1) Recent development of QCD applications in HEP.

a) Non-global log resummation. (BMS equation at large Nc)

Color density matrix (Caron-Huot '15).

Dressed gluon expansion (Larkoski, Moult, Neill, '15, '16)

Multi-Wilson-line structure in SCET. (Becher, Neubert, Rothen, DYS, '15, '16)

Collinear logs improved BMS eq. (Mueller. et. al, '17).

Soft (Glauber) gluon evolution at amplitude level. (Plätzer, Seymour, '18).

Reduced density matrix (Neill & Vaidya, '18).

b). QCD factorization including Glauber gluons.

#. EFT for forward scattering & factorization violation (Rothstein & Stewart, '16).

Cancellation of glauber gluon exchange in double Drell-Yan process (M. Diehl, '16)

Glauber operators & quark Reggeization (Moult, Stewart, et.al, '17)

Factorization violation & scale invariance. (Schwartz. K. Yan, H.X. Zhu, '17, '18).

- when initial and final-state particles are not collinear, then the Glauber gluon contribution is entirly contained in the soft function.

- The contribution from Glauber gluons are necessarily non-analytic functions of external momentum, with non-analyticity arising from the rapidity regulator.

- Factorization violating effects in hadron scattering are mainly due to the spectator-spectator interactions.

- For pure Glauber ladder graphs, all amplitude-level factorization violating effects cancel completely for any single-scale observable, due to scale invariance of two-to-two scattering amplitudes.

  (hadronic transverse energy, beam thrust).

  (arxiv: 1801.01138).

#. super-leading logarithms. (enhanced NGLs due to glauber effects)

(Finite N_c) + Glauber + non-global = super-leading log

(Forshaw, Kyrieleis, Seymour, arxiv: 0808.1269).

C) Subleading power corrections to QCD processes

\# Subleading power operator basis. (Arjol, Stewart, '03, Beneke, Manel, et.al, '04)

(also. I. Stewart. et. al. '17).

\# next-to-eikonal corrections to threshold resummation for the Drell-Yan & DIS cross sections

(E. Laenen. Magnea, Stavenga, '08, '09, '10, '16).

\# Subleading power corrections to sub-leading power N-jet processes.

(Beneke. et. al, '17, I. Moult, ..., H. X. Zhu, '16, Boughezal, X. H. Liu, ..., '16).

\# Subleading power corrections to B-meson decays.

- inclusive $B \to X_u \ell \nu$, $B \to X_s \ell \ell$ decays (Neubert. et. al, '04,

Stewart. et. al, '04, Beneke, et. al, '04)

- Subleading power corrections to exclusive B-meson decays

① $B \to \gamma \ell \nu$. (dispersion analysis, QCDSR .....).

② $B \to \pi \ell \nu$. (3-particle DAs' effects @ LO)

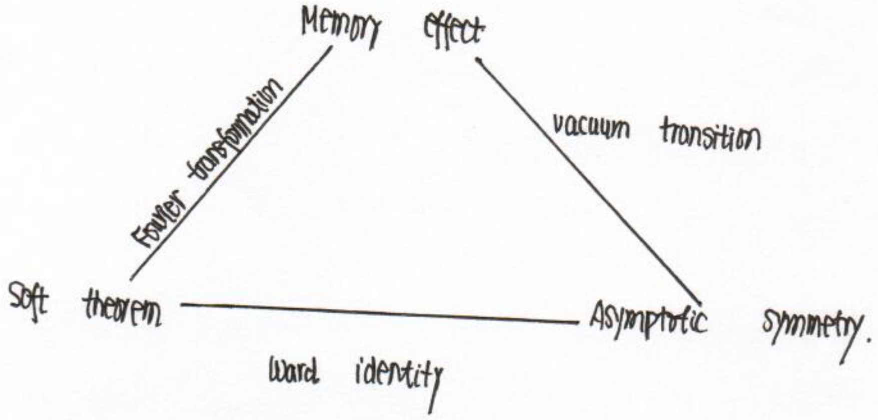
③ $B \to K^* \ell \ell$. (charm-loop soft corrections, et. al.).

d) NLP soft theorem (extending Low - Burnett - Kroll theorem).

