Standard Parton Physics

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量子色动力学与有效理论暑期学校

2018.7.3-7.20， 上海交通大学

	上午 9:00-11:30	下午 2:00-4:30	下午 4:30-6:00
7.3 星期二	马建平 量子色动力学基础 1	冯旭 格点QCD基础 1	
7.4 星期三	马建平 量子色动力学基础 2	冯旭 格点QCD基础 2	
7.5 星期四	马建平 量子色动力学基础 3	冯旭 格点QCD基础 3	
7.6 星期五	曹庆宏 对撞机物理导论 1	L. Maiani, TBA 1	马建平 量子色动力学基础 4
7.7 星期六	曹庆宏 对撞机物理导论 2	马建平 量子色动力学基础 5	
7.8 星期日			
7.9 星期一	冯旭 格点QCD基础 4	季向东 TBA 1	
7.10 星期二	冯旭 格点QCD基础 5	L. Maiani, TBA 2	季向东 TBA 2
7.11 星期三	贾宇 NRQCD 1	季向东 TBA 3	
7.12 星期四	季向东 TBA 4	L. Maiani, TBA 3	
7.13 星期五	贾宇 NRQCD 2	贾宇 NRQCD 3	
7.14 星期六			
7.15 星期日			
7.16 星期一	袁烽 Hadron&GPD 1	朱华星 QCD@LHC 1	
7.17 星期二	袁烽 Hadron&GPD 2	袁烽 Hadron&GPD 3	
7.18 星期三	朱华星 QCD@LHC 2	朱华星 QCD@LHC 3	
7.19 星期四	王玉明 Frontier of B-physics 1	王玉明 Frontier of B-physics 2	
7.20 星期五	袁烽 Hadron&GPD 4	王玉明 Frontier of B-physics 3	

Outline

- Review earlier lectures (in particular, by JP Ma) and introduction
- Introduce GPDs, connection to TMDs, and Wigner Distribution
- Small-x physics

Content:

1. QCD Lagrangian

2. Divergences in QCD and $e^+e^- \to$ hadrons

3. DIS and QCD Factorization

4. QCD Factorization in $e^+e^- \to h + X$

5. TMD Factorization for SIDIS

6. SCET

7. ....

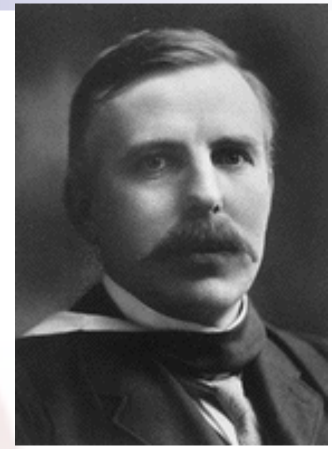
BERKELEY LAB

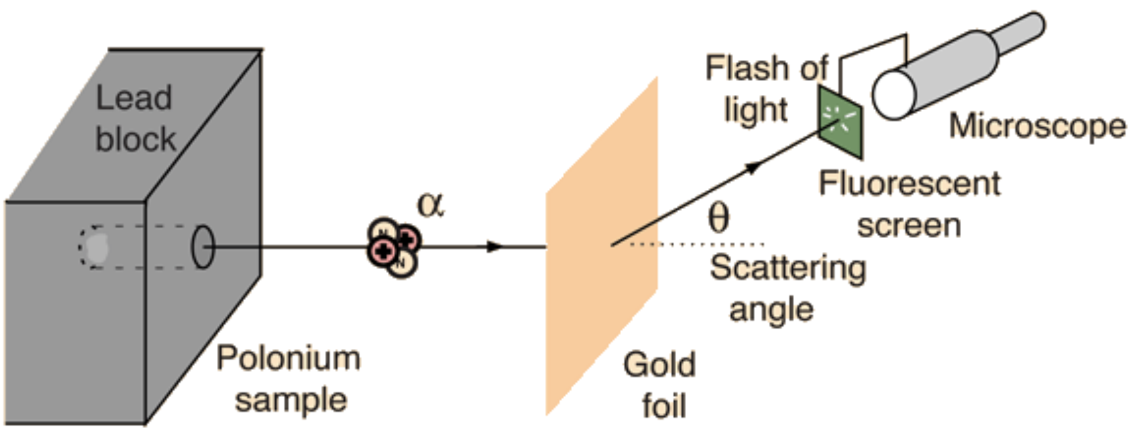
- Parton Physics
  - G. Sterman, *Partons, Factorization and Resummation*, hep-ph/9606312
  - John Collins, *The Foundations of Perturbative QCD*, published by Cambridge, 2011
  - CTEQ, *Handbook of perturbative QCD*, Rev. Mod. Phys. 67, 157 (1995).
- General references
  - CTEQ web site: http://www.phys.psu.edu/~cteq/

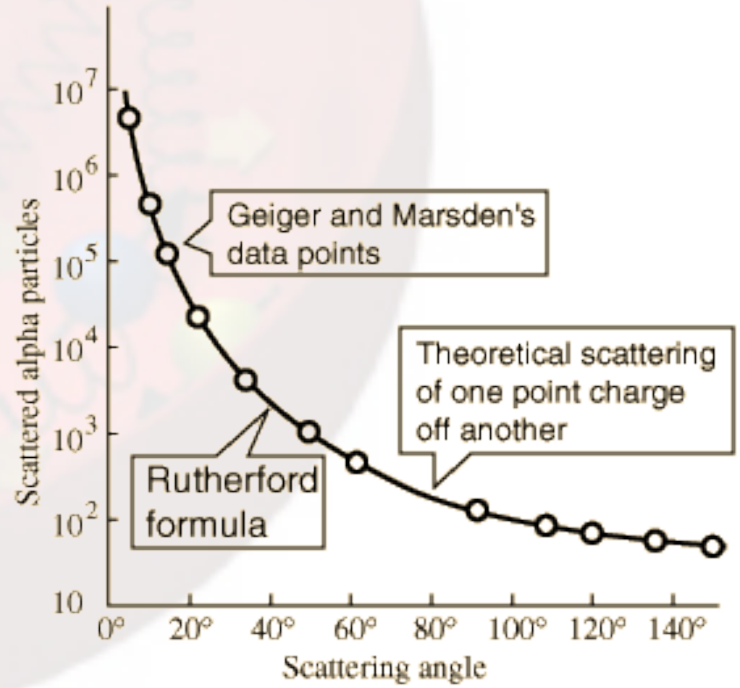
Rutherford scattering

The Scattering of α and β Particles by Matter and the Structure of the Atom

E. Rutherford, F.R.S.*
Philosophical Magazine
Series 6, vol. 21
May 1911, p. 669-688

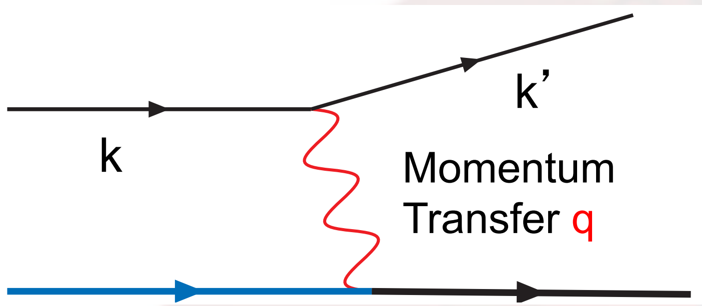






Discovery of Nuclei

Power counting analysis



$$2E_{k'}\frac{d\sigma}{d^3k'} \propto |\mathcal{M}|^2 \quad \mathcal{M} \propto \frac{1}{q^2}$$

$$q^2 = -Q^2 \approx E_k E_k' \sin^2\frac{\theta}{2}$$

- EM interaction perturbation, leading order dominance, potential~1/r
- Point-like structure
- Powerful tool to study inner structure

Basic idea of nuclear science

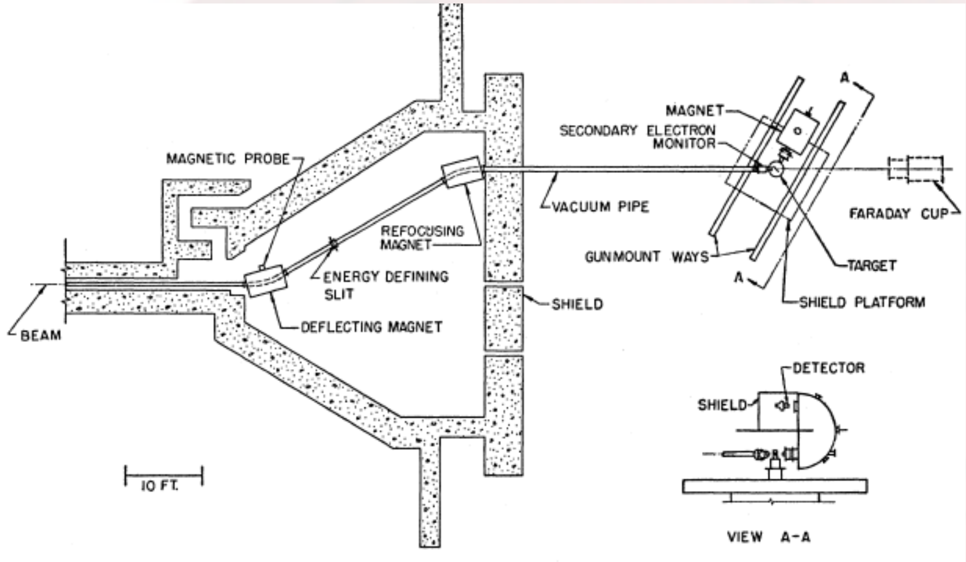
Since the α and β particles traverse the atom, it should be possible from a close study of the nature of the deflexion to form some idea of the constitution of the atom to produce the effects observed. In fact, the scattering of high-speed charged particles by the atoms of matter is one of the most promising methods of attack of this problem. The develop-

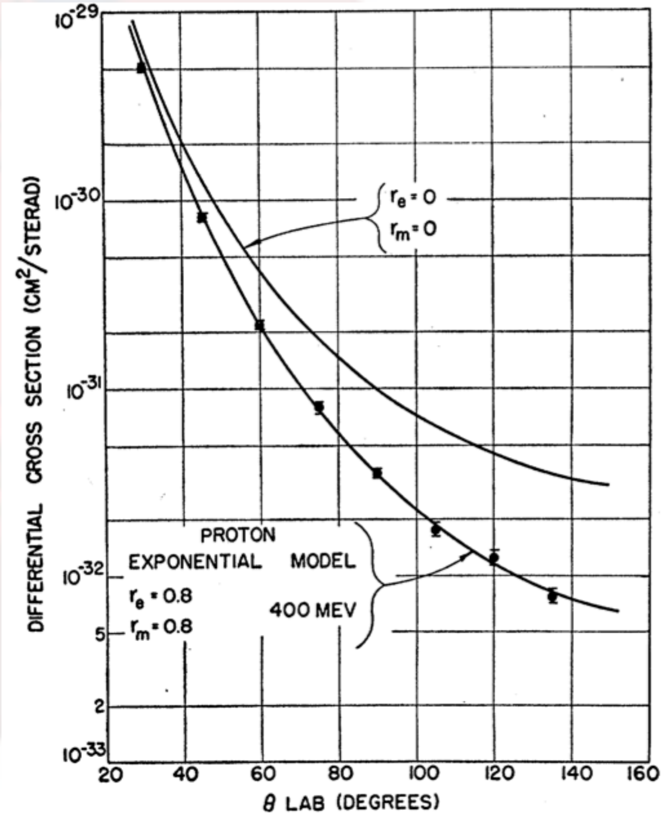
Rutherford, 1911

Finite size of nucleon (charge radius)

- Rutherford scattering with electron


Hofstadter





Renewed interest on proton radius:
μ-Atom vs e-Atom (EM-form factor)

RevModPhys.28.214

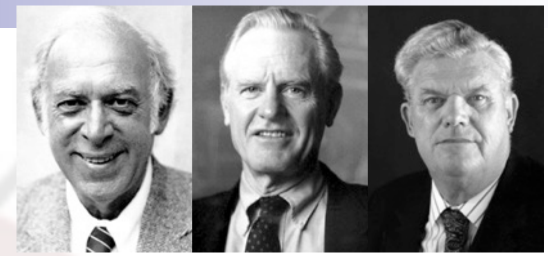
Quark model


Gell-Man

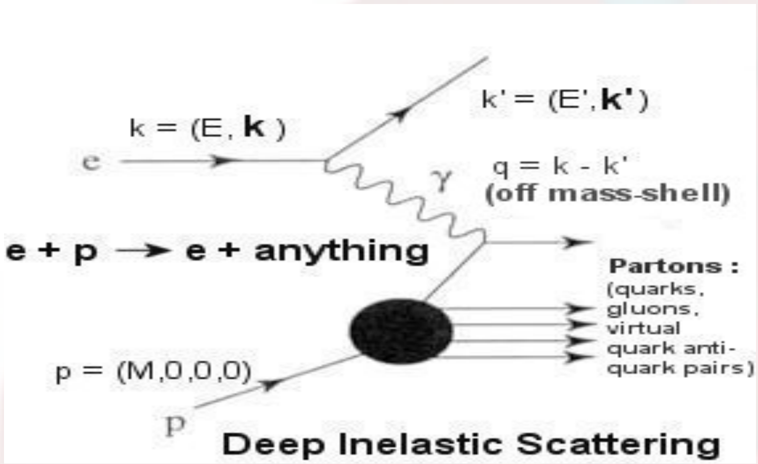
- Nucleons, and other hadrons are not fundamental particles, they have constituents
- Gell-Man Quark Model
  - Quark: spin ½
    - Charges: up (2/3), down (-1/3), strange (-1/3)
  - Flavor symmetry to classify the hadrons
    - Mesons: quark-antiquark
    - Baryons: three-quark
    - Gell-Man-Okubo Formula

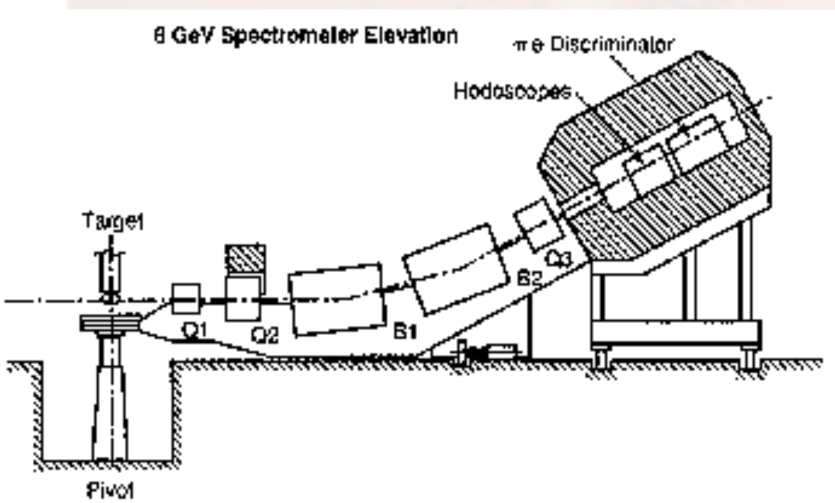
Deep Inelastic Scattering

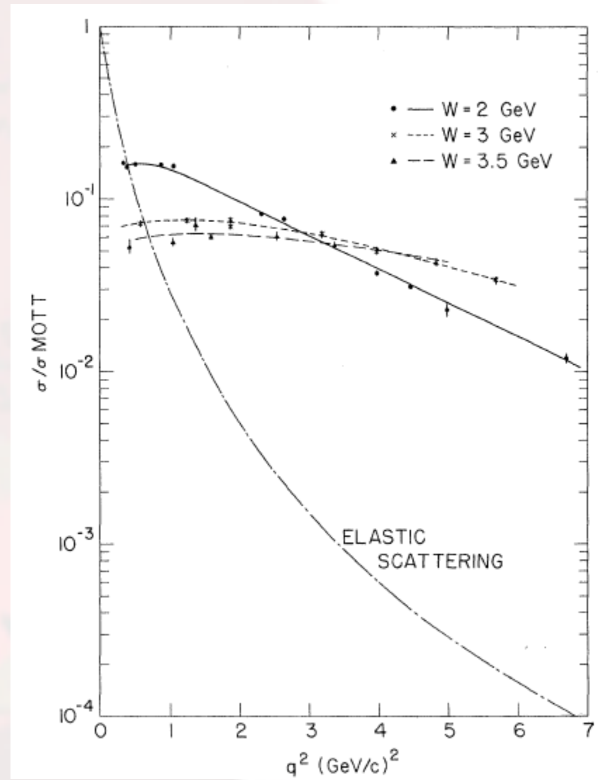
Discovery of Quarks



Friedman Kendall Taylor





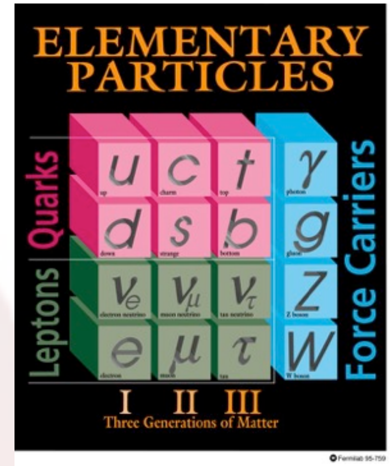


Bjorken Scaling: $Q^2 \rightarrow$ Infinity
Feynman Parton Model:
Point-like structure in Nucleon

Understanding the scaling

- Weak interactions at high momentum transfer
  - Rutherford formula rules
- Strong interaction at long distance
  - Form factors behavior
  - No free constituent found in experiment
- Strong interaction dynamics is different from previous theory

