

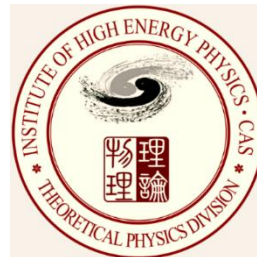
Charm physics in lattice QCD

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BELLE, LHCb

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Charm physics in lattice QCD

- 格点QCD
- 粲物理相关的格点QCD计算
 - 衰变常数
 - 形状因子(半轻树图过程)
 - 粲夸克质量
 - D-Dbar混合
- 小结

格点量子色动力学(Lattice QCD)

- 高能实验和微扰计算告诉我们，QCD是描写夸克、胶子之间强相互作用的正确理论
- 但怎么将强子与QCD的基本自由度夸克、胶子联系起来？
- 低能标下QCD不能微扰求解，需要非微扰的办法
- 格点QCD用数值模拟的办法，能非微扰地研究低能QCD性质
- 固定参数 α_s 和夸克(流)质量后，即可给出其他物理量

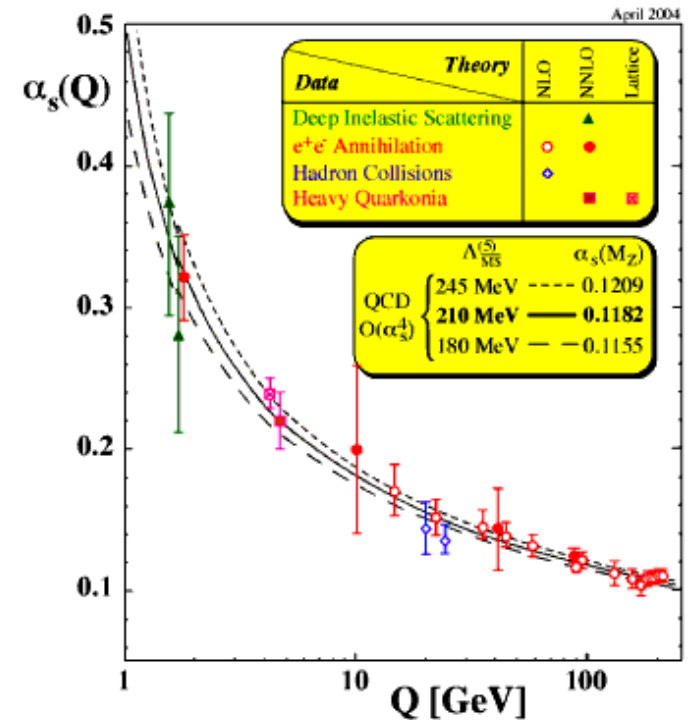


FIG. 1: QCD running coupling constant

格点QCD

- 1973年Kenneth G. Wilson提出
- 1979年M. Creutz首个计算机上的数值计算
- 1980年开始制造并行计算机专用于LQCD计算

Edison, NERSC



2.57 Pflops 2014

QCD on chips(QCDOC)

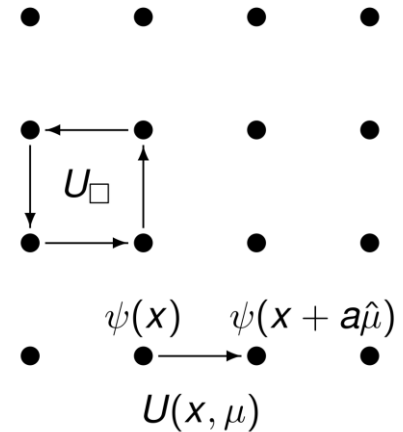


2X10 Tflops 2005

IBM (BG/L)



格点QCD



- LQCD is based on the path integral formalism in Euclidean space.

$$\langle \mathcal{O} \rangle = \frac{\int DA_{\mu} D\bar{\psi} D\psi \mathcal{O}[A, \bar{\psi}, \psi] e^{-\int \mathcal{L}_{QCD} d^4x}}{\int DA_{\mu} D\bar{\psi} D\psi e^{-\int \mathcal{L}_{QCD} d^4x}},$$

where $\mathcal{L}_{QCD} = \bar{\psi} \mathbf{M}[A] \psi + \mathcal{L}_G$, $\mathbf{M} = \boldsymbol{\gamma} \cdot \mathbf{D} + m_q$

- On the 4D lattice, the A fields are put on the links

$$U(x, \mu) = e^{igaA_{\mu}}.$$

- 完成费米子场的积分:

$$\langle \mathcal{O} \rangle = \frac{\int DU_{\mu} \mathcal{O}[U, \mathbf{M}^{-1}[U]] \text{Det}[\mathbf{M}[U]] e^{-S_G}}{\int DU_{\mu} \text{Det}[\mathbf{M}[U]] e^{-S_G}}$$

Lattice QCD

- $\langle O \rangle = \frac{\int DU_\mu O[U, M^{-1}[U]] \text{Det}[M[U]] e^{-S_G}}{\int DU_\mu \text{Det}[M[U]] e^{-S_G}}$

- 价夸克传播子 $M^{-1}[U]$

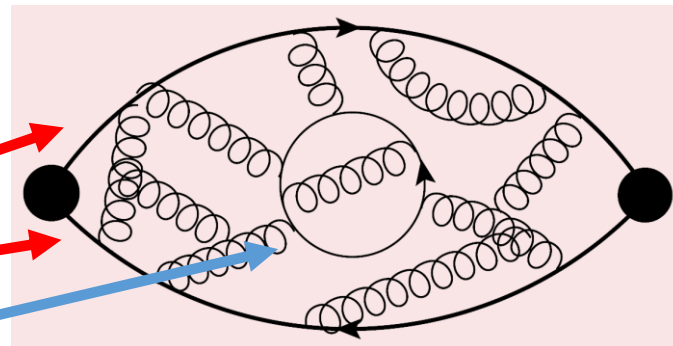
- 海夸克效应 $\text{Det}[M[U]]$

- 用重点抽样产生组态: gluon fields U (最耗费机时的部分)

- 计算价夸克传播子和强子关联函数 (很耗费机时, 大量硬盘空间)

- 数据分析和拟合: 强子质量及矩阵元

- 确定格距 a 和夸克质量 m_q , 给出其他物理结果



按几率分布产生组态
 $P[U] \propto \text{Det}[M[U]] e^{-S_G}$

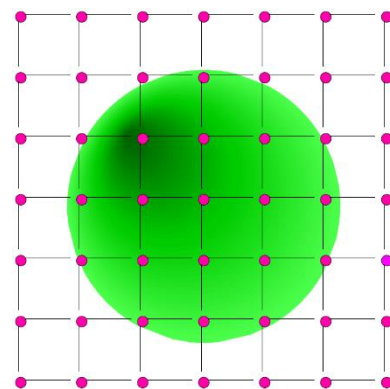
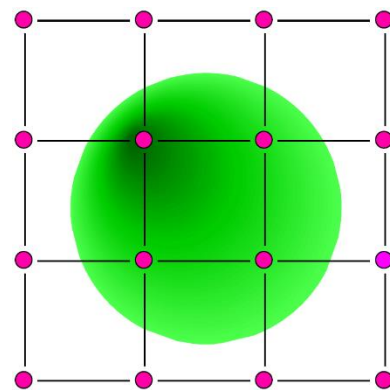
对规范场的路径积分变为对组态的统计平均

$$\langle O \rangle = \frac{1}{N} \sum_{i=1}^N O_i$$

统计误差: N 有限

格点QCD：误差

- 除了统计误差，还有系统误差
 - 非零格距 a 引入离散误差，连续极限 $a \rightarrow 0$
 - 多数情况下 $m_\pi > 135 \text{ MeV}$ ，手征外推
 - 有限体积，格子边长 L 需大于 π 介子康普顿波长 $1/m_\pi$
- 在多个不同的 a 、 L 、 m_π 下作计算，量化和控制系统误差
- 其他误差来源：重整化常数，.....
- 目前最小的 $a \approx 0.05 \text{ fm}$ (a 越小，同样体积需要格子数越多)
- 海夸克味道数 $N_f = 2 + 1 + 1, 2 + 1, 2, 0$ (淬火近似)
- 已开始有 $m_\pi = 135 \text{ MeV}$ 的组态产生



味物理与LQCD

- LQCD can calculate form factors and meson decay constants appearing in weak decays of hadrons
- Combined with experiments, they can give us CKM matrix elements
- Test the SM (is CKM unitary?)
- Or use V_{ab} from elsewhere to compare QCD/SM results with experiments