\# Soft theorem beyond the tree level. (Larkoski, Neill, Stewart, '15)

\# Soft theorem for the graviton. (Di Vecchia, Marotta, Mojaza, '15, '16).

\# Infrared structure of gravity & gauge theory. (connection to soft theorem).



(Andrew Strominger, arxiv: 1703. 05448)

\# New symmetries of QED (Kapec, Pate, Strominger, arxiv: 1506. 02906)

Soft photon theorem in U(1) gauge theories with only massless charged particles is due to the ward identity of an infinite-dimensional asymptotic symmetry group.

e) Exotic topics:

SCET without modes, (M. Luke '18), ... SUSY SCET, ----, Factorization, Renormalization, Effective Field Theory. April 22, 2018

1) General background: (Notes P_7)

EFTs in heavy quark physics. NP → EW SM → weak eff. lagran. → QCD scale.

2). Why we need QCDF? (or why HQET is not sufficient for heavy quark decays?)
(Notes P_8).

3) Different complexity for different processes? (Notes P_9).

4) Modern understanding of QFT & Technical issues. (Notes P_9)

5) QCD factorization for pion-photon form factor.

- History: # related to $\pi^0 \to 2\gamma$, # Braaten's NLO calculation.
  # Hard-collinear factorization (similar to DIS).
  # relation between ERBL & DGLAP.
- Detailed calculation: Note P_{11}

Lecture I: General aspects of EFTs

Aug. 09. 2017

1) What are EFTs ?

- Modern viewpoint: Most theories are probably effective theories, and non-renormalizable
- With EFTs, we seperate a set of phenomena from all the rest, so that we can describe it withouting understanding everything.
- It provides an "appropriate" description of the "important" physics.

2) Different types of EFTs:

Unknown fundamental theory

Known fundemental theory

"bottom-up"

"top down"

Effective Theory

"bottom-up":

- The SM.
- Einstein gravity
- Higher-dimensional gauge theories

⋮

"top down":

- Fermi theory.
- Chiral Lagrangian

⋮

3) Why EFTs are useful?

- fundamental theory too difficult. (QCD, ...).
- emergent symmetries at $E \ll M$ (heavy quark symmetry, large recoil symmetry, ...)
- summation of $(\lambda \ln \frac{E}{M})^n$, even for $\lambda \ll 1$.

4). An example: Integrating out top-quarks in QCD

$$\mathcal{L} = -\frac{1}{4} G^A_{\mu\nu} G^{A,\mu\nu} + \sum_{f=1}^{5} \bar{\psi}_f (i\not{D} - m_f) \psi_f + \bar{Q} (i\not{D} - m_Q) Q \qquad (1)$$

top field.

- Assumption: $p_i \cdot p_j \ll m_t^2$, for all external momenta

$\Rightarrow$ No external Q lines.

- Field decomposition:

$$A_\mu = A_\mu^{(L)} + A_\mu^{(H)},$$

(low and high frequencies).

$$\psi_f = \psi_f^{(L)} + \psi_f^{(H)}, \qquad (2)$$

• Everything about the theory can be derived from vacuum correlation functions, which can be further computed from the corresponding generating function.

$$\langle 0 | T\{ \phi_L(x_1), \cdots \phi_L(x_n) \} | 0 \rangle = \frac{1}{Z[0]} \left(-i \frac{\delta}{\delta J_L(x_1)}\right) \cdots \left(-i \frac{\delta}{\delta J_L(x_n)}\right) Z[J_L] \Big|_{J_L = 0} \qquad (3)$$

with

$= \int d^D x\, \mathcal{L}(x)$ is the action.

$$Z[J_L] = \int D\phi^{(L)}\, D\phi^{(H)}\, \exp\left[i S(\phi^{(L)}, \phi^{(H)}) + i \int d^D x\, J_L(x)\, \phi^{(L)}(x) \right]. \qquad (4)$$