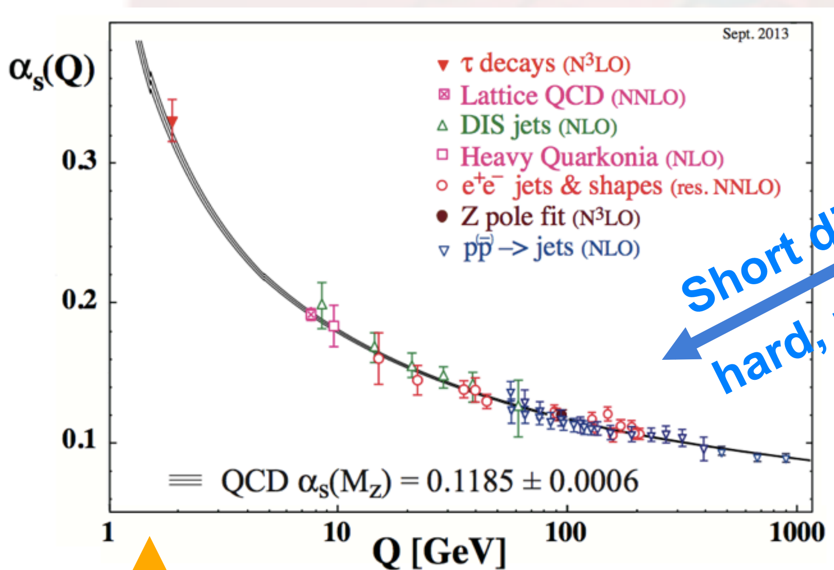
QCD and Strong-Interactions



- QCD: Non-Abelian gauge theory
  - Building blocks: quarks (spin½, m_q, 3 colors; gluons: spin 1, massless, 3^2-1 colors)

$$L = \bar{\psi}(i\gamma \cdot \partial - m_q)\psi - \frac{1}{4}F^{\mu\nu a}F_{\mu\nu a} - g_s\bar{\psi}\gamma \cdot A\psi$$

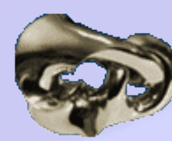
- Asymptotic freedom and confinement



~1/Length

Short distance hard, perturbative

Long distance:? Soft, non-perturbative



Clay Mathematics Institute Millennium Prize Problem

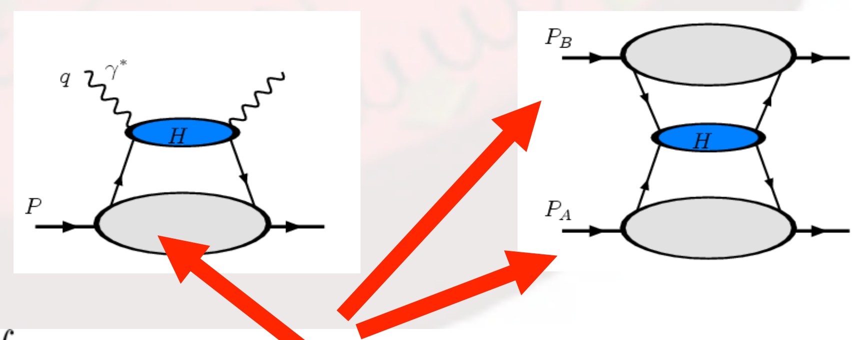
Nonperturbative scale Λ_{QCD}~1GeV

Quantum Chromodynamics

- There is no doubt that QCD is the right theory for hadron physics
- However, many fundamental questions…
- How does the **nucleon mass**?
- Why quarks and gluons are **confined** inside the nucleon?
- How do the fundamental **nuclear forces** arise from QCD?
- We don't have a **comprehensive picture** of the nucleon structure as we don't have an approximate QCD nucleon wave function
- …

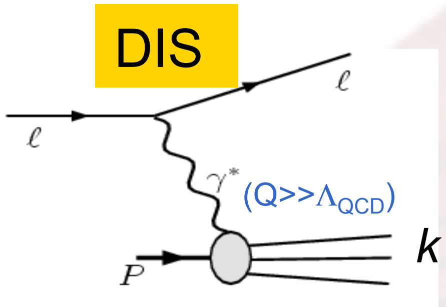
Feynman's parton language and QCD Factorization

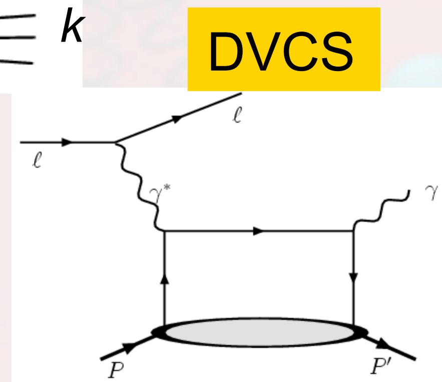
- If a hadron is involved in high-energy scattering, the physics simplifies in the infinite momentum frame (Feynman's Parton Picture)
- The scattering can be decomposed into a convolution of parton scattering and parton density (distribution), or wave function or correlations
  - QCD Factorization!

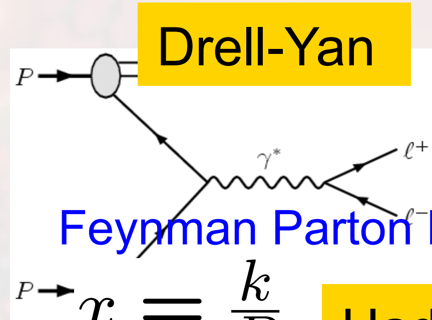


$$\sim \int \text{Parton Distributions} \otimes \text{Hard Partonic Cross Section}$$

High energy scattering as a probe to the nucleon structure

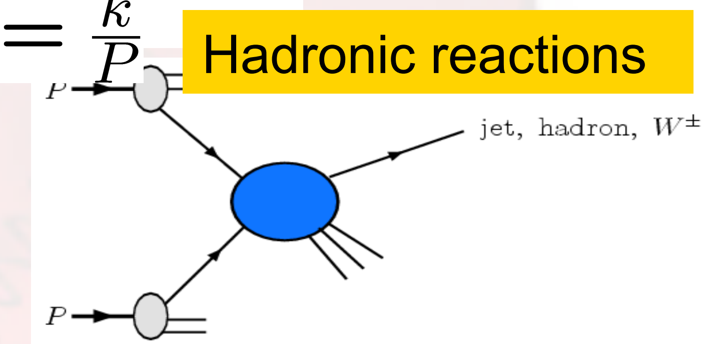






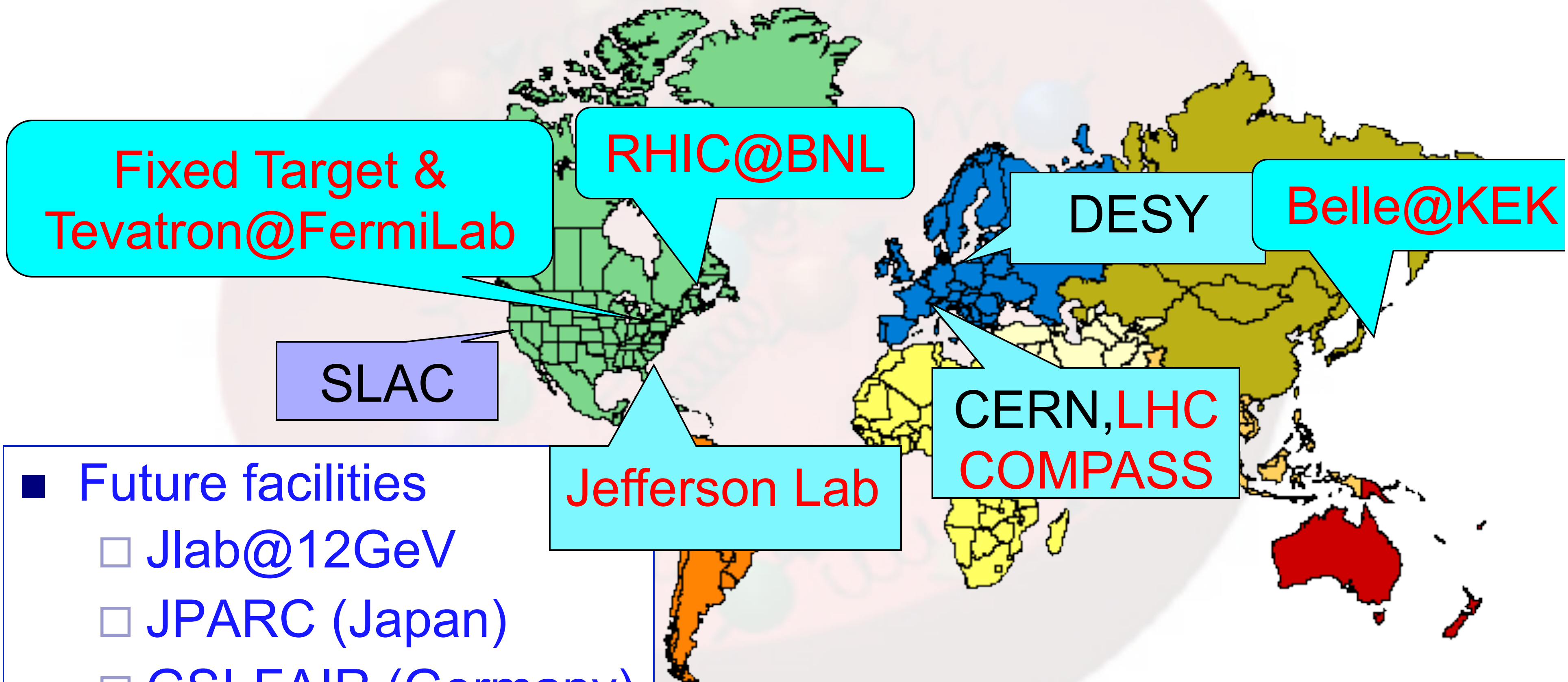
Feynman Parton Momentum fraction

$$x = \frac{k}{P}$$



- Many processes: Deep Inelastic Scattering, Deeply-virtual compton scattering, Drell-Yan lepton pair production, pp→jet+X
  - Momentum distribution: Parton Distribution
  - Spin density: polarized parton distribution
  - Wave function in infinite momentum frame: Generalized Parton Distributions

Exploring the partonic structure of nucleon worldwide



- Future facilities
  - Jlab@12GeV
  - JPARC (Japan)
  - GSI-FAIR (Germany)
  - **EIC, LHeC, …**

Perturbative Computations

- Singularities in higher order calculations
- Dimension regularization
  - n<4 for UV divergence
  - n>4 for IR divergence
  - MS ($\overline{MS}$) scheme for UV divergence

$$\int \frac{d^n k}{k^4} \rightarrow \int \frac{dk}{k} k^{n-4}$$

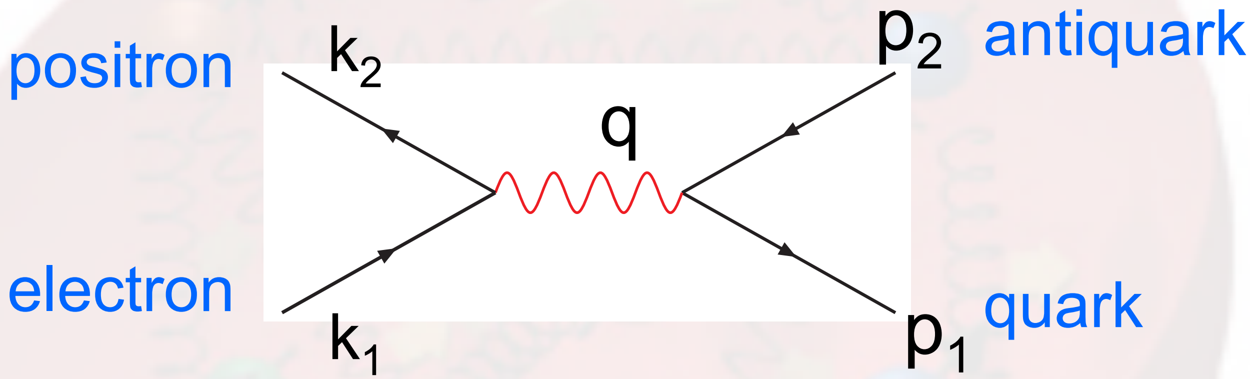
- pQCD predictions rely on Infrared safety of the particular calculation

pQCD predictions

- Infrared safe observables
  - Total cross section in e+e-→hadrons
  - EW decays, tau, Z, …
- Factorizable hard processes: parton distributions/fragmentation functions
  - Deep Inelastic Scattering
  - Drell-Yan Lepton pair production
  - Inclusive process in ep, ee, pp scattering, W, Higgs, jets, hadrons, …

Infrared safe: $e^+e^- \rightarrow$ hadrons

- Leading order



  - Electron-positron annihilate into virtual photon, and decays into quark-antiquark pair, or muon pair
  - Quark-antiquark pair hadronize

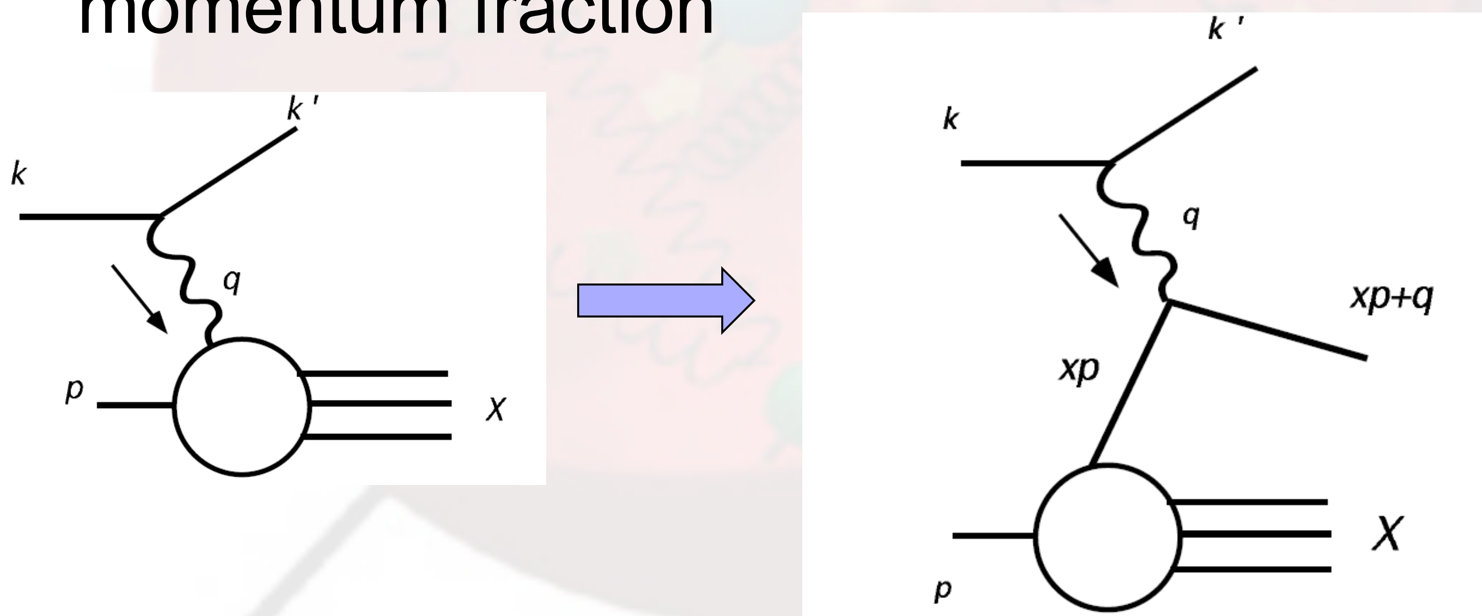
Long distance physics (factorization)

- Not every quantities calculated in perturbative QCD are infrared safe
  - Hadrons in the initial/final states, e.g.
- Factorization guarantee that we can safely separate the long distance physics from short one
- There are counter examples where the factorization does not work

Naïve Parton Model

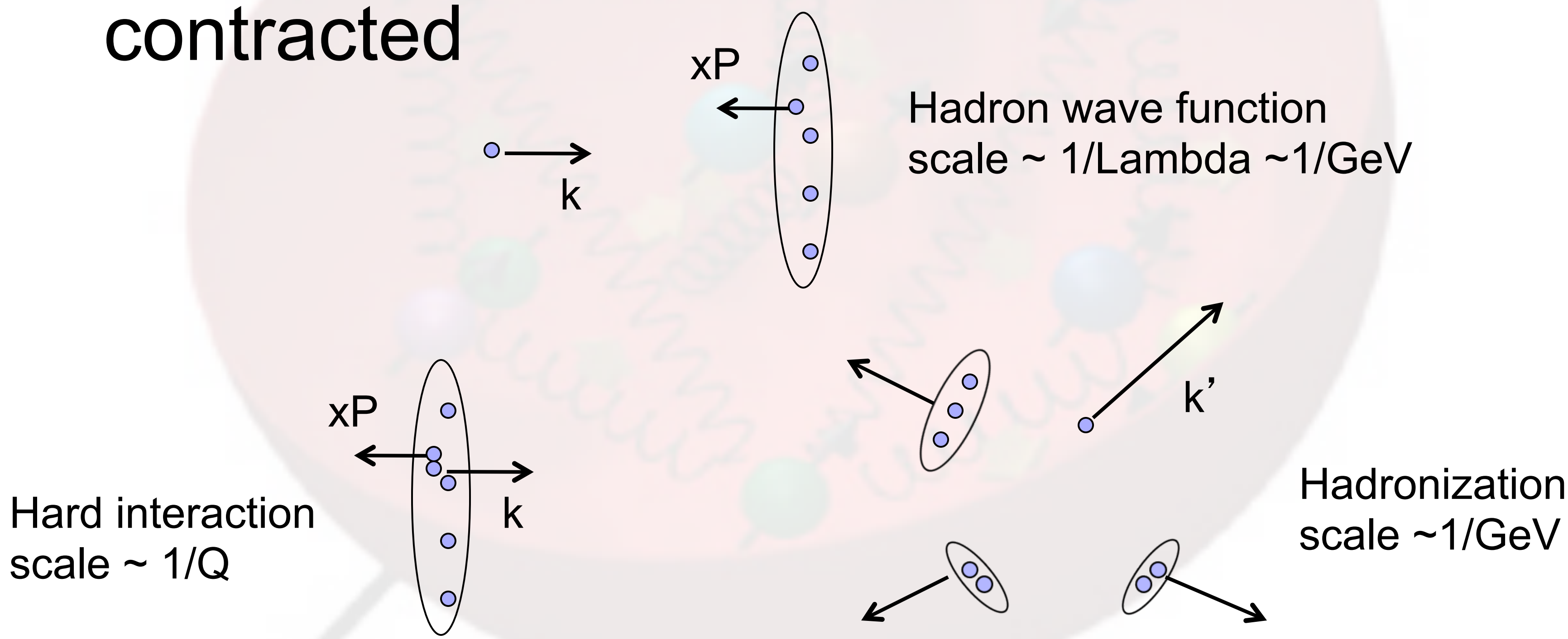
$$d\sigma^{(\ell N)}(p,q) = \sum_f \int_0^1 d\xi \; d\sigma_{\text{Born}}{}^{(\ell f)}(\xi p, q)\phi_{f/N}(\xi)$$

