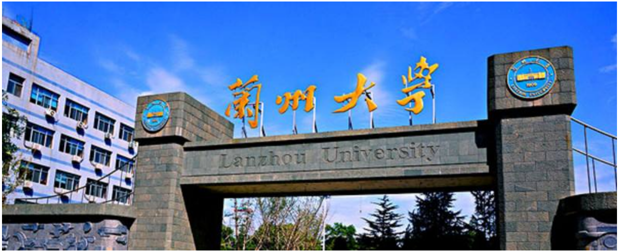
Searching for stable tetraquarks

Wei Chen

Sun Yat-Sen University

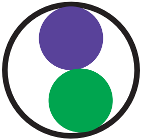
Lanzhou University, Mar. 31-April. 01, 2018



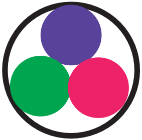
Outline

1. Background of the exotic hadron states
2. Briefly Introduction of QCD Sum Rules
3. Tetraquark sum rule analyses for mass spectra
   - For the doubly charmed/bottom $QQ\bar{q}_i\bar{q}_j$ tetraquarks
   - For the fully-heavy $QQ\bar{Q}\bar{Q}$ tetraquarks
4. Decay properties of the $QQ\bar{Q}\bar{Q}$ tetraquarks
5. Summary

Quark model is our pride



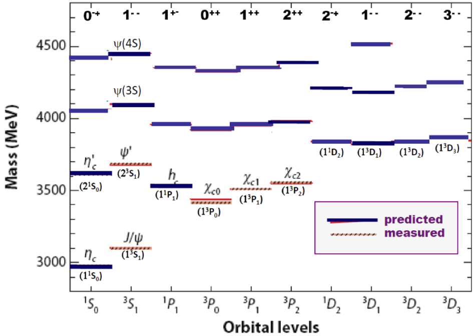
meson($q\bar{q}$)

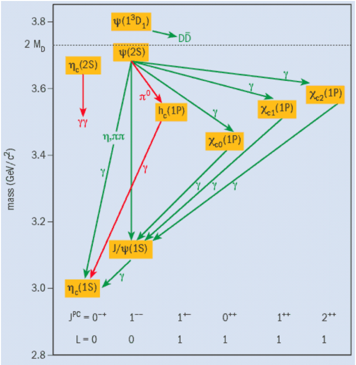


baryon(qqq)

Gell-Mann and Zweig

- **Quark model** is established to classify hadrons: mesons ($q\bar{q}$) and baryons (qqq).
- The **charmonium** ($c\bar{c}$) spectrum is a strong support for QM!
- The masses of the assigned states match theory predictions.
- Transitions between charmonium states are in reasonably good agreement with theoretical expectations.



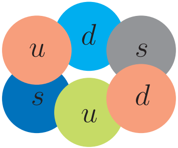


E1 transitions

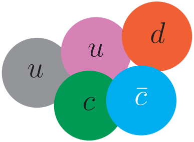
Transition	Γ_γ	*Th.*	Γ_γ (keV) *Expt*
$2\,{}^3S \to {}^3P_2$	$5 I_1 \alpha k^3$	*24*	*27 ± 4*
$\to {}^3P_1$	$3 I_1 \alpha k^3$	*29*	*27 ± 3*
$\to {}^3P_0$	$1 I_1 \alpha k^3$	*26*	*27 ± 3*
${}^3P_2 \to 1\,{}^3S$	$I_2 \alpha k^3$	*313*	*426 ± 51*
${}^3P_1 \to 1\,{}^3S$	$I_2 \alpha k^3$	*239*	*291 ± 48*
${}^3P_0 \to 1\,{}^3S$	$I_2 \alpha k^3$	*114*	*110 + 19*

Exotic hadrons in QCD

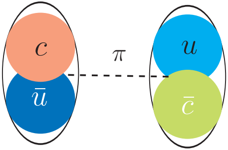
- Hadron structures are more complicated in **QCD**: $N_{\text{quarks}} \neq 2, 3$



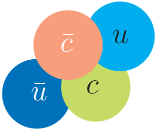
dibaryon



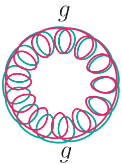
pentaquark



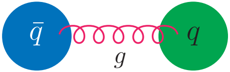
molecule



tetraquark



glueball



hybrid

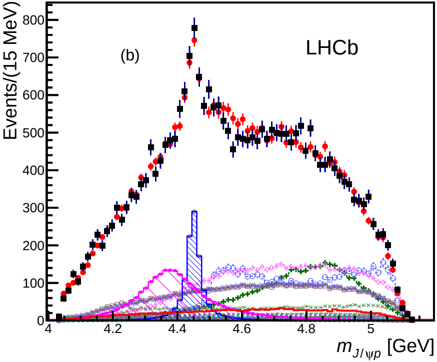
Searching for exotica

- **Hybrid** candidates: $\pi_1(1400)$, $\pi_1(1600)$ and $\pi_1(2015)$ (dispute)
- **Deuteron**: H states, $d^*(2380)$
- No solid evidence on existence of **glueball**: $a_0(980)$ and $f_0(980)$
- **Pentaquark**: $\Theta^+(uudd\bar{s})$ (long story of appeared and disappeared)
  The 2008 Review of Particle Physics:

  There are two or three recent experiments that find weak evidence for signals near the nominal masses, but there is simply no point in tabulating them in view of the overwhelming evidence that the claimed pentaquarks do not exist... The whole story–the discoveries themselves, the tidal wave of papers by theorists and phenomenologists that followed, and the eventual "undiscovery"–is a curious episode in the history of science.
- **F. Wilczek**: "The story of pentaquark shows how poorly we understand QCD."
- $P_c(4380)$ and $P_c(4450)$: Hidden-charm pentaquark states.
- **What is the next story for exotic hadrons?**