For QCD, we have

$$Z[J, \eta, \bar{\eta}] = N \int D[A, \psi_f, \bar{\psi}_f, Q, \bar{Q}]\, \exp\left[i \int d^D x \left(\underline{\mathcal{L}} + \boxed{J^\mu A_\mu^{(L)} + \bar{\eta}\, \psi_f^{(L)} + \bar{\psi}_f^{(L)} \eta} \right) \right]$$

$$= N' \int D[A^{(L)}, \psi_f^{(L)}, \bar{\psi}_f^{(L)}]\, \exp\left[i \int d^D x \left(\underline{\mathcal{L}_{eff}} + \boxed{J^\mu A_\mu^{(L)} + \bar{\eta}\, \psi_f^{(L)} + \bar{\psi}_f^{(L)} \eta} \right) \right] \qquad (5)$$

with

integrating out the heavy fields & high energy modes.

$$\exp\left[i S_{eff}(A^{(L)}, \psi_f^{(L)}, \bar{\psi}_f^{(L)}) \right] = \frac{N}{N'} \int D[A^{(H)}, \psi_f^{(H)}, \bar{\psi}_f^{(H)}, Q, \bar{Q}] \cdot \exp\left[i S(A, \psi_f, \bar{\psi}_f, Q, \bar{Q}) \right]. \qquad (6)$$

can be expanded in local operators.

In most cases, S_{eff} can be constructed only perturbatively.

• Matching condition:

$A^{(4)}$ $A^{(4)}$

$\overset{!}{=}$

Choose the

short-distance functions

λ_i so that this is true! ——— (7)

• Consider the gluon 2-point function

A) <u>1st type diagrams.</u>

Such contributions can be exactly reproduced for $\mathcal{L}_{eff} = -\frac{1}{4} G^2 + \sum_f \bar{q}_f (i\not{D} - m_f) q_f$.

<u>Comments:</u> ① Renormalize $\mathcal{L}$ and $\mathcal{L}_{eff}$ in $\overline{MS}$ after using dim. reg., hence no explicit high-frequency cut-off.

② High frequency modes of A, q_f appear only in diagrams with Q-lines that contain the scale m_t.

B) 2nd type diagrams

+ counter term.

$$= i\,(q^2 g_{\mu\nu} - q_\mu q_\nu)\ \delta^{AB}\ \Pi(q^2) \qquad \text{——— (7)}$$

The explicit expression of $\Pi(q^2)$ is given by

$$\Pi(q^2) = \frac{2T_f \alpha_s}{\pi} \cdot \int_0^1 dx\ x(1-x)\ \ln \frac{m_f^2 - x(1-x)q^2}{\mu^2} \qquad (\text{with } T_f = 1/2).$$

$$= \frac{\alpha_s T_f}{3\pi} \ln \frac{m_f^2}{\mu^2} \quad - \quad \underline{\frac{\alpha_s T_f}{15\pi}\ \frac{q^2}{m_f^2}} \quad + O(q^4/m_f^4). \qquad \text{——— (8)}$$

$O_1 = -\frac{1}{4} G^2$

$O_2 = G\, D_\mu D^\mu\, G$

$$\mathcal{L}_{eff} = \underline{-\frac{1}{4}\left(1 - \frac{\alpha_s T_f}{3\pi} \cdot \ln \frac{m_f^2}{\mu^2}\right) G^A_{\mu\nu}\, G^{A\mu\nu}} \quad + \sum_f \bar{q}_f (i\not{D} - m_f)\, q_f$$

D=4 terms modified

$$+ \underline{\frac{\alpha_s T_f}{60\pi m_f^2} \cdot G^A_{\mu\nu}\, D^2 \cdot G^{A\mu\nu}} \quad + \cdots \qquad \text{——— (9)}$$

D=6 non-renormalized interaction generated !

To recover the kinetic term, we redefine the gluon field:

$$\hat{A} = \left(1 - \frac{\alpha_s T_f}{6\pi} \ln\frac{m_t^2}{\mu^2}\right) A, \qquad (10)$$

However,

$$g_s \bar{q}_f A q_f = \underbrace{\frac{g_s}{1 - \frac{\alpha_s T_f}{6\pi} \ln\frac{m_t^2}{\mu^2}}}_{\equiv \hat{g}_s,} \bar{q}_f \hat{A} q_f \qquad (11)$$

$\equiv \hat{g}_s,$ (strong coupling in the effective theory)

scale

- use $\mathcal{L}$, $\boxed{\frac{d\alpha_s}{d\ln\mu} = -2\beta_0 \frac{\alpha_s^2}{4\pi},}$ with $\underline{\underline{\beta_0 = 11 - \frac{4}{3} \cdot \overset{\overset{\text{\# of flavours}}{\downarrow}}{6}}}$.