- $\phi_{f/N}(\xi)$ the parton distribution describes the probability that the quark carries nucleon momentum fraction



Intuitive argument for the factorization (DIS)

- In the Bjorken limit, nucleon is Lorentz contracted



Factorization formula

$$F_2{}^{(h)}(x,Q^2) = \sum_{i=f,\bar{f},G} \int_x^1 d\xi\; C_2{}^{(i)}\left(\frac{x}{\xi},\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)\phi_{i/h}(\xi,\mu^2)$$

$$F_1{}^{(h)}(x,Q^2) = \sum_{i=f,\bar{f},G} \int_x^1 \frac{d\xi}{\xi}\; C_1{}^{(i)}\left(\frac{x}{\xi},\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)\phi_{i/h}(\xi,\mu^2)$$

- Factorization → scale dependence

$$\mu\frac{d^2}{d\mu^2}\phi_{i/h}(x,\mu^2) = \sum_{j=f,\bar{f},G}\int_x^1 \frac{d\xi}{\xi}P_{ij}(\frac{x}{\xi},\alpha_s(\mu^2))\phi_{j/h}(\xi,\mu^2)$$

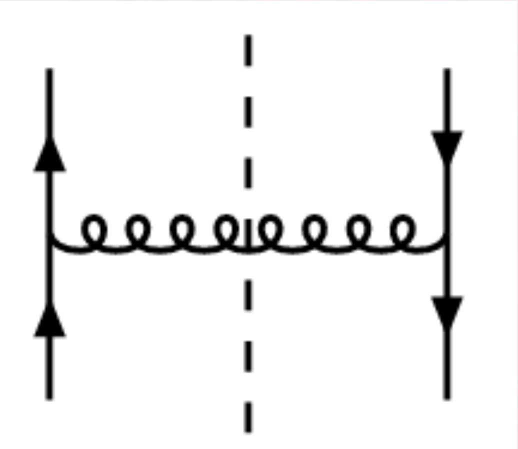
$$\mu\frac{d}{d\mu}\ln\bar{\phi}\left(n,\alpha_s(\mu^2)\right) = -\gamma_n\left(\alpha_s(\mu^2)\right) \qquad \bar{f}(n) \equiv \int_0^1 dx\; x^{n-1} f(x)$$

- Scale dependence → resummation

$$\bar{\phi}^{(\mathrm{val})}(n,\mu^2) = \bar{\phi}^{(\mathrm{val})}(n,\mu_0{}^2)\exp\left\{-\frac{1}{2}\int_0^{\ln\mu^2/\mu_0{}^2} dt\; \gamma_n\left(\alpha_s(\mu_0{}^2 e^t)\right)\right\}$$

anomalous dimension: $\int_0^1 d\xi\, \xi^{n-1} P_{ij}(\xi,\alpha_s) = -\gamma_{ij}(n)$

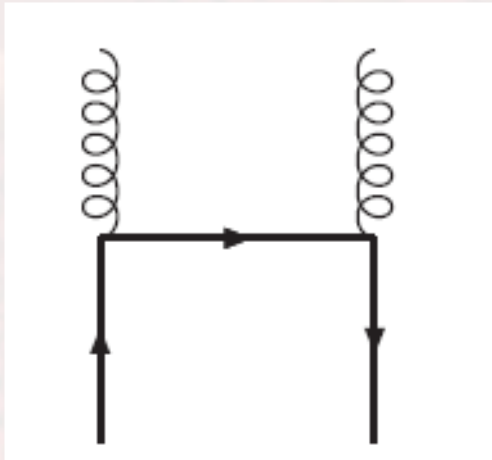
Quark-quark splitting



- Physical polarization for the radiation gluon
- Incoming quark on-shell, outgoing quark off-shell

$$\mathcal{P}_{qq} = C_F \left[\frac{1+x^2}{(1-x)_+} + \delta(1-x) \right]$$

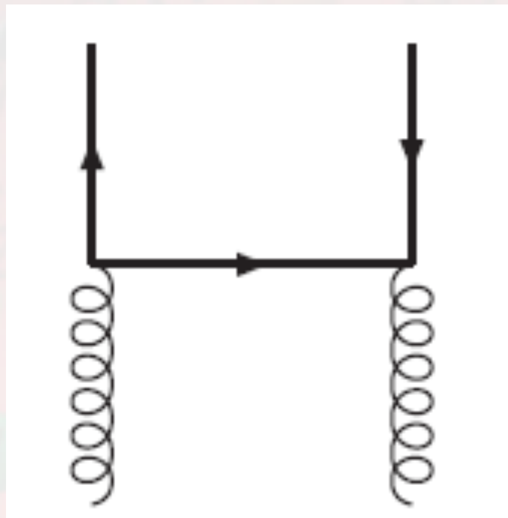
Quark-gluon splitting



- Incoming quark on-shell, gluon is off-shell

$$\mathcal{P}_{g/q} = C_F \left[\frac{1+(1-x)^2}{x} \right]$$

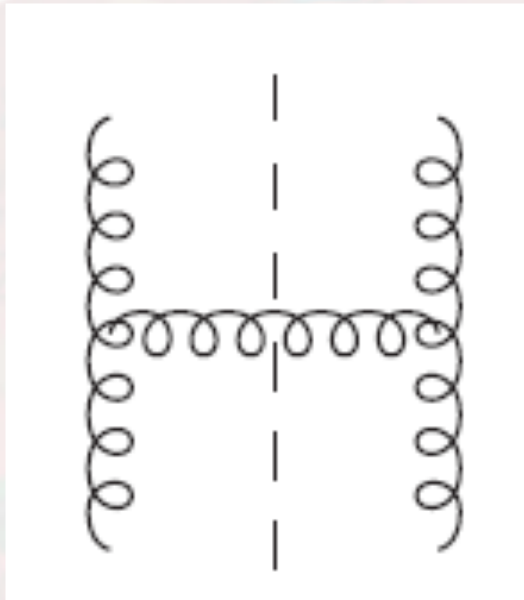
Gluon-quark splitting



- Incoming gluon is on-shell, physical polarization

$$\mathcal{P}_{q/g} = T_F \left[(1-x)^2 + x^2\right]$$

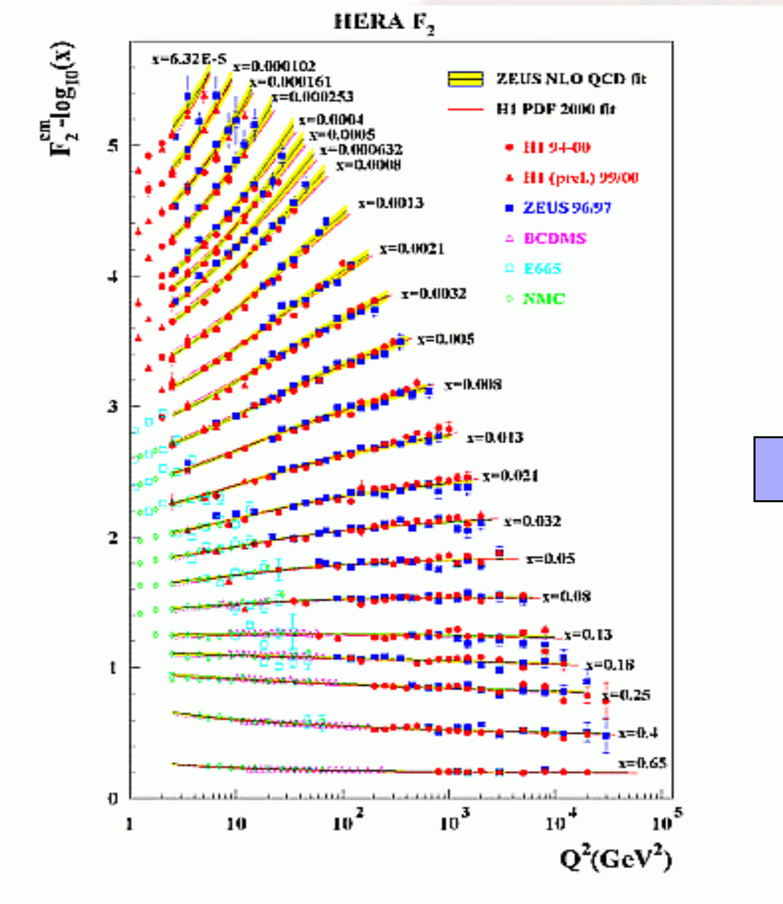
Gluon-gluon splitting

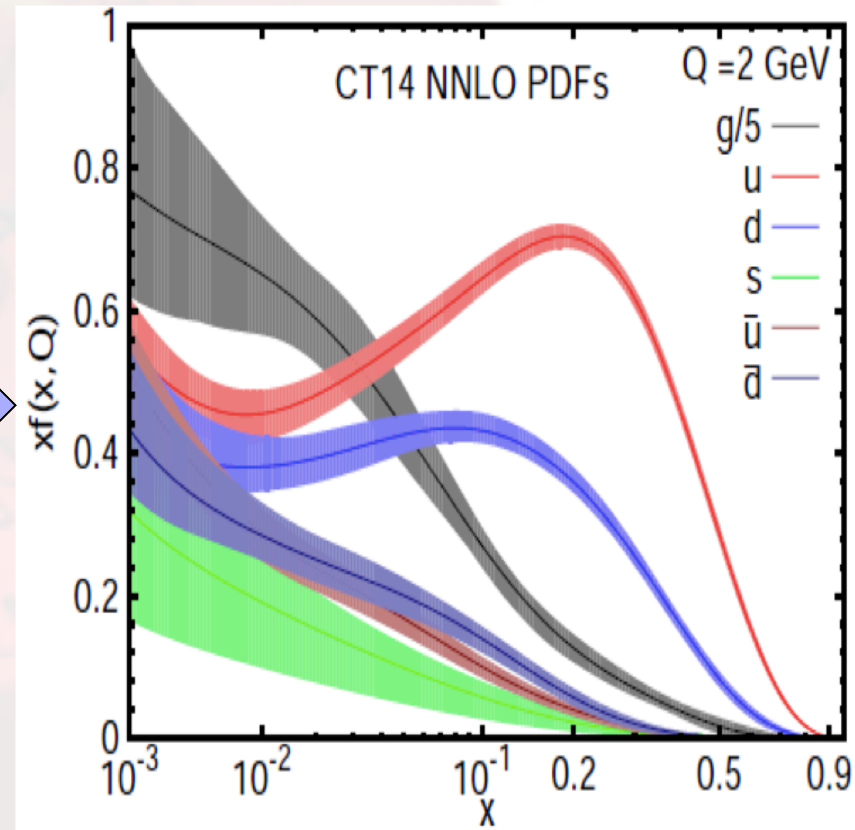


- Physical polarizations for the gluons

$$\mathcal{P}_{gg}(x) = \frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) + \delta(x-1)\beta_0$$

These evolutions describe the HERA data





Reverse the DIS:
Drell-Yan

MASSIVE LEPTON-PAIR PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES*

Sidney D. Drell and Tung-Mow Yan
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 25 May 1970)

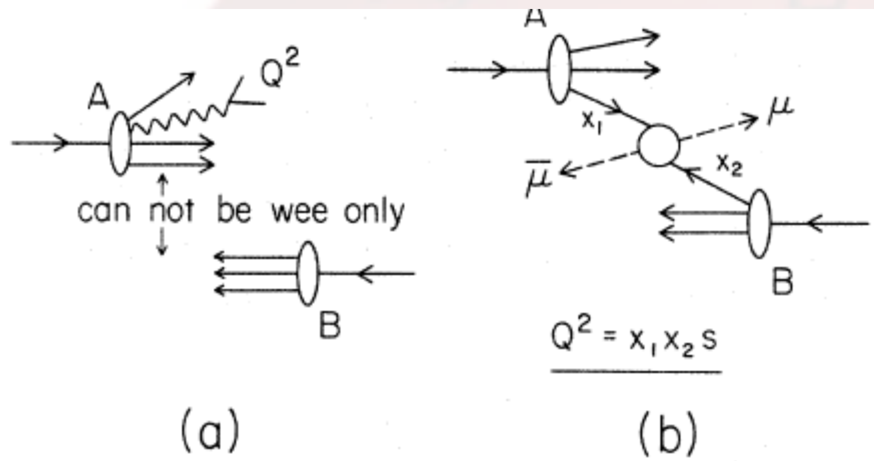
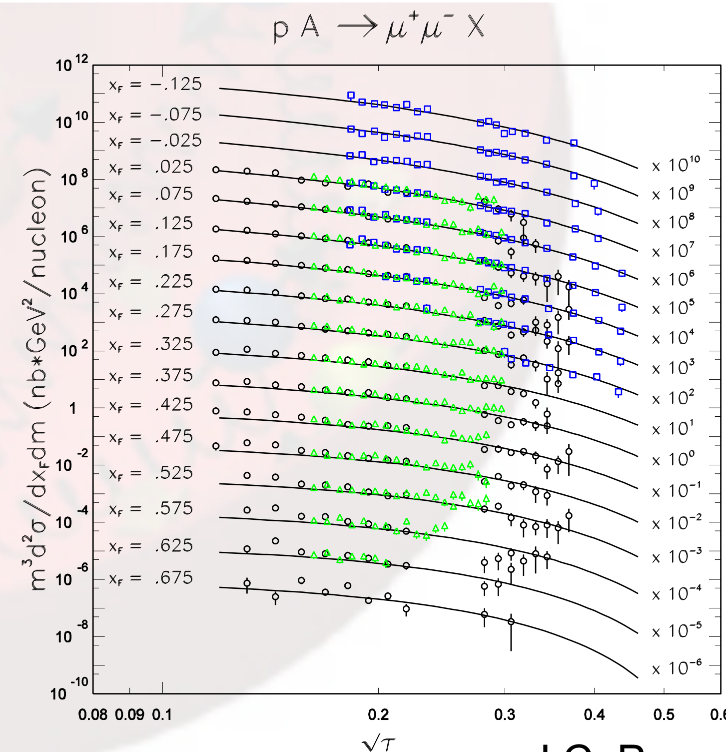
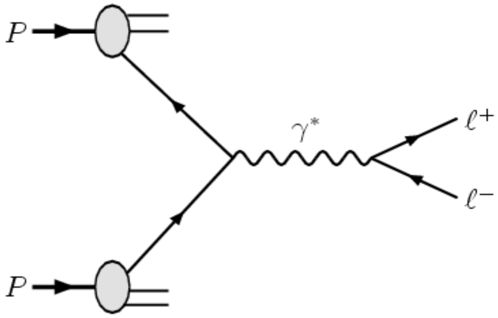


FIG. 1. (a) Production of a massive pair Q^2 from one of the hadrons in a high-energy collision. In this case it is kinematically impossible to exchange "wee" partons only. (b) Production of a massive pair by parton-antiparton annihilation.



