D^0-$\overline{D^0}$ Mixing and CP Violation Measurement at Belle

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Outline

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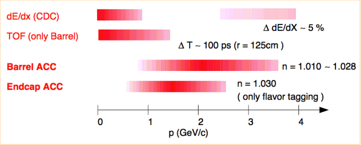
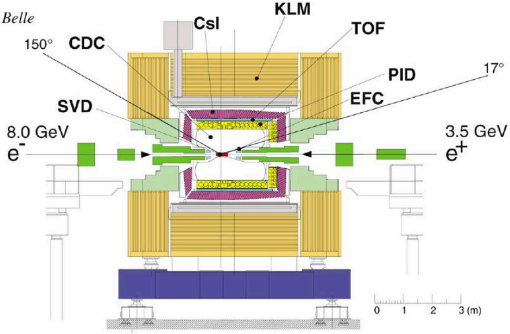
Belle experiment at KEKB

✓ KEK: 高エネルギー加速器研究機構 @ 筑波市
✓ B factory: $\sqrt{s} = 10.58GeV$: $\Upsilon(4S) \to B\bar{B}$
✓ highest peak lumin. $\mathcal{L} = 2.1 \times 10^{34}\ cm^{-2}s^{-1}$
✓ Belle Collab.: 536 colleag. 91 institut., 20 countries/regions
✓ China(5): IHEP, USTC, PKU, BUAA, Fudan.





- Belle detector:
  - ☺ good $\vec{p}$ / vtx. resolution
  - ☺ high rec. eff. for $\gamma/K^0_S/\pi^0$
  - ☺ low bkg, high trigger eff.
- $\int \mathcal{L}dt \sim 1\ ab^{-1}$ dataset provides
  - $\sim 1.1 \times 10^9\ B\bar{B} \Rightarrow$ a B-factory;
  - $\sim 1.3 \times 10^9\ c\bar{c} \Rightarrow$ a charm factory;
  - $\sim 0.9 \times 10^9\ \tau^+\tau^- \Rightarrow$ a τ factory;
  - wide $E^{eff}_{c.m}$=[0.5-10] GeV via ISR.

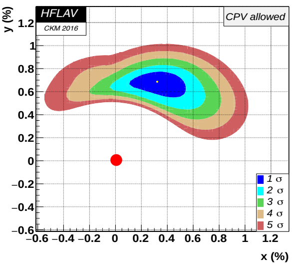
Available Charm Samples from Charm factories, B-factories, hadron colliders

Experiment	Machine	C.M	Lumin.	N(D)	efficiency	advantage/disadvantage
CLEO	CESR (e^+e^-)	3.77 GeV	0.8 fb^{-1}	2.9×10^6 $2.3 \times 10^6 (D^\pm)$	~10-30%	☺ extremely clean enviroment ☺ pure D-beam, almost no bkg ☺ quantum coherence ☹ no CM boost, no T-dep analyses
		4.17 GeV	0.6 fb^{-1}	0.6×10^6		
BESIII	BEPC-II (e^+e^-)	3.77 GeV	2.92 fb^{-1}	10.5×10^6 8.4×10^6 $(D^\pm)$		
		4.18 GeV	3 fb^{-1}	3×10^6		
				★	★★★	
BELLE	KEKB (e^+e^-)	10.58 GeV	1 ab^{-1}	1.3×10^9	~5-10%	☺ clear event environment ☺ high trigger efficiency ☺ high-efficiency detection of neutrals ☺ many high-statistics control samples ☺ time-dependent analysis ☹ smaller cross-section than pp colliders
BaBar	PEP-II (e^+e^-)	10.58 GeV	0.5 ab^{-1}	6.5×10^8		
				★★	★★	
CDF	Tevatron ($p\bar{p}$)	1.96 TeV	9.6 fb^{-1}	1.3×10^{11}	<0.5%	☺ large production cross-section ☺ large boost: excellent time resolution ☹ dedicated trigger required ☹ hard to do neutrals and neutrinos
LHCb	LHC (pp)	7 TeV 8 TeV	1.0 fb^{-1} 2.0 fb^{-1}	5.0×10^{12}		
				★★★	★	

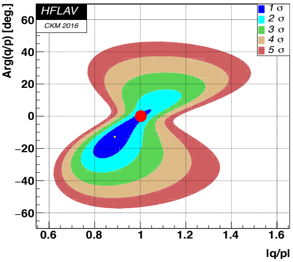
here we used $\sigma(D^0\bar{D}^0 @3.77\ GeV)$=3.61 nb, $\sigma(D^+D^- @3.77\ GeV)$=2.88 nb, $\sigma(D^* D_s @4.17\ GeV)$=0.967 nb, $\sigma(c\bar{c} @10.58\ GeV)$=1.3 nb, $\sigma(D^0 @LHCb)$=1.661 nb. This table mainly refers to IJMP **A 29** (2014) 24, 14300518 and G. Casarosa's report at SLAC experimental seminar 2016.

Status of D^0-$\overline{D}^0$ mixing and CP violation [mainly ref. charm physics at HFLAV]

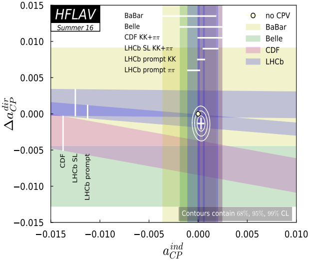
D^0-$\overline{D}^0$ mixing



indirect CP violation



CP violation in $D^0 \to KK/\pi\pi$



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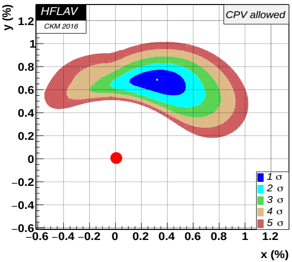
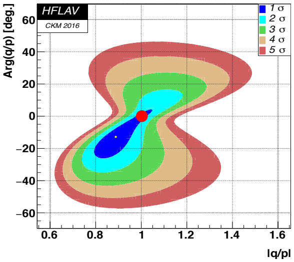
D^0-$\overline{D}^0$ mixing



indirect CP violation



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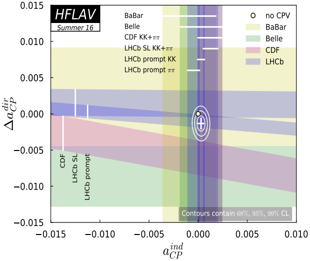
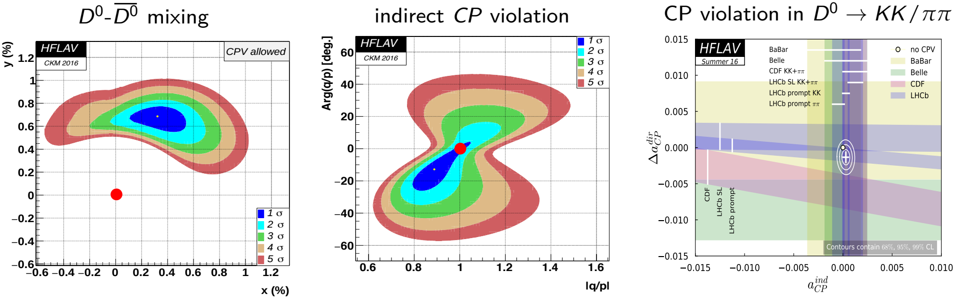