m_t

- near m_t, relate $\hat{\alpha}_s = \dfrac{\alpha_s}{1 - \frac{\alpha_s T_f}{6\pi} \ln\frac{m_t^2}{\mu^2}}$ (*) $\longrightarrow$ matching condition

$\Rightarrow$ $\boxed{\frac{d\hat{\alpha}_s}{d\ln\mu} = -2\beta_0^{(5)} \frac{\hat{\alpha}_s^2}{4\pi},}$ $\underline{\underline{\beta_0^{(5)} = 11 - \frac{4}{3} \cdot 5}}$.

- far below m_t, MUST use $\mathcal{L}_{eff}$, otherwise we get $\alpha_s \ln\frac{m_t^2}{P^2}$; for $\mu \sim P \ll m_t$.

  and perturbation theory breaks down

- In $\mathcal{L}_{eff}$ these high energy logs are absorbed in $\hat{\alpha}_s(P)$, with

P

  the RGE $\frac{d\hat{\alpha}}{d\ln\mu} = -2\beta(\hat{\alpha})$, and the initial condition (*)

- Also for 3-gluon vertex:

$$\Rightarrow \quad \frac{C}{m_t^2} f_{ABC}\, G^A_{\mu\nu}\, G^B_{\mu\alpha}\, G^C_{\nu\alpha} \qquad (12)$$

- Below m_b, we have two situations.

A) no external bottom lines $\Rightarrow$ repeat the above procedure.

B) external bottom lines must be conserved below m_b

$\Rightarrow$ NRQCD, HQET, SCET, ...

5) EFTs in the heavy quark physics

New physics (?) $\qquad \mathcal{L}_{\text{unknowns}}$ (SMEFT, ...).

$\downarrow$

Electroweak scale m_W $\qquad \mathcal{L}_{SM}$ + Higher-dim. operators.

$\downarrow \qquad\qquad \downarrow$

$D=4$, Flavor change only at EW scale

Heavy-quark scale m_b

$$\mathcal{L}_{eff} = -\frac{G_F}{\sqrt{2}} \sum_i C_i \cdot Q_i + \text{higher. dim. operators} + \mathcal{L}_{QCD+QED}$$

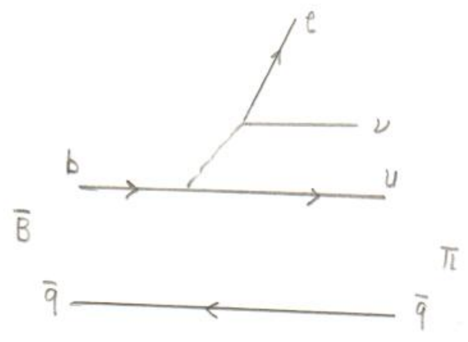
$\downarrow$

key task: $\langle f \mid Q_i \mid \bar{B} \rangle = ?$ $\qquad \boxed{\text{QCDF + EFTs}}$

multi-scale problems, m_b, $(m_b \Lambda)^{1/2}$, Λ.

QCD scale Λ_{QCD} $\qquad \mathcal{L}_{eff}$ depends on processes. (heavy quarks as external lines)

6) Why do we need QCDF ?



$P_\pi = (P_0, \vec{P})$,

- For $P_\pi \sim O(\Lambda)$, $\rightarrow$ soft pion.

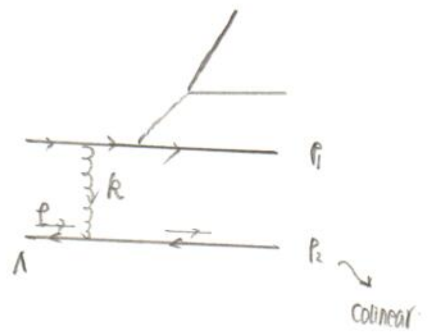
  heavy-to-light transitions in HQET.