J.C. Peng

Drell-Yan lepton pair production



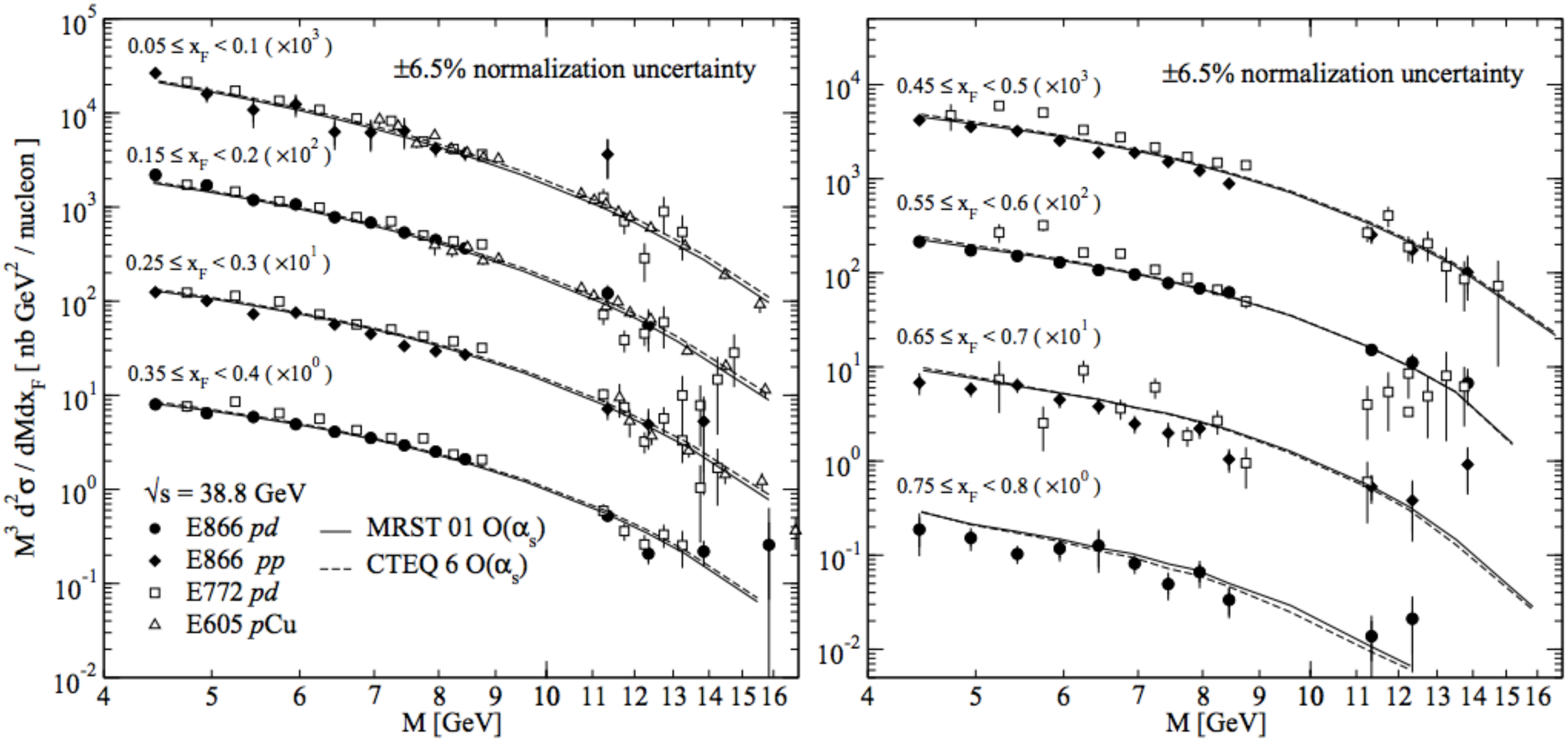
$$\sigma(pp \to \ell^+\ell^- + X) = \int dx_1 dx_2 \phi_{q/p}(x_1)\phi_{\bar{q}/p}(x_2)\hat{\sigma}(q\bar{q} \to \ell^+\ell^-)$$

- The same parton distributions as DIS
  - Universality
- Partonic cross section

$$\sigma(e^+e^- \to q\bar{q}) = N_c \frac{4\pi}{3}\frac{\alpha^2}{Q^2}e_q^2$$

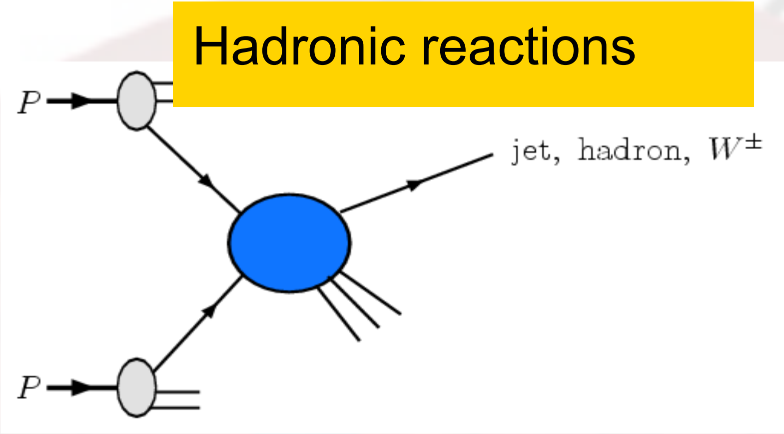
$$\hat{\sigma}(q\bar{q} \to \ell^+\ell^-) = \frac{4\pi}{3}\frac{\alpha^2}{Q^2}e_q^2\frac{1}{N_c}$$

Profound results



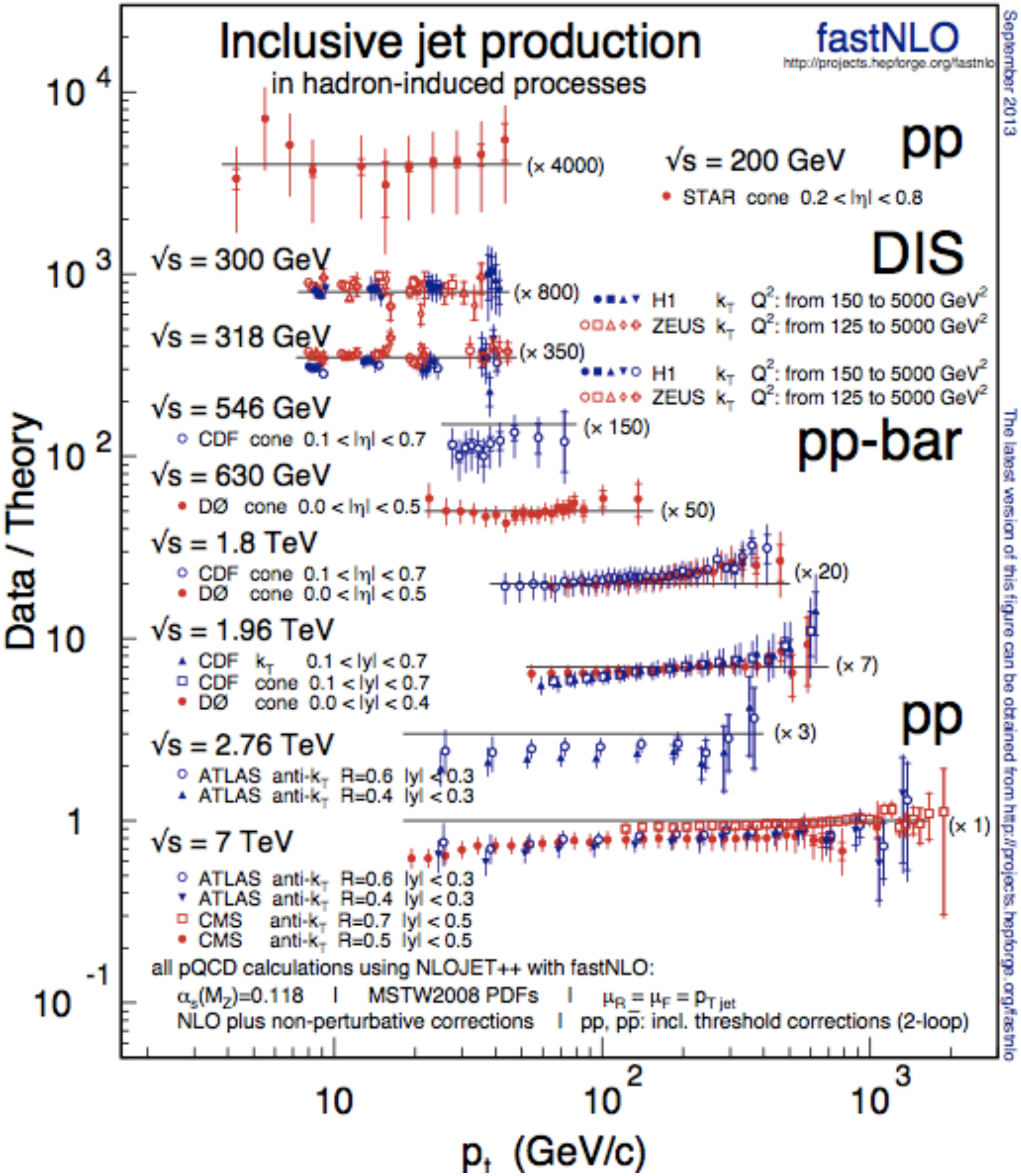
- Universality
- Perturbative QCD at work

More general hadronic process



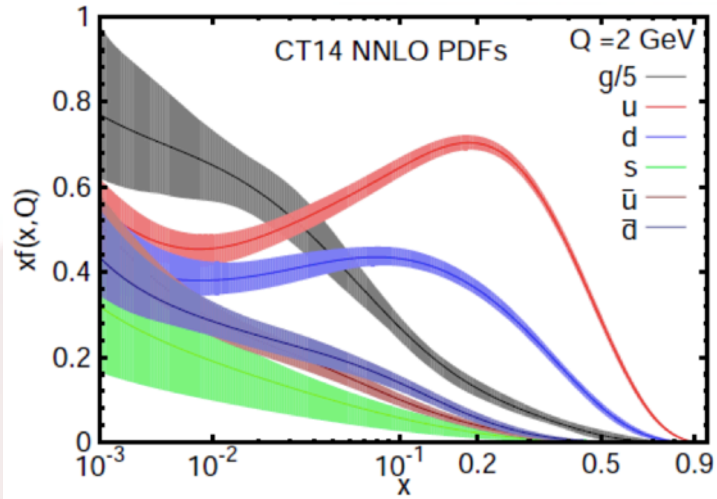
$$\sigma(pp \to c + X) = \int dx_1 dx_2 \phi_{a/p}(x_1)\phi_{b/p}(x_2)\hat{\sigma}(ab \to c + X)$$

- All these processes have been computed up to next-to-leading order, some at NNLO, few at N^3LO


Inclusive jet production
in hadron-induced processes
fastNLO
http://projects.hepforge.org/fastnlo
September 2013
pp
√s = 200 GeV
STAR cone 0.2 < |η| < 0.8
(× 4000)
DIS
√s = 300 GeV
(× 800)
H1 k_T Q²: from 150 to 5000 GeV²
ZEUS k_T Q²: from 125 to 5000 GeV²
√s = 318 GeV
(× 350)
H1 k_T Q²: from 150 to 5000 GeV²
ZEUS k_T Q²: from 125 to 5000 GeV²
pp-bar
√s = 546 GeV
CDF cone 0.1 < |η| < 0.7
(× 150)
√s = 630 GeV
DØ cone 0.0 < |η| < 0.5
(× 50)
√s = 1.8 TeV
CDF cone 0.1 < |η| < 0.7
DØ cone 0.0 < |η| < 0.5
(× 20)
√s = 1.96 TeV
CDF k_T 0.1 < |y| < 0.7
CDF cone 0.1 < |y| < 0.7
DØ cone 0.0 < |y| < 0.4
(× 7)
pp
√s = 2.76 TeV
ATLAS anti-k_T R=0.6 |y| < 0.3
ATLAS anti-k_T R=0.4 |y| < 0.3
(× 3)
√s = 7 TeV
ATLAS anti-k_T R=0.6 |y| < 0.3
ATLAS anti-k_T R=0.4 |y| < 0.3
CMS anti-k_T R=0.7 |y| < 0.5
CMS anti-k_T R=0.5 |y| < 0.5
(× 1)
all pQCD calculations using NLOJET++ with fastNLO:
α_s(M_Z)=0.118 | MSTW2008 PDFs | μ_R = μ_F = p_T jet
NLO plus non-perturbative corrections | pp, pp̄: incl. threshold corrections (2-loop)
The latest version of this figure can be obtained from http://projects.hepforge.org/fastnlo
Data / Theory
p_t (GeV/c)
10
10²
10³
10⁴
1
10⁻¹

PDG2014

Parton picture of the nucleon



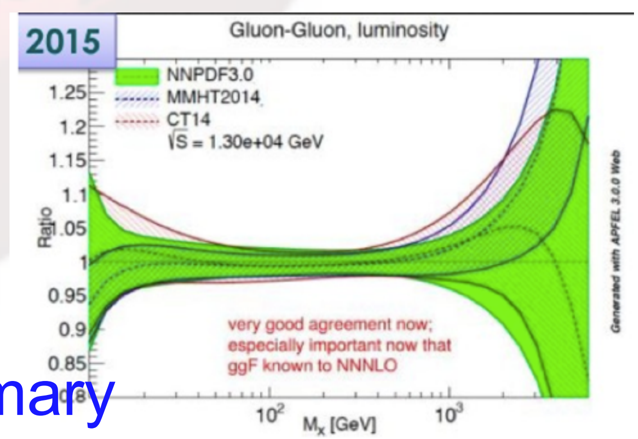
C.-P.Yuan@DIS15

- Beside valence quarks, there are sea and gluons
- Precisions on the PDFs are very much relevant for LHC physics: SM/New Physics

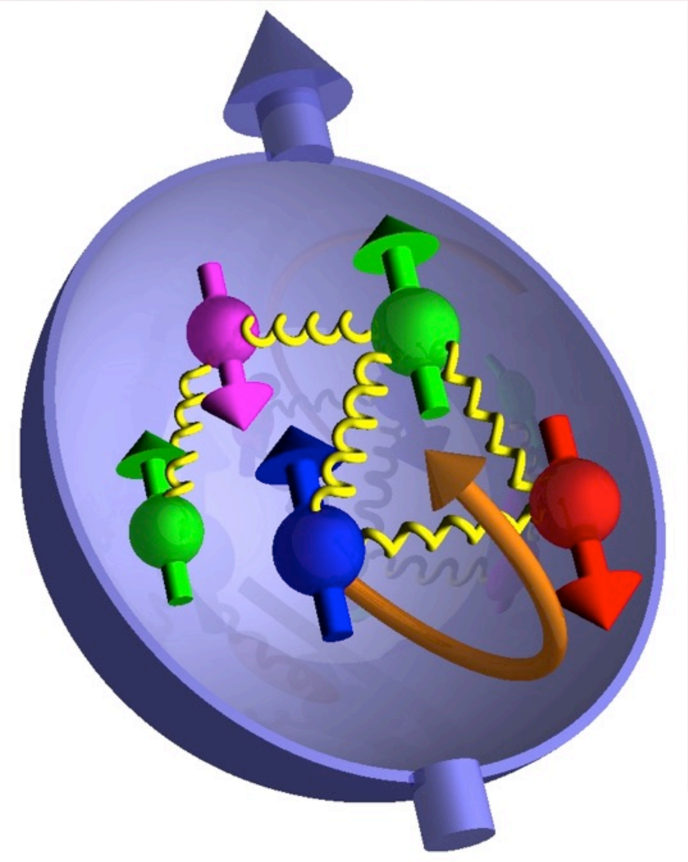
$$\sigma(gg \to H), \sqrt{(s)} = 13\text{TeV}$$

CT14	MMHT2014	NNPDF3.0
42.68 pb	42.70 pb	42.97 pb
+2.0% -2.4%	+1.3% -1.8%	+1.9% -1.9%

DIS summary



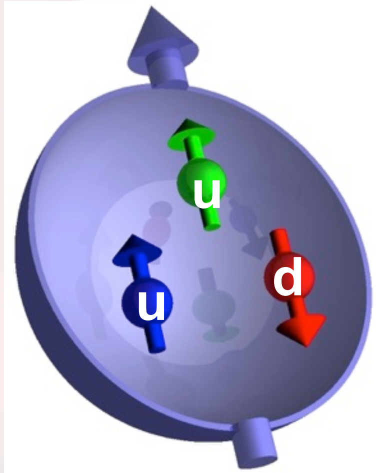
Parton distribution when nucleon is polarized?



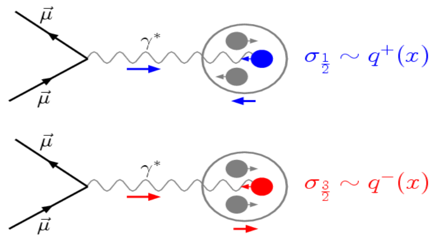
Proton Spin

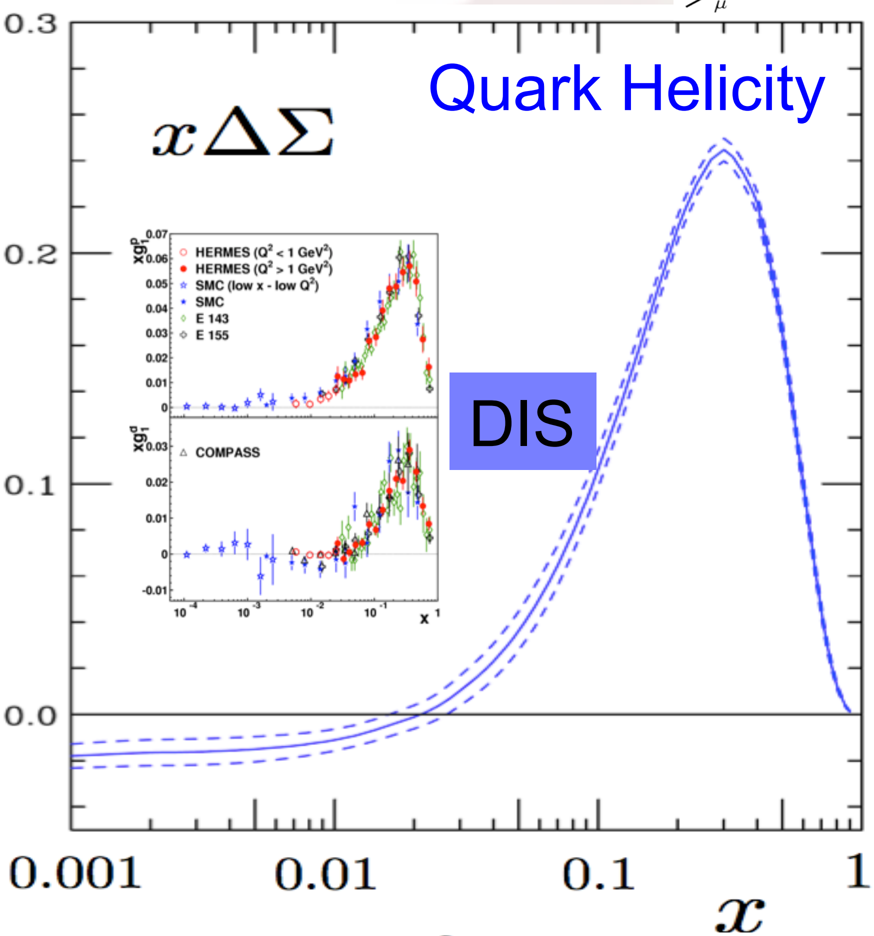
- The story of the proton spin began with the quark model in 60's
- In the simple Quark Model, the nucleon is made of three quarks (nothing else)
- Because all the quarks are in the s-orbital, its spin (½) should be carried by the three quarks
- European Muon Collaboration: 1988

