

Precise measurement of W mass

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Motivation

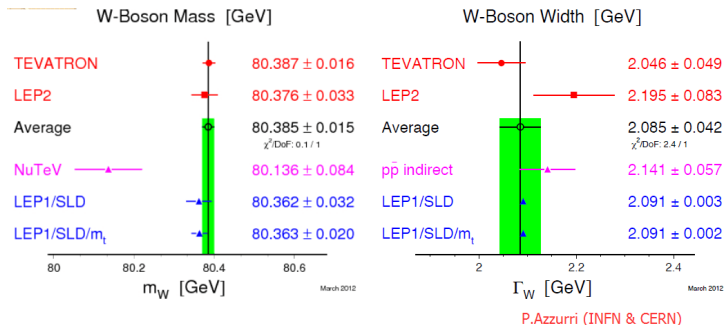
- The mass of W boson plays a central role in precision EW measurements and in constraints on the SM model through global fit.

$$m_W^2 \left(1 - \frac{m_W^2}{m_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_\mu} (1 + \Delta r) \quad (1)$$

Improving the precision of m_W is important for testing the overall consistency of the SM.

- The direct measurement of m_W by reconstruction with its daughters suffer the large systematic uncertainty, such as the radiative correction, modeling of hadronization.
- The threshold scan method is more sensitive to the statistic of data and accelerator performance (this study).

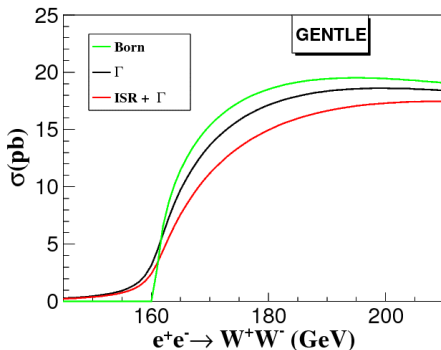
Status and goal



- ★ Using the threshold scan method, **2.5 MeV** for total uncertainty for m_W can be achieved with **500 fb⁻¹** integrated luminosity at CEPC (Pre-CDR).

Theoretical tool

- ▶ The $\sigma_{W^+W^-}$ is a function of \sqrt{s} , M_W , Γ_W , which is calculated with the GENTLE package in this study.
- ▶ The ISR correction is also calculated by convoluting the Born cross section with ISR radiator, <https://arxiv.org/abs/hep-ph/9910523v1> with the radiator up to order α^2 correction.



Statistic uncertainty for m_W

$\Delta\sigma_{W^+W^-}, \Delta M_W, \Delta\Gamma_W$ (Stat.)

$$\begin{aligned}
\Delta\sigma_{W^+W^-} &= \sigma_{W^+W^-} \times \frac{\Delta N_{W^+W^-}}{N_{W^+W^-}} \\
&= \sigma_{W^+W^-} \times \frac{\sqrt{N_{W^+W^-} + N_{bkg}}}{N_{W^+W^-}} \\
&= \sqrt{\frac{\sigma_{W^+W^-}}{\mathcal{L}\epsilon P}} \quad \left(P = \frac{N_{W^+W^-}}{N_{W^+W^-} + N_{bkg}}\right)
\end{aligned} \tag{2}$$

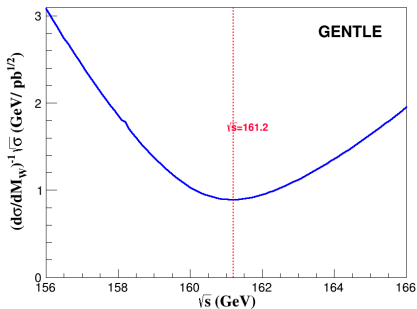
$$\Delta M_W = \left(\frac{\partial\sigma_{W^+W^-}}{\partial M_W}\right)^{-1} \times \sqrt{\frac{\sigma_{W^+W^-}}{\mathcal{L}\epsilon P}} \tag{3}$$

$$\Delta\Gamma_W = \left(\frac{\partial\sigma_{W^+W^-}}{\partial\Gamma_W}\right)^{-1} \times \sqrt{\frac{\sigma_{WW}}{\mathcal{L}\epsilon P}} \tag{4}$$

$\Delta\sigma_{W^+W^-}, \Delta M_W, \Delta\Gamma_W$ (Stat.)