- For $P_0 \sim m_b$, but $P_\pi^2 = m_\pi^2 \ll m_b^2$,

  $\Rightarrow$ energetic pion, no such field in HQET.

$$\mathcal{L}_{HQET} = \sum_{Q=b,c} \bar{h}_Q \, i v_Q \cdot D_s \, h_Q + \mathcal{L}_{gluon} + \sum_{q\ light} \bar{q} \, i \not{D}_s \, q. \qquad (13)$$

$P_\pi \sim (m_b, \Lambda, \Lambda, m_b)$ $\Rightarrow$ "collinear" mode. (almost light-like)



$$\begin{cases} k^2 = (P_2 - l)^2 \sim m_b \cdot \Lambda \\ k^0 \sim m_b \end{cases}$$

$\Rightarrow$ "hard-collinear" mode.

7) • QCDF for B decays treats processes involving energetic light Particles (hadrons).

$B \to \gamma \ell \nu$

π, γ energetic

$B \to \pi \ell \nu$.

Do not involve four-quark operators

$\Rightarrow$ QCD form factors.

$B \to D \pi$,

$B \to \pi\pi, \cdots$ (charmless).

$B \to K^* \gamma^{(*)}$
$\hookrightarrow e^+e^-$

involve four-quark operators.

(strong final state interaction)

• Modern language:

QCDF $\hat{=}$ soft-collinear effective theory for heavy quark physics.

[extends HQET by including (hard)-collinear modes]

• Techniques / Difficulties.

① Renormalization-scheme dependence.
- Projection scheme,
- γ_5 scheme
- Subtraction scheme. (MS, $\overline{MS}$, ----).

② Evanescent operators in dim. reg..

[no Fierz transformation in D-dimensions].

- Long history of $\gamma^* \gamma \to \pi$ (before QCD).

- Braaten's QCD calculation (γ_5). [standard method of physics]

- Hard - collinear faction; similar to DIS.

  but only non-forward kinematics.

- also π DA evolution related to DGLAP eq.

Lecture II: QCDF for $\gamma^*\gamma \to \pi$ at $O(\alpha_s)$ Aug. 14. 2017.

1) Definition.

$$\langle \pi(p) | j_\mu^{em} | \gamma(p') \rangle = g_{em}^2 \, \epsilon_{\mu\nu\alpha\beta} \, q^\alpha \, p^\beta \, \epsilon^{\nu}(p') \, F_{\gamma^*\gamma\to\pi}(Q^2)$$

$$\int d^4x \, e^{-iq\cdot x} \langle \pi(p) | T\{ j_\mu^{em}(x), j_\nu^{em}(0) \} | 0 \rangle$$

$$= g_{em}^2 \cdot \epsilon_{\mu\nu\alpha\beta} \, q^\alpha \, p^\beta \, \epsilon^{\nu}(p') \, F_{\gamma^*\gamma\to\pi}(Q^2), \qquad (14)$$

photon momentum.

- $p' = p - q, \quad p'^2 = 0, \quad Q^2 = -q^2 > 0. \qquad (15)$

see next page for Feynman diagrams.

- Conventions:

$$j_\mu^{em} = \sum_q g_{em} \cdot Q_q \, \bar{q} \gamma_\mu q, \qquad \epsilon_{0123} = -1. \qquad (16)$$

- Power counting.

$$\bar{n}\cdot p \sim n\cdot p' \sim O(\sqrt{Q^2}), \qquad n\cdot p \sim O(\Lambda^2/\sqrt{Q^2}). \qquad (17)$$

π momentum. real photon momentum.

2) Factorization at tree level.

- Consider the four-point QCD matrix element

$$\Pi_\mu = \langle q(xp) \, \bar{q}(\bar{x}p) | j_\mu^{em} | \gamma(p') \rangle \qquad (18)$$

Comments: ① Factorization property refers to the QCD correlation function in the framework of perturbative factorization approach.

② Factorization property independent of the external states, since the Wilson coefficient is independent of the long-distance physics.

③ A definite power counting scheme must be established for the factorization proof systematically.