- $\gg 11.5\sigma$ to exclude no mixing (x,y)=(0,0) with CPV-allowed

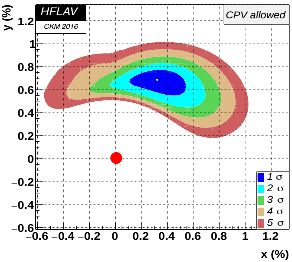
Status of D^0-$\overline{D}^0$ mixing and CP violation [mainly ref. charm physics at HFLAV]



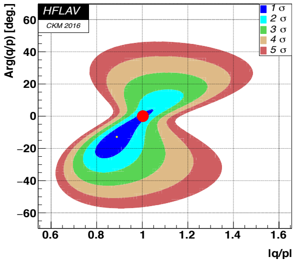
- $\gg 11.5\sigma$ to exclude no mixing (x,y)=(0,0) with CPV-allowed
- No hints for indirect CPV $\Leftarrow$ no direct CPV ($|q/p|,\phi$)=(1,0) at C.L=40%

Status of D^0-$\overline{D}^0$ mixing and CP violation [mainly ref. charm physics at HFLAV]

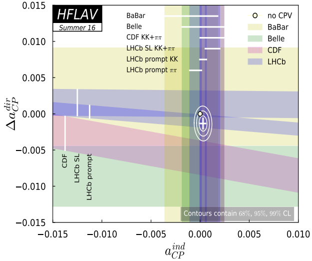
D^0-$\overline{D}^0$ mixing



indirect CP violation



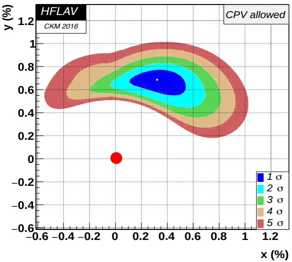
CP violation in $D^0 \to KK/\pi\pi$



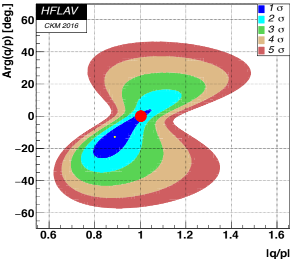
- $\gg$ 11.5σ to exclude no mixing (x,y)=(0,0) with CPV-allowed
- No hints for indirect CPV $\Leftarrow$ no direct CPV (|q/p|,ϕ)=(1,0) at C.L=40%
- No clear evidence of direct CPV $\Leftarrow$ no CPV at C.L=9.3%

Status of D^0-$\overline{D}^0$ mixing and CP violation [mainly ref. charm physics at HFLAV]

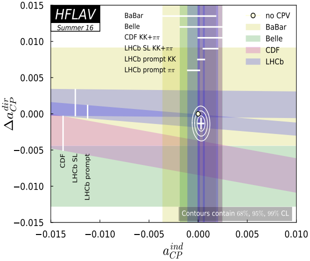
D^0-$\overline{D}^0$ mixing



indirect CP violation



CP violation in $D^0 \to KK/\pi\pi$



- $\gg 11.5\sigma$ to exclude no mixing (x,y)=(0,0) with CPV-allowed
- No hints for indirect CPV $\Leftarrow$ no direct CPV (|q/p|,ϕ)=(1,0) at C.L=40%
- No clear evidence of direct CPV $\Leftarrow$ no CPV at C.L=9.3%

D^0-$\overline{D}^0$ mixing observation in more channels, and CPV searches are two of most important physical goals for Charm WG at our Belle/Belle II experiment.

Status of D^0-$\overline{D}^0$ mixing and CPV [mainly ref. charm physics at HFLAV]

Decay Type	Final State	BELLE	BaBar	LHCb	CDF	CLEO	BESIII
DCS 2-body(WS)	$K^+\pi^-$	★	☆	★[a]	★	✓	✓ $_{\delta K\pi}$
DCS 3-body(WS)	$K^+\pi^-\pi^0$	○[c]	☆			✓ A_{CP}	○ $_{\delta}$
CP-eigenstate	(even) h^+h^-	☆	☆	☆[b] A_{CP}	✓ A_{CP}	✓	
	(odd) $K_S^0\phi$	✓					
Self-conj. 3-body decay	$K_S^0\pi^+\pi^-$	✓	✓	✓	✓ A_{CP}	✓	○ $_{\delta}$
	$K_S^0K^+K^-$	○	✓	○			○ $_{\delta}$
	$K_S^0\pi^0\pi^0$					✓ Dalitz	○ y_{CP}
Self-conj. SCS 3-body decay	$\pi^+\pi^-\pi^0$	✓ A_{CP}	✓ mixing A_{CP}	✓ A_{CP}			○ $_{\delta}$
	$K^+K^-\pi^0$		✓ A_{CP}				○ $_{\delta}$
SCS 3-body	$K_S^0K^\pm\pi^\mp$			✓ A_{CP}		✓ $_{\delta}$	○ $_{\delta}$
Semileptonic decay	$K^+\ell^-\nu_\ell$	✓	✓			✓	
Multi-body(n≥4)	$K^+\pi^-\pi^+\pi^-$	✓ R_{WS}	✓	★			○ $_{\delta RS}$
	$\pi^+\pi^-\pi^+\pi^-$	○ A_{CP}		✓[d] A_{CP}			
	$K^+K^-\pi^+\pi^-$	○ A_T	✓ A_T	✓[e] A_{CP}		✓ A_{CP}	○
	$K_S^0\pi^+\pi^-\pi^0$	✓ A_T					
$\psi(3770)\to D^0\overline{D}^0$ via correlations						✓ $_{\delta K\pi}$	✓ y_{CP}

In D^0-$\overline{D}^0$ mixing measurements: ★ for observation ($>5\sigma$); ☆ for evidence ($>3\sigma$); ✓ for measurement published; ○ for analysis on going. A_T stands for measuring CP asymmetry using T-odd correlations.

(a) LHCb gave the measurement of charm mixing and CP violation in $D^0\to K^\pm h^\mp$ decay in PRD **95**, 052004 (2017).

(b) LHCb gave the measurements of CP violation in $D^0\to h^-h^+$ decay in PRL **112**, 041801 (2014) and PRL **118**, 261803 (2017).

(c) Belle measured WS-to-RS ratio R_{WS} and A_{CP} in $D^0\to K^\mp\pi^\pm\pi^0$ in PRL **95**, 231801 (2005).

(d) LHCb also searched for CP violation in phase space of $D^0\to\pi^+\pi^-\pi^+\pi^-$ decays in PLB **769**(2017) 345.

(e) LHCb also searched for CP violation using T-odd correlations in $D^0\to K^+K^-\pi^+\pi^-$ decays in JHEP **10**(2014)005.