Pentaquarks: $P_c(4380)$ and $P_c(4450)$

In 2015, LHCb reported two hidden-charm pentaquark states $P_c(4380)$ and $P_c(4450)$ in $J/\psi p$ invariant mass distribution via $\Lambda_b^0 \to J/\psi K^- p$ decay (PRL115, 072001(2015))



$$M_1 = (4380 \pm 8 \pm 29)\,\mathrm{MeV}\,,$$
$$\Gamma_1 = (205 \pm 18 \pm 86)\,\mathrm{MeV}\,,$$
$$M_2 = (4449.8 \pm 1.7 \pm 2.5)\,\mathrm{MeV}\,,$$
$$\Gamma_2 = (39 \pm 5 \pm 19)\,\mathrm{MeV}\,.$$

$$J^P\,(P_c(4380), P_c(4450)) = \left(\frac{3}{2}^-, \frac{5}{2}^+\right), \left(\frac{3}{2}^+, \frac{5}{2}^-\right) \text{ or } \left(\frac{5}{2}^+, \frac{3}{2}^-\right)$$

Searching for exotica

- **Hybrid** candidates: $\pi_1(1400)$, $\pi_1(1600)$ and $\pi_1(2015)$ (dispute)
- **Deuteron**: H states, $d^*(2380)$
- No solid evidence on existence of **glueball**: $a_0(980)$ and $f_0(980)$
- **Pentaquark**: $\Theta^+(uudd\bar{s})$ (long story of appeared and disappeared)
  The 2008 Review of Particle Physics:

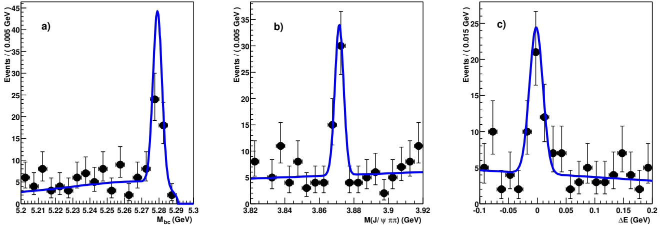
  There are two or three recent experiments that find weak evidence for signals near the nominal masses, but there is simply no point in tabulating them in view of the overwhelming evidence that the claimed pentaquarks do not exist... The whole story–the discoveries themselves, the tidal wave of papers by theorists and phenomenologists that followed, and the eventual "undiscovery"–is a curious episode in the history of science.
- **F. Wilczek**: "The story of pentaquark shows how poorly we understand QCD."
- $P_c(4380)$ and $P_c(4450)$: Hidden-charm pentaquark states.
- **What is the next story for exotic hadrons?**

The First XYZ State: $X(3872)$

- In 2003, Belle discovered $X(3872)$ in $B^+ \to K^+ J/\psi \pi^+ \pi^-$ with

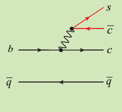
  $m = 3872.0 \pm 0.6 \pm 0.5\text{MeV}, \quad \Gamma < 2.3\text{MeV}.$ (PRL91, 262001(2003))

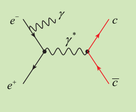
- In 2013, LHCb (PRL110, 222001(2013)) determined the quantum numbers of $X(3872)$: $J^{PC} = 1^{++}$

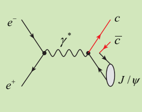


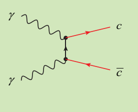
- Lots of XYZ states have been observed after $X(3872)$.

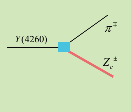
Overview of XYZ States







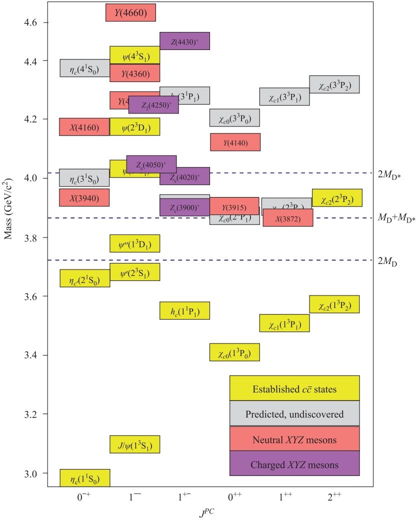




$X(3872)$	$Y(4260)$	$X(3940)$	$X(3915)$	$Z_c(3900)$
$Y(3940)$	$Y(4008)$	$X(4160)$	$X(4350)$	$Z_c(4025)$
$Z^+(4430)$	$Y(4360)$		$Z(3930)$	$Z_c(4020)$
$Z^+(4051)$	$Y(4630)$			$Z_c(3885)$
$Z^+(4248)$	$Y(4660)$			
$Y(4140)$				
$Y(4274)$				
$Z_c^+(4200)$				
$Z^+(4240)$				
$X(3823)$				

H.X.Chen, W.Chen, X.Liu, S.L.Zhu, Phys.Rept.639(2016) 1-121.

Overview of XYZ States



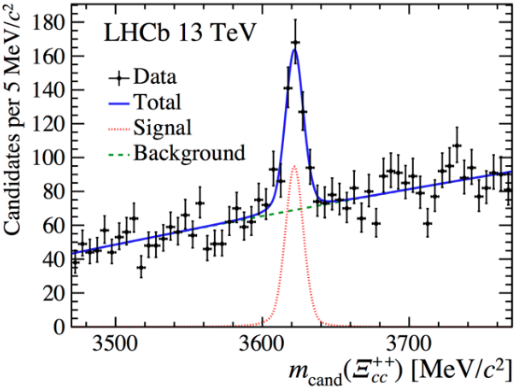
S. L. Olsen, Front. Phys. 10 (2015) 101401

- Many charmonium-like states were discovered above the open-charm thresholds.
- Their masses and decay modes are different from the pure $c\bar{c}$ charmonium states.
- Some charged Z_c states were observed, which are evidences for four-quark states ($c\bar{c}u\bar{d}$).
- They are good candidates for exotic hadron states!