- With $\mathcal{L} = 500 \text{ fb}^{-1}$, $\epsilon = 0.8$, $P = 0.9$:

$$\Delta M_W = \left(\frac{\partial\sigma_{W^+W^-}}{\partial M_W} \right)^{-1} \times \sqrt{\frac{\sigma_{W^+W^-}}{\mathcal{L}\epsilon P}} \approx 1.5 \text{ MeV.}$$

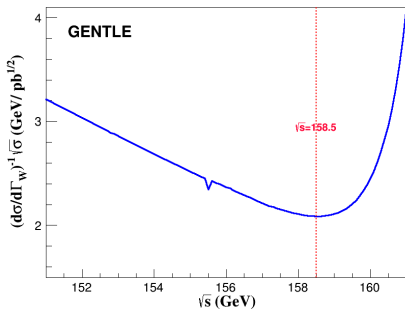


Max stat. sensitivity at $\sqrt{s} \sim 2m_W + 0.4 \text{ GeV}$

$\Delta\sigma_{W^+W^-}, \Delta M_W, \Delta\Gamma_W$ (Stat.)

- With $\mathcal{L} = 500 \text{ fb}^{-1}$, $\epsilon = 0.8$, $P = 0.9$:

$$\Delta\Gamma_W = \left(\frac{\partial\sigma_{W^+W^-}}{\partial\Gamma_W}\right)^{-1} \times \sqrt{\frac{\sigma_{W^+W^-}}{\mathcal{L}\epsilon P}} \approx 3.5 \text{ MeV.}$$



Max stat. sensitivity at $\sqrt{s} \sim 2m_W - 3.3 \text{ GeV}$

Data taken for the measurement of M_W

If we just consider the M_W , with Γ_W fixed to PDG value:

- ▶ One point at $\sqrt{s} = 161.2$ GeV, $\Delta M_W \approx 1.5$ MeV
- ▶ Two or three points around $\sqrt{s} = 161.2$ GeV, ΔM_W doesn't change much.
- ▶ ΔM_{W^\pm} increases when there are more than four points.

Systematic uncertainty for m_W

Beam energy spread

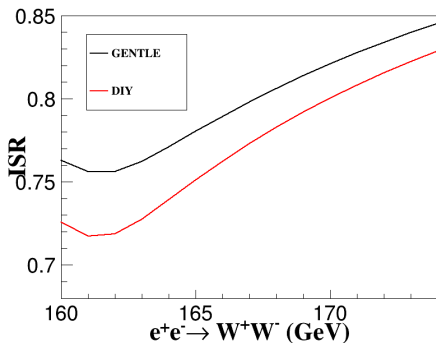
With the beam energy spread, the σ_{W+W^-} becomes:

$$\begin{aligned} \sigma_{W+W^-}(E) &= \int_0^{\infty} \sigma(E') \times G(E, E') dE' \\ &\approx \int_{E-6\sqrt{2}\Delta \cdot E}^{E+6\sqrt{2}\Delta \cdot E} \sigma(E') \times \frac{1}{\sqrt{2\pi}\sqrt{2\Delta \cdot E}} e^{\frac{-(E-E')^2}{2(\sqrt{2}\Delta \cdot E)^2}} dE' \end{aligned} \quad (5)$$

Δ (%)	Δm_W (MeV)
2	0.11
1	0.07
0.16	0.06

ISR factor $(1 + \delta)$

- ▶ The ISR factor is calculated by convoluting the Gentle's results (no ISR) and ISR radiator.
- ▶ Actually, the difference between the results from Gentle (with ISR) and our method will not contribute to Δm_W , but the accuracy of radiator we used does.



Luminosity \mathcal{L}

Considering the $\Delta\mathcal{L}$, the luminosity becomes :

$$\mathcal{L} \sim G(\mathcal{L}_0, \Delta\mathcal{L}) \quad (6)$$

If just taking data at one energy point, we simulate data with \mathcal{L} and use \mathcal{L}_0 in fit. By 500 samplings, the $\Delta m_W \propto \Delta\mathcal{L}$:

\mathcal{L} (‰)	Δm_W (MeV)
1.0	1.70
0.5	0.80
0.1	0.16

So corresponding Δm_W is very large if just taking data at one energy point. Instead, the contribution from $\Delta\mathcal{L}$ can be added in the χ^2 construction when there are more than one energy point.

ISR factor $(1 + \delta)$ and luminosity \mathcal{L}

For fake data, $\mathcal{L} = G(\mathcal{L}_0, \Delta\mathcal{L}_0)$. For fit, χ^2 is defined as

$$\chi^2 = \sum_i \frac{(y_i - h \cdot x_i)^2}{\delta_i^2} + \frac{(h - 1)^2}{\delta_c^2} \quad (7)$$

Here, y_i, x_i are the true and fit results at scan point i , h is a free parameter, δ_i and δ_c are the independent and correlated uncertainty, respectively.