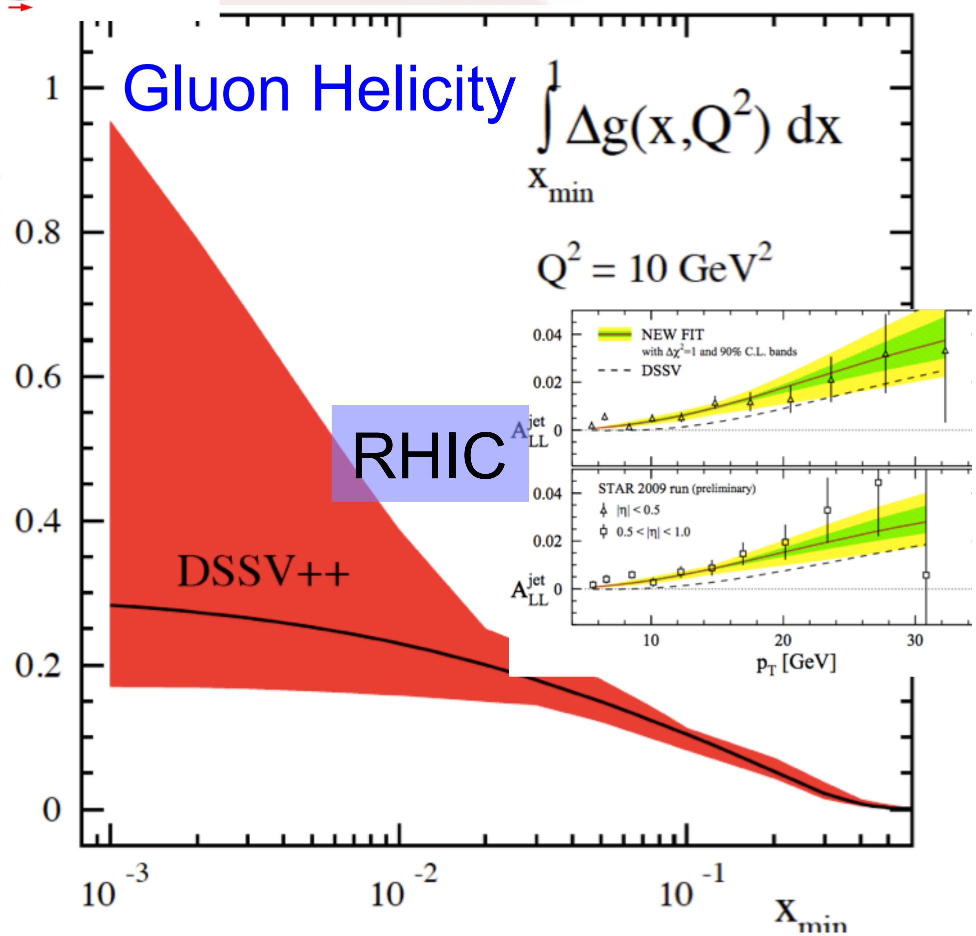
  "Spin Crisis"--- proton spin carried by quark spin is rather small



Parton distributions in a polarized nucleon







$Q^2 = 5\,\mathrm{GeV}^2$ de Florian-Sassot-Stratmann-Vogelsang, 2014

Proton spin: $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L$

emerging phenomena?

- We know fairly well how much quark helicity contributions, $\Delta\Sigma=0.3\pm0.05$
- With large errors we know gluon helicity contribution plays an important role
- No direct information on quark and gluon orbital angular momentum contributions

The orbital motion:

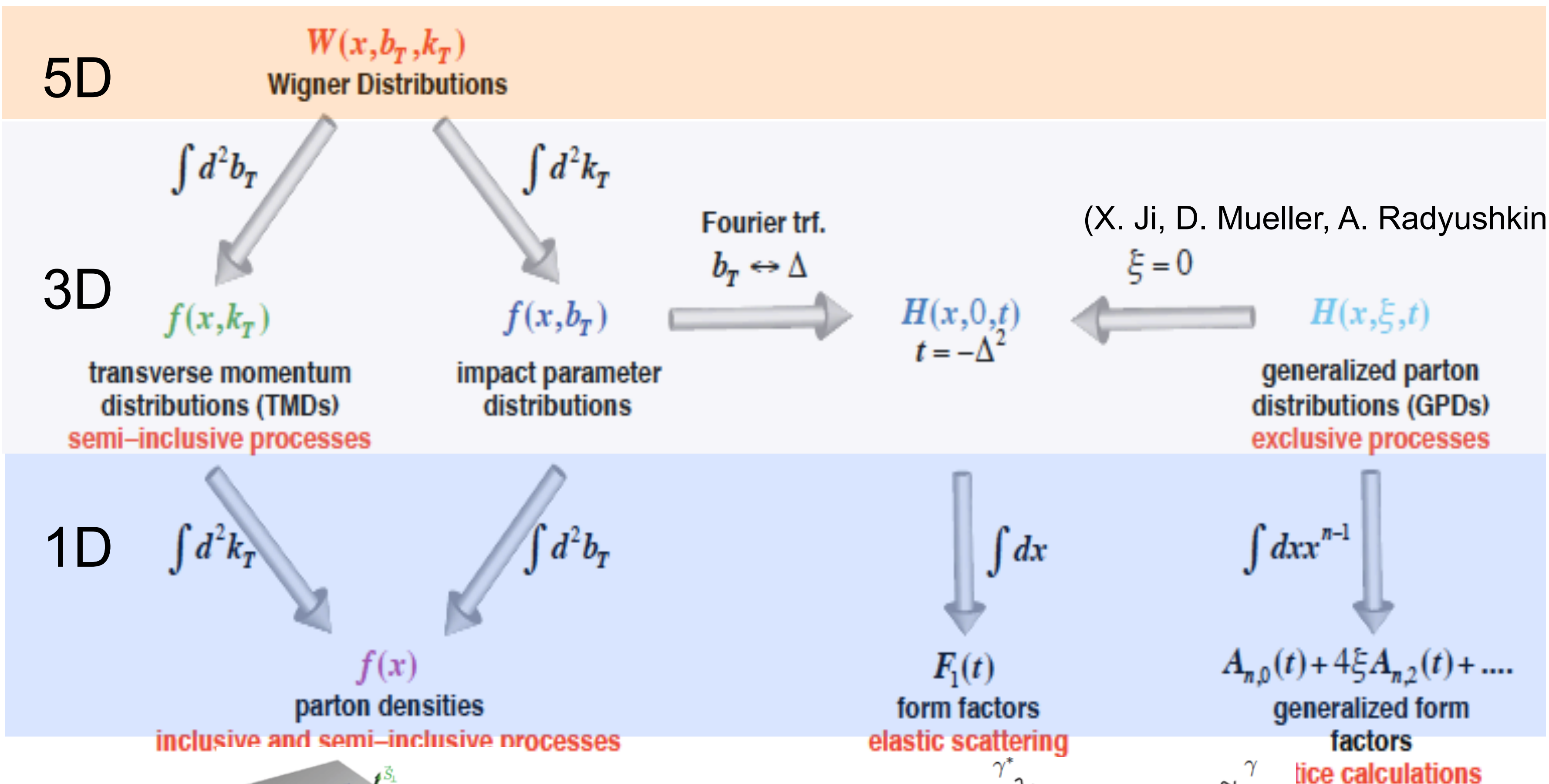
- Orbital motion of quarks and gluons must be significant inside the nucleons!
  - This is in contrast to the naive non-relativistic quark model
- Orbital motion shall generate direct orbital Angular Momentum which must contribute to the spin of the proton
- Orbital motion can also give rise to a range of interesting physical effects (Single Spin Asymmetries)

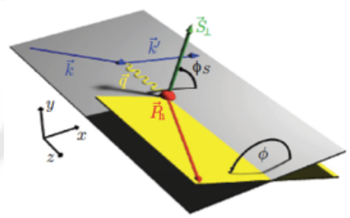
New ways to look at partons

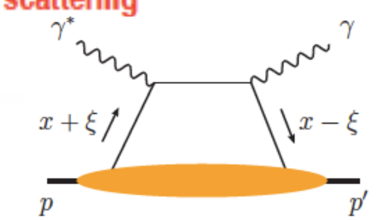
- We not only need to know that partons have long. momentum, but must have transverse degrees of freedom as well
- Partons in transverse coordinate space
  - Generalized parton distributions (GPDs)
- Partons in transverse momentum space
  - Transverse-momentum distributions (TMDs)
- Both? Wigner distributions!

Unified view of the Nucleon

- **Wigner distributions (Belitsky, Ji, Yuan)**



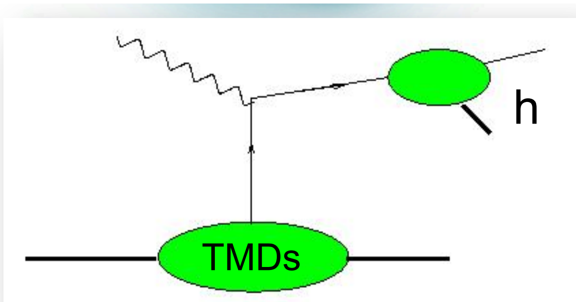


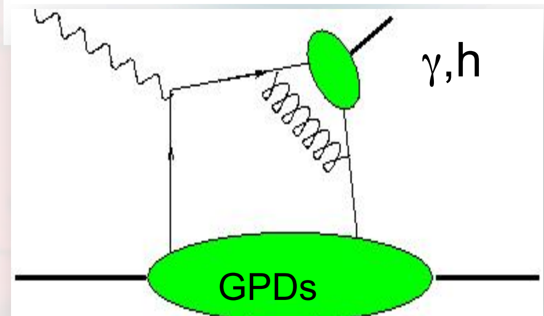


Zoo of TMDs & GPDs

	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	$h_1, h_{1T}^{\perp}$

	U	L	T
U	H		$\mathcal{E}_T$
L		$\widetilde{H}$	
T	E		$H_T, \widetilde{H}_T$



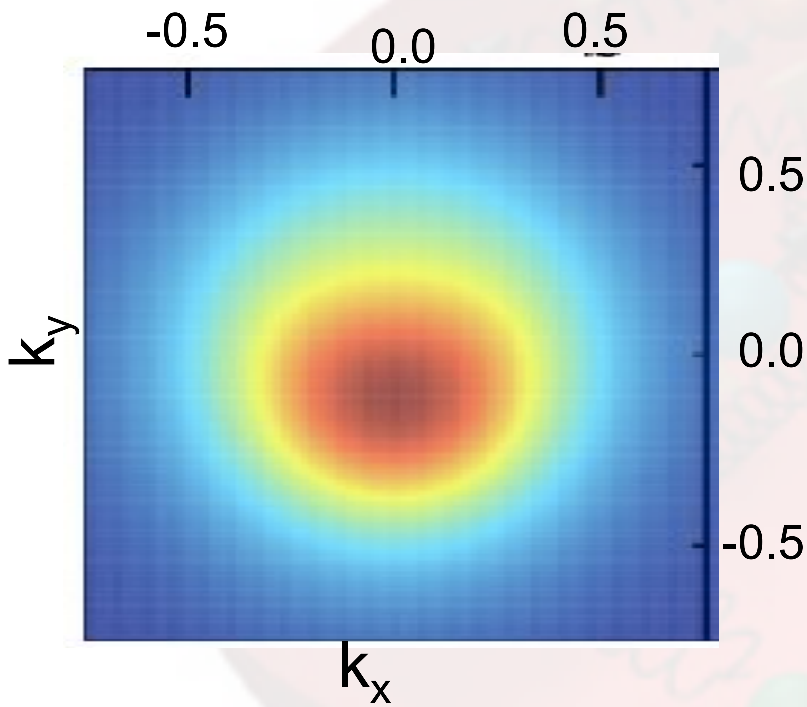


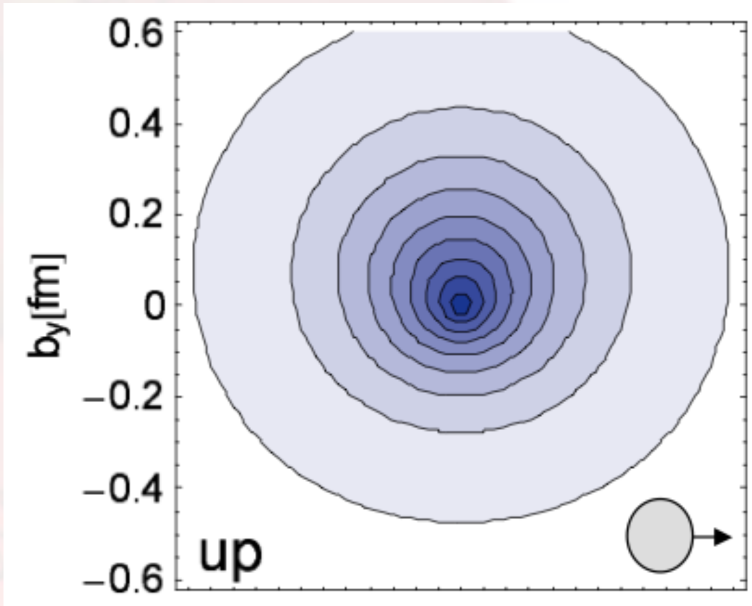
- NOT directly accessible
- Their extractions require measurements of x-sections and asymmetries in **a large kinematic domain of x_B, t, Q^2** (GPD) and **x_B, P_T, Q^2, z** (TMD)

What can we learn

- 3D Imaging of partons inside the nucleon (non-trivial correlations)
  - Try to answer more detailed questions as Rutherford was doing 100 years ago
- QCD dynamics involved in these processes
  - Transverse momentum distributions: universality, factorization, evolutions,…
  - Small-x: BFKL vs Sudakov?

Deformation when nucleon is transversely polarized





Quark Sivers function fit to the SIDIS Data, Anselmino, et al. 2009

Lattice Calculation of the transvese density Of Up quark, QCDSF/UKQCD Coll., 2006

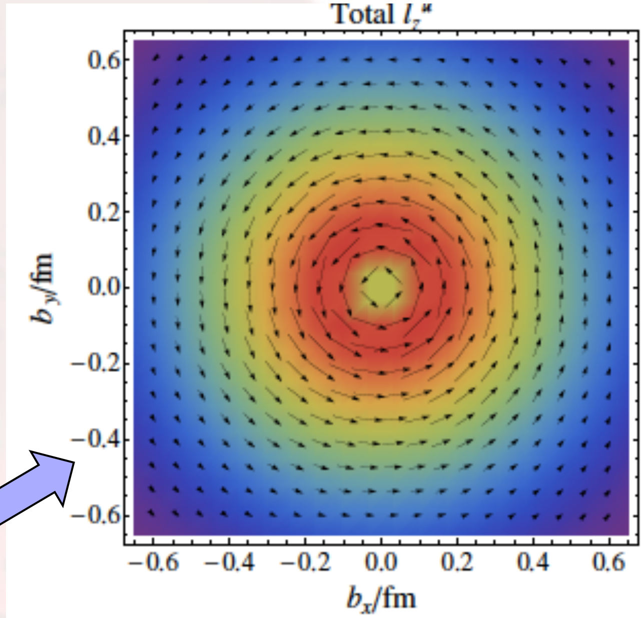
Parton's orbital motion through the Wigner Distributions

Phase space distribution:

Projection onto p (x) to get the momentum (probability) density

Quark orbital angular momentum

$$L(x) = \int (\vec{b}_\perp \times \vec{k}_\perp) W(x, \vec{b}_\perp, \vec{k}_\perp) d^2\vec{b}_\perp d^2\vec{k}_\perp$$



Well defined in QCD:

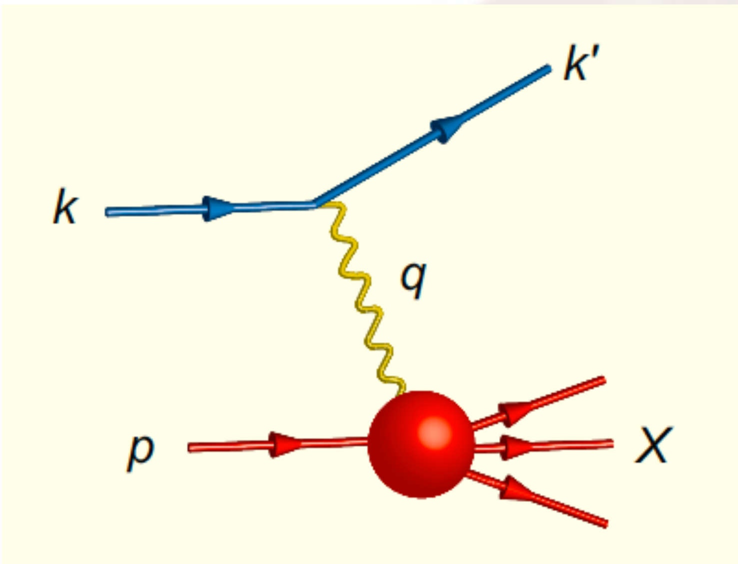
Ji, Xiong, Yuan, PRL, 2012; PRD, 2013

Lorce, Pasquini, Xiong, Yuan, PRD, 2012

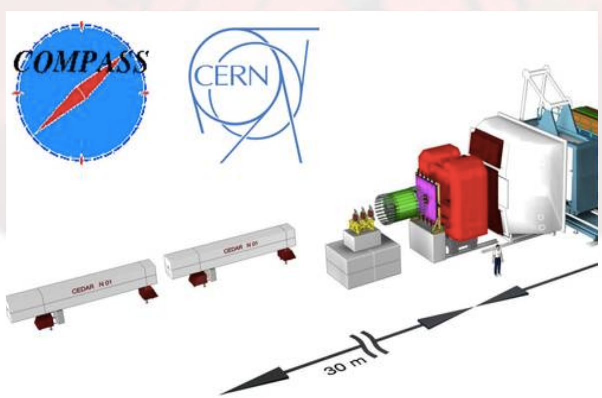
Lorce-Pasquini 2011

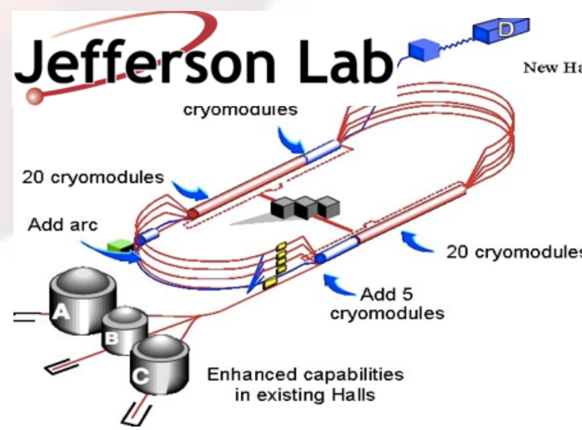
Hatta 2011

Where can we study: Deep Inelastic Scattering



- Inclusive DIS
  - Parton distributions
- Semi-inclusive DIS, measure additional hadron in final state
  - Kt-dependence
- Exclusive Processes, measure recoiled nucleon
  - Nucleon tomography