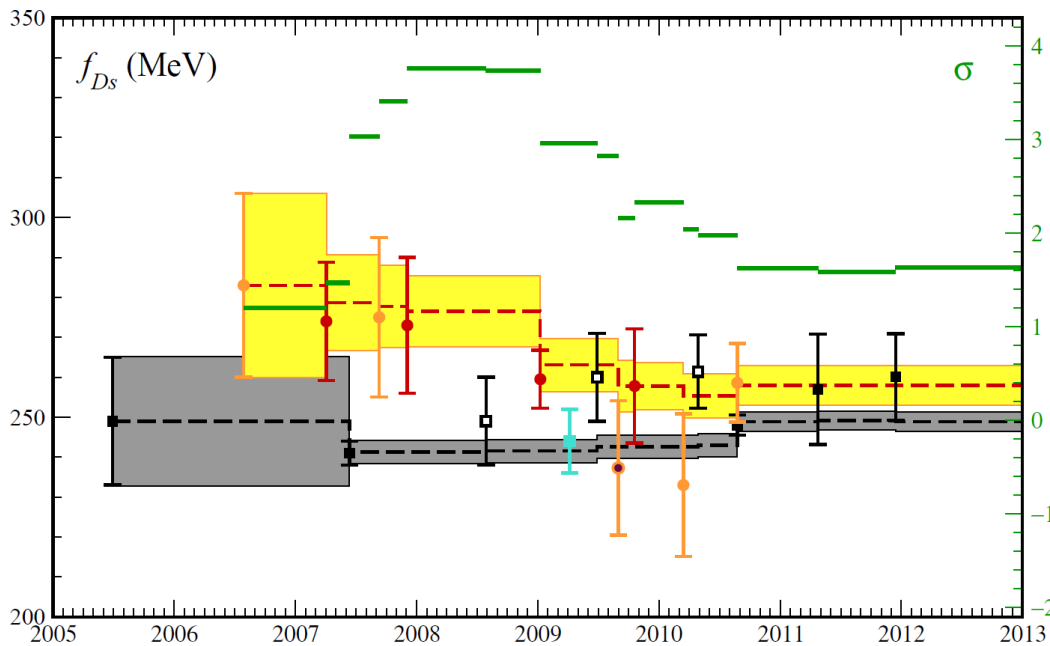
$$\begin{pmatrix}
 V_{ud} & V_{us} & V_{ub} \\
 \pi \rightarrow \ell\nu & K \rightarrow \ell\nu & B \rightarrow \pi\ell\nu \\
 & K \rightarrow \pi\ell\nu & \\
 \hline
 V_{cd} & V_{cs} & V_{cb} \\
 D \rightarrow \ell\nu & D_s \rightarrow \ell\nu & B \rightarrow D\ell\nu \\
 D \rightarrow \pi\ell\nu & D \rightarrow K\ell\nu & B \rightarrow D^*\ell\nu \\
 V_{td} & V_{ts} & V_{tb} \\
 B_d \leftrightarrow \bar{B}_d & B_s \leftrightarrow \bar{B}_s &
 \end{pmatrix}$$

Can be calculated by LQCD

例如:
$$\Gamma(P \rightarrow \ell\nu) = \frac{G_F^2 |V_{q_1 q_2}|^2 f_P^2 m_\ell^2 M_P \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2}{8\pi}$$

衰变常数，例如 f_{D_s}

- 衰变常数与纯轻衰变的实验测量结合起来可确定 $|V_{cd}|$ 和 $|V_{cs}|$



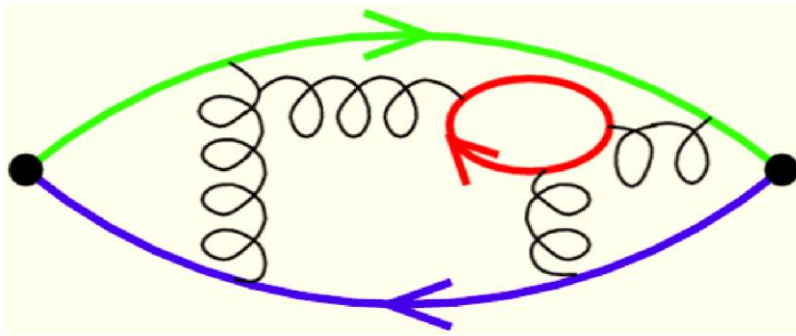
- 2013年之前的情况
- 灰颜色：2+1味格点平均(FNAL/MILC, HPQCD, PACS-CS)
- 黄颜色：实验平均(BaBar, Belle, CLEO-c)
- 青色点：ETMC 2味格点计算, 未计入平均

Kronfeld, 1203.1204

赝标介子衰变常数

- $\langle 0 | \bar{q} \gamma_\mu \gamma_5 c | P(p) \rangle = f_P p_\mu \quad q = d, s$
- Let $O = \bar{q} \gamma_0 \gamma_5 c$, its matrix element can be obtained from

$$C(t) = \langle 0 | O(t) O^\dagger(0) | 0 \rangle \xrightarrow{t \rightarrow \infty} |\langle 0 | O | P \rangle|^2 e^{-m_P t}$$



Heavier hadrons with same quantum numbers as O are suppressed at large t .

f_{D_s}

χ QCD Collaboration:

Y.-B. Yang et al., PRD92, 2015

ZL et al., PRD90, 2014 ($Z_m, Z_A...$)

- $\langle 0 | \bar{s} \gamma_\mu \gamma_5 c | D_s \rangle = f_{D_s} p_\mu$
- The local current $\bar{s} \gamma_\mu \gamma_5 c$ on the lattice needs a (re)normalization Z_A
- Using the PCAC relation, one can also get f_{D_s} by

$$(m_s + m_c) \langle 0 | \bar{s} \gamma_5 c | D_s \rangle = f_{D_s} M_{D_s}^2$$

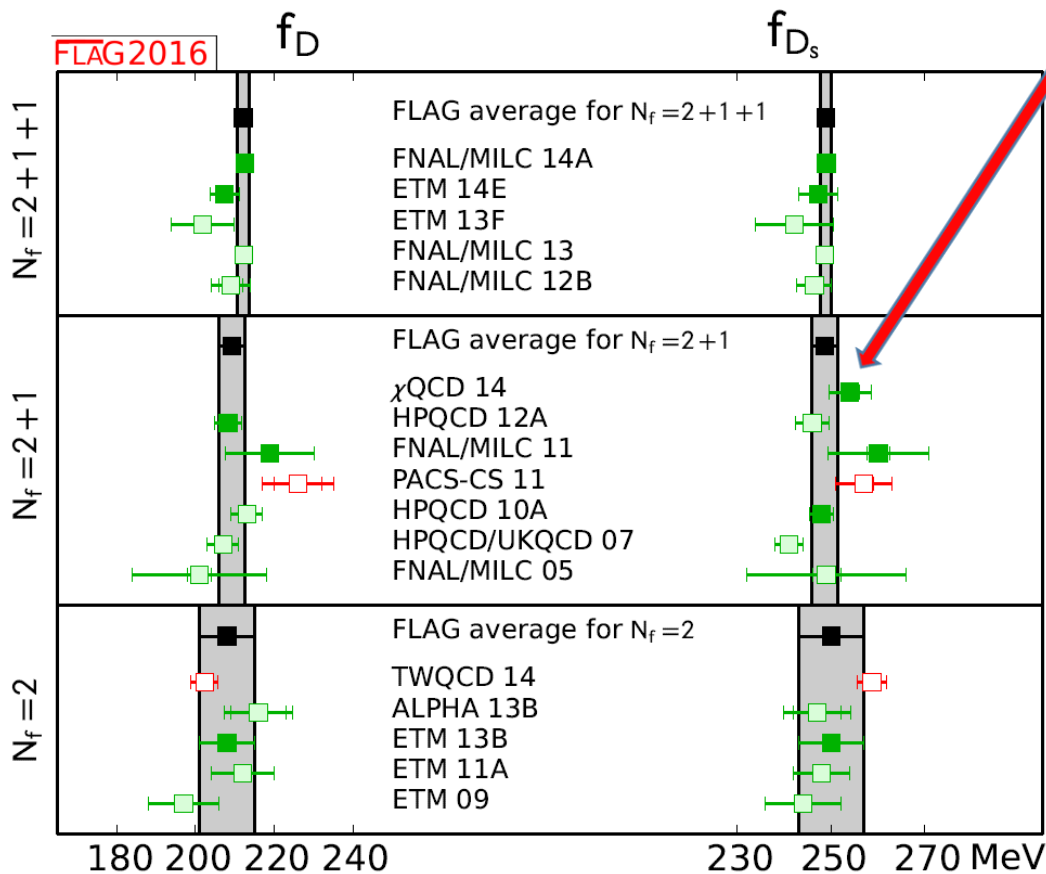
No renormalization is needed due to $Z_P Z_m = 1$ (for chiral fermions on the lattice)

- f_{D_s} are obtained from both methods in our work (2+1味).
- We find $f_{D_s} = 254(2)(4) \text{ MeV}$

f_D 和 f_{D_s}

ZL et al., PRD90, 2014

Y.-B. Yang et al., PRD92, 2015



- $N_f = 2 + 1$:

$$f_{D_s} = 249.8(2.3) \text{ MeV}$$

- $N_f = 2 + 1 + 1$:

$$f_{D_s} = 248.83(1.27) \text{ MeV}$$

- **PDG2016 (CPC40,100001):**

$$f_{D_s^+}^{exp} = 257.8(4.1) \text{ MeV}$$

- 格点结果与实验在 2σ 之内一致

- 预期2020年实验精度将与理论精度持平 (BESIII和 BelleII)

FLAG2016, arXiv:1607.00299

f_{D_s} 和 f_D

N_f	Collaboration	f_D [MeV]	f_{D_s} [MeV]	m_π^{\min} [MeV]	a [fm]	# a
2+1	RBC/UKQCD [2]	208.7(2.8) $\left(\begin{smallmatrix} +2.1 \\ -1.8 \end{smallmatrix}\right)$	246.4(1.3) $\left(\begin{smallmatrix} +1.3 \\ -1.9 \end{smallmatrix}\right)$	139	0.07-0.11	3
	JLQCD [3] ¹	212.8(1.7)(3.6)	244.0(0.8)(4.1)	230	0.044-0.08	3
	χ QCD [4]		254(2)(4)	300	0.08-0.11	2
	HPQCD [5]	208.3(1.0)(3.3)	246.0(0.7)(3.5)	245	0.08-0.12	2
	HPQCD [6]		248.0(2.5)	260	0.045-0.15	5
	FNAL/MILC [7]	218.9(11.3)	260.1(10.8)	230	0.09-0.15	3
2+1+1	FNAL/MILC [8]	212.6(0.4) $\left(\begin{smallmatrix} +1.0 \\ -1.2 \end{smallmatrix}\right)$	249.0(0.3) $\left(\begin{smallmatrix} +1.1 \\ -1.5 \end{smallmatrix}\right)$	130	0.06-0.15	4
	ETM [9]	207.4(3.8)	247.2(4.1)	210	0.06-0.09	3