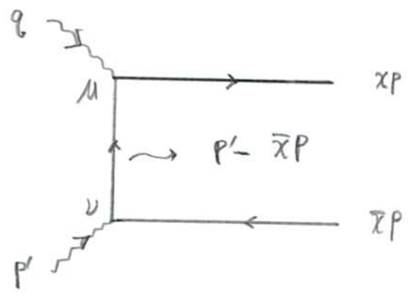
pion mom.

$$P_\mu = \frac{\bar n\cdot P}{2}\, n_\mu + \overbrace{\frac{n\cdot P}{2}\, \bar n_\mu}^{O(\Lambda^2/\sqrt{Q^2})} .$$

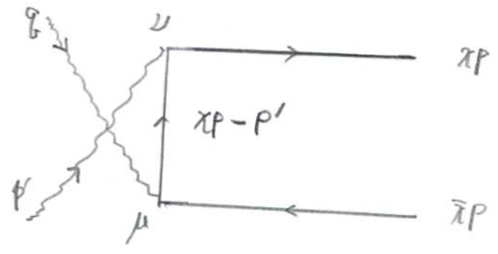
$$P'_\mu = \frac{n\cdot P'}{2}\, \bar n_\mu .$$

real photon mom.

- Tree-level diagrams.



(a)



(b)

$$Q^2 = -(P-P')^2 \simeq 2P\cdot P' \simeq \bar n\cdot P\; n\cdot P'$$

Evaluating the above two diagrams yields

$$\Pi_\mu^{(0)} = -\frac{i\, g_{em}^2 \left(Q_u^2 - Q_d^2\right)}{2\sqrt{2}\; \bar n\cdot P}\, \epsilon^{\nu}(P') \left[\frac{\bar u(xP)\, \gamma_{\mu,\perp}\, \not{n}\, \gamma_{\nu,\perp}\, v(\bar x P)}{\bar x} - \frac{\bar u(xP)\, \gamma_{\nu,\perp}\, \not{n}\, \gamma_{\mu,\perp}\, v(\bar x P)}{x} \right] \quad (19)$$

from the flavour structure of pion $\frac{u\bar u - d\bar d}{\sqrt 2}$

we can rewrite eq. (19) as the matrix elements of light-cone operators.

$$\Pi_\mu^{(0)} = -\frac{i\, g_{em}^2 (Q_u^2 - Q_d^2)}{2\sqrt{2}\ \bar{n}\cdot p}\left[\frac{1}{\bar{x}'} * \langle O_{A,\mu\nu}(x,x')\rangle^{(0)} - \frac{1}{x'} * \langle O_{B,\mu\nu}(x,x')\rangle^{(0)}\right] \qquad (20)$$

convolution integral

$$\langle O_{j,\mu\nu}(x,x')\rangle \equiv \langle q(xp)\,\bar{q}(\bar{x}p)\,|\,O_{j,\mu\nu}(x')\,|0\rangle = \bar{\xi}(xp)\,\Gamma\,\xi(\bar{x}p)\,\delta(x-x') + O(\alpha_s)$$

light-cone operator (SCET operator)

collinear spinor

$$\xi = \frac{n\!\!\!/\,\bar{n}\!\!\!/}{4}\,u$$

the definition of the SCET operator in the momentum space is.

$$O_{j,\mu\nu}(x') = \frac{\bar{n}\cdot p}{2\pi}\int d\tau\ e^{-i x' \tau\, \bar{n}\cdot p}\ \bar{\xi}(\tau\bar{n})\,W_c(\tau\bar{n},0)\,\Gamma_{j,\mu\nu}\,\xi(0). \qquad (21)$$

$$W_c \equiv P\left\{\exp\left[i g_s \int_0^\tau d\lambda\ \bar{n}\cdot A_c(\lambda\bar{n})\right]\right]$$

collinear wilson line.

- $\Gamma_{A,\mu\nu} = \gamma_{\mu,\perp}\,\bar{n}\!\!\!/\,\gamma_{\nu,\perp}, \qquad \Gamma_{B,\mu\nu} = \gamma_{\nu\perp}\,\bar{n}\!\!\!/\,\gamma_{\mu\perp}.$ — (22)

<u>Comments</u>

① Eq. (20) can be already viewed as tree-level factorization formula.

② But we do not define the pion LCDA in terms of operators displyed in eq. (22) Instead, we define pion DAs via the axial-vector light-cone current, involving "γ_5". ⇒ "γ_5" scheme dependence!