With $\delta\mathcal{L} = 0.1\%$, $\Delta m_W = 0.4$ MeV.

Since the uncertainties of \mathcal{L} and ISR correction affect the Δm_W in same way, the situation for ISR correction is similar.

Beam energy uncertainty ΔE

With the ΔE , the total energy becomes:

$$E = G(E_p, \Delta E) + G(E_m, \Delta E) \quad (8)$$

By 500 samplings, the corresponding ΔM_W is:

ΔE (MeV)	ΔM_W (MeV)
2.0	1.54
1.5	1.03
1.0	0.74
0.5	0.36

MC simulation and Event selection ($\mu\nu_\mu qq$)

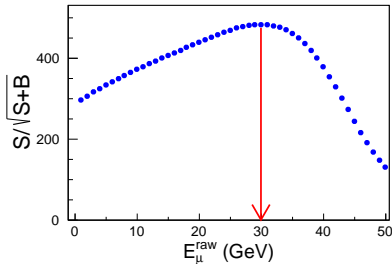
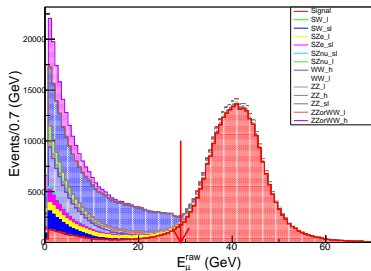
MC samples

		N_G	N_P	N_S	Scale Factor
Signal		300857	300202	272251	1.00
Bkg.	ZZ_l	5000000	120292	14932	0.11
	ZZ_{sl}	614909	300454	13299	0.41
	WW_l	100000	15367	14366	0.50
	SZe_l	693376	36559	1847	0.46
	$ZZ(WW)_l$	200000	4877	548	0.35
	ZZ_h	400000	86214	497	0.16
	SZe_{sl}	200000	19841	121	0.46
	$SZ\nu_l$	200000	3295	89	0.30
	SW_l	200000	107	82	0.48
	WW_h	823843	111109	41	0.28
	$SZ\nu_{sl}$	200000	19001	14	0.05
	$ZZ(WW)_h$	393463	35280	3	1.00
	SW_{sl}	285715	13498	2	1.00

Here, the N_G is the generated number of events, N_P and N_S are the ones passing preliminary and final event selections.

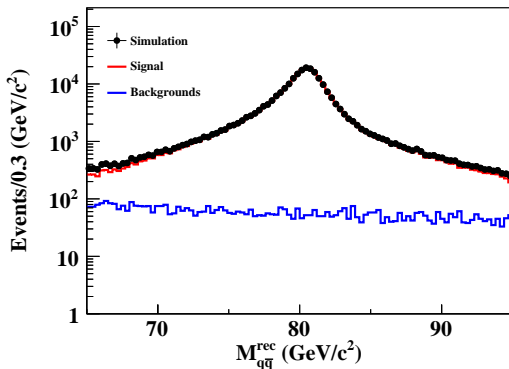
Event selection

- ▶ The signal events are selected with one lepton (μ), two jets, and one missing neutrino.
- ▶ To reject backgrounds, the $E_\mu^{\text{raw}} > 30$ GeV is performed. This cut is optimized with: $S/\sqrt{S+B}$, where S and B are the number of signal and background events.

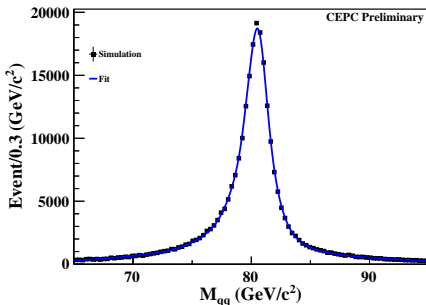


Signal and backgrounds

The distributions of $M_{q\bar{q}}^{\text{rec}}$ after the E_μ^{raw} cut:



Signal yields



Signal PDF: signal shape (RooKeysPdf)

Background PDF: 2-nd Chebychev function.