[1702.05360]

- 2015年后的两个计算：RBC/UKQCD (1701.02644), JLQCD
- JLQCD (lattice2016, 1702.02303): 系统误差尚只考虑了格距标定。

f_{D^*} 和 $f_{D_s^*}$

- 矢量介子衰变常数的定义:

$$\langle 0 | \bar{q}(0) \gamma^\mu q'(0) | V(p, \lambda) \rangle = f_V m_V e_\lambda^\mu$$

- 矢量介子的纯轻衰变宽度, 如 D_s^* (尚无实验测量)

$$\Gamma_{(D_s^* \rightarrow \ell \nu)} = \frac{G_F^2}{12\pi} |V_{cs}|^2 f_{D_s^*}^2 M_{D_s^*}^3 \left(1 - \frac{m_\ell^2}{M_{D_s^*}^2}\right)^2 \left(1 + \frac{m_\ell^2}{2M_{D_s^*}^2}\right)$$

- 用因子化方法研究 $B \rightarrow D^{(*)} M$ 等过程时也需要用到 f_{D^*} 等衰变常数

f_{D^*} 和 $f_{D_S^*}$

- 我们正在做的一个计算(overlap on domain wall)

$1/a(\text{GeV})$	label	am_{sea}	volume
1.730(4)	48I	0.00078/0.0362	$48^3 \times 96$

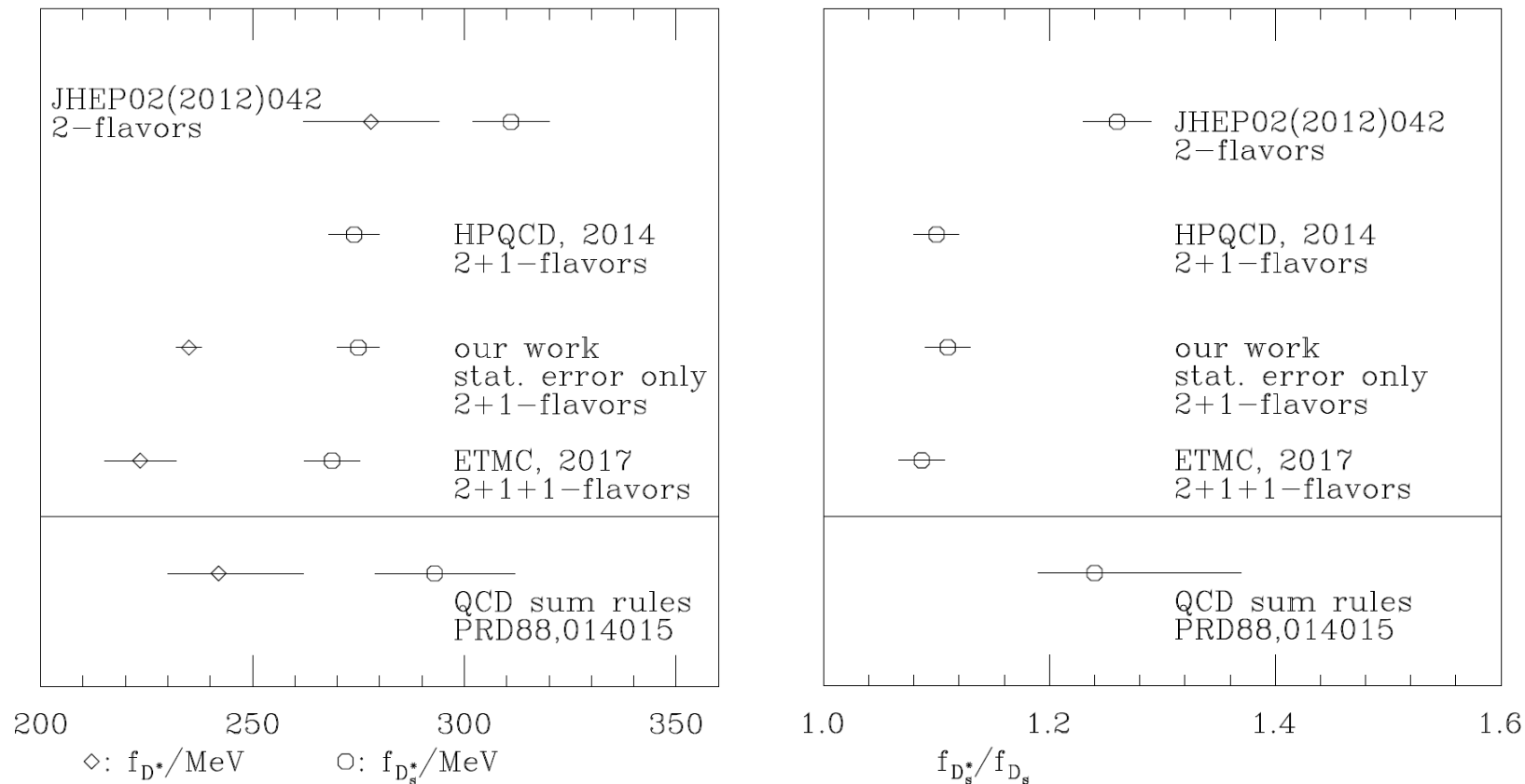
- 轻夸克质量接近物理点: $m_\pi = 139.2(4) \text{ MeV}$
- 2+1味组态(RBC/UKQCD合作组产生)
- 初步结果(尚无系统误差)

	D	D^*	D_S	D_S^*
f_M/MeV	213(1)	235(3)	248(2)	275(5)

$$f_{D^*}/f_D = 1.10(3)$$

$$f_{D_S^*}/f_{D_S} = 1.11(2)$$

f_{D^*} 和 $f_{D_s^*}$



- HPQCD, PRL112, 212002 (2014) 2个格距, 2+1味
- ETMC, PRD96, 034524 (2017) 3个格距, 2+1+1味
- Becirevic et al., JHEP02 (2012) 042 4个格距, 2味
- Large sea quark effects?

粲介子半轻衰变（树图过程）

- $D \rightarrow \pi l \nu$, $D \rightarrow K l \nu$ 可用于确定 $|V_{cd}|$ 和 $|V_{cs}|$

$$\frac{d\Gamma(D \rightarrow K l \nu)}{dq^2} = (\text{known}) |\mathbf{p}_K|^3 |V_{cs}|^2 |f_+^{D \rightarrow K}(q^2)|^2$$

- 需要形状因子 f_+

$$\langle K | V^\mu | D \rangle = f_+(q^2) \left(p_D^\mu + p_K^\mu - \frac{m_D^2 - m_K^2}{q^2} q^\mu \right) + f_0(q^2) \frac{m_D^2 - m_K^2}{q^2} q^\mu$$

(对于 $l = e, \mu$, 形状因子 f_0 的贡献可忽略)

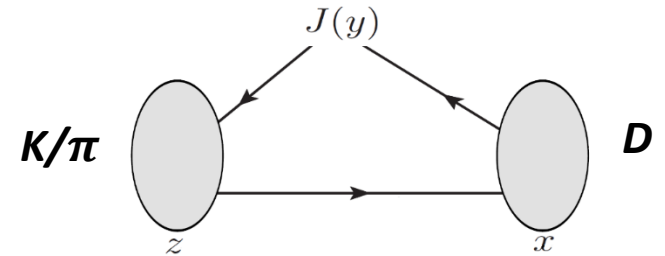
- 其他道: $D_s \rightarrow \phi l \nu$, $D_s \rightarrow \eta^{(\prime)} l \nu$

形状因子的格点计算

- 计算三点函数, 例如:

$$C_3(\vec{p}, \vec{p}', T, t) = \sum_{\vec{x}} \sum_{\vec{y}} \langle 0 | O_D(\vec{x}, T) J(\vec{y}, t) O_P^\dagger(\vec{z}, 0) | 0 \rangle e^{-i\vec{p} \cdot \vec{x}} e^{i\vec{q} \cdot \vec{y}}$$

$$\xrightarrow[\substack{t \rightarrow \infty \\ T \rightarrow \infty \\ (T-t) \rightarrow \infty}]{\text{}} \langle 0 | O_D | D \rangle \langle D | J | P \rangle \langle P | O_P^\dagger | 0 \rangle e^{-m_P t} e^{-m_D (T-t)}$$



- $q^2 = (p - p')^2$, 格点计算中3-动量取分立值(周期性BC)
- 对于 $D \rightarrow K/\pi$, $q^2 = 0$ 附近有数值结果, 可内插, 对 q^2 依赖的参数化形式不敏感
- 或者用 **twisted BC** 直接在 $q^2 = 0$ 点计算