Outline

Formalism of D^0-$\overline{D}^0$ mixing

- Open-flavor neutral meson transforms to its anti-meson and vice versa:

$$K^0 \Leftrightarrow \overline{K^0},\ B^0_d \Leftrightarrow \overline{B^0_d},\ B^0_s \Leftrightarrow \overline{B^0_s},\ D^0 \Leftrightarrow \overline{D^0}$$

- Flavor eigenstate ($|D^0\rangle$, $|\overline{D^0}\rangle$) $\neq$ mass eigenstate $|D_{1,2}\rangle$ with $M_{1,2}$ and $\Gamma_{1,2}$)

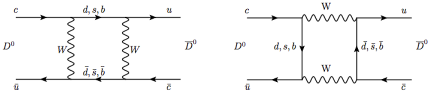
$$|D_{1,2}\rangle \equiv p|D^0\rangle \pm q|\overline{D^0}\rangle \text{ (CPT: } p^2+q^2=1)$$

- Mixing parameters definition:

$$\mathbf{x} \equiv \frac{M_1 - M_2}{\Gamma},\quad \mathbf{y} \equiv \frac{\Gamma_1 - \Gamma_2}{2\Gamma},\quad \Gamma \equiv \frac{\Gamma_1 + \Gamma_2}{2}$$

- under phase convention $CP|D^0\rangle = |\overline{D^0}\rangle$, $CP|\overline{D^0}\rangle = |D^0\rangle$,
- with CP conservation ($q = p = 1/\sqrt{2}$): $|D_{1,2}\rangle = |D_{+,-}\rangle$ (CP eigenstates)

- Unique: only the up-type meson for mixing
- Standard Model predicts: $\sim \mathcal{O}(1\%)$



(1) short distance ($<0.1\%$ by CKM and GIM)

(2) long distance ($\sim 1\%$)

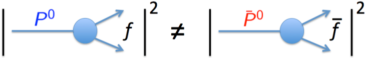
- Precise measurement of x, y: effectively limit New Physics(NP) modes;
- search for NP, eg: $|x| \gg |y|$

Formalism of CP violation

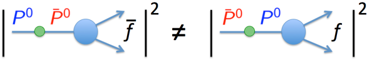
- CPV: Charged-conjugated-Parity combined symmetry Violation

$$A_{CP}^f = \frac{\Gamma(D \to f) - \Gamma(\bar{D} \to \bar{f})}{\Gamma(D \to f) + \Gamma(\bar{D} \to \bar{f})} = a_d^f + a_m^f + a_i^f$$

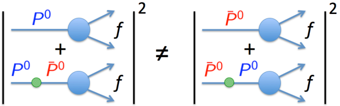
(1) a_d^f: (direct CPV) CPV in decay $|\bar{A}_{\bar{f}}/A_f| \neq 1$



(2) a_m^f: CPV in mixing with $r_m = |q/p| \neq 1$



(3) a_i^f: CPV in interference with $\arg(q/p) \neq 0$



- Standard Model supplies only a CPV source: the phase in CKM matrix
- in charm sector, it's predicted at $\sim \mathcal{O}(10^{-3})$
- under current experiment sensitivity $\sim 1\%$, to observe CPV $\to$ NP

Formalism for time evolution

- Time evolution of D^0-$\overline{D^0}$ system:

$$i\frac{\partial}{\partial t}\begin{pmatrix} D^0(t) \\ \overline{D^0}(t) \end{pmatrix} = (M - \frac{i}{2}\Gamma)\begin{pmatrix} D^0(t) \\ \overline{D^0}(t) \end{pmatrix}$$

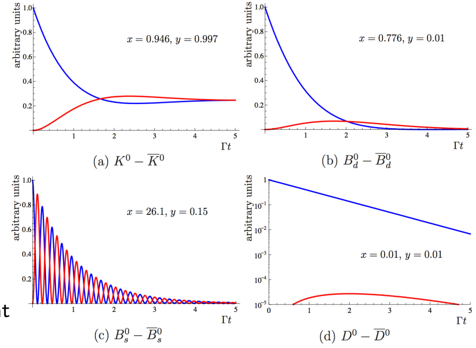
diagonal: $D \to D$,non-diagonal: $D \to \bar{D}$.

- time evolution related to (x,y) and (q/p)

$$|D^0(t)\rangle = g_+(t)|D^0\rangle + \frac{q}{p}g_-(t)|\overline{D^0}\rangle$$
$$|\overline{D^0}(t)\rangle = \frac{p}{q}g_-(t)|D^0\rangle + g_+(t)|\overline{D^0}\rangle$$
$$g_+(t) = e^{(-iM-\frac{1}{2}\Gamma)t}\cosh\left(-\frac{i\mathbf{x}+\mathbf{y}}{2}\Gamma t\right)$$
$$g_-(t) = e^{(-iM-\frac{1}{2}\Gamma)t}\sinh\left(-\frac{i\mathbf{x}+\mathbf{y}}{2}\Gamma t\right)$$

- Probability that the flavor is/is not changed at time t with a pure flavor state $|D^0\rangle$

$$|\langle D^0|D^0(t)\rangle|^2 = \frac{1}{2}e^{-\Gamma t}\left(\cosh(y\Gamma t) + \cos(x\Gamma t)\right)$$
$$|\langle D^0|\overline{D^0}(t)\rangle|^2 = \frac{1}{2}\left|\frac{q}{p}\right|^2 e^{-\Gamma t}\left(\cosh(y\Gamma t) - \cos(x\Gamma t)\right)$$

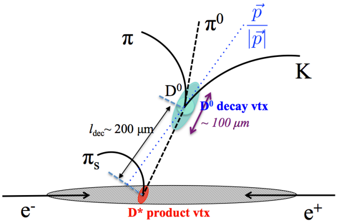


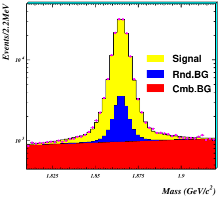
y effects lifetime in amplitude; x: brings a sine oscillating.

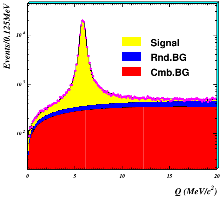
- ▶ D^0-$\overline{D^0}$ mixing measurement is most difficult.

General experiment remarks at Belle

- tag flavor of D^0 using charge of π_s from D^*: $D^{*\pm} \to D^0\pi_s^+ / \overline{D}^0\pi_s^-$ with $D \to f$.
- veto signals from B decays: $p^*(D^*) > 2.5/3.1$ GeV$/c$ for $\Upsilon(4S/5S)$.
- D^0 lifetime: $t_{D^0} = \frac{m_{D^0}}{cp}(\overrightarrow{r_{dec}} - \overrightarrow{r_{prod}}) \cdot \frac{\vec{p}}{p}$;
- lifetime error σ_t by error matrix of production vertex, decay vertex and momentum.
- extract signal and BKG fraction by M-Q 2-dimensional fit:
  - M: invariant mass of reconstructed D^0: $M = M_{D^0 \to f}$
  - Q: release energy of D^* decay: $Q = M_{D^*} - M_{D^0} - m_{\pi_s}$ / $\Delta M = M_{D^*} - M_{D^0}$







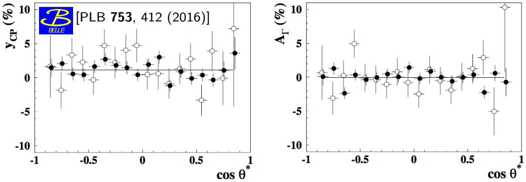
mixing and CPV in CP-eigenstate decays

- Mixing and CPV: using CP eigenstates D^0 lifetime analysis relative to non-CP eigenstates:

$$y_{cp} = \frac{\tau_{K\pi}}{<\tau_{hh}>} - 1 \qquad A_\Gamma = \frac{\tau(\bar{D}^0 \to hh) - \tau(D^0 \to hh)}{\tau(\bar{D}^0 \to hh) + \tau(D^0 \to hh)} \qquad \text{(here } h = K/\pi\text{)}$$