Doubly charmed baryon Ξ_{cc}^{++}

LHCb discovery of Ξ_{cc}^{++}

R. Aaij, et. al. (LHCb) , PRL **119** 112001 (2017)

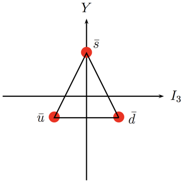


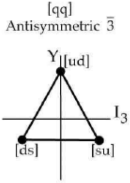
$\Lambda_c^+ K^- \pi^+ \pi^+$

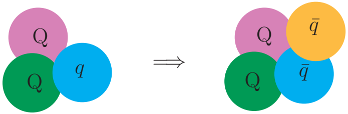
$$m_{\Xi_{cc}^{++}} = 3621.4 \pm 0.72 \pm 0.27 \pm 0.14 \,\text{MeV}$$

Doubly bottom tetraquarks

- Ξ_{cc}^{++} contains quark contents: *ccu*
- Attractive diquark channel: $[qq]$ with $\bar{3}_c$, $\bar{3}_f$ and $J^P = 0^+$.
- Characterizing diquarks: $qq \leftrightarrow \bar{q}$, $\bar{q}\bar{q} \leftrightarrow q$ (R. Jaffe, Phys.Rept., 2005, 409, 1-45)







- Stable tetraquarks $bb\bar{q}_i\bar{q}_j$, recently motivated vigorous theoretical interest! (Phys.Rev.Lett. 119 (2017), 202001;Phys.Rev.Lett. 119 (2017), 202002).
- Our previous studies of the $QQ'\bar{q}\bar{q}$ states in **Phys.Rev., D87, 014003 (2013);Phys.Rev., D89, 054037 (2014)**.

QCD Sum Rules

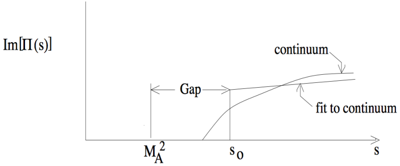
- Study two-point correlation function of current $J_\mu(x)$ with the same quantum numbers with hadron state:

$$\Pi_{\mu\nu}(q^2) = i\int d^4x e^{iq\cdot x}\langle\Omega|T[J_\mu(x)J_\nu^\dagger(0)]|\Omega\rangle$$

- Classify states $|X\rangle$ by coupling to current $\langle\Omega|J_\mu(x)|X\rangle \neq 0$
- Hadron level: described by the dispersion relation

$$\Pi(q^2) = \frac{(q^2)^N}{\pi}\int\frac{\mathrm{Im}\Pi(s)}{s^N(s-q^2-i\epsilon)}ds + \sum_{n=0}^{N-1} b_n(q^2)^n,$$

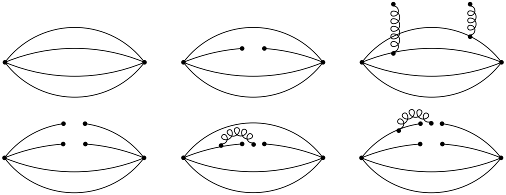
$$\rho(s) = \frac{1}{\pi}\mathrm{Im}\Pi(s) = \sum_n \delta(s-m_n^2)\langle 0|J|n\rangle\langle n|J^\dagger|0\rangle$$



QCD Sum Rule

- Quark-gluon level: evaluated via operator product expansion(OPE)

$$\rho(s)=\rho^{pert}(s)+\rho^{\langle\bar{q}q\rangle}(s)+\rho^{\langle GG\rangle}(s)+\rho^{\langle\bar{q}q\rangle^2}(s)+\rho^{\langle\bar{q}Gq\rangle}(s)+...,$$



- Apply Borel transform to correlation functions
- Quark-hadron duality: Laplace Sum Rules with QCD spectral function

$$\mathcal{L}_k\left(s_0, M_B^2\right)=\int_{4m_Q^2}^{s_0} dse^{-s/M_B^2}\rho(s)s^k=f_X^2 m_X^{2k}e^{-m_X^2/M_B^2}\,.$$

Diquark-antidiquark currents

$q\Gamma q$	J^P	States	(Flavor, Color)
$q_a^T C\gamma_5 q_b$	0^+	$^1S_0$	$(\mathbf{6_f}, \mathbf{6_c}), (\mathbf{\bar{3}_f}, \mathbf{\bar{3}_c})$
$q_a^T C q_b$	0^-	$^3P_0$	$(\mathbf{6_f}, \mathbf{6_c}), (\mathbf{\bar{3}_f}, \mathbf{\bar{3}_c})$
$q_a^T C\gamma_\mu\gamma_5 q_b$	1^-	$^3P_1$	$(\mathbf{6_f}, \mathbf{6_c}), (\mathbf{\bar{3}_f}, \mathbf{\bar{3}_c})$
$q_a^T C\gamma_\mu q_b$	1^+	$^3S_1$	$(\mathbf{6_f}, \mathbf{\bar{3}_c}), (\mathbf{\bar{3}_f}, \mathbf{6_c})$
$q_a^T C\sigma_{\mu\nu} q_b$	$\begin{cases} 1^-, & \text{for } \mu,\nu=1,2,3 \\ 1^+, & \text{for } \mu=0,\nu=1,2,3 \end{cases}$	$^1P_1$ $^3S_1$	$(\mathbf{6_f}, \mathbf{\bar{3}_c}), (\mathbf{\bar{3}_f}, \mathbf{6_c})$
$q_a^T C\sigma_{\mu\nu}\gamma_5 q_b$	$\begin{cases} 1^+, & \text{for } \mu,\nu=1,2,3 \\ 1^-, & \text{for } \mu=0,\nu=1,2,3 \end{cases}$	$^3S_1$ $^1P_1$	$(\mathbf{6_f}, \mathbf{\bar{3}_c}), (\mathbf{\bar{3}_f}, \mathbf{6_c})$