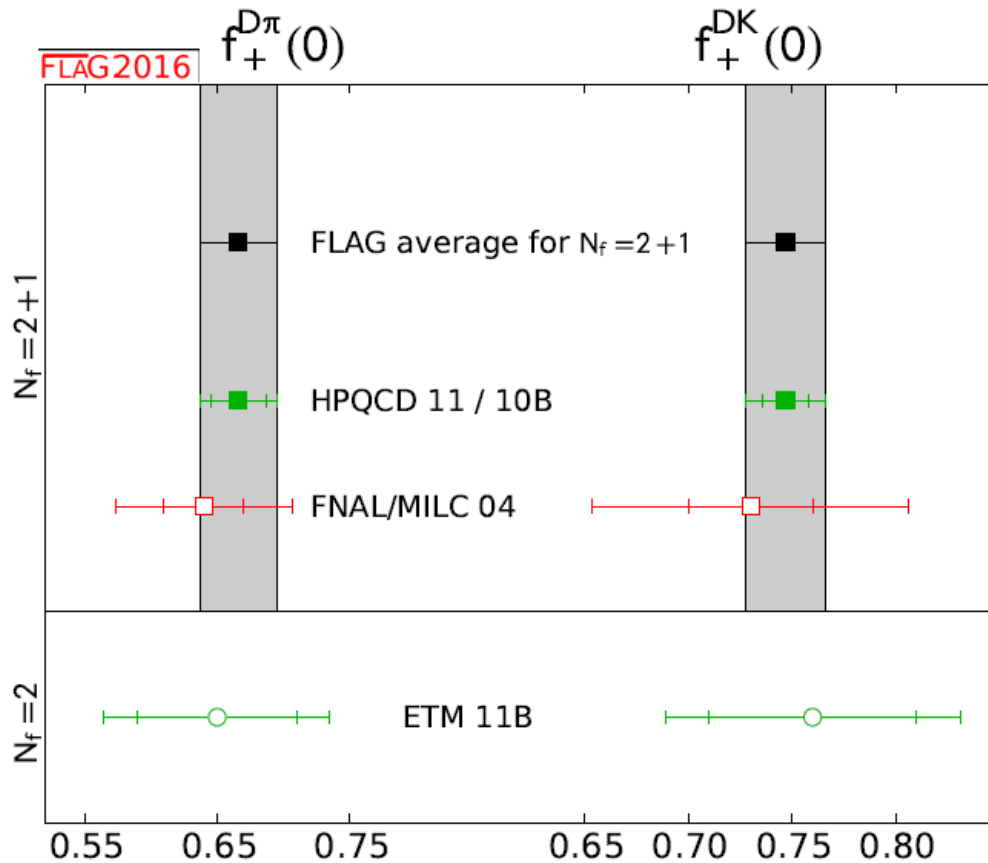
形状因子的格点计算

- 直接算 f_+ 需要 V^μ 的重整化常数, 引入额外误差
- **HPQCD**计算 $f_+(\mathbf{0}) = f_0(\mathbf{0})$
- 利用PCVC: $\partial_\mu V^\mu = (m_c - m_s)S$, 可得

$$\langle K|S|D\rangle = f_0^{D\rightarrow K}(q^2) \frac{M_D^2 - M_K^2}{m_c - m_s}$$

- 手征格点费米子有 $Z_S Z_m = 1$, 无需计算重整化常数
- 但不能考察 f_+ 随 q^2 的变化行为

$f_+(q^2 = 0)$



- FNAL/MILC/HPQCD 04:
一个格距, m_π 大于 500 MeV

- $f_+^{D\pi}(0) = 0.666(29)$
[HPQCD, PRD84,114505, 1109.1501]

- $f_+^{DK}(0) = 0.747(19)$
[HPQCD, PRD82,114506, 1008.4562]

ETMC, LAT2010

$|V_{cd}|$ 和 $|V_{cs}|$

- **PDG2016(CPC40,100001):**

$$f_D |V_{cd}| = 45.91(1.05) \text{ MeV}$$

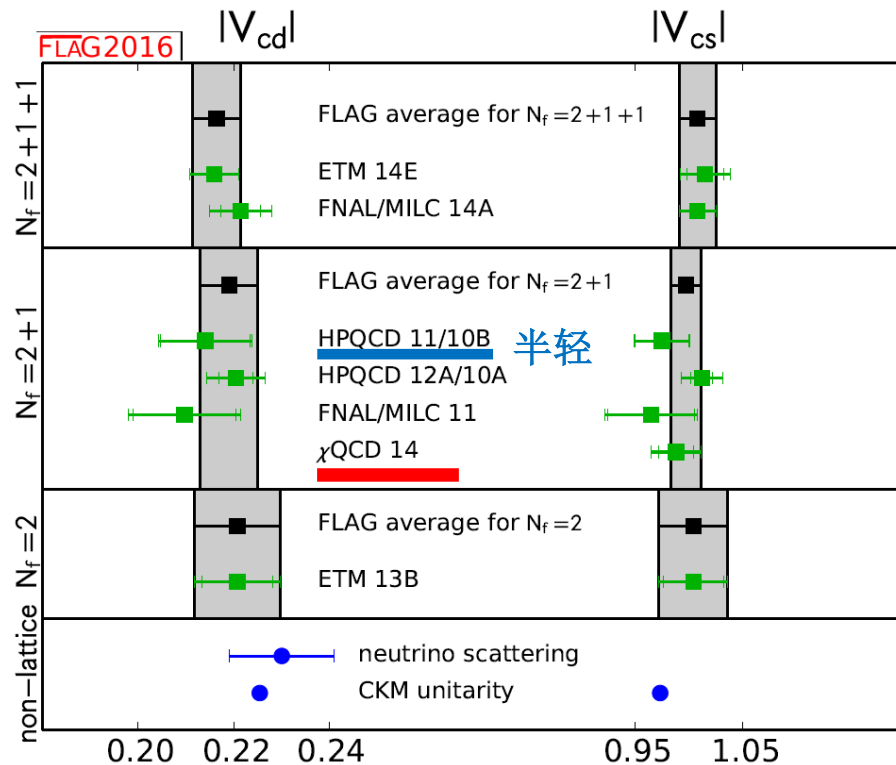
$$f_{D_s} |V_{cs}| = 250.9(4.0) \text{ MeV}$$

- **HFAG2014:**

$$f_+^{D\pi}(0) |V_{cd}| = 0.1425(19)$$

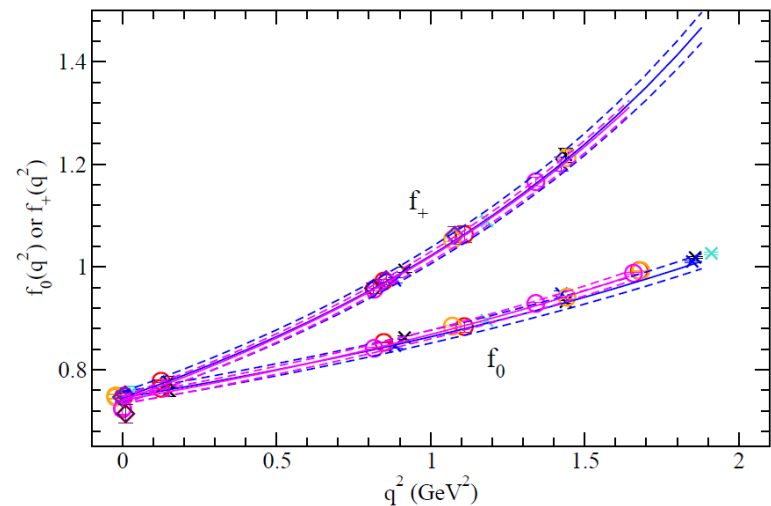
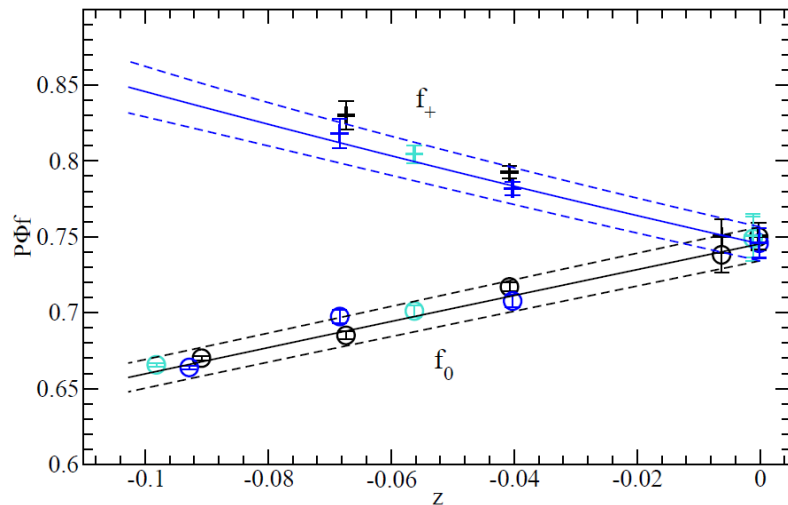
$$f_+^{DK}(0) |V_{cs}| = 0.728(5)$$

- **LQCD** 误差比实验误差大 (半轻)
- 和CKM幺正性结果 $\sim 2\sigma$ 一致



Form factor shapes

- Form factor shape对参数化形式有依赖，现在一般用z-expansion
- HPQCD, LAT2011(1111.0225), Charm2012(1208.6242), [1305.1462](#)
- $f_+^{DK}(0) = 0.745(11)$