- Belle: the first evidence (540 fb^{-1}): $y_{CP} = (+1.31 \pm 0.32 \pm 0.25)\%$ [PRL **98**, 211801 (2007)]
- Belle updated result (976 fb^{-1})

  y_{CP}=[1.11±0.22±0.09]% (4.7σ)
  A_Γ=[-0.03±0.20±0.07]%
- Belle II sensitivity (50 ab^{-1}):
  $\sigma_{y_{CP}} \approx 0.06\%$, $\sigma_{A_\Gamma} \approx 0.04\%$
  One order of magnitude improvement



open(full) circles for 3(4)-layer SVD

- similar analysis y_{CP} in $D^0 \to K^0_S\omega$ is on-going at Belle
- y_{CP} in $D^0 \to K^0_S\phi$ at Belle with untagged D^0 sample: 72K(ON)+62K(OFF) signals

$$y_{CP} = \frac{1}{f_{ON} - f_{OFF}} \left(\frac{\tau_{OFF} - \tau_{ON}}{\tau_{OFF} + \tau_{ON}} \right) = (+0.11 \pm 0.61 \pm 0.52)\% \text{ [PRD 80,052006 (2009)]}$$

- y_{CP} in $D^0 \to \pi^0\phi$ (~40K tagged signals) (wait for somebody to study at Belle)

Hadronic WS decay $D^0 \to K^+\pi^-$

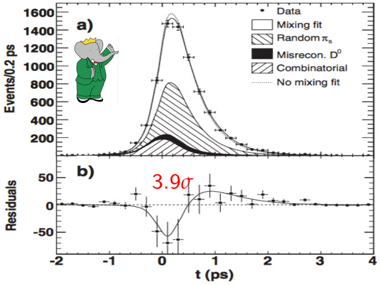
first evidence for D^0-$\overline{D^0}$ mixing

[B. Aubert *et al.* PRL **98**, 211802 (2007)]

- fitting D^0 proper time distribution of WS sample (384 fb^{-1})

$$\frac{T_{WS}(t)}{e^{-\Gamma t}} \propto R_D + \sqrt{R_D}y'\Gamma t + \frac{x'^2+y'^2}{4}(\Gamma t)^2$$

- fit results:
  - R_D=(0.303±0.016(stat)±0.010(syst))%
  - x'^2=(-0.22±0.30(stat)±0.21(syst))×10^{-3}
  - y'=(9.7±4.4(stat)±3.1(syst))×10^{-3}
  - (x'^2, y') with correlation -0.95



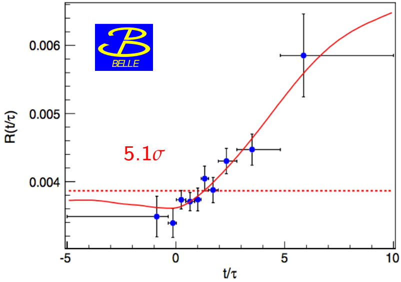
first observation in e^+e^- collisions

[B.R. Ko *et al.* PRL **112**, 111801 (2014)]

- fitting time-dependent ratio of WS-to-RS decay (976 fb^{-1})

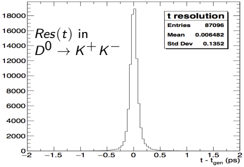
$$R_{WS}(t) = \frac{\Gamma_{WS}(t)}{\Gamma_{RS}(t)} \approx R_D + \sqrt{R_D}y'\Gamma t + \frac{x'^2+y'^2}{4}(\Gamma t)^2$$

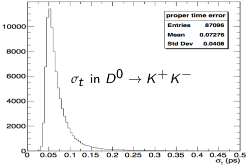
- fit results:
  - R_D=(0.353±0.013(stat+syst))%
  - x'^2=(0.09±0.22(stat+syst))×10^{-3}
  - y'=(4.6±3.4(stat+syst))×10^{-3}
  - (x'^2, y') with correlation -0.948



prospect at Belle II for $D^0 \to K^+\pi^-$

- Time resolution at Belle II based on MC study:





Belle II detector
- ✓ 4-layer SVD + 2-layer PXD
- ✓ smaller innermost layer radius
- ✓ larger outmost layer radius
- ✓ squeezed beams at IP

- ▶ time resolution = 140 fs: 2× better than BaBar (270 fs)
- ▶ time error σ_t: factor 3 improvement; and RMS(σ_t): reduced by a factor 2.
  - $Res = Gauss(\mu, k\sigma_t)$, so reduced RMS($\sigma_t$) (higher weight in the fit) results in an increased statistics

- Belle II sensitivity estimation based on ToyMC
  - Smear decay time with Gauss (σ =140 fs) for 1000 experiments
  - Obtain sensitivities by RMS of residuals distribution of parameters

Parameter		Belle	Belle II		
		976 /fb	5 /ab	20 /ab	50 /ab
no CPV	$\sigma(x'^2)(10^{-5})$	22	7.5	3.7	2.3
	$\sigma(y')(\%)$	0.34	0.11	0.056	0.035
CPV-allowed	$\sigma(x')(\%)$		0.37	0.23	0.15
	$\sigma(y')(\%)$		0.26	0.17	0.10
	$\sigma(\|q/p\|)$		0.197	0.089	0.051
	$\sigma(\phi)(^o)$		15.5	9.2	5.7

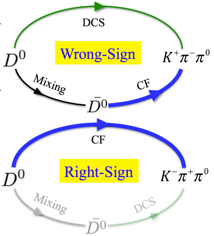
one order of magnitude better than Belle

Three-body WS decay $D^0 \to K^+\pi^-\pi^0$

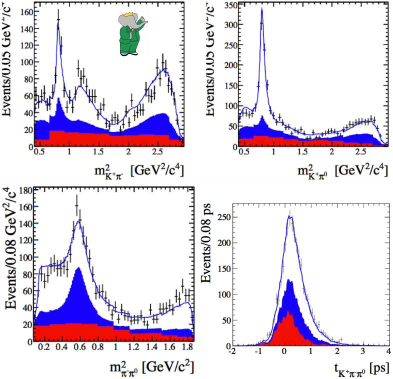
- time-dependent Dalitz analyses(TDDA): an essential tool to study mixir
  $\Rightarrow$ the only method has linear order sensitive on both x and y.
- TDDA in WS 3-body decay $D^0 \to \bar{f}$ (assuming no CPV and $|x|,|y| \ll 1$

$$|\mathcal{A}_{\bar{f}}|^2 = \left[|\mathcal{A}_{\bar{f}}^{DCS}|^2 e^{-\Gamma t} + \frac{(x''^2+y''^2)}{4r_0^2}|\bar{\mathcal{A}}_{\bar{f}}^{CF}|^2(\Gamma t)^2 e^{-\Gamma t} + \left(\frac{y''}{r_0}\mathrm{Re}[\mathcal{A}_{\bar{f}}^{DCS}\bar{\mathcal{A}}_{\bar{f}}^{*CF}] + \frac{x''}{r_0}\mathrm{Im}[\mathcal{A}_{\bar{f}}^{DCS}\bar{\mathcal{A}}_{\bar{f}}^{*CF}]\right)(\Gamma t)e^{-\Gamma t}\right]$$