- Interpolating currents with $J^P = 0^+$ for $QQ\bar{q}\bar{q}$ systems

$$
\begin{aligned}
\eta_1 &= Q_a^T C Q_b (\bar{q}_a C \bar{q}_b^T + \bar{q}_b C \bar{q}_b^T), \\
\eta_2 &= Q_a^T C\gamma_5 Q_b (\bar{q}_a \gamma_5 C \bar{q}_b^T + \bar{q}_b \gamma_5 C \bar{q}_b^T), \\
\eta_3 &= Q_a^T C\gamma_\mu Q_b (\bar{q}_a \gamma^\mu C \bar{q}_b^T - \bar{q}_b \gamma^\mu C \bar{q}_b^T), \\
\eta_4 &= Q_a^T C\gamma_\mu\gamma_5 Q_b (\bar{q}_a \gamma^\mu\gamma_5 C \bar{q}_b^T + \bar{q}_b \gamma^\mu\gamma_5 C \bar{q}_b^T), \\
\eta_5 &= Q_a^T C\sigma_{\mu\nu} Q_b (\bar{q}_a \sigma^{\mu\nu} C \bar{q}_b^T - \bar{q}_b \sigma^{\mu\nu} C \bar{q}_a^T).
\end{aligned}
$$

- The tetraquark interpolating currents with $J^P = 1^+$ are

$$
\begin{aligned}
\eta_1 &= Q_a^T C\gamma_\mu\gamma_5 Q_b(\bar{q}_a C\bar{q}_b^T + \bar{q}_b C\bar{q}_a^T),\\
\eta_2 &= Q_a^T C Q_b(\bar{q}_a\gamma_\mu\gamma_5 C\bar{q}_b^T + \bar{q}_b\gamma_\mu\gamma_5 C\bar{q}_a^T),\\
\eta_3 &= Q_a^T C\sigma_{\mu\nu}\gamma_5 Q_b(\bar{q}_a\gamma^\nu C\bar{q}_b^T - \bar{q}_b\gamma^\nu C\bar{q}_a^T),\\
\eta_4 &= Q_a^T C\gamma^\nu Q_b(\bar{q}_a\sigma_{\mu\nu}\gamma_5 C\bar{q}_b^T - \bar{q}_b\sigma_{\mu\nu}\gamma_5 C\bar{q}_a^T),\\
\eta_5 &= Q_a^T C\gamma_\mu Q_b(\bar{q}_a\gamma_5 C\bar{q}_b^T - \bar{q}_b\gamma_5 C\bar{q}_a^T),\\
\eta_6 &= Q_a^T C\gamma_5 Q_b(\bar{q}_a\gamma_\mu C\bar{q}_b^T + \bar{q}_b\gamma_\mu C\bar{q}_a^T),\\
\eta_7 &= Q_a^T C\sigma_{\mu\nu} Q_b(\bar{q}_a\gamma^\nu\gamma_5 C\bar{q}_b^T - \bar{q}_b\gamma^\nu\gamma_5 C\bar{q}_a^T),\\
\eta_8 &= Q_a^T C\gamma^\nu\gamma_5 Q_b(\bar{q}_a\sigma_{\mu\nu} C\bar{q}_b^T + \bar{q}_b\sigma_{\mu\nu} C\bar{q}_a^T).
\end{aligned}
$$

Quark Content	$[\bar{q}\bar{q}]_f$	I	$J^P = 0^-$	$J^P = 0^+$	$J^P = 1^-$	$J^P = 1^+$
$QQ\bar{q}\bar{q}$	$\bar{\mathbf{6}}_f$	1	η_1, η_2, η_3	$\eta_1, \eta_2, \eta_3, \eta_4, \eta_5$	$\eta_1, \eta_2, \eta_3, \eta_4$	$\eta_1, \eta_2, \eta_3, \eta_4$
$QQ\bar{s}\bar{s}$	$\bar{\mathbf{6}}_f$	0	η_1, η_2, η_3	$\eta_1, \eta_2, \eta_3, \eta_4, \eta_5$	$\eta_1, \eta_2, \eta_3, \eta_4$	$\eta_1, \eta_2, \eta_3, \eta_4$
$QQ\bar{q}\bar{s}$	$\bar{\mathbf{6}}_f$	1/2	$\eta_1, \eta_2, \eta_3,$	$\eta_1, \eta_2, \eta_3, \eta_4, \eta_5$	$\eta_1, \eta_2, \eta_3, \eta_4,$	$\eta_1, \eta_2, \eta_3, \eta_4,$
	$\mathbf{3}_f$		η_4, η_5		$\eta_5, \eta_6, \eta_7, \eta_8$	$\eta_5, \eta_6, \eta_7, \eta_8$
$QQ\bar{u}\bar{d}$	$\mathbf{3}_f$	0	η_4, η_5	−	$\eta_5, \eta_6, \eta_7, \eta_8$	$\eta_5, \eta_6, \eta_7, \eta_8$