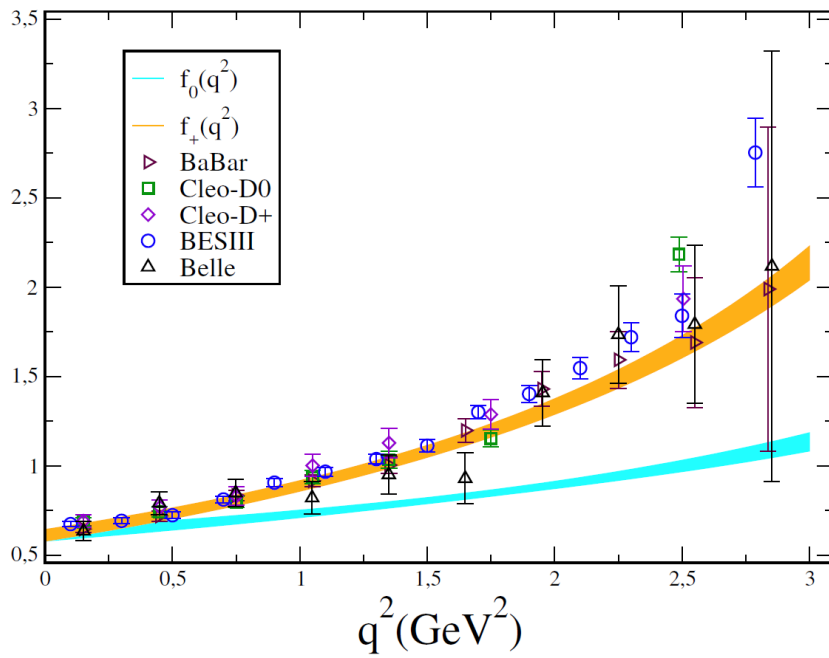


$D \rightarrow Kl\nu$, HPQCD [1305.1462](#)

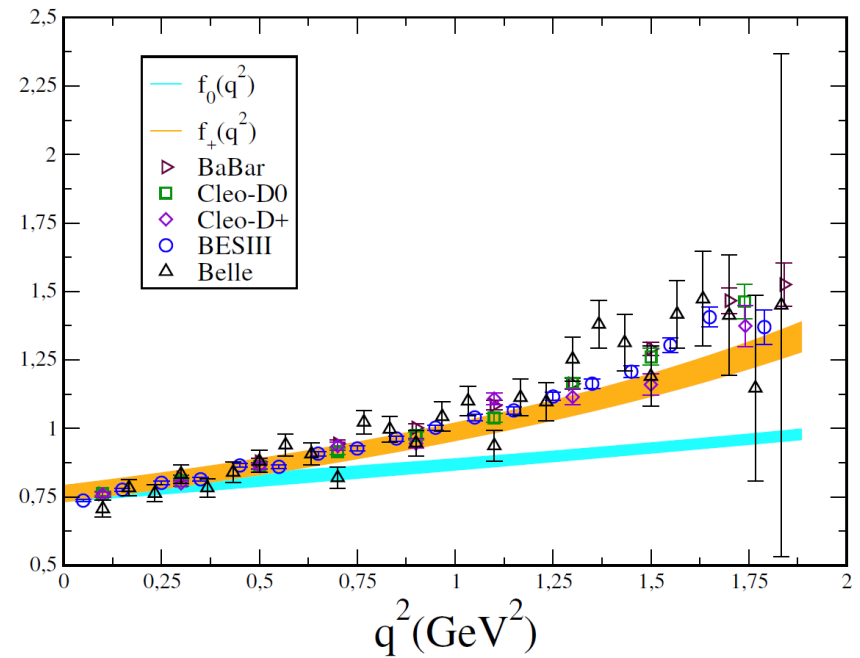
$f_+^{D\pi(K)}$ 和 $f_0^{D\pi(K)}$: 2015年后的计算

- JLQCD, Lattice2016, [1701.00942](#), 初步结果
 - 3个格距, 2+1味, Mobius domain wall fermions
 - $f_+^{D\pi}(0) = 0.644(49)(27)$, $f_+^{DK}(0) = 0.701(46)(33)$
- ETMC, [1706.03017](#)
 - 3个格距, 2+1+1味, Twisted mass fermions

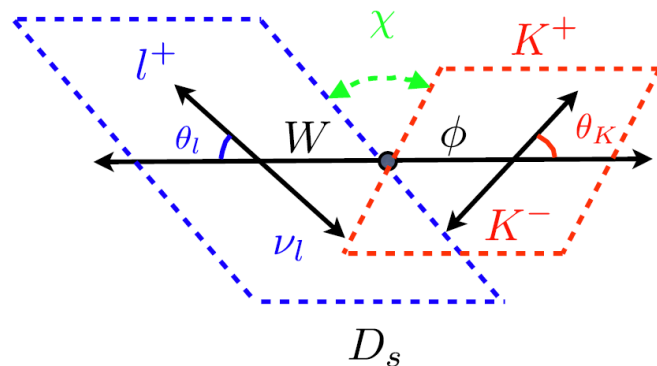
$D \rightarrow \pi$



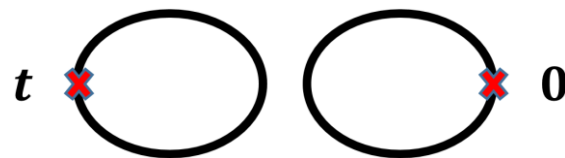
$D \rightarrow K$



其他衰变道



- D_s 的半轻衰变主要是到 ϕ 、 $\eta^{(\prime)}$
- 格点计算牵涉到(费米子线)非连通图, 更困难
- $D_s \rightarrow \phi l \nu$ [HPQCD, PRD90, 1311.6669]
 - 除了在 q^2 -bin 中比较截面, 还有更多观测量, $\phi \rightarrow K\bar{K}$
 - 计算忽略非连通图贡献(OZI压低)
 - 数据中 ϕ 不衰变; ϕ 宽度较窄, 预期阈效应小
 - 形状因子 V, A_2, A_1, A_0 ; z-expansion
 - 2+1味; 两个格距; HISQ价夸克作用量
 - 和 BABAR, CLEO $D \rightarrow K^*$ 的结果做了比较 (假设 spectator quark 的影响很小)



其他衰变道

- $D_s \rightarrow \eta^{(\prime)} l \nu$ [G. S. Bali, S. Collins, S. Durr, I. Kanamori, PRD91,014503 (2015) 1406.5449]
 - 2+1味; 一个格距: ~ 0.075 fm
 - 两个轻夸克质量点: $m_\pi = 370$ 和 470 MeV
 - 计算了非连通图
 - 用标量算符计算 f_0 ;
 - 单极点ansatz参数化 q^2 依赖
- $\Lambda_c \rightarrow \Lambda l \nu$: [S. Meinel, PRL (2017) 1611.09696]
 - 2+1味; 两个格距; $m_\pi = 139$ -- 352 MeV(4个取值);
 - (轴)矢量流: 6个形状因子
 - z-expansion

$\Lambda_c \rightarrow \Lambda l \nu$

$$\mathcal{B}(\Lambda_c \rightarrow \Lambda l^+ \nu_\ell) = \begin{cases} 0.0380(19)_{\text{LQCD}(11)\tau_{\Lambda_c}}, & \ell = e, \\ 0.0369(19)_{\text{LQCD}(11)\tau_{\Lambda_c}}, & \ell = \mu \end{cases}$$