Input: $N_{sig} = 259570$, $N_{bkg} = 5762$

Fit: $N_{sig} = 259573.0 \pm 695.0$, $N_{bkg} = 5758.4 \pm 470.6$

Summary and Questions

- ▶ Using the threshold scan method, we study the measurement of m_W .
- ▶ With 500 fb^{-1} integrated luminosity, a precision of 2 MeV can be achieved in CEPC with 2 energy points ($\Delta\mathcal{L} \leq 0.1\%$, $\Delta E \leq 1.5 \text{ MeV}$, $\epsilon_P = 0.72$).
- ▶ The event selection for process $e^+e^- \rightarrow W^+W^- \rightarrow \mu\nu_\mu qq$ is simulated, the event select efficiency is about 0.9.
- For theoretical uncertainty of σ_{WW} , we just consider the ISR correction. But how about others, e.g., the modeling of hadronization?

Thank you!

backup

Backup

Energy spread

The effect of energy spread should be very small (**compute precision**). To check this, we use 100 times (**10000 steps**), the results are:

Mean (GeV)	80.3848	80.3849	80.3850	80.3851	80.3852
N	1	17	60	12	1

Theoretical error $\Delta\sigma_{WW}$

For ISR, the σ_{WW} is calculated with different options(different $O(\alpha^2)$).

For IZERO:

$$\bullet S = \frac{3}{4}\beta_e + \frac{\alpha}{\pi} \left(\frac{\pi^2}{3} - \frac{1}{2} \right) \times \text{IZERO} + \dots$$

For IQEDHS:

- -1, $e^{O(\alpha)}$ - constant terms (a'la WWGENPV?)
- 0, $e^{O(\alpha)}$ + constant terms (a'la BBOR, universal?)
- 1, $e^{O(\alpha)} + L^2$ of $O(\alpha^2)$
- 2, $e^{O(\alpha)} + L^2 + L$ of $O(\alpha^2)$
- 3, $e^{O(\alpha)} + L^2 + L + 1$ of $O(\alpha^2)$ (recommended)

IZERO/IQEDHS	-1	0	1	2	3
0	4.105	4.456	4.438	4.443	4.443
1	4.105	4.483	4.465	4.470	4.469

Energy spread (1-D)

To consider the effect of energy spread ($\Delta_{E_{tot}} = \sqrt{\Delta_{E_p} + \Delta_{E_m}} = \sqrt{2}\Delta$, **ID assumption**), the experimental $\sigma_{W^+W^-}$ become:

$$\begin{aligned} \sigma_{W^+W^-}(E) &= \int_0^{\infty} \sigma(E') \times G(E, E') dE' \\ &\approx \int_{E-6\sqrt{2}\Delta \cdot E}^{E+6\sqrt{2}\Delta \cdot E} \sigma(E') \times \frac{1}{\sqrt{2\pi}\sqrt{2\Delta \cdot E}} e^{-\frac{(E-E')^2}{2(\sqrt{2}\Delta \cdot E)^2}} dE' \end{aligned} \quad (9)$$

Here, $\sqrt{2}\Delta \cdot E$ is the energy spread, and Δ is 0.16% (preCDR). To save compute time, we use the region $[E - 6\sqrt{2}\Delta \cdot E, E + 6\sqrt{2}\Delta \cdot E]$.

Input (GeV)	80.385
Fit (GeV)	80.3851

Energy spread (2-D?)

The σ_{W+W^-} with the 2-D convolution with $\Delta_{E_p}, \Delta_{E_m}$:

$$\sigma_{W+W^-}(E_p, E_m) = \int_0^\infty \int_0^\infty \sigma(E'_p + E'_m) \times G_1(E_p, E'_p) dE'_p \times G_2(E_m, E'_m) dE'_m \quad (10)$$

Do we need to use the 2-D formula? **Very slow but without assumption!**

Beam energy measurement uncertainty ΔE

Considering the ΔE , the total energy become (ID assumption):

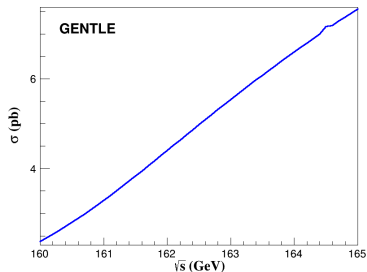
$$E = N(E_p, \Delta E^2) + N(E_m, \Delta E^2) \quad (11)$$

By 500 samplings, the corresponding ΔM_{W^\pm} is:

ΔE (MeV)	ΔM_{W^\pm} (MeV)
2.0	1.54
1.5	1.03
1.0	0.74
0.5	0.36

Uncertainty from luminosity ΔL (more points)

The cross sections around the most sensitive region are almost linear. So we take more points in this region (average luminosity).



N_{pt}	1	2	3	4	5	6	7
$\Delta M_W (MeV)$	1.70	1.23	1.17	...			