Mass spectra for $QQ\bar{q}_i\bar{q}_j$ tetraquarks: 1^+

	Current	s_0 (GeV^2)	$[M^2_{Bmin}, M^2_{Bmax}]$ (GeV^2)	M^2_B (GeV^2)	m_X (GeV)	PC (%)	f_X (GeV^5)	open charm/bottom threshold (GeV)
	η_2	27	$3.1 \sim 4.0$	3.6	4.87 ± 0.11	38.5	0.0726	
$cc\bar{q}\bar{s}$	η_3	21	$2.4 \sim 3.4$	2.8	4.12 ± 0.17	47.5	0.0571	
	η_4	21	$2.5 \sim 3.4$	2.8	4.13 ± 0.16	47.9	0.0574	3.975
	η_5	21	$2.8 \sim 3.7$	3.2	4.12 ± 0.16	41.7	0.0378	
	η_1	29	$3.2 \sim 4.5$	3.8	5.03 ± 0.13	42.5	0.138	
	η_2	30	$3.2 \sim 4.6$	3.8	5.12 ± 0.14	45.9	0.150	4.081
$cc\bar{s}\bar{s}$	η_3	21	$2.2 \sim 3.4$	2.8	4.17 ± 0.16	45.4	0.0838	
	η_4	21	$2.2 \sim 3.4$	2.8	4.19 ± 0.16	45.7	0.0849	
$bb\bar{q}\bar{q}$	η_3	115	$6.5 \sim 8.8$	7.8	10.2 ± 0.3	41.4	0.459	
	η_4	115	$6.8 \sim 8.8$	7.8	10.2 ± 0.3	41.7	0.454	
$bb\bar{u}\bar{d}$	η_5	115	$7.0 \sim 9.0$	8.0	10.2 ± 0.3	42.8	0.215	10.60
	η_6	115	$7.0 \sim 9.2$	8.0	10.2 ± 0.3	42.0	0.304	
	η_7	115	$6.5 \sim 8.6$	7.6	10.2 ± 0.3	43.2	0.241	
	η_8	115	$6.8 \sim 8.8$	7.6	10.2 ± 0.3	41.7	0.343	
$bb\bar{q}\bar{s}$	η_3	120	$6.2 \sim 9.8$	8.0	10.4 ± 0.3	48.9	0.452	
	η_4	120	$6.5 \sim 9.8$	8.0	10.4 ± 0.3	49.3	0.446	10.69
	η_5	120	$6.6 \sim 9.8$	8.0	10.3 ± 0.3	52.3	0.298	
	η_6	120	$6.6 \sim 9.8$	8.0	10.3 ± 0.4	52.1	0.418	
	η_7	120	$6.2 \sim 9.6$	8.0	10.4 ± 0.3	48.1	0.342	
	η_8	120	$5.8 \sim 9.6$	8.0	10.4 ± 0.3	46.3	0.491	
$bb\bar{s}\bar{s}$	η_3	120	$6.2 \sim 9.8$	8.0	10.4 ± 0.3	48.1	0.657	10.78
	η_4	120	$6.2 \sim 9.8$	8.0	10.4 ± 0.3	48.5	0.651	

Mass spectra for $QQ\bar{q}_i\bar{q}_j$ tetraquarks: 0^+

	Current	s_0 (GeV^2)	$[M^2_{Bmin}, M^2_{Bmax}]$ (GeV^2)	M^2_B (GeV^2)	m_X (GeV)	PC (%)	f_X (GeV^5)	open charm/bottom threshold (GeV)
$cc\bar{q}\bar{s}$	η_2	22	$2.8 \sim 3.6$	3.2	4.16 ± 0.14	39.0	0.0548	3.833
	η_3	20	$2.6 \sim 3.4$	3.0	4.02 ± 0.18	39.3	0.0561	
$cc\bar{s}\bar{s}$	η_1	28	$3.2 \sim 4.1$	3.4	5.05 ± 0.15	43.3	0.136	3.937
	η_2	22	$2.6 \sim 3.8$	3.2	4.27 ± 0.11	43.2	0.0933	
	η_2	120	$7.0 \sim 9.8$	8.2	10.3 ± 0.3	48.2	0.590	
$bb\bar{q}\bar{q}$	η_3	115	$6.9 \sim 9.0$	8.0	10.2 ± 0.3	40.3	0.539	10.56
	η_5	115	$6.7 \sim 8.8$	8.0	10.2 ± 0.3	39.4	1.10	
	η_3	115	$6.5 \sim 8.8$	8.0	10.2 ± 0.3	40.3	0.398	
$bb\bar{q}\bar{s}$	η_4	115	$5.8 \sim 8.6$	7.2	10.2 ± 0.3	45.6	0.337	10.65
	η_5	120	$6.2 \sim 9.8$	8.0	10.3 ± 0.3	49.3	0.806	
	η_1	130	$7.5 \sim 9.8$	8.5	11.0 ± 0.2	41.4	0.391	
	η_2	120	$6.4 \sim 9.8$	8.0	10.4 ± 0.3	49.7	0.632	
$bb\bar{s}\bar{s}$	η_3	115	$6.3 \sim 9.0$	8.0	10.2 ± 0.3	40.5	0.560	10.73
	η_4	120	$6.2 \sim 8.4$	8.0	10.4 ± 0.3	41.9	0.486	
	η_5	115	$6.2 \sim 8.8$	8.0	10.2 ± 0.3	38.9	1.14	

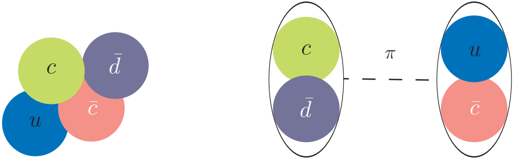
- These $bb\bar{q}_i\bar{q}_j$ tetraquarks lie below thresholds of two-bottom mesons, two-bottom baryons.
- They are thus very stable and narrow!

Mass spectra for $bc\bar{q}\bar{q}$ tetraquarks: 0^+ and 1^+

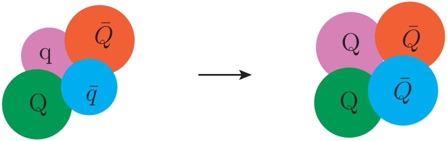
System	J^P	Current	$s_0(\text{GeV}^2)$	$[M^2_{\text{min}}, M^2_{\text{max}}](\text{GeV}^2)$	$m_X(\text{GeV})$	PC(%)	$D^{(*)}_{(s)}\bar{B}^{(*)}_{(s)}$ threshold (GeV)
$bc\bar{q}\bar{q}$	0^+	J_1	60 ± 2	$5.4 - 6.2$	7.27 ± 0.19	35.5	7.15
		J_2	59 ± 2	$6.1 - 6.4$	7.16 ± 0.16	32.9	
		J_3	58 ± 2	$5.4 - 6.0$	7.14 ± 0.16	33.9	
		J_4	60 ± 2	$6.1 - 6.4$	7.23 ± 0.19	33.5	
$bc\bar{s}\bar{s}$	0^+	J_1	61 ± 2	$4.9 - 6.4$	7.35 ± 0.17	39.1	7.34
		J_4	60 ± 2	$5.6 - 6.5$	7.26 ± 0.24	36.7	
$bc\bar{q}\bar{q}$	1^+	$J_{1\mu}$	59 ± 2	$5.5 - 6.1$	7.21 ± 0.16	34.7	7.19
		$J_{2\mu}$	60 ± 2	$5.3 - 6.2$	7.27 ± 0.20	37.5	
		$J_{3\mu}$	60 ± 2	$5.4 - 6.3$	7.26 ± 0.19	36.8	
		$J_{4\mu}$	58 ± 2	$5.3 - 6.0$	7.13 ± 0.17	35.7	
$bc\bar{s}\bar{s}$	1^+	$J_{2\mu}$	61 ± 2	$4.9 - 6.4$	7.35 ± 0.22	41.2	7.38
		$J_{3\mu}$	61 ± 2	$4.9 - 6.4$	7.34 ± 0.22	42.1	