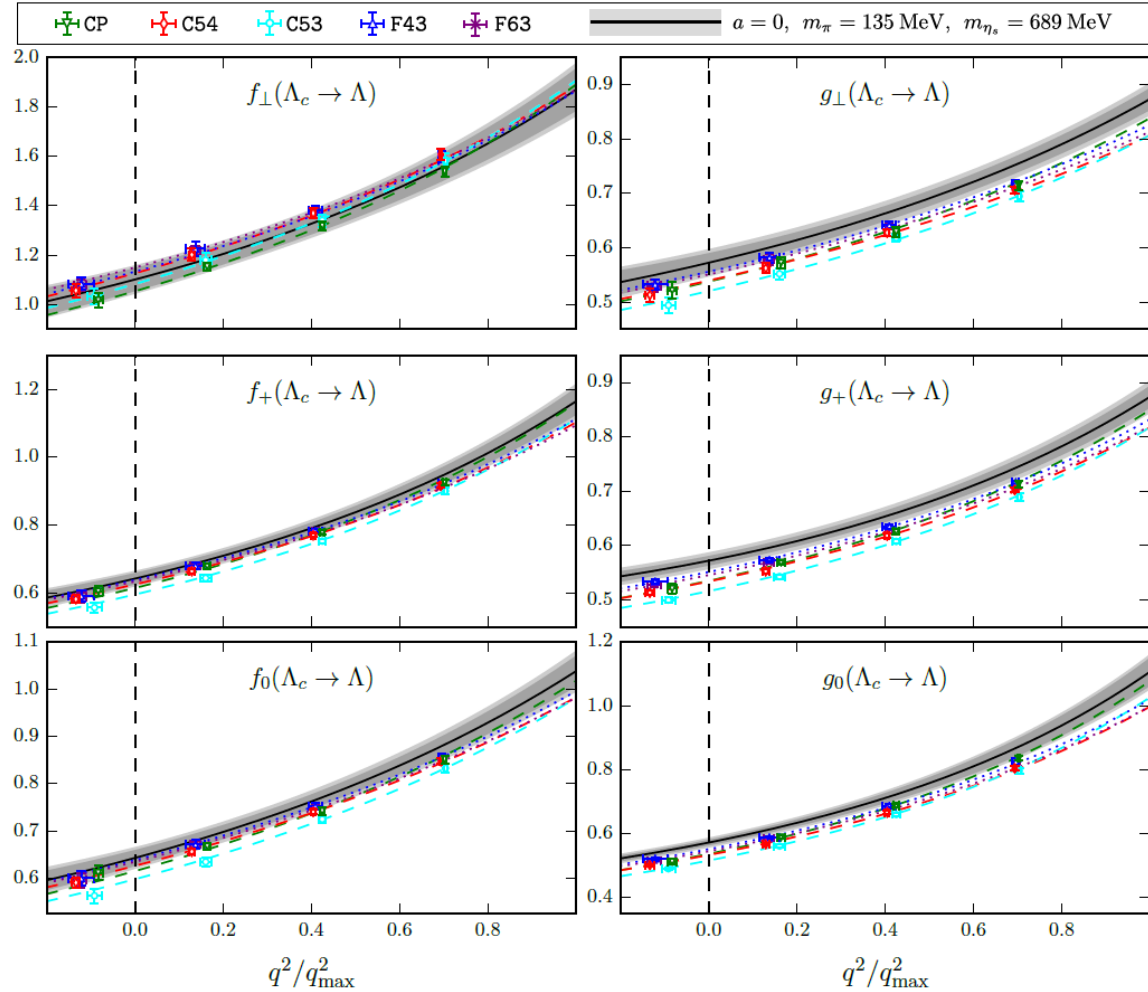
S. Meinel, PRL(2017) 1611.09696

- 得到的 $|V_{cs}|$ 和从 $D_s \rightarrow l \nu$ 得到的一致

$$\mathcal{B}(\Lambda_c \rightarrow \Lambda l^+ \nu_\ell) = \begin{cases} 0.0363(38)(20), & \ell = e, \\ 0.0349(46)(27), & \ell = \mu. \end{cases}$$

BESIII, PRL(2015) 1510.02610

BESIII, PLB(2017) 1611.04382



小结

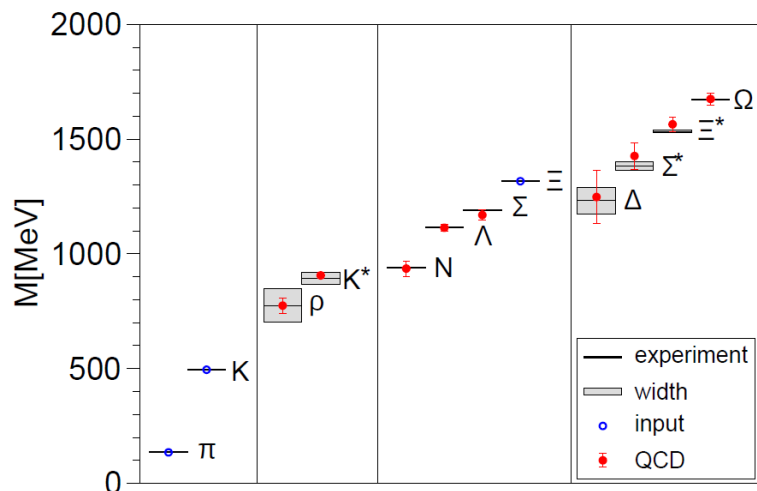
- 粲物理相关的格点QCD计算有助于精确检验标准模型
 - 衰变常数
 - 形状因子
 - 粲夸克质量
 - D-Dbar混合
- 结果正变得越来越精确(误差百分之几)
- 相关强子矩阵元的格点计算需要大量计算资源
($\sim O(10 - 100)$ 百万 cpu-hours/project, 不含组态产生)

请大家多多支持CLQCD!

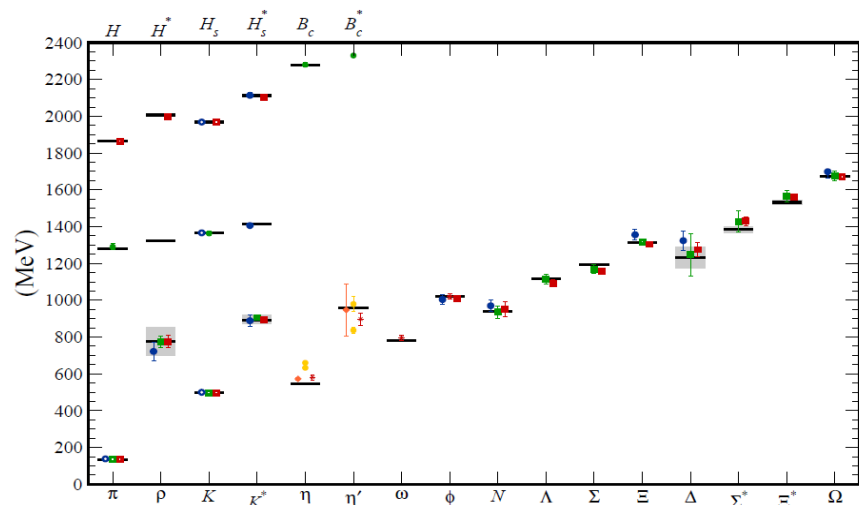
谢谢!

格点QCD

- 强子谱：实验输入(如 m_π, m_K, \dots)定出 a 和 m_q ，输出其他强子质量
- 不同格点作用量给出一致结果；预言了 B_c 和 B_c^* 的质量(HPQCD)



Science 2008, BMW

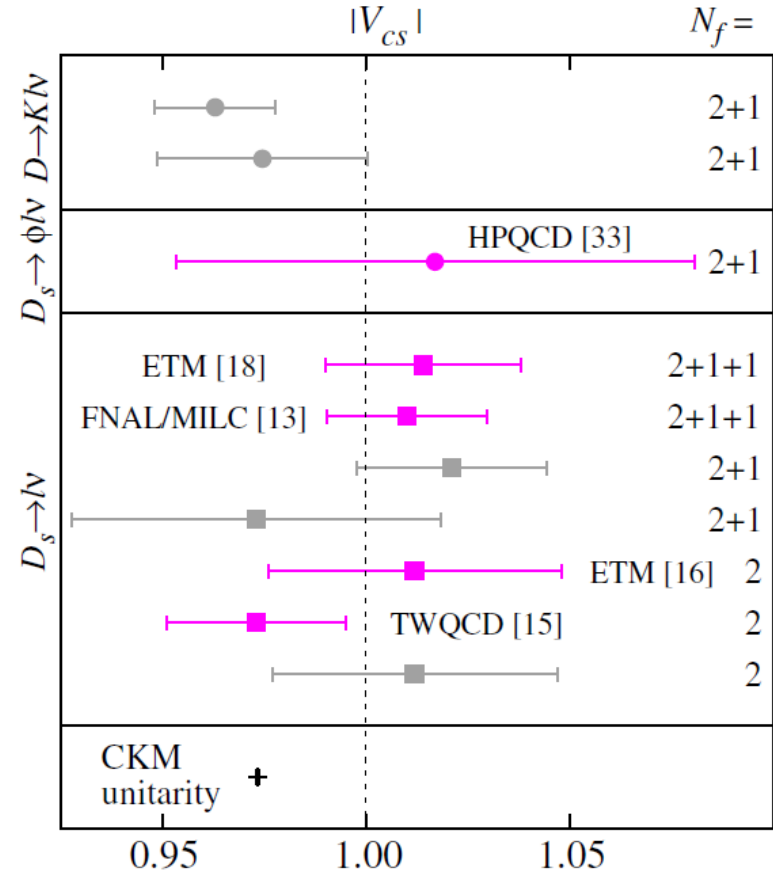
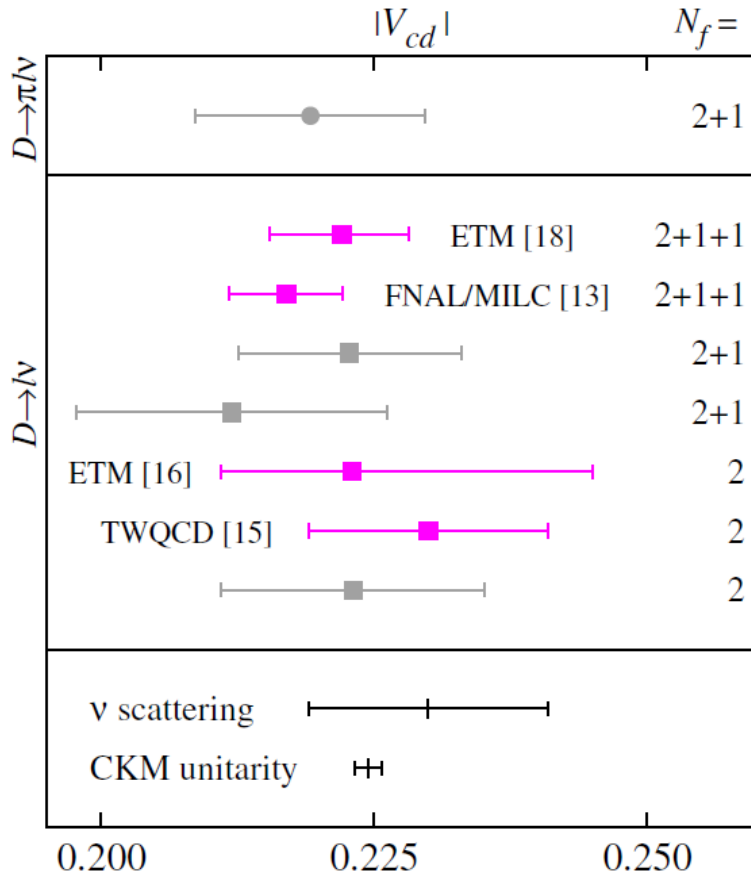