What we have learned

- Unpolarized transverse momentum (coordinate space) distributions from, mainly, DIS, Drell-Yan, W/Z boson productions, (HERA exp.)
- Indications of polarized quark distributions from low energy DIS experiments (HERMES, COMPASS, JLab)

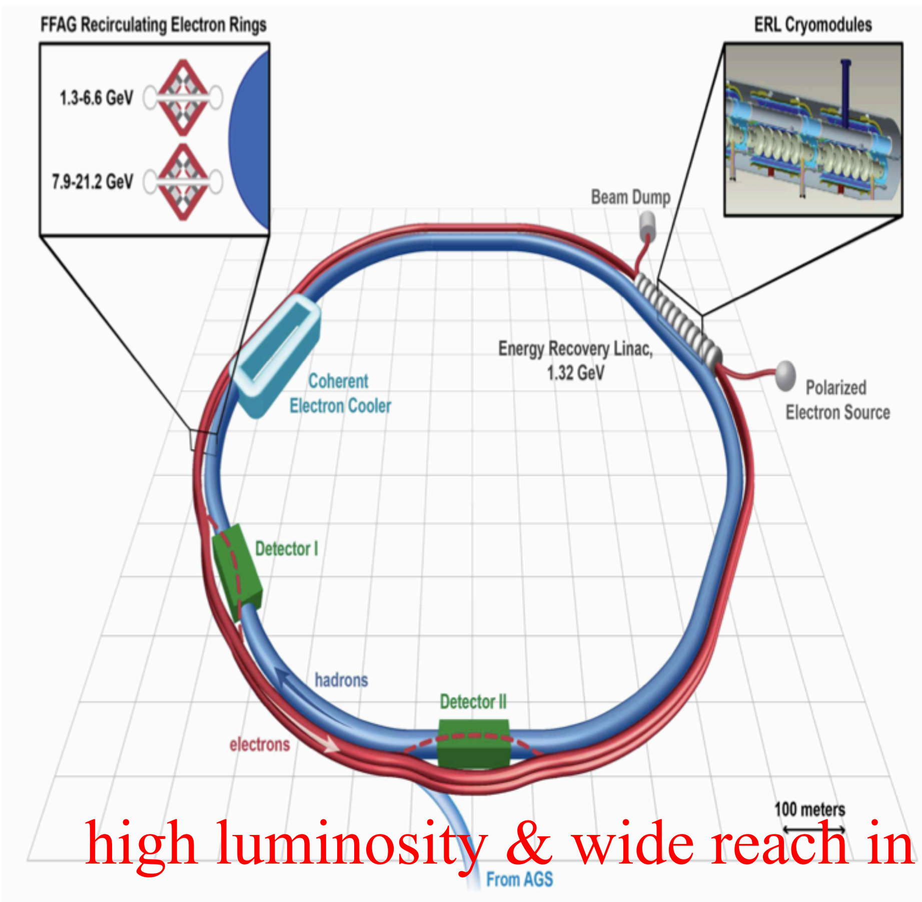
What we are missing

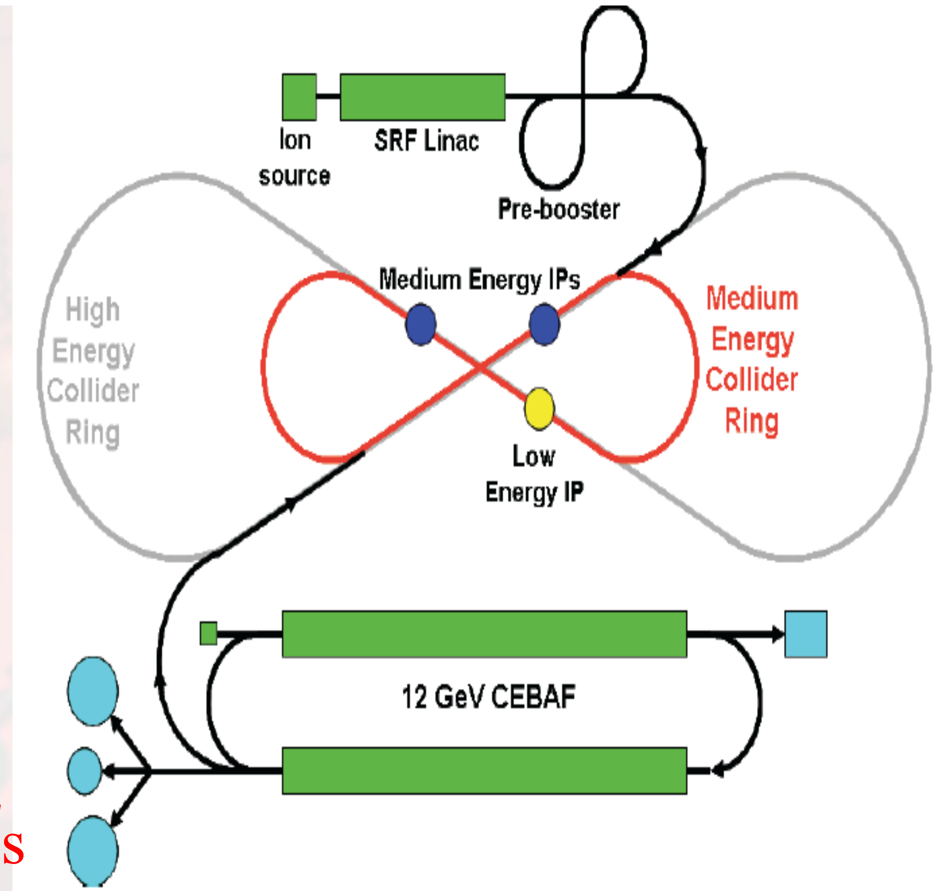
- Precise, detailed, mapping of polarized quark/gluon distribution
  - Universality/evolution more evident
- Spin correlation in momentum and coordinate space/tomography
  - Crucial for orbital motion
- Small-x: links to other hot fields (Color-Glass-Condensate)

Perspectives

- HERA (ep collider) is limited by the statistics, and is not polarized
- Existing fixed target experiments are limited by statistics and kinematics
- JLab 12 will provide un-precedent data with high luminosity
- Ultimate machine will be the Electron-Ion-Collider (EIC): kinematic coverage with high luminosity

We need a new machine: EIC Proposals in US





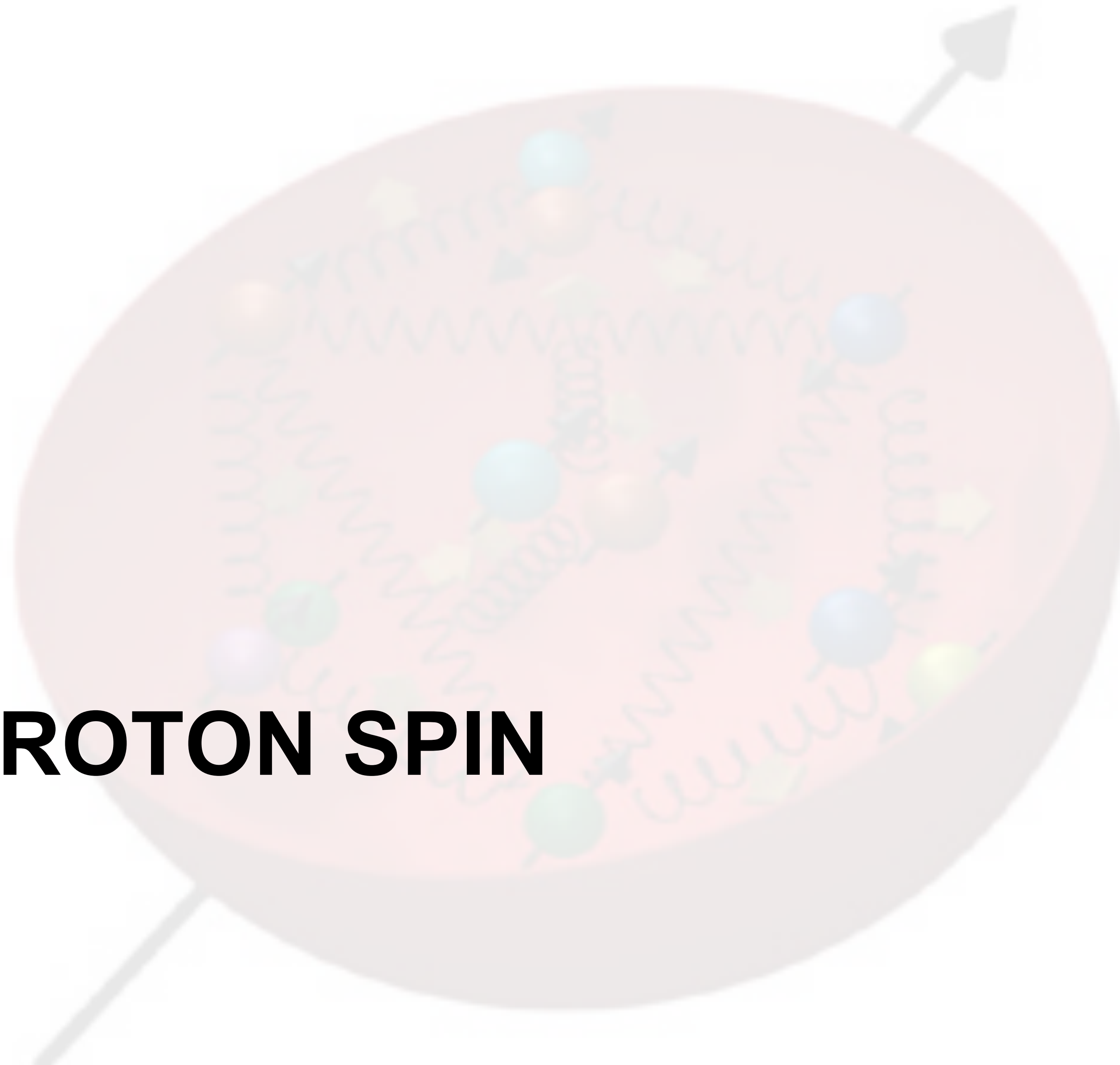
high luminosity & wide reach in √s

polarized lepton & hadron beams

nuclear beams

arXiv: 1108.1713, arXiv: 1212.1701

PROTON SPIN



Proton Spin

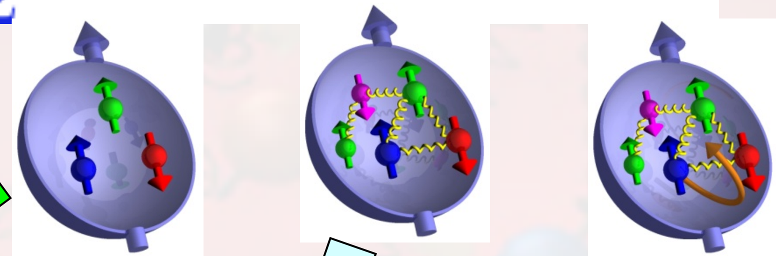
- Emerging property of the fundamental building block of the universe
  - Spin sum rule in parton model and QCD
  - Exp. vs Lattice
- Emerging phenomena
  - Parity violating, electro-weak interaction, SM
  - (naïve) time-reversal odd Single trandverse spin asymmetries
  - Under extreme conditions: small vs large x

Ultimate goal of spin physics?

- Spin sum rule

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L$$

Jaffe-Manohar, 90
Ji, 96



Quark spin,
Best known

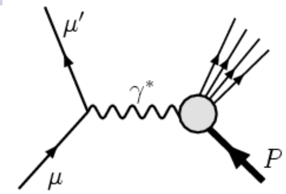
$$\frac{1}{2}\int dx \left(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right)$$

Gluon spin,
Start to know

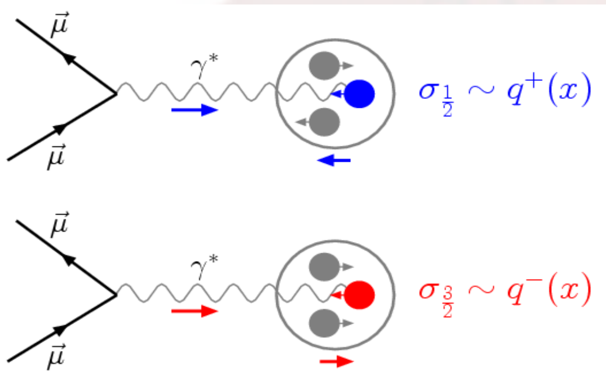
$$\Delta G = \int dx \Delta g(x)$$

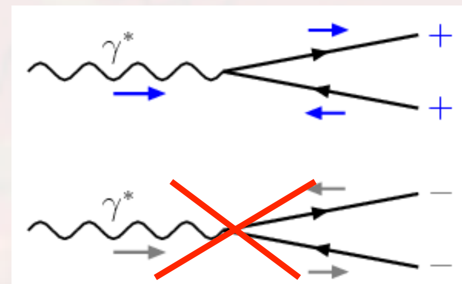
Orbital Angular Momentum of quarks and gluons,
Little known

EMC experiment at CERN



- Polarized muon + p deep inelastic scattering,



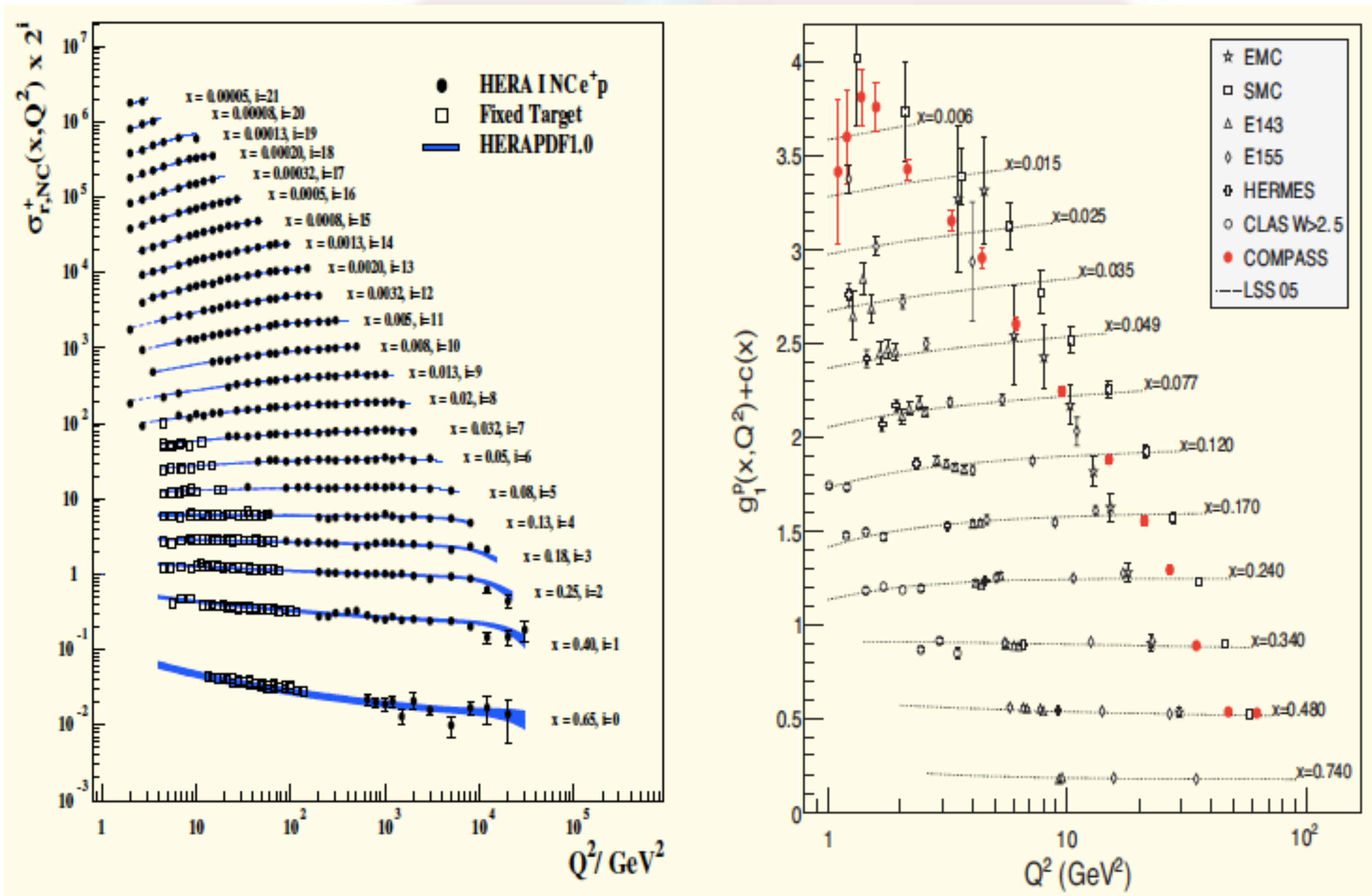


  - Virtual photon can only couple to quarks with opposite spin, because of angular momentum conservation
  - Select $q^+(x)$ or $q^-(x)$ by changing the spin direction of the nucleon or the incident lepton
  - The polarized structure function measures the quark spin density

$$g_1(x) \sim \left(\sigma_{\frac{1}{2}} - \sigma_{\frac{3}{2}}\right) \propto \sum_q e_q^2 \left(q^+(x) - q^-(x)\right)$$