- The $bc\bar{q}\bar{q}$ tetraquarks are very close to the $D^{(*)}\bar{B}^{(*)}$ thresholds.
- The $bc\bar{s}\bar{s}$ tetraquarks lie slightly below the $D_s^{(*)}\bar{B}_s^{(*)}$ thresholds.
- They are probably stable and narrow!

- Theoretical configurations for XYZ: tetraquark, molecule, hybrid,...



- What happens as the mass of the light quarks is raised? Binding becomes stronger?

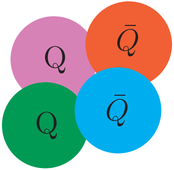


- QED analog: molecular positronium Ps_2 (bound state of $e^+e^-e^+e^-$) discovered in 2007 Nature 449 (09, 2007) 195–197.

Doubly hidden-flavor tetraquarks: $QQ\bar{Q}\bar{Q}$

$QQ\bar{Q}\bar{Q}$ Tetraquarks:

- They are far away from the mass range of the observed conventional $q\bar{q}$ hadrons and XYZ states.
- Can be clearly distinguished experimentally from the normal states.
- The light mesons $(\pi, \rho, \omega, \sigma ...)$ can not be exchanged between two charmonia/bottomonia.
- The binding force comes from the short-range gluon exchange.
- A molecule configuration is not favored and thus the $QQ\bar{Q}\bar{Q}$ is a good candidate for compact tetraquark.



Experimental events:

- $J/\psi J/\psi$ pairs: Phys. Lett. B707, 52 (2012) (LHCb); JHEP 1409, 094(2014) (CMS); Phys. Rev. D90, 111101 (2014) (D0).
- $J/\psi\Upsilon(1S)$ events: Phys. Rev. Lett. 116, 082002 (2016) (D0); K. Dilsiz's talk at APS April Meeting 2016 on behalf of CMS, see https://absuploads.aps.org/presentation.cfm?pid=11931.
- $\Upsilon(1S)\Upsilon(1S)$ pairs: JHEP 05, 013 (2017) (CMS).

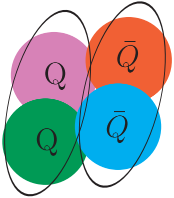
Theoretical works:

- Quark-Gluon models: Prog. Theor. Phys. 54, 492 (1975); Zeit. Phys. C7, 317 (1981).
- Potential model: Phys.Rev. D25, 2370 (1982); Phys. Lett. B123, 449 (1983).
- MIT bag model: Phys. Rev. D32, 755 (1985).
- Hyperspherical harmonic formalism: Phys. Rev. D73, 054004 (2006).
- BS or Schroedinger Eqs: Phys.Rev.D86, 034004 (2012); Phys.Lett.B718, 545 (2012).
- Recent studies: PRD95, 034011 (2017); EPJC77, 432 (2017);PRD97, 054505 (2018); arXiv:1605.01134; 1612.00012; 1706.07553; 1709.09605; 1710.02540.
- **Our study:** Phys.Lett. B773 (2017) 247-251, by using moment sum rules.

- Study two-point correlation function of current $J(x)$ with the same quantum numbers with hadron state:

$$\Pi(q^2) = i \int d^4x e^{iq\cdot x} \langle \Omega | T[J(x) J^\dagger(0)] | \Omega \rangle$$

- Classify states $|X\rangle$ by coupling to current $\langle \Omega | J(x) | X \rangle \neq 0$
- Currents are probes of spectrum and might not overlap with state



Interpolating currents with $J^{PC} = 0^{++}$:

$$\begin{aligned}
J_1 &= Q_a^T C\gamma_5 Q_b \bar{Q}_a \gamma_5 C \bar{Q}_b^T \,, \\
J_2 &= Q_a^T C\gamma_\mu\gamma_5 Q_b \bar{Q}_a \gamma^\mu \gamma_5 C \bar{Q}_b^T \,, \\
J_3 &= Q_a^T C\sigma_{\mu\nu} Q_b \bar{Q}_a \sigma^{\mu\nu} C \bar{Q}_b^T \,, \\
J_4 &= Q_a^T C\gamma_\mu Q_b \bar{Q}_a \gamma^\mu C \bar{Q}_b^T \,, \\
J_5 &= Q_a^T C Q_b \bar{Q}_a C \bar{Q}_b^T \,,
\end{aligned}$$