MILC, PACS-CS, BMW, RBC-UKQCD....., arXiv:1203.1204

- 胶球， J/psi辐射衰变
- D介子低能散射的研究：XYZ相关
- 强子结构：PDF， 质子自旋
- 有限温度有限密度QCD
-

$|V_{cd}|$ 和 $|V_{cs}|$

[Bouchard, Lattice2014]



$\Lambda_c \rightarrow \Lambda lv$

- 6个形状因子
- **b is to be replaced by c at below**

$$\begin{aligned} \langle \Lambda(p', s') | \bar{s} \gamma^\mu b | \Lambda_b(p, s) \rangle &= \bar{u}_\Lambda(p', s') \left[f_0(q^2) (m_{\Lambda_b} - m_\Lambda) \frac{q^\mu}{q^2} + f_+(q^2) \frac{m_{\Lambda_b} + m_\Lambda}{s_+} \left(p^\mu + p'^\mu - (m_{\Lambda_b}^2 - m_\Lambda^2) \frac{q^\mu}{q^2} \right) \right. \\ &\quad \left. + f_\perp(q^2) \left(\gamma^\mu - \frac{2m_\Lambda}{s_+} p^\mu - \frac{2m_{\Lambda_b}}{s_+} p'^\mu \right) \right] u_{\Lambda_b}(p, s), \end{aligned}$$

$$\begin{aligned} \langle \Lambda(p', s') | \bar{s} \gamma^\mu \gamma_5 b | \Lambda_b(p, s) \rangle &= -\bar{u}_\Lambda(p', s') \gamma_5 \left[g_0(q^2) (m_{\Lambda_b} + m_\Lambda) \frac{q^\mu}{q^2} + g_+(q^2) \frac{m_{\Lambda_b} - m_\Lambda}{s_-} \left(p^\mu + p'^\mu - (m_{\Lambda_b}^2 - m_\Lambda^2) \frac{q^\mu}{q^2} \right) \right. \\ &\quad \left. + g_\perp(q^2) \left(\gamma^\mu + \frac{2m_\Lambda}{s_-} p^\mu - \frac{2m_{\Lambda_b}}{s_-} p'^\mu \right) \right] u_{\Lambda_b}(p, s), \end{aligned}$$

FLAG: 味物理格点计算平均

- 始于2007: <http://itpwiki.unibe.ch/flag>
- FLAG-2, Eur. J. Phys. C74 (2014) 2890
- FLAG-3, arXiv:1607.00299
- **Continuum** extrapolation(# a 's, <0.1 fm?)
- **Chiral** extrapolation of light quark masses ($m_{\pi, \min} < 200$ MeV, 400 MeV?)
- **Finite volume** ($m_{\pi, \min} L > 4$, or 3 ?)
- **Renormalization** (non-perturbative? 1-loop?)
- **Heavy quark treatment** ($O(a)$ improved?)
- Publication status (published, preprint, conference)

Color-coding of systematic errors:

★ has been estimated in a satisfactory manner.

○ reasonable, could be improved.

■ no estimation, or unsatisfactory.

■ Included in the average.

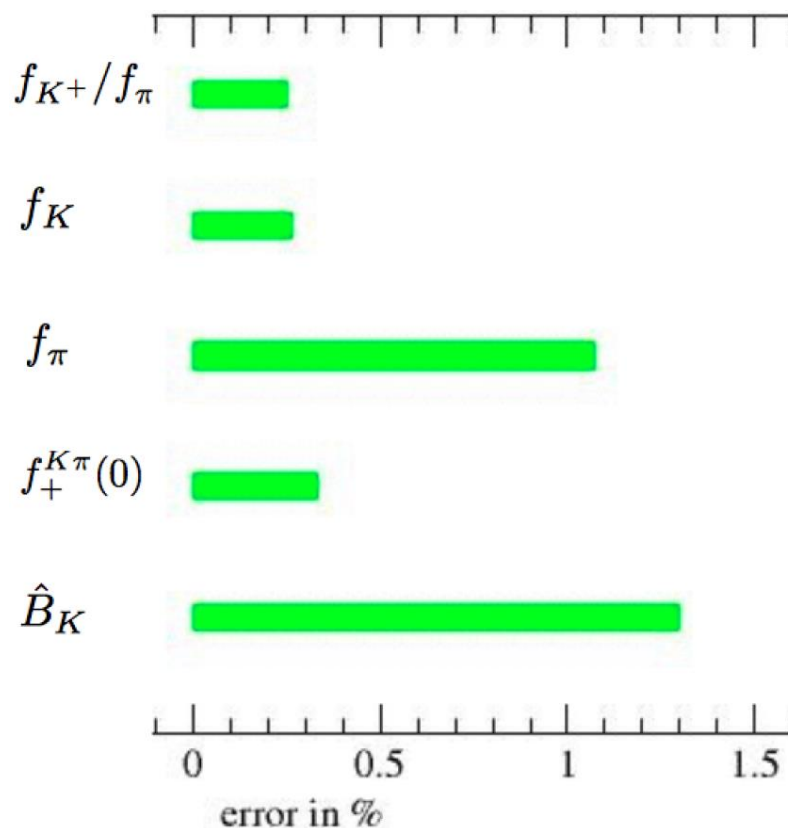
□ Not included, but pass quality criteria.

■ Other results.

轻介子衰变常数与形状因子

- Flavor Lattice Averaging Group (FLAG)
[arXiv:1607.00299](https://arxiv.org/abs/1607.00299)

- 截止到2015年11月30日



粲夸克质量

- 夸克质量是标准模型的基本参数之一
- LQCD计算含粲夸克的强子质量, 如 D , D_s ,
- 输入强子质量的实验值定出 m_c
- 需要质量重整化常数 Z_m
- LQCD计算moments of pseudoscalar-pseudoscalar correlators, 微扰可算, 解出 m_c
- 可不用计算 Z_m ($Z_m Z_P = 1$)

Inputs: $M_{D_s} = 1.9685$ GeV

$M_{D_s^*} - M_{D_s} = 0.1438$ GeV

$M_{J/\psi} = 3.0969$ GeV

Outputs: $r_0 = 0.465(4)(9)$ fm

$m_s^{\overline{MS}}(2 \text{ GeV}) = 0.101(3)(6)$ GeV

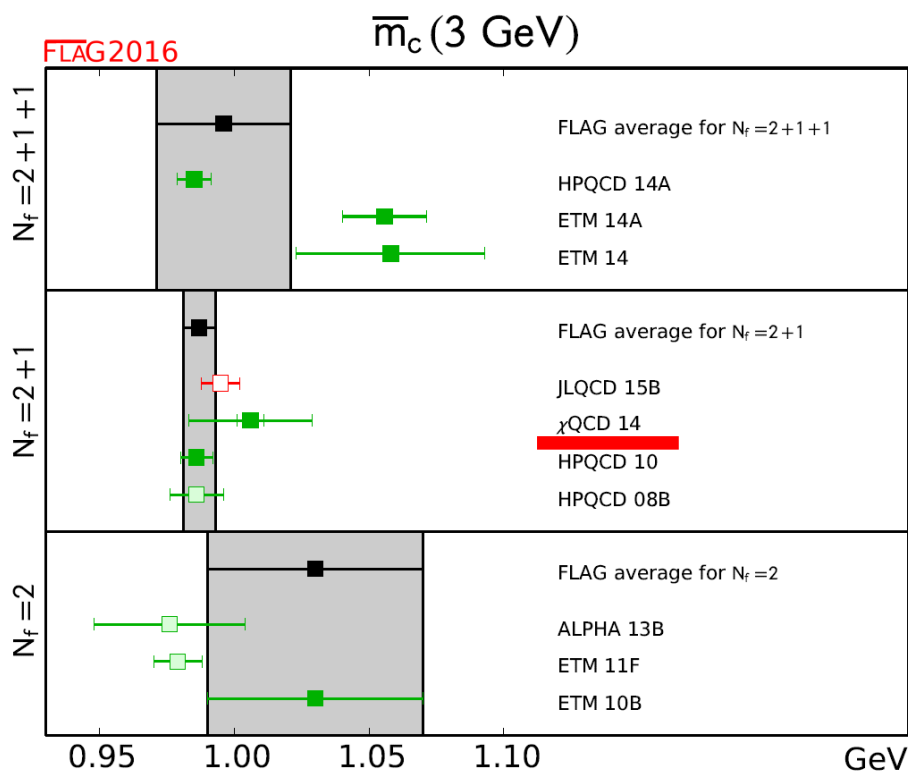
$m_c^{\overline{MS}}(2 \text{ GeV}) = 1.118(6)(24)$ GeV

χ QCD Collaboration:

Y.-B. Yang et al., PRD92, 2015

ZL et al., PRD90, 2014 ($Z_m, Z_A...$)

粲夸克质量格点计算平均值

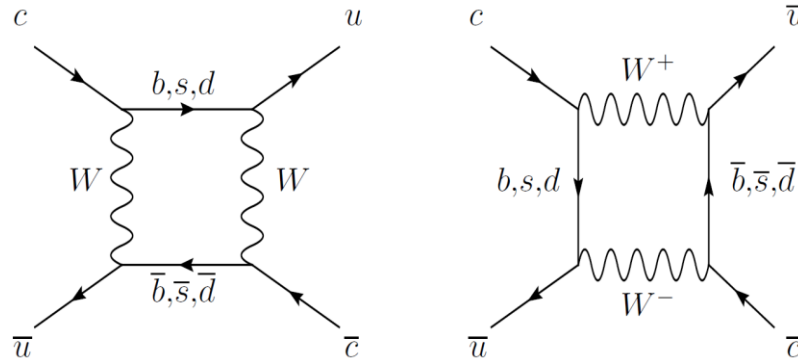


- $N_f = 2 + 1 + 1$:
 $\bar{m}_c(3 \text{ GeV}) = 0.996(25) \text{ GeV}$
- $N_f = 2 + 1$:
 $\bar{m}_c(3 \text{ GeV}) = 0.987(6) \text{ GeV}$
- $N_f = 2$:
 $\bar{m}_c(3 \text{ GeV}) = 1.03(4) \text{ GeV}$