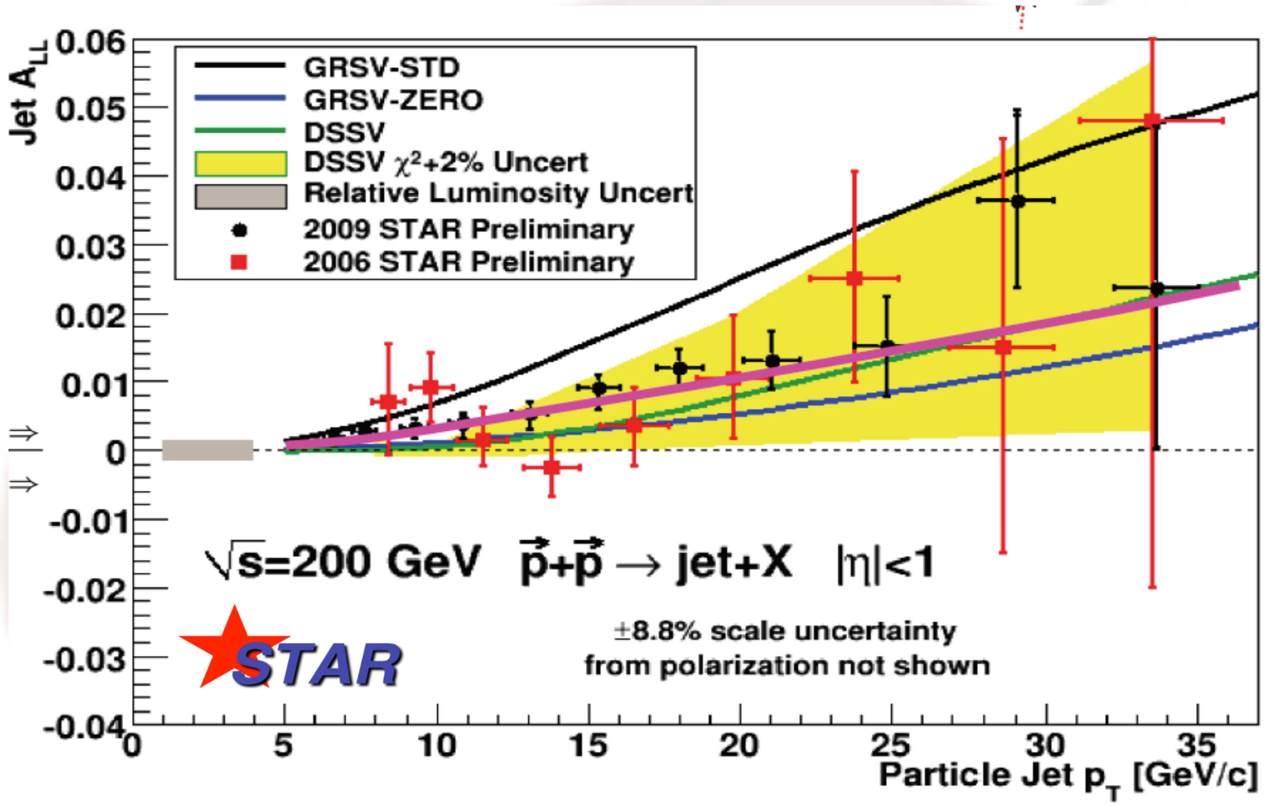
$$\Delta q(x)$$

Summary of the polarized DIS data



$$\Delta\Sigma = \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}$$

$$\approx \ 0.25$$



$$\int_{0.05}^{0.2} dx\Delta g \sim 0.1$$

How to access the OAM

- Generalized Parton Distributions
- Transverse Momentum Dependent Distributions
- Wigner Distributions

Hunting for L_q:

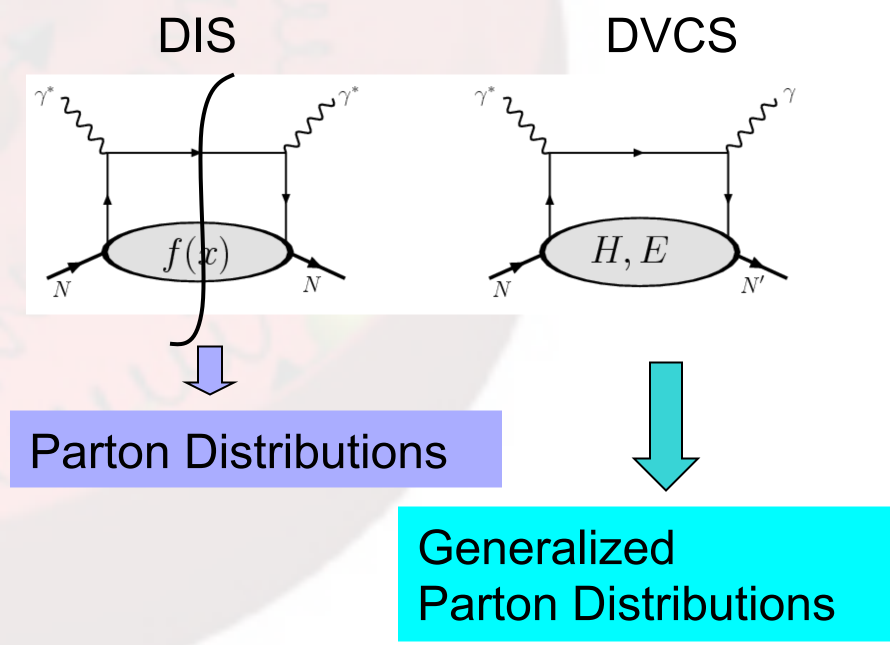
Generalised Parton Distributions (GPDs)

$$\int (H + E)\, x\, dx = J_q = 1/2\ \Delta\Sigma + L_z$$

Ji,96

$\Delta\Sigma \rightarrow$ 30%(DIS)

- A new type of parton "distributions" contains much more information
  - Can be measured in deeply virtual compton scattering and other hard exclusive processes
  - **Related to form factors and parton distributions**



Mueller et al., 94; Ji, 96; Radyushkin, 96

General Comments

- Gauge invariant
- Frame independent
- Works for L/T polarizations of nucleon
- Physical accessible

Proton spin decomposition

- Angular momentum density → spin vector

$$J^i = \frac{1}{2}\epsilon^{ijk}\int d^3x M^{0jk}$$

$$\begin{aligned} M^{\alpha\mu\nu} &= T^{\alpha\nu}x^{\mu} - T^{\alpha\mu}x^{\nu} \\ T_q^{\mu\nu} &= \frac{1}{2}\left[\bar{\psi}\gamma^{(\mu}i\overrightarrow{D^{\nu)}}\psi + \bar{\psi}\gamma^{(\mu}i\overleftarrow{D^{\nu)}}\psi\right] \\ T_g^{\mu\nu} &= \frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F_{\alpha}{}^{\nu} \end{aligned}$$

$$\begin{aligned} \vec{J}_q &= \int d^3x \left\{\psi^{\dagger}\vec{\gamma}\gamma_5\psi + \psi^{\dagger}(\vec{x}\times i\vec{D})\psi\right\} \\ \vec{J}_g &= \int d^3x \left(\vec{x}\times(\vec{E}\times\vec{B})\right) \end{aligned}$$

$$J_{q,g}(Q^2)2\vec{S} = \langle PS|\vec{J}_{q,g}(Q^2)|PS\rangle$$

Angular momentum density

$$\langle PS|\int d^4\xi M^{\mu\alpha\beta}(\xi)|PS\rangle = J\frac{2S_\rho P_\sigma}{M^2}(2\pi)^4\delta^4(0)$$
$$\left(\epsilon^{\alpha\beta\rho\sigma}P^\mu + \epsilon^{[\alpha\mu\rho\sigma}P^{\beta]} - (\text{trace})\right) + \cdots\ ,$$

- Partonic interpretation works in the infinite momentum frame (IMF)
- In this frame, the leading component is P^+, S^+
- Next-to-leading component, S^T

Leading component M^{++T}

$$\langle PS| \int d^4\xi M^{++\perp}|PS\rangle = J\left[\frac{3(P^+)^2 S^{\perp'}}{M^2}\right](2\pi)^4\delta^4(0)$$

- Because of antisymmetric of α,β. The leading term is α=+,β=T, which related to the transverse spin of the nucleon
- Transverse spin of nucleon has leading-twist interpretation in parton language
- However, individual spin is obscure

Next-to-leading: M^{+TT}

$$\langle PS|\int d^3\vec{\xi}M^{+12}|PS\rangle = J(2S^+)(2\pi)^3\delta^3(0)$$

- Because of two transverse indices, it inevitably involves twist-three operators
- However, it does lead to the individual spin contribution, e.g., from the quark
  - Jaffe-Manohar spin decomposition

Angular Momentum density (T)

- Define the momentum density

$$\rho^+(x,\xi,S^\perp) = x\int \frac{d\lambda}{4\pi} e^{i\lambda x}\langle PS^\perp|\overline{\psi}(-\frac{\lambda n}{2},\xi)\gamma^+\psi(\frac{\lambda n}{2},\xi)|PS^\perp\rangle$$

- AM depending momentum fraction x,

$$J_q(x) = \frac{M^2}{2(P^+)^2 S^{\perp\prime}(2\pi)^2\delta^2(0)}\int d^2\xi\xi^\perp\rho^+(x,\xi,S^\perp) = \frac{x}{2}(q(x)+E(x))$$

Which gives the angular momentum density for quark with longitudinal momentum x

In more detail

- Calculate $\rho^+(x,\xi,S^T)$

$$\rho^+(x,\xi,S^\perp)/P^+ = xq(x) + \frac{1}{2}x\left(q(x)+E(x)\right)\lim_{\Delta_\perp\to 0}\frac{S^{\perp\prime}}{M^2}\partial^{\perp_\xi}e^{i\xi_\perp\Delta_\perp}$$

- Integrate out ξ, second term drops out, we obtain the momentum density
- Integral with weight ξ_T', the first term drops out, → Angular Momentum density

Longitudinal (helicity)

$$J^3 = \int d^3\vec{\xi} M^{+12}(\xi)$$
$$= \int d^3\vec{\xi} \left[\overline{\psi}\gamma^+ (\frac{\Sigma^3}{2})\psi + \overline{\psi}\gamma^+ \left(\xi^1(iD^2) - \xi^2(iD^1)\right) \psi \right]$$

- Quark spin explicitly
- OAM, twist-three nature

Wigner function: Phase Space Distributions

- Define as

Wigner 1933

$$W(x,p) = \int \psi^*(x-\eta/2)\psi(x+\eta/2)e^{ip\eta}d\eta \ ,$$

  - When integrated over x (p), one gets the momentum (probability) density
  - Not positive definite in general, but is in classical limit
  - Any dynamical variable can be calculated as

$$\langle O(x,p)\rangle = \int dxdp O(x,p) W(x,p)$$

Wigner distribution for the quark

- The quark operator

Ji: PRL91,062001(2003)

$$\hat{\mathcal{W}}_\Gamma(\vec{r},k) = \int \overline{\Psi}(\vec{r}-\eta/2)\Gamma\Psi(\vec{r}+\eta/2)e^{ik\cdot\eta}d^4\eta$$

- Wigner distributions

$$\begin{aligned} W_\Gamma(\vec{r},k) &= \frac{1}{2M_N}\int\frac{d^3\vec{q}}{(2\pi)^3}\left\langle \vec{q}/2\left|\hat{\mathcal{W}}_\Gamma(\vec{r},k)\right|-\vec{q}/2\right\rangle \\ &= \frac{1}{2M_N}\int\frac{d^3\vec{q}}{(2\pi)^3}\mathrm{e}^{-i\vec{q}\cdot\vec{r}}\left\langle \vec{q}/2\left|\hat{\mathcal{W}}_\Gamma(0,k)\right|-\vec{q}/2\right\rangle \end{aligned}$$

After integrating over r, one gets TMD

After integrating over k, one gets Fourier transform of GPDs

Importance of the gauge links

- Gauge invariance
- Depends on the processes
- Comes from the QCD factorization

$$\Psi_{LC}(\xi) = P\left[\exp\left(-ig\int_0^\infty d\lambda n \cdot A(\lambda n + \xi)\right)\right]\psi(\xi)$$

- And partonic interpretation as well

Fixed point gauge link

$$\Psi_{FS}(\xi) = P\left[\exp\left(-ig\int_0^\infty d\lambda \xi \cdot A(\lambda\xi)\right)\right]\psi(\xi)$$

- Becomes unit in ξ.A=0 gauge
- Moment gives the quark OAM

$$L(x) = \int (\vec{b}_\perp \times \vec{k}_\perp) W(x, \vec{b}_\perp, \vec{k}_\perp) d^2\vec{b}_\perp d^2\vec{k}_\perp$$

- OPE

$$\int x^{n-1} L_{FP}(x) dx = \langle PS| \int d^3\vec{r} \sum_{i=0}^{n-1} \frac{1}{n} \overline{\psi}(\vec{r})(in \cdot D)^i \times (\vec{r}_\perp \times i\vec{D}_\perp)(in \cdot D)^{n-1-i} \psi(\vec{r})|PS\rangle \ . \qquad (16)$$

Quark OAM

- Any smooth gauge link results the same OAM for the partons

$$\frac{\langle PS|\int d^3\vec{r}\,\overline{\psi}(\vec{r})\gamma^+(\vec{r}_\perp\times i\vec{D}_\perp)\psi(\vec{r})|PS\rangle}{\langle PS|PS\rangle}$$

$$=\int(\vec{b}_\perp\times\vec{k}_\perp)W_{FS}(x,\vec{b}_\perp,\vec{k}_\perp)dxd^2\vec{b}_\perp d^2\vec{k}_\perp$$

Light-cone gauge link

$$\Psi_{LC}(\xi) = P\left[\exp\left(-ig\int_0^\infty d\lambda n\cdot A(\lambda n+\xi)\right)\right]\psi(\xi)$$

- it comes from the physical processes
  - DIS: future pointing
  - Drell-Yan: to -∞
- Cautious: have light-cone singularities, and need to regulate
- Moments related to twist-three PDFs, and GPDs

Light-cone decomposition

$$J^3 = \int d^3\vec{\xi}\left[\overline{\psi}\gamma^+(\vec{\xi}\times i\vec{\partial})^3\psi + \frac{1}{2}\overline{\psi}\gamma^+\Sigma^3\psi + E^i(\vec{\xi}\times\vec{\partial})^3A^i + (\vec{E}\times\vec{A})^3\right],$$

- Quark OAM only contains the partial derivative

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \tilde{L}_q + \Delta G + \tilde{L}_g .$$

$$\tilde{L}^q(x) = \int \frac{d\lambda}{2\pi} e^{ix\lambda} d^2\xi \langle PS|\overline{\psi}(-\frac{\lambda n}{2},\xi)\gamma^+ \times(\xi^1 i\partial^2 - \xi^2 i\partial^1)\psi(\frac{\lambda n}{2},\xi)|PS\rangle$$

Gauge Invariant Extension

- GIE is not unique

$$i\partial_\xi^\perp = iD_\xi^\perp + \int^{\xi^-} d\eta^- L_{[\xi^-,\eta^-]} g F^{+\perp}(\eta^-,\xi_\perp) L_{[\eta^-,\xi^-]}$$

- Canonical OAM can be calculated

$$\tilde{L}_q = \frac{\langle PS|\int d^3\vec{r}\,\overline{\psi}(\vec{r})\gamma^+(\vec{r}_\perp \times i\vec{\partial}_\perp)\psi(\vec{r})|PS\rangle}{\langle PS|PS\rangle}$$

$$= \int (\vec{b}_\perp \times \vec{k}_\perp) W_{LC}(x,\vec{b}_\perp,\vec{k}_\perp)\, dx\, d^2\vec{b}_\perp\, d^2\vec{k}_\perp \ .$$

OAM from Wigner distribution

$$\tilde{L}_q(x) = \int (\vec{b}_\perp \times \vec{k}_\perp) W_{LC}(x, \vec{b}_\perp, \vec{k}_\perp) d^2\vec{b}_\perp d^2\vec{k}_\perp$$

- Can be measured from hard processes
- Moments access to the canonical OAM
- In the end of day, depends on twist-3 GPDs
  - Might be studied in many processes