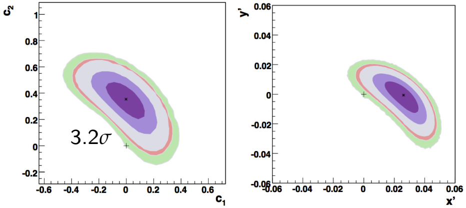
$$x'' = x\cos\delta_{K\rho} + y\sin\delta_{K\rho},\ y'' = y\cos\delta_{K\rho} - x\sin\delta_{K\rho},\ r_0 = |A^{CF}|/|A^{DCS}|$$



- BaBar: the first evidence at D^0 three-body decay. [PRL 103, 211801 (2009)]

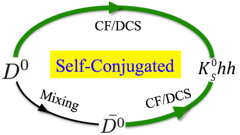


$x'' = [2.61^{+0.57}_{-0.68} \pm 0.39]\%; \quad y'' = [-0.06^{+0.55}_{-0.64} \pm 0.34]\%$

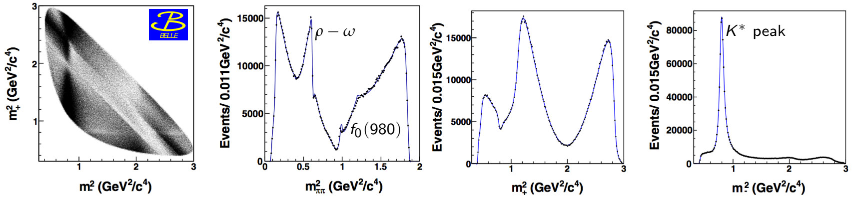


Self-conjugated decay $D^0 \to K_S^0\pi^+\pi^-$ [T. Peng *et al.* PRD **89**, 091103(R) (2014)]

- TDDA in self-conjugated decays:
  (1) direct measurement for x and y; (2) search for CPV: $q/p \neq 1$
- $D^0 \to K_S^0\pi^+\pi^-$ with qusai-two-body decays with difference physics process:
  RS: $\mathcal{A}_f = \langle f|\mathcal{H}|D^0\rangle$; $\frac{q}{p}\frac{\bar{A}_f}{A_f} = \left|\frac{\bar{A}_f}{A_f}\right| \frac{1-\epsilon}{1+\epsilon} e^{i(\delta+\phi)}$; eg: $D^0 \to K^{*-}\pi^+$ etc.
  WS: $\mathcal{A}_{\bar{f}} = \langle \bar{f}|\mathcal{H}|D^0\rangle$; $\frac{q}{p}\frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}} = \left|\frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}}\right| \frac{1-\epsilon}{1+\epsilon} e^{-i(\delta-\phi)}$; eg: $D^0 \to K^{*+}\pi^-$ etc.
  CP: $\mathcal{A}_+ = \langle +|\mathcal{H}|D^0\rangle$ $\frac{q}{p}\frac{\bar{A}_+}{A_+} = +\frac{1-\epsilon}{1+\epsilon} e^{+i\phi}$; eg: $D^0 \to K_S^0 f_0$ etc.
  CP: $\mathcal{A}_- = \langle -|\mathcal{H}|D^0\rangle$ $\frac{q}{p}\frac{\bar{A}_-}{A_-} = -\frac{1-\epsilon}{1+\epsilon} e^{-i\phi}$; eg: $D^0 \to K_S^0\rho/K_S^0\omega$ etc.



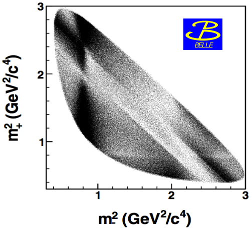
- DP Model with Isobar: 12 BW+K-matrix($\pi\pi$ S-wave)+LASS($K\pi$ S-wave)
- DP $m_-^2 = m^2_{K_S^0\pi^-}$, $m_+^2 = m^2_{K_S^0\pi^+}$ for D^0, exchange for $\overline{D^0}$.

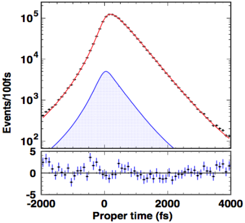


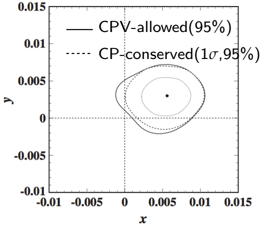
Self-conjugated decay $D^0 \to K_S^0\pi^+\pi^-$ [T. Peng *et al.* PRD **89**, 091103(R) (2014)]

- Time-dependent DP fit on (m_-^2, m_+^2, t) to extract (x,y) and $(|q/p|, \arg(q/p))$

$$|\mathcal{M}(f,t)|^2 = \frac{e^{-\Gamma t}}{2}\left[(|\mathcal{A}_f|^2 + |\tfrac{q}{p}|^2|\mathcal{A}_{\bar{f}}|^2)\cosh(y\Gamma t) + (|\mathcal{A}_f|^2 - |\tfrac{q}{p}|^2|\mathcal{A}_{\bar{f}}|^2)\cos(x\Gamma t)\right.$$
$$\left. +2\,\mathrm{Re}[\tfrac{q}{p}\mathcal{A}_{\bar{f}}\mathcal{A}_f^*]\sinh(y\Gamma t) + 2\,\mathrm{Im}[\tfrac{q}{p}\mathcal{A}_{\bar{f}}\mathcal{A}_f^*]\sin(x\Gamma t)\right]$$





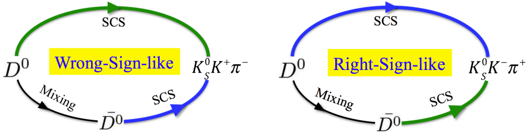


- fit results and prospect estimation:

Fit type	Para.	Belle Fit result 921 fb^{-1}	Belle II prospect 5 ab^{-1}	Belle II prospect 50 ab^{-1}	model-indept. 100 M signals	LHCb 50 fb^{-1}
No CPV	x(%)	$0.56 \pm 0.19^{+0.03+0.06}_{-0.09-0.09}$	$\pm 0.08 \pm 0.11$	$\pm 0.03 \pm 0.11$	± 0.017	
	y(%)	$0.30 \pm 0.15^{+0.04+0.03}_{-0.05-0.06}$	$\pm 0.06 \pm 0.05$	$\pm 0.02 \pm 0.04$	± 0.019	
indirect CPV	x(%)	$0.56 \pm 0.19^{+0.04+0.06}_{-0.08-0.08}$	$\pm 0.08 \pm 0.11$	$\pm 0.03 \pm 0.11$		0.04
	y(%)	$0.30 \pm 0.15^{+0.04+0.03}_{-0.05-0.07}$	$\pm 0.06 \pm 0.05$	$\pm 0.02 \pm 0.04$		0.004
	$\|q/p\|$	$0.90^{+0.16+0.05+0.06}_{-0.15-0.04-0.05}$	$\pm 0.069 \pm 0.073$	$\pm 0.022 \pm 0.069$		0.04
	$\arg(q/p)(^o)$	$-6 \pm 11 \pm 3^{+3}_{-4}$	$\pm 4.7 \pm 4.2$	$\pm 1.5 \pm 3.8$		3

SCS non-self-conjugated decay $D^0 \to K_S^0 K^{\mp}\pi^{\pm}$

- decay rate of WS-like and RS-like:



  - $\Gamma(D^0 \to K_S^0 K^+\pi^-) = e^{-\Gamma t}\left[r_D^2 + r_D R_D y'\Gamma t + \frac{(1-r_D^2)x'^2+(1+r_D^2)y'^2}{4}(\Gamma t)^2\right]$
  - $\Gamma(D^0 \to K_S^0 K^-\pi^+) = e^{-\Gamma t}\left[1 + r_D R_D y'\Gamma t + \frac{(1+r_D^2)x'^2-(1-r_D^2)y'^2}{4}(\Gamma t)^2\right]$

  where $r_D = A_{sup.}/\bar{A}_{fav.}$, $y' = y\cos\delta_D - x\sin\delta_D$, coherence factor R_D: $A_f\bar{A}_f R_D e^{-i\delta_D} = A_f\bar{A}_f^*$

- Comparing with WS decays $D^0 \to K^+\pi^-$: $r_D^{K\pi} \ll 1$ Vs. $r_D^{K_S^0 K\pi} \sim 1$
- $D^0 \to K_S^0 K\pi$: effectively sensitive to y'(large R_D); higher purity (large r_D)
- CLEO gave in $D^0 \to K_S^0 K\pi$, $R_D = 0.73 \pm 0.09$ and $\delta_D = (8.2 \pm 15.2)^o$ [PRD 85, 092016(2012)]
- sensitivity estimation of mixing: $\sigma(y') = 0.55\%$ (80K signals) [PLB 701(2011)353]
- LHCb has performed Dalitz plot fit on these two decays. [PRD 93, 052018 (2016)]
- mixing and CPV measurement in these decays......

Outline

Time-integrated CP asymmetry in $D^0 \to K_S^0 K_S^0$ [arXiv:1705.05966]

- time-integrated CP asymmetry measurement based on the partial decay rates:

$$A_{CP}^f = \frac{\Gamma(D \to f) - \Gamma(\overline{D} \to \overline{f})}{\Gamma(D \to f) + \Gamma(\overline{D} \to \overline{f})} = a_d^f + a_m^f + a_{inf}^f$$

- A_{CP} at Belle [Vs. Belle II precision $\sim \mathcal{O}(0.01\%)$]

Channel	$\mathcal{L}$(/fb)	value(%)	References
$D^0 \to \pi^+\pi^-$	976	$+0.55 \pm 0.36 \pm 0.09$	PoS ICHEP2012 (2013) 353
$D^0 \to K^+K^-$	976	$-0.32 \pm 0.21 \pm 0.09$	PoS ICHEP2012 (2013) 353
$D^0 \to \pi^0\pi^0$	966	$-0.03 \pm 0.64 \pm 0.10$	PRL **112**, 211601 (2014)
$D^0 \to K_S^0 K_S^0$	921	$-0.02 \pm 1.53 \pm 0.17$	arXiv:1705.05966
$D^0 \to K_S^0 \pi^0$	966	$-0.21 \pm 0.16 \pm 0.07$	PRL **112**, 211601 (2014)
$D^0 \to K_S^0 \eta$	791	$+0.54 \pm 0.51 \pm 0.16$	PRL **106**, 211801 (2011)
$D^0 \to K_S^0 \eta'$	791	$+0.98 \pm 0.67 \pm 0.14$	PRL **106**, 211801 (2011)
$D^0 \to \pi^+\pi^-\pi^0$	532	$+0.43 \pm 0.41 \pm 1.23$	PLB **662**, 102 (2008)
$D^0 \to K^+\pi^-\pi^0$	281	-0.60 ± 5.30	PRL **95**, 231801 (2005)
$D^0 \to K^+\pi^-\pi^+\pi^-$	281	$+0.43 \pm 1.30$	PRL **95**, 231801 (2005)
$D^+ \to \pi^0\pi^+$	921	$+0.89 \pm 1.98 \pm 0.22$	Belle Preliminary
$D^+ \to \phi\pi^+$	955	$+0.51 \pm 0.28 \pm 0.05$	PRL **108**, 071801 (2012)
$D^+ \to \eta\pi^+$	791	$+1.74 \pm 1.13 \pm 0.19$	PRL **107**, 221801 (2011)
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$D^+ \to K_S^0\pi^+$	977	$-0.363 \pm 0.094 \pm 0.067 (3.2\sigma)$	PRL **109**, 021601 (2012)
$D^+ \to K_S^0 K^+$	977	$-0.25 \pm 0.28 \pm 0.14$	JHEP **02** (2013) 098
$D_s^+ \to K_S^0\pi^+$	673	$+5.45 \pm 2.50 \pm 0.33$	PRL **104**, 181602 (2010)
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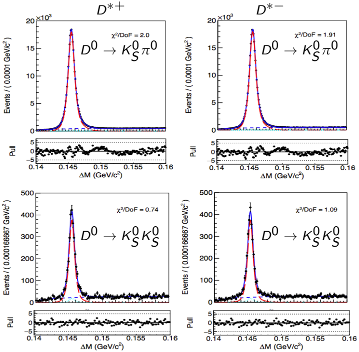


- eg: $D^0 \to K_S^0 K_S^0$(sig), respect to $D^0 \to K_S^0 \pi^0$(resp), measure A_{CP} and branching fraction

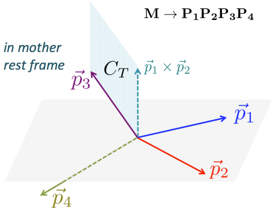
$$A_{raw} = \frac{N(D^0) - N(\bar{D}^0)}{N(D^0) + N(\bar{D}^0)} = A_{CP} + A_{FB} + A_\epsilon^\pm + A_\epsilon^K$$
$$A_{CP}^{sig} = A_{raw}^{sig} - A_{raw}^{resp} + A_{CP}^{resp} + A_\epsilon^K$$
$$\frac{\mathcal{B}(D^0 \to K_S^0 K_S^0)}{\mathcal{B}(D^0 \to K_S^0 \pi^0)} = \frac{(N/\epsilon)_{D^0 \to K_S^0 K_S^0}}{(N/\epsilon)_{D^0 \to K_S^0 \pi^0}}$$

- most precise measurement on both A_{CP}: consistent with SM expectation; and $BR = (1.32 \pm 0.02 \pm 0.04 \pm 0.04) \times 10^{-4}$: consistent with PDG $(1.8 \pm 0.4) \times 10^{-4}$, but 2.3σ away from BESIII $(1.67 \pm 0.11 \pm 0.11) \times 10^{-4}$ [PLB 765(2017)231]

T-odd asymmetry measurement in $D^0 \to K_S^0\pi^+\pi^-\pi^0$

- T-odd correlations provides a powerful tool to indirectly search for CP violation:
  (1) a triple product of momenta; (2) assuming CPT symmetry conservation
- Parity-odd observable $C_T = (\vec{p}_1 \times \vec{p}_2) \cdot \vec{p}_3$ and its CP-conjugated observable $\overline{C}_T$

$$A_T = \frac{\Gamma(C_T > 0) - \Gamma(C_T < 0)}{\Gamma(C_T > 0) + \Gamma(C_T < 0)} \quad \bar{A}_T = \frac{\Gamma(-\overline{C}_T > 0) - \Gamma(-\overline{C}_T < 0)}{\Gamma(-\overline{C}_T > 0) + \Gamma(-\overline{C}_T < 0)}$$



- T-odd asymmetry: remove strong phase introduced by FSI

$$a_{CP}^{\text{T-odd}} = \frac{1}{2}(A_T - \bar{A}_T)$$

- Observing a T-odd asymmetry would be a signal for processes beyond the SM.
- Status of T-odd asymmetries in charmed mesons decay-rates:

$D^0 \to K_S^0\pi^+\pi^-\pi^0$	$a_{CP}^{\text{T-odd}} = (-0.28 \pm 1.38^{+0.23}_{-0.76}) \times 10^{-3}$	Belle[1]
$D^0 \to K^+K^-\pi^+\pi^-$	$a_{CP}^{\text{T-odd}} = (+1.7 \pm 2.7) \times 10^{-3}$	LHCb[2], BaBar[3], Focus[4]
$D^+ \to K_S^0 K^+\pi^+\pi^-$	$a_{CP}^{\text{T-odd}} = (-1.10 \pm 1.09) \times 10^{-2}$	BaBar[5], Focus[4]
$D_s^+ \to K_S^0 K^+\pi^+\pi^-$	$a_{CP}^{\text{T-odd}} = (-1.39 \pm 0.84) \times 10^{-2}$	BaBar[5], Focus[4]

[1] K. Prasanth *et al.*(Belle Collab.), Phys. Rev. D **95**, 091101(R) (2017)
[2] R. Aaij *et al.*(LHCb Collab.), JHEP **10**, 5 (2014)
[3] P. del Amo Sanchez *et al.*(BaBar Collab.), Phys. Rev. D **81**, 111103(R) (2010)
[4] J.M. Link *et al.*(FOCUS Collab.), Phys. Lett. B **622**, 239 (2005)
[5] J.P. Lees *et al.*(BaBar Collab.), Phys. Rev. D **84**, 031103(R) (2011)

Belle II could improve these results with more precision benefited from the increased dataset.

Outline

Summary

- Many measurements of D^0-$\overline{D^0}$ mixing and CP violation at Belle has been performed, in different physical process or different technique.
  - only one channel gives the observation of D^0-$\overline{D^0}$ mixing: $D^0 \to K^+\pi^-$ (in e^+e^- Collider)
  - lots of measurements are under the limit of statistics.
  - in some channels, stat. error comparable with syst. error model-independent method has prospect.
  - no hints for indirect CPV in charm
- Time-integrated CP asymmetries have been studies with many channels, but no significant signal for direct CP violation in charm.
- For more information on prospect at Belle II, please keep mind of talk by Prof. Chengping SHEN tomorrow.

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Back up

Thank you for your attention.

谢谢！

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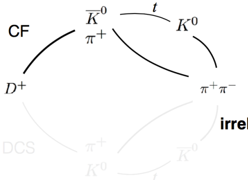
A_{CP} in charm decays into neutral kaons [ref. Fu-Sheng Yu et al., arXiv:1707.09297]

- A new CP violation effect in charm decays into neutral kaons, e.g: $D^+ \to K_S^0 \pi^0$ with evidence for CP violation: $A_{CP} = (-0.363 \pm 0.094 \pm 0.067)\%$

Indirect CPV in kaon mixing

$$A_{CP}(t) \simeq \left[A_{CP}^{\overline{K}^0}(t) + A_{CP}^{dir}(t) + A_{CP}^{int}(t)\right] / D(t)$$

$$A_{CP}^{\overline{K}^0}(t) = 2e^{-t/\tau_S}\left[\mathcal{R}e(\epsilon) - e^{\Delta\Gamma t/2}\left(\mathcal{R}e(\epsilon)\cos(\Delta m t) + \mathcal{I}m(\epsilon)\sin(\Delta m t)\right)\right] \mathbf{(10^{-3})}$$



Dominant **but** **irrelevant to charm**

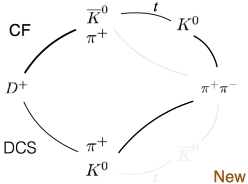
Report at Beihang Univ. at 22nd Jun. 2017

CPV in interference between kaon mixing and charm decays

$$A_{CP}(t) \simeq \left[A_{CP}^{\overline{K}^0}(t) + A_{CP}^{dir}(t) + A_{CP}^{int}(t)\right] / D(t)$$

$$A_{CP}^{int}(t) = e^{-t/\tau_S}\, 4 r_f \cos\phi \sin\delta_f \left[-\mathcal{I}m(\epsilon) + e^{\Delta\Gamma t/2}\left(\mathcal{I}m(\epsilon)\cos(\Delta m t) - \mathcal{R}e(\epsilon)\sin(\Delta m t)\right)\right]$$

$(10^{-4\sim-3})$

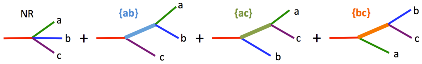


Non-negligible and important

New findings

Dalitz plot(DP) analysis with Isobar model

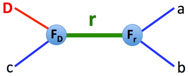
- DP of 3-body decay described by Isobar model:
  Phys. Rev. **123**, 333 (1961)



$$\mathcal{M}(m^2_{ab}, m^2_{bc}) = \sum_r a_r e^{i\phi_r} \mathcal{A}_r(m^2_{ab}, m^2_{bc}) + a_{NR} e^{i\phi_{NR}}$$

here $a_r(\phi_r)$ is magnitude (phase) of resonance r.

- $\mathcal{A}_r$: dynamics of $D \to (r \to ab)c$
  PRD **63**, 092001 (2001)



$$\mathcal{A}_r(m^2_{ab}, m^2_{bc}) = F_D \times F_r \times T_r \times W_r$$

▷ T_r: resonance lineship

usually use relativistic Breit-Wigner with mass dept. width
For wide width or special resonances:
mass-threshold: Flatté model, eg: $f_0(980)(KK)$ / $a_0(980)(KK/\eta'\pi)$
$\pi\pi$ S-wave overlapping res.: K-matrix model   EPJ **A16** (2003) 229-258
$\pi\pi$ P-wave: ρ with Gounaris-Sakurai (GS) model   PRL **24**,244(1968)
$K\pi$ S-wave: $K_0^*(1430)$ with LASS model   EPJ **C74** (2014): 3026

▷ W_r angular distribution:

(1)Helicity form   [PRD **78**, 052001 (2008)]
(2)Zemach covariant tensor form   [PR **133**, B1201 (1964), PR **140**, B109 (1965)]

▷ F_r, F_D form factor: PR **D 63**, 092001 (2001)

using Blatt-Weisskopt Barrier form factor, depend on orbital angular momentum ℓ(here $\ell = J$(spin of res.)

$$F_{J=0} = 1$$

$$F_{J=1} = \frac{\sqrt{1+z_r}}{\sqrt{1+z_{ab}}}$$

$$F_{J=2} = \frac{\sqrt{(z_r-3)^2+9z_r}}{\sqrt{(z_{ab}-3)^2+9z_{ab}}}$$

$$F_{J=3} = \frac{\sqrt{z_r(z_r-15)^2+9(2z_r-5)}}{\sqrt{z_{ab}(z_{ab}-15)^2+9(2z_{ab}-5)}}$$