FLAG2016, arXiv:1607.00299

D-meson mixing

- D-mixing 被 doubly Cabibbo 压低 ($|V_{ub}V_{cb}^*|^2$), long distance contribution dominates



- 但标准模型里 long distance contribution 中的 CP 破坏可忽略, 所以可以考虑新物理中 local 有效算符带来的 CP 破坏
- 最一般的 $\Delta C = 2$ 有效哈密顿量 (dimension-6):

$$H_{\text{eff}}^{\Delta C=2} = \frac{1}{4} \sum_{i=1}^5 C_i(\mu) Q_i(\mu)$$

D-meson mixing

- In the SUSY basis, they are: $Q_1 = [\bar{c}^a \gamma_\mu (1 - \gamma_5) \ell^a] [\bar{c}^b \gamma_\mu (1 - \gamma_5) \ell^b]$,

标准模型里，只有
 Q_1 有贡献

$$Q_2 = [\bar{c}^a (1 - \gamma_5) \ell^a] [\bar{c}^b (1 - \gamma_5) \ell^b],$$

$$Q_3 = [\bar{c}^a (1 - \gamma_5) \ell^b] [\bar{c}^b (1 - \gamma_5) \ell^a],$$

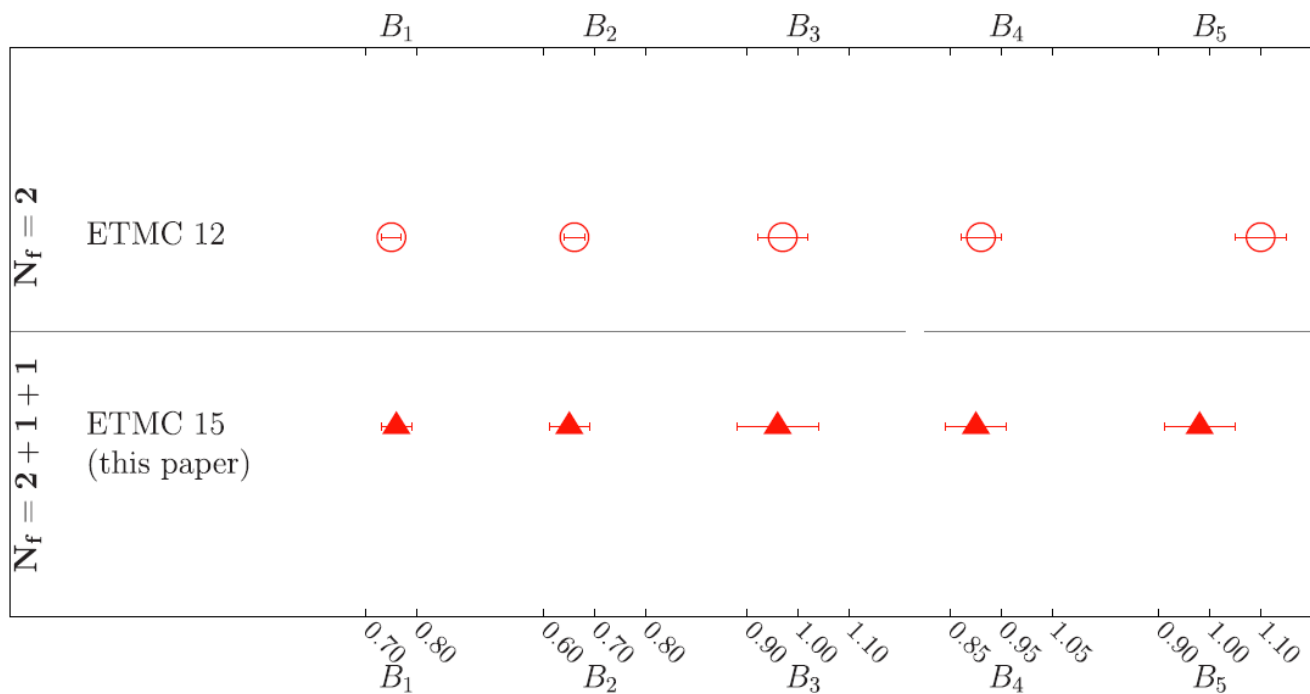
$$Q_4 = [\bar{c}^a (1 - \gamma_5) \ell^a] [\bar{c}^b (1 + \gamma_5) \ell^b],$$

$$Q_5 = [\bar{c}^a (1 - \gamma_5) \ell^b] [\bar{c}^b (1 + \gamma_5) \ell^a],$$

- 5个算符矩阵元的格点QCD计算有两家(unquenched)
- ETMC, 2味(2014)和2+1+1味(2015)
- FNAL/MILC (2+1味, 4a's, preliminary, [LAT2014,2015](#))
- LQCD结果比实验精度高; 估计到2020年两者精度持平

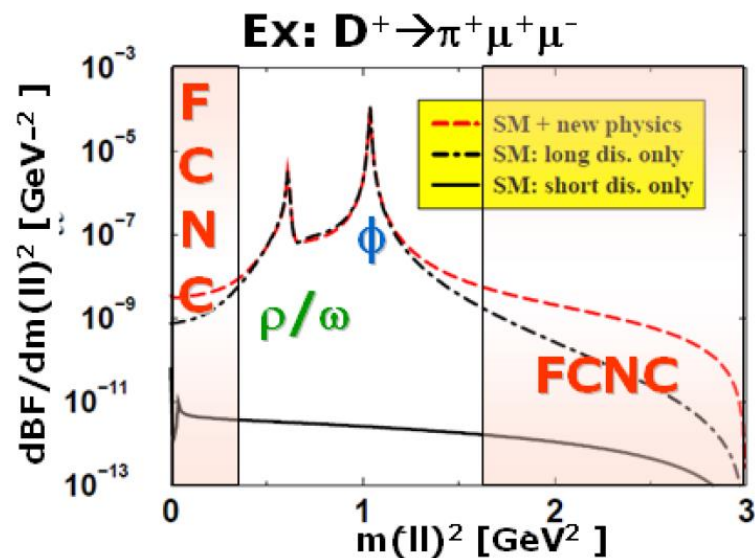
D-meson mixing

- ETMC (2味, 精度3-5%, bag parameters, 4a's; [PRD90\(2014\), 1403.7302](#))
- ETMC (2+1+1味, 4-8%, 3a's; [PRD92\(2015\), 1505.06639](#))



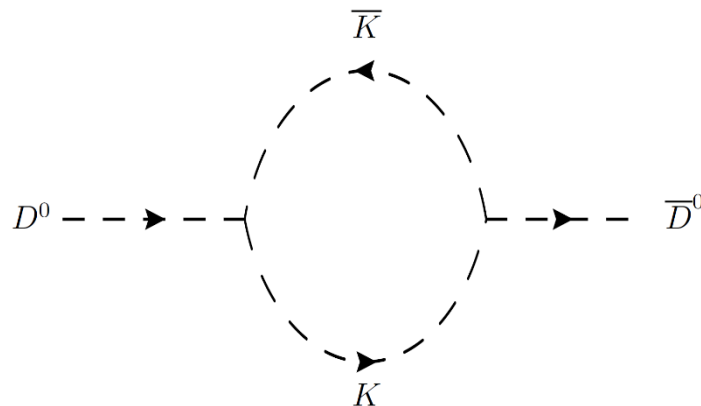
D介子稀有衰变 (FCNC: $c \rightarrow ull$)

- The total rate of $D \rightarrow Xl^+l^-$ is dominated by long-distance resonant contributions at dilepton mass $m_{ll} = m_\rho, m_\omega, m_\phi$
- New physics could only modify the distribution below ρ or above ϕ
- There is a broad kinematical region above ϕ resonance in $D \rightarrow \pi l^+l^-$
- $D \rightarrow \pi l^+l^-$ and $D^0 \rightarrow \rho^0 l^+l^-$ are the best exclusive decays in experimental searches
- 尚无有关D介子稀有衰变的格点QCD计算



Long distance contribution to D-mixing

- The long distance contributions are nonperturbative
- Coming from transitions to final states that are accessible to both D^0 and \bar{D}^0
- Down and strange quarks circulating in the box diagrams



LQCD和超级计算机

NERSC: National Energy Research Scientific C...

<https://www.nersc.gov/>

- 2011年11月Top500排名第8。
153,408个核，峰值
1.28Pflops



Powering Scientific Discovery Since 1974

Computing at NERSC

OUR GETTING DOCUMENT LIVE
SYSTEMS STARTED FOR STATUS

Now Computing

A small sample of massively parallel scientific computing jobs running right now at NERSC.

Project	Machine	CPU Cores	CPU Core Hours Used
Quantum Chromodynamics with four flavors of dynamical quarks PI: Doug Toussaint, University of Arizona	Hopper	18,432	128,947
QCD Thermodynamics at High Temperature PI: Alexei Bazavov, Brookhaven National Lab	Hopper	18,432	115,779
Large Scale 3D Geophysical Inversion & Imaging PI: Gregory A. Newman, Lawrence Berkeley National Laboratory	Hopper	16,896	27,386.1