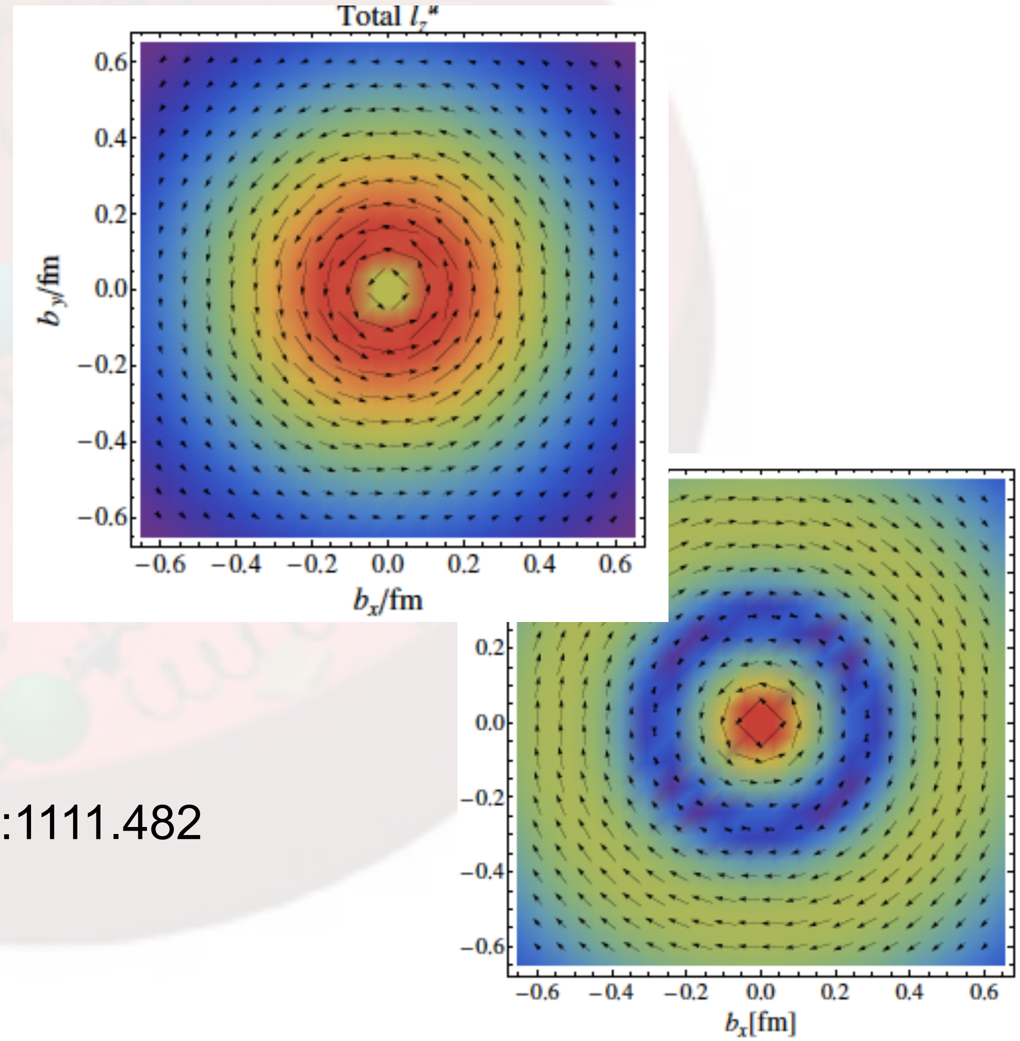
Wigner Distributions

Define the net momentum projection

$$\mathcal{K}(\vec{r}_\perp) = \int d^2k_\perp \vec{k}_\perp \mathcal{H}(\vec{r}_\perp, \vec{k}_\perp)$$

Quark oribital angular momentum

$$L_q = \int d^2r_\perp d^2k_\perp \vec{r}_\perp \times \vec{k}_\perp \mathcal{H}(\vec{r}_\perp, \vec{k}_\perp)$$



Lorce, Pasquini, arXiv:1106.0139
Lorce, Pasquini, Xiong, Yuan, arXiv:1111.482
Hatta, arXiv:1111.3547

OAMs: Light-cone Wave Functions

- They are building blocks for the hadron structure

$$|P\rangle = \sum_{n,\lambda_i} \int \Pi_i \frac{dx_i d^2 k_{\perp i}}{\sqrt{x_i} 16\pi^3} \phi_n(x_i, k_{\perp i}, \lambda_i) |n : x_i, k_{\perp i}, \lambda_i\rangle$$

- Which can be used to calculate the integrated parton distributions, GPDs, and hard exclusive scattering amplitudes, including the Compton scattering amplitudes

General Structure

- Starting from any general structure for a Fock state, $l_z+\lambda=\Lambda$, with $l_z=\sum_{i=1}^{n-1} l_{zi}$

$$\int \prod_{i=1}^{n} d[i] \quad (k_{1\perp}^{\pm})^{|l_{z1}|}(k_{2\perp}^{\pm})^{|l_{z2}|}...(k_{(n-1)\perp}^{\pm})^{|l_{z(n-1)}|}$$

$$\times \psi_n(x_i, k_{\perp i}, \lambda_i, l_{zi}) \; a_1^\dagger a_2^\dagger ... a_n^\dagger |0\rangle \; ,$$

- We will get

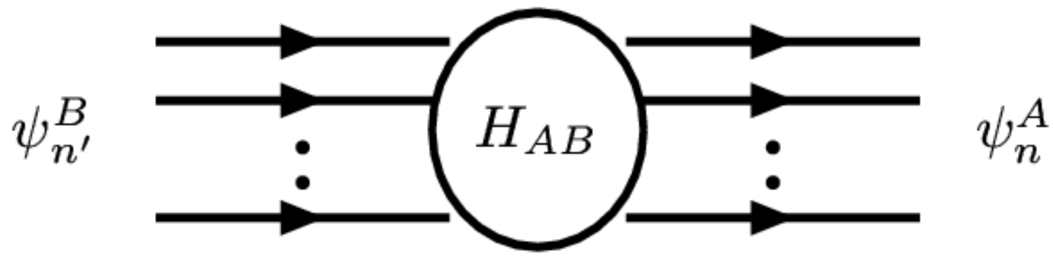
$$\int \prod_{i=1}^{n} d[i] \quad (k_{1\perp}^{+})^{l_{z1}}(k_{2\perp}^{+})^{l_{z2}}...(k_{(n-1)\perp}^{+})^{l_{z(n-1)}}$$

$$\times \left(\psi_n + \sum_{i<j=1|_{l_{zi}=l_{zj}=0}}^{n-1} i\epsilon^{\alpha\beta} k_{i\alpha} k_{j\beta} \psi_{n(ij)} \right) \; a_1^\dagger a_2^\dagger ... a_n^\dagger |0\rangle$$

Asymptotic Behavior

- The asymptotic behavior for the light-cone wave function can be studied from hard diagrams

$$\psi_n^A(x_i, k_{i\perp}, l_{zi}) = \int H_{AB} \otimes \psi_{n'}^B(y_i, k'_{i\perp}, l'_{zi}),$$



$$\psi_n^{(A)}(x_i, k_{\perp i}, l_{zi}) \sim \frac{1}{(k_\perp^2)^{[n+|l_z|+\min(n'+|l'_z|)]/2-1}} ,$$

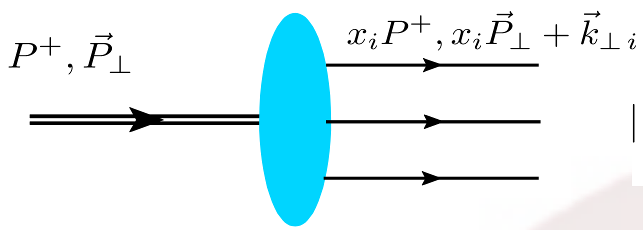
Nucleon's 3-quarks WF

- According to the general structure, there are six independent light-cone wave functions for three quarks component: L_z=0 (2), L_z=1 (3), L_z=2 (1)
- The power counting rule gives, asymptotically,

$$\psi\, |_{l_z=0} \sim 1/k_T^4$$

$$\psi\, |_{l_z=1} \sim 1/k_T^6$$

$$\psi\, |_{l_z=2} \sim 1/k_T^8$$



$$|P,\lambda\rangle = \sum_\beta \int d[1]d[2]d[3]\ \Psi^f_{\lambda,\beta}(x_i,\vec{k}_{\perp,i})\ \frac{\varepsilon^{ijk}}{\sqrt{6}}\ u^\dagger_{i\lambda_1}(1)u^\dagger_{j\lambda_2}(2)d^\dagger_{k\lambda_3}(3)|\,0\rangle$$

❖ classification of LCWFs in angular momentum components

[Ji, J.P. Ma, Yuan, 03; Burkardt, Ji, Yuan, 02]

$$|P,\uparrow\rangle = |P,\uparrow\rangle^{L_z=2}_{-\frac{3}{2}} + |P,\uparrow\rangle^{L_z=1}_{-\frac{1}{2}} + |P,\uparrow\rangle^{L_z=0}_{\frac{1}{2}} + |P,\uparrow\rangle^{L_z=-1}_{\frac{3}{2}}$$

total quark helicity J^q

$J_z = J_z^q + L_z^q$

$L_z^q = -1$	$L_z^q = 0$	$L_z^q = 1$	$L_z^q = 2$
$(\uparrow\uparrow\uparrow)_{LC}$	$(\uparrow\uparrow\downarrow)_{LC}$	$(\uparrow\downarrow\downarrow)_{LC}$	$(\downarrow\downarrow\downarrow)_{LC}$

$$\langle 0\,|\epsilon^{ijk}\,u^\dagger_{i\lambda_i}(1)\,\Gamma\,u^\dagger_{j\lambda_j}(2)d^\dagger_{k\lambda_k}(3)|\,P\rangle$$

⇓ parity, time reversal, Isospin symmetry

6 independent wave function $\psi^{(i)}\ i = 1,..,6$

$L_z^q = -1$

$$|P\uparrow\rangle^{L_z=-1}_{\frac{3}{2}} = \int d[1]d[2]d[3]\ (k_2^x - ik_2^y)\psi^{(5)}(1,2,3)^{6)}(1,2,3)^{(2)}(1,2,3)\Big)\,2,3)\Big)$$
$$\times\frac{\epsilon^{ijk}}{\sqrt{6}}u^\dagger_{i\uparrow}(1)\left(u^\dagger_{j\uparrow}(2)d^\dagger_{k\uparrow}(3) - d^\dagger_{j\uparrow}(2)u^\dagger_{k\uparrow}(3)\right)|0\rangle\Big)\,|0\rangle$$

Quark OAM (Jaffe-Manohar)

- Definition

$$\mathcal{L}_q(x) = \int \frac{d\xi^-}{2\pi} e^{ik^+\xi^-} \int d^2r \langle P|\psi(0) i \left(r^1\partial_\perp^2 - r^2\partial_\perp^2\right)\psi(\xi^-)|P\rangle$$

- Using light-cone quantization

$$\psi(\xi) = \int \frac{d^2k_\perp}{(2\pi)^3} \frac{dk^+}{2k^+} u_\lambda(k) d_\lambda(k) e^{-ik\cdot\xi}$$

$$\mathcal{L}_q(x) = \int \frac{d^2k_{1\perp} dk_1^+}{(2\pi)^3 2k_1^+} \frac{d^2k_{2\perp} dk_2^+}{(2\pi)^3 2k_2^+} \delta(k^+ - k_2^+) \int d^2r_\perp e^{-i(k_{2\perp}-k_{1\perp})\cdot r_\perp}$$
$$u_{\lambda_1}^\dagger(k_1)\left(-r^x k_{2\perp}^y + r^y k_{2\perp}^x\right) u_{\lambda_2}(k_2) \langle P|d_{\lambda_1}^\dagger(k_1) d_{\lambda_2}(k_2)|P\rangle \ .$$

Distribution in x of Orbital Angular Momentum

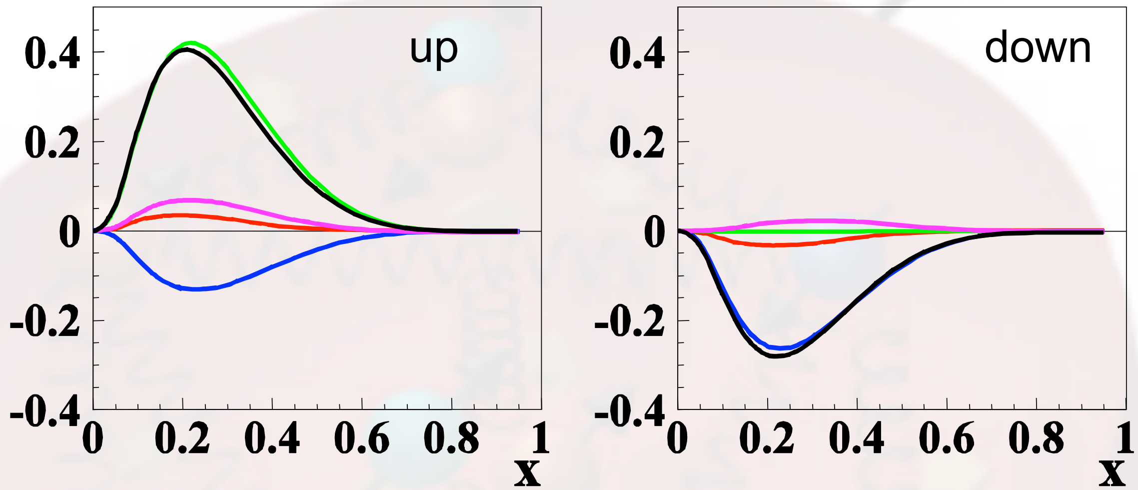
Definition of Jaffe and Manohar: contribution from different partial waves

$L_z=0$

$L_z=-1$

$L_z=+1$

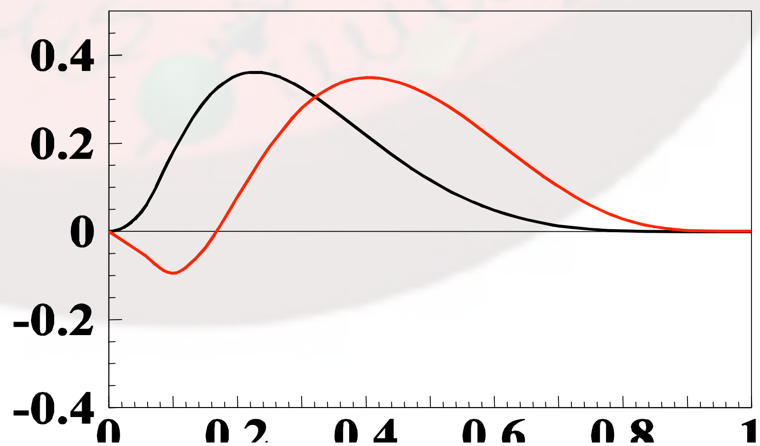
$L_z=+2$



Comparison between the results with the Jaffe-Manohar definiton and the results with the Ji definition (total results for the sum of up and down quark contribution)

Jaffe-Manohar

Ji



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Definition of Jaffe and Manohar: contribution from different par

$$\langle P\uparrow|\sum_q L^q|P\uparrow\rangle = {}^{L_z=0}\langle P,\uparrow|\sum_q L^q|P\uparrow\rangle^{L_z=0} + {}^{L_z=-1}\langle P,\uparrow|\sum_q L^q|P\uparrow\rangle^{L_z=-1}$$

$$+{}^{L_z=+1}\langle P,\uparrow|\sum_q L^q|P\uparrow\rangle^{L_z=+1} + {}^{L_z=+2}\langle P,\uparrow|\sum_q L^q|P\uparrow\rangle^{L_z=+2}$$

$$= 0\cdot +{}^{L_z=0}\langle P,\uparrow|P\uparrow\rangle^{L_z=0} + (-1)\cdot{}^{L_z=-1}\langle P,\uparrow|P\uparrow\rangle^{L_z=-1}$$

$$+(+1)\cdot{}^{L_z=+1}\langle P,\uparrow|P\uparrow\rangle^{L_z=+1} + (+2)\cdot{}^{L_z=+2}\langle P,\uparrow|P\uparrow\rangle^{L_z=+2}$$

= 0 x 0.62 + (-1) x 0.14 + (+1) x 0.23 + (+2) x 0.018 = 0.126

Definition of Ji:

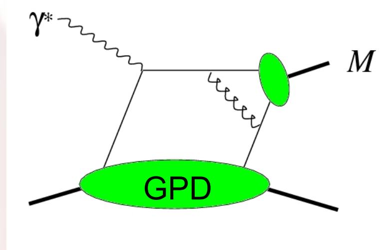
$$\sum_q L^q = \frac{1}{2}\sum_q\left[\int \mathrm{d}x\,(xH^q + xE^q) - \Sigma^q\right]$$

$$= \frac{1}{2}\left[1 + 0 - 0.74\right] = 0.126$$

Wigner Distribution
$W(x,r,k_t)$

d^3r → Transverse Momentum Dependent PDF **$f(x,k_t)$**

$d^2k_t dz$ F.T. → Generalized Parton Distr. **$H(x,\xi,t)$**



Transverse Momentum Dependent PDF **$f(x,k_t)$** — d^2k_t → PDF **$f(x)$**

Generalized Parton Distr. **$H(x,\xi,t)$** — dx → Form Factors **$F_1(Q),F_2(Q)$**

TMD Parton Distributions

- The definition contains explicitly the gauge links

Collins-Soper 1981,
Collins 2002,
Belitsky-Ji-Yuan 2002

$$f(x,k_\perp) = \frac{1}{2}\int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{-i(\xi^- k^+ - \vec{\xi}_\perp \cdot \vec{k}_\perp)} \times \langle PS|\overline{\psi}(\xi^-,\xi_\perp) L^\dagger_{\xi_\perp}(\xi^-)\gamma^+ L_0(0)\psi(0)|PS\rangle$$

- The polarization and kt dependence provide rich structure in the quark and gluon distributions
  - Mulders-Tangerman 95, Boer-Mulders 98

Generalized Parton Distributions

Mueller, et al. 1994; Ji, 1996, Radyushkin 1996

- Off-diagonal matrix elements of the quark operator (along light-cone)

$$F_q(x,\xi,t)=\frac{1}{2}\int\frac{d\lambda}{2\pi}e^{i\lambda x}\left\langle P'\left|\overline{\psi}_q\left(-\frac{\lambda}{2}n\right)\not{n}\mathcal{P}e^{-ig\int_{\lambda/2}^{-\lambda/2}d\alpha\, n\cdot A(\alpha n)}\psi_q\left(\frac{\lambda}{2}n\right)\right|P\right\rangle$$
$$=H_q(x,\xi,t)\,\frac{1}{2}\overline{U}(P')\not{n}U(P)+E_q(x,\xi,t)\,\frac{1}{2}\overline{U}(P')\frac{i\sigma^{\mu\nu}n_\mu\Delta_\nu}{2M}U(P)\,.$$

- It depends on quark momentum fraction x and skewness ξ, and nucleon momentum transfer t

$$\xi=-n\cdot(P'-P)/2$$
$$t=\Delta^2=(P-P')^2$$