- To achieve the factorization at tree level, we introduce the SCET operator basis

$\{ O_{1,\mu\nu}, \quad O_{2,\mu\nu}, \quad O_{E,\mu\nu} \}$ with

evanescent operator

$$\Gamma_{1,\mu\nu} = g^{\perp}_{\mu\nu} \bar{n}\!\!\!/ , \qquad \Gamma_{2,\mu\nu} = i\, \epsilon^{\perp}_{\mu\nu} \bar{n}\!\!\!/ \gamma_5 , \qquad \Gamma_{E,\mu\nu} = \bar{n}\!\!\!/ \left(\frac{[\gamma_{\mu\perp}\, \gamma_{\nu\perp}]}{2} - i\, \epsilon^{\perp}_{\mu\nu} \gamma_5 \right) \tag{23}$$

$= \frac{1}{2} \epsilon_{\mu\nu\alpha\beta}\, \bar{n}^{\alpha} n^{\beta}$

vanishes in $D=4$.

Making use of the operator identities,

$$O_{A,\mu\nu} = -\left(O_{1,\mu\nu} + O_{2,\mu\nu} + O_{E,\mu\nu} \right),$$

$$O_{B,\mu\nu} = -\left(O_{1,\mu\nu} - O_{2,\mu\nu} - O_{E,\mu\nu} \right), \tag{24}$$

- Matching equation with evanescent operator:

$$\Pi_{\mu} = \left[\frac{i\, g^2_{em} (Q_u^2 - Q_d^2)}{2\sqrt{2}\; \bar{n}\cdot p} \epsilon^{*\nu}(p') \right] \cdot \sum_i T_i(x') * \langle O_{i,\mu\nu}(x, x') \rangle \tag{25}$$

$= \Pi^{(0)}_{\mu} + \Pi^{(1)}_{\mu} + \cdots,$ (expanded in α_s)

At tree level, we obtain

$$T_1^{(0)}(x') = \frac{1}{x'} - \frac{1}{\bar{x}'}, \qquad T_2^{(0)}(x') = T_E^{(0)}(x') = \frac{1}{x'} + \frac{1}{\bar{x}'}, \tag{26}$$

Only the operator $O_{E,\mu\nu}$ can couple to pion and $\langle O_{E,\mu\nu}\rangle^{(0)} = 0$, we obtain

$$F^{(0)}_{\gamma^*\gamma\to\pi} = \frac{\sqrt{2}\,(Q_u^2 - Q_d^2)\, f_\pi}{Q^2} \int_0^1 dx \; T_2^{(0)}(x)\; \phi_\pi(x,\mu) + O(\delta). \qquad (27).$$

where the twist-2 pion DA is given by

$$\langle \pi(p)\,|\,\bar{\xi}(y)\, W_c(y,0)\, \gamma_\mu \gamma_5\, \xi(0)\,|0\rangle$$

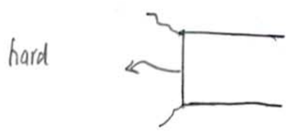
$$= -i f_\pi\, p_\mu \int_0^1 du \; e^{i u p\cdot y}\; \phi_\pi(u,\mu) + O(y^2). \qquad (28)$$

Comments:

① Evanescent operator plays no role at tree level.

② Hard-collinear factorization holds at leading twist.

③ light-cone OPE is guaranteed by the hard fluctuation of the internal quark propagator.

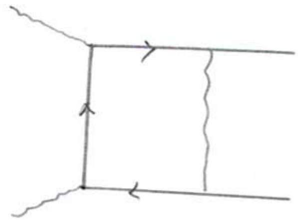


④ In general, there's no correspondence between the power expansion & the twist expansion.

⑤ Many different sources of power corrections.