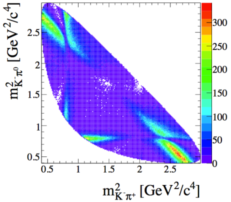
$$F_{J=4} = \frac{\sqrt{(z_r^2-45z_r+105)^2+15z_r(21z_r-21)^2}}{\sqrt{(z_{ab}^2-45z_{ab}+105)^2+15z_{ab}(21z_{ab}-21)^2}}$$

here $z = (R \cdot q)^2$, R is the radius of D or res. r

Dalitz amplitude analysis

- Dalitz plot(DP) fit
  - p.d.f of signal DP:
    (1) efficiency plane ϵ correction
    (2) considering mass resolution $Res(m)$
    (3) normalization

$$p_{\rm sig}(m^2_{12,i}, m^2_{23,i}) = \frac{|\mathcal{M}(m^2_{12,i},m^2_{23,i})|^2 \otimes_m Res(m)\cdot\epsilon(m^2_{12,i},m^2_{23,i})}{\iint_{DP} dm^2_{12}dm^2_{23}\ |\mathcal{M}(m^2_{12},m^2_{23})|^2\otimes_m Res(m)\cdot\epsilon(m^2_{12},m^2_{23})}$$



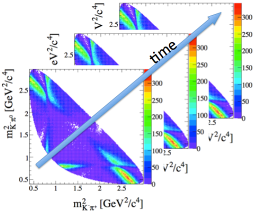
  - fit method: unbinned maximum likelihood (UML)

$$2\ln\mathcal{L} = 2\sum_{i=1}^{n}\ln\left[f^i_{sig}p_{\rm sig}(m^2_{12,i}, m^2_{23,i}) + \sum_{X=bg} f^i_X p_X(m^2_{12,i}, m^2_{23,i})\right]$$

signal-to-bkg f^i determined by kinematic variable fit result, like M-Q。

- time-dependent Dalitz plot(TDDP) fit
  - p.d.f of signal TDDP with considering time resolution : $R_{sig}(t)$

$$p_{\rm sig}(m^2_{12,i}, m^2_{23,i}, t_i, \sigma^i_t) = \frac{\int dt' R_{\rm sig}(t_i - t', \sigma^i_t)\left|\mathcal{M}_f(m^2_{12,i},m^2_{23,i},t')\right|^2\cdot\epsilon(m^2_{12,i},m^2_{23,i})}{\int dt \iint_{DP} dm^2_{12}dm^2_{23}\ \left|\mathcal{M}_f(m^2_{12},m^2_{23},t)\right|^2\epsilon(m^2_{12},m^2_{23})}$$



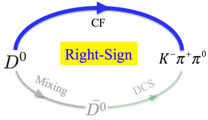
  - fit method: unbinned maximum likelihood (UML)

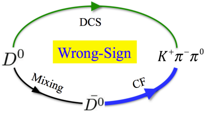
$$2\ln\mathcal{L} = 2\sum_i^n\{\ln(f^i_{\rm sig}p_{\rm sig}(m^2_{12,i}, m^2_{23,i}, t_i, \sigma^i_t; x, y)p^{nc}_{sig}(\sigma^i_t) + \sum_{x=bg} f^i_x p_x(m^2_{12,i}, m^2_{23,i}, t_i)p^{nc}_x(\sigma^i_t))\}$$

here $p^{nc}_x(\sigma^i_t)$ is global function for time error, independent on others.

Time-dependent amplitude of D^0 3-body hadronic decays

- Right-Sign(RS) abd Wrong-Sign(WS) decays



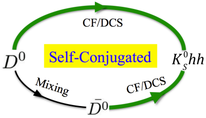


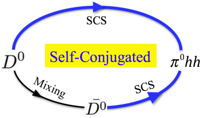
$|\mathcal{M}_f(RS,t)|^2 = e^{-\Gamma t}|\bar{\mathcal{A}}_f^{CF}|^2$

$\mathcal{A}_{\bar{f}}/\bar{\mathcal{A}}_{\bar{f}} = -\sqrt{R_D}e^{-i\delta}$; $x'' = x\cos\delta + y\sin\delta$; $y'' = y\cos\delta - x\sin\delta$

$|\mathcal{M}_{\bar{f}}(WS,t)|^2 = e^{-\Gamma t}\Big\{|\mathcal{A}_{\bar{f}}^{DCS}|^2 - (\frac{y''}{r_0}\,\mathrm{Re}[\mathcal{A}_{\bar{f}}^{DCS}\bar{\mathcal{A}}_{\bar{f}}^{CF*}] + \frac{x''}{r_0}\,\mathrm{Im}[\mathcal{A}_{\bar{f}}^{DCS}\bar{\mathcal{A}}_{\bar{f}}^{CF*}])\Gamma t + \frac{x''^2+y''^2}{4r_0^2}|\bar{\mathcal{A}}_{\bar{f}}^{CF}|^2(\Gamma t)^2\Big\}$

- Self-conjugated CF/SCS decays





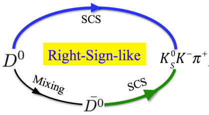
RS: $\mathcal{A}_f = \langle f|\mathcal{H}|D^0\rangle$; WS: $\mathcal{A}_{\bar{f}} = \langle \bar{f}|\mathcal{H}|D^0\rangle$; CP: $\mathcal{A}_\pm = \langle \pm|\mathcal{H}|D^0\rangle$

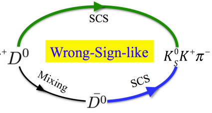
D^0: $m_+^2 = m^2_{K_S^0h^+}$; $m_-^2 = m_{K_S^0h^-}$; $\overline{D^0}$: $m_-^2 = m^2_{K_S^0h^+}$; $m_+^2 = m_{K_S^0h^-}$

$|\mathcal{M}_f|^2 = \Big\{|A_1|^2e^{-y\Gamma t} + |A_2|^2e^{y\Gamma t} + 2\,\mathrm{Re}[A_1A_2^*]\cos(x\Gamma t) + 2\,\mathrm{Im}[A_1A_2^*]\sin(x\Gamma t)\Big\}e^{-\Gamma t}$

$|\bar{\mathcal{M}}_f|^2 = \Big\{|\bar{A}_1|^2e^{-y\Gamma t} + |\bar{A}_2|^2e^{y\Gamma t} + 2\,\mathrm{Re}[\bar{A}_1\bar{A}_2^*]\cos(x\Gamma t) + 2\,\mathrm{Im}[\bar{A}_1\bar{A}_2^*]\sin(x\Gamma t)\Big\}e^{-\Gamma t}$

- SCS non-self-conjugated decays





(very difficult! No one experiment gives time-dept. amplitude analysis to date.)

$A = \langle K_S^0K^-\pi^+|\mathcal{H}|D^0\rangle = \langle K_S^0K^+\pi^-|\mathcal{H}|\bar{D}^0\rangle$; $B = \langle K_S^0K^+\pi^-|\mathcal{H}|D^0\rangle = \langle K_S^0K^-\pi^+|\mathcal{H}|\bar{D}^0\rangle$

$|\mathcal{M}(RS,t)|^2 = e^{-\Gamma t}\Big\{|A|^2 + \frac{x^2+y^2}{4}r_D^2|B|^2(\Gamma t)^2 + r_D(y'\,\mathrm{Re}[AB^*] + x'\,\mathrm{Im}[AB^*])\Gamma t\Big\}$

$|\mathcal{M}(WS,t)|^2 = e^{-\Gamma t}\Big\{r_D^2|B|^2 + \frac{x^2+y^2}{4}|A|^2(\Gamma t)^2 + r_D\cdot(y'\,\mathrm{Re}[BA^*] + x'\,\mathrm{Im}[BA^*])\Gamma t\Big\}$