- Hadron level: described by the dispersion relation

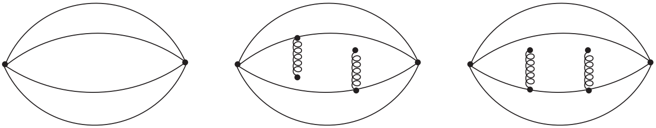
$$\Pi(q^2) = \frac{(q^2)^N}{\pi}\int \frac{\mathrm{Im}\Pi(s)}{s^N(s-q^2-i\epsilon)}ds + \sum_{n=0}^{N-1} b_n(q^2)^n,$$

$$\rho(s) = \frac{1}{\pi}\mathrm{Im}\Pi(s) = \sum_n \delta(s-m_n^2)\langle 0|J|n\rangle\langle n|J^\dagger|0\rangle$$

$$= f_X^2\delta(s-m_X^2) + \text{continuum},$$

- Quark-gluon level: evaluated via operator product expansion(OPE)

$$\Pi(s) = \Pi^{pert}(s) + \Pi^{\langle GG\rangle}(s) + ...,$$



- Define moments in Euclidean region $Q^2 = -q^2 > 0$:

$$M_n(Q_0^2) = \frac{1}{n!}\left(-\frac{d}{dQ^2}\right)^n \Pi(Q^2)|_{Q^2=Q_0^2}$$
$$= \int_{m_H^2}^{\infty} \frac{\rho(s)}{(s+Q_0^2)^{n+1}} ds = \frac{f_X^2}{(m_X^2+Q_0^2)^{n+1}}\left[1+\delta_n(Q_0^2)\right],$$

where $\delta_n(Q_0^2)$ contains the higher states and continuum.

- Ratio of the moments

$$r(n, Q_0^2) \equiv \frac{M_n(Q_0^2)}{M_{n+1}(Q_0^2)} = \left(m_X^2+Q_0^2\right)\frac{1+\delta_n(Q_0^2)}{1+\delta_{n+1}(Q_0^2)}.$$

- Predict hadron mass

$$m_X = \sqrt{r(n, Q_0^2) - Q_0^2}$$

for sufficiently large n when $\delta_n(Q_0^2) \cong \delta_{n+1}(Q_0^2)$ for convergence.

Limitations for (n, ξ) parameter space:

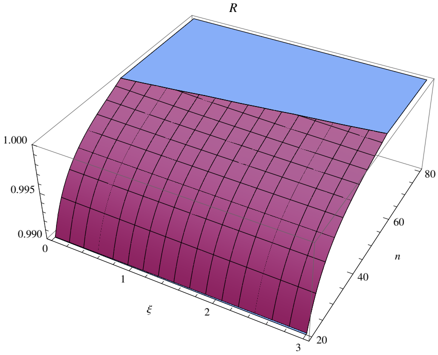
$$\xi = Q_0^2/16m_c^2, \text{ for } cc\bar{c}\bar{c} \text{ system;}$$
$$\xi = Q_0^2/m_b^2, \text{ for } bb\bar{b}\bar{b} \text{ system.}$$

- Small ξ: higher dimensional condensates give large contributions to $M_n(Q_0^2)$, leading to bad OPE convergence.
- Large ξ: slower convergence of $\delta_n(Q_0^2)$. This can be compensated by taking higher derivative n for the lowest lying resonance to dominate.
- Large n: moving further away from the asymptotically free region. The OPE convergence would also become bad.
- Requiring $\Pi^{\langle GG\rangle}(s) \leq \Pi^{pert}(s)$ to obtain an upper limit n_{max}, which will increase with respect to ξ.
- Good (n, ξ) region: the lowest lying resonance dominates the moments while the OPE series has good convergence.

$$n_{max} = 75, 76, 77, 78 \text{ for } \xi = 0.2, 0.4, 0.6, 0.8$$

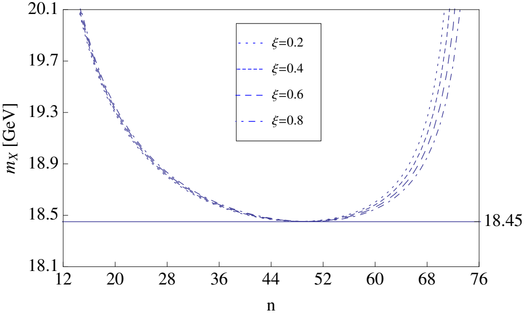
Hölder's inequality:

$$R = \frac{M_n(Q_0^2)^2}{M_r(Q_0^2)M_{2n-r}(Q_0^2)} \leq 1\,,$$



The boundary gives $(n, \xi) = (48, 0.2), (49, 0.4), (49, 0.6), (50, 0.8)$.

Mass for scalar $bb\bar{b}\bar{b}$ tetraquark: mass curves have plateaus at $(n, \xi) = (48, 0.2), (49, 0.4), (49, 0.6), (50, 0.8)$

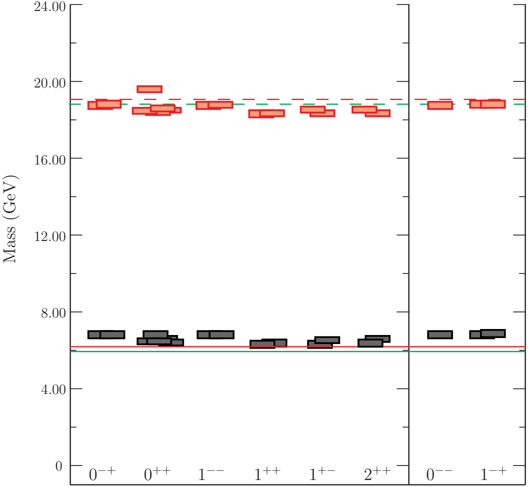


$$m_{X_b} = (18.45 \pm 0.15)\,\text{GeV}.$$

Mass spectra for the $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ tetraquarks:

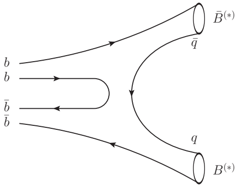
J^{PC}	Currents	m_{X_c}(GeV)	m_{X_b}(GeV)
0^{++}	J_1	6.44 ± 0.15	18.45 ± 0.15
	J_2	6.59 ± 0.17	18.59 ± 0.17
	J_3	6.47 ± 0.16	18.49 ± 0.16
	J_4	6.46 ± 0.16	18.46 ± 0.14
	J_5	6.82 ± 0.18	19.64 ± 0.14
1^{++}	$J_{1\mu}^+$	6.40 ± 0.19	18.33 ± 0.17
	$J_{2\mu}^+$	6.34 ± 0.19	18.32 ± 0.18
1^{+-}	$J_{1\mu}^-$	6.37 ± 0.18	18.32 ± 0.17
	$J_{2\mu}^+$	6.51 ± 0.15	18.54 ± 0.15
2^{++}	$J_{1\mu\nu}$	6.51 ± 0.15	18.53 ± 0.15
	$J_{2\mu\nu}$	6.37 ± 0.19	18.32 ± 0.17
0^{-+}	J_1^+	6.84 ± 0.18	18.77 ± 0.18
	J_2^+	6.85 ± 0.18	18.79 ± 0.18
0^{--}	J_1^-	6.84 ± 0.18	18.77 ± 0.18
1^{-+}	$J_{1\mu}^+$	6.84 ± 0.18	18.80 ± 0.18
	$J_{2\mu}^+$	6.88 ± 0.18	18.83 ± 0.18
1^{--}	$J_{1\mu}^-$	6.84 ± 0.18	18.77 ± 0.18
	$J_{2\mu}^-$	6.83 ± 0.18	18.77 ± 0.16

Spontaneous dissociation thresholds:



Decay behavior: $bb\bar{b}\bar{b}$ tetraquarks

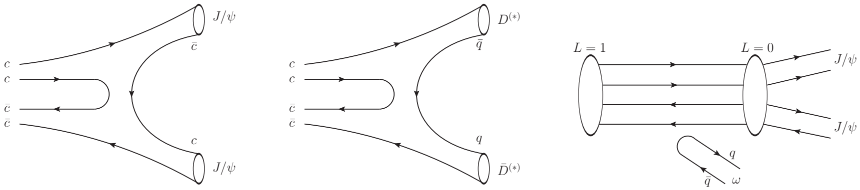
- $X_{bb\bar{b}\bar{b}} \to (b\bar{b}) + (b\bar{b})$: **kinematically forbidden**.
- $X_{bb\bar{b}\bar{b}} \to (bbq) + (\bar{b}\bar{b}\bar{q})$: **kinematically forbidden**.
- $X_{bb\bar{b}\bar{b}} \to (bqq) + (\bar{b}\bar{q}\bar{q})$: **suppressed** by two light quark pair creation.
- $X_{bb\bar{b}\bar{b}} \to (q\bar{b}) + (b\bar{q})$: **possible** in $B^{(*)}\bar{B}^{(*)}$ final states, with large phase space.
- $X_{bb\bar{b}\bar{b}} \to (b\bar{b}) + \gamma$: electromagnetic decay via $b\gamma_\mu\bar{b} \to \gamma$.
- $X_{bb\bar{b}\bar{b}} \to \Upsilon(1S)X \to l^+l^-l^+l^-$: multi-lepton final states could provide clean signals although the branching fraction may be small.



- **These $bb\bar{b}\bar{b}$ states are expected to be very narrow.** They are good candidates for compact tetraquarks, if they do exist.

Decay behavior: $cc\bar{c}\bar{c}$ tetraquarks

- $cc\bar{c}\bar{c} \to (ccq) + (\bar{c}\bar{c}\bar{q})$: kinematically forbidden.
- $cc\bar{c}\bar{c} \to (cqq) + (\bar{c}\bar{q}\bar{q})$: suppressed by two light quark pair creation.
- $cc\bar{c}\bar{c} \to (c\bar{c}) + (c\bar{c})$: charm quark pair rearrangement or annihilation (suppressed). Phase space is small.
- $cc\bar{c}\bar{c} \to (q\bar{c}) + (c\bar{q})$: possible via a heavy quark pair annihilation and a light quark pair creation, with large phase space.
- $cc\bar{c}\bar{c}(L=1) \to cc\bar{c}\bar{c}(L=0) + (q\bar{q})_{I=0}$: OZI forbidden.



Spontaneous dissociations

J^{PC}	S-wave	P-wave
0^{++}	$\eta_c(1S)\eta_c(1S)$, $J/\psi J/\psi$	$\eta_c(1S)\chi_{c1}(1P)$, $J/\psi h_c(1P)$
0^{-+}	$\eta_c(1S)\chi_{c0}(1P)$, $J/\psi h_c(1P)$	$J/\psi J/\psi$
0^{--}	$J/\psi \chi_{c1}(1P)$	$J/\psi \eta_c(1S)$
1^{++}	$J/\psi J/\psi$	$J/\psi h_c(1P)$, $\eta_c(1S)\chi_{c1}(1P)$, $\eta_c(1S)\chi_{c0}(1P)$
1^{+-}	$J/\psi \eta_c(1S)$	$J/\psi \chi_{c0}(1P)$, $J/\psi \chi_{c1}(1P)$, $\eta_c(1S)h_c(1P)$
1^{-+}	$J/\psi h_c(1P)$, $\eta_c(1S)\chi_{c1}(1P)$	–
1^{--}	$J/\psi \chi_{c0}(1P)$, $J/\psi \chi_{c1}(1P)$, $\eta_c(1S)h_c(1P)$	$J/\psi \eta_c(1S)$

Summary

- We have calculated the mass spectra for the $QQ\bar{q}_i\bar{q}_j$, $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ tetraquark states.
- These $bb\bar{q}_i\bar{q}_j$ tetraquarks lie below thresholds of two-bottom mesons, two-bottom baryons. They are thus very stable and narrow!
- The $cc\bar{c}\bar{c}$ states lie above two charmonium thresholds and thus mainly decay via spontaneous dissociations.
- The $bb\bar{b}\bar{b}$ states are below $\eta_b\eta_b$ threshold, expected to be narrow. They are good candidate compact tetraquarks.
- They could be searched for in final states $B^{(*)}\bar{B}^{(*)}$, bottomonia$+\gamma$, $l^+l^-l^+l^-$.

Thank you for your attention!