Omega}{\delta\sigma} = \xi$$



Summary

- Nonequilibrium models are necessary to understand CP signals from HICs
- CP signals influenced by critical slowing down
- First-order phase transition produces interesting effects in nonequilibrium
- Widened critical region is found in NχFD model
- Possibility for signal in net-proton cumulants