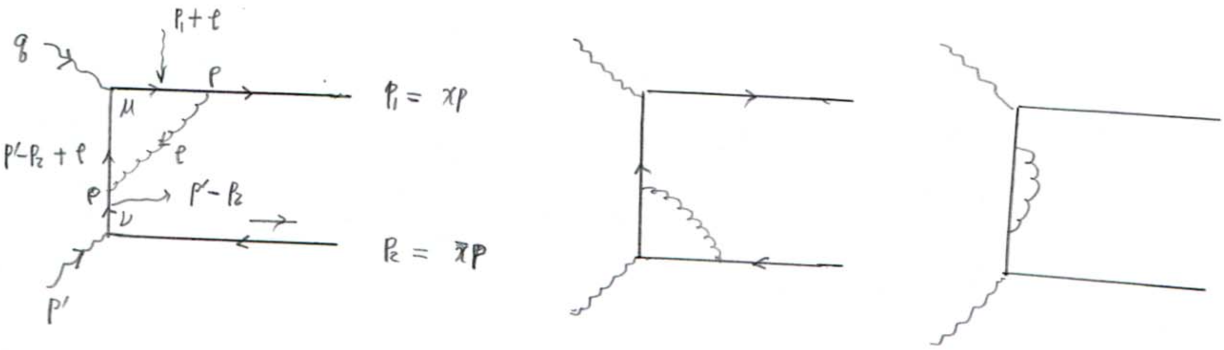
light-cone corrections, subleading twists, quark-hadron duality approximation, ...

3) Factorization at $O(\alpha_s)$.



(d)

- One-loop diagrams.



(a) (b) (c)

$$p_\mu = \frac{\bar{n}\cdot p}{2} n_\mu + \underbrace{\frac{n\cdot p}{2} \bar{n}_\mu}_{\to\, O(\Lambda^2/\sqrt{Q^2})}, \qquad p'_\mu = \frac{n\cdot p'}{2} \bar{n}_\mu$$

$$\boxed{\begin{aligned} p_\mu &= (n\cdot p,\ \bar{n}\cdot p,\ p_{\perp\mu}) \\ &= \sqrt{Q^2}\,(\lambda^2,\ 1,\ \lambda) \\ p' &= (n\cdot p',\ \bar{n}\cdot p',\ p'_{\perp\mu}) \\ &= \sqrt{Q^2}\,(1,\ \lambda^2,\ \lambda) \end{aligned}}$$

- Evaluating the diagram (a) with the method of region gives.

$$\Pi_\mu^{(1a)}(p,q) = g_{em}^2 \cdot \frac{Q_u^2 - Q_d^2}{\sqrt{2}}\, g_s^2 \cdot C_F \cdot \frac{1}{(p'-p_2)^2+i0}\, \mu^{2\epsilon} \int \frac{d^D\ell}{(2\pi)^D} \cdot \frac{1}{[(p_1+\ell)^2+i0]\,[(p'-p_2+\ell)^2+i0]\,[\ell^2+i0]}$$

$$\bar{u}(p_1)\,\gamma_\rho\,(\not{p}_1+\not{\ell})\,\gamma_\mu^\perp\,(\not{p}'-\not{p}_2+\not{\ell})\,\gamma^\rho\,(\not{p}'-\not{p}_2)\,\gamma_\nu^\perp\, v(p_2) \qquad \text{—— (29)}$$

To establish the LP contributions, we consider the scalar integral.

$$I_1 = \int [d\ell]\, \frac{1}{[(p_1+\ell)^2+i0]\,[(p'-p_2+\ell)^2+i0]\,[\ell^2+i0]} \qquad \text{—— (30)}$$

- $\ell \sim$ hard, $\quad I_1 \sim \lambda^0, \quad (\lambda \sim \Lambda/\sqrt{Q^2})$.

- $\ell \sim$ collinear $\quad (n\cdot\ell,\ \bar{n}\cdot\ell,\ \ell_\perp) \sim (1,\ \lambda^2,\ \lambda)$

  ($\ell \parallel p'$)

  $$I_1 \sim \lambda^2 \Rightarrow \text{power suppressed.}$$

- $\ell \sim$ anti-collinear $\quad (n\cdot\ell,\ \bar{n}\cdot\ell,\ \ell_\perp) \sim (\lambda^2,\ 1,\ \lambda)$.

  ($\ell \parallel p$)

  $$I_1 \sim \lambda^0 \Rightarrow \text{LP contribution}$$

- $\ell \sim$ soft $\quad \ell_\mu \sim \lambda, \quad I_1 \sim \lambda \Rightarrow$ power suppressed

Hence, only the hard and (anti)-collinear contributions are relevant at LP. The (anti)-collinear contribution vanishes due to $p^2 = 0$, when applying the dim. reg..

Evaluating the hard contribution to the diagram gives

$$\Pi_\mu^{(1a)} = \frac{i\, g_{em}^2 \cdot (Q_u^2 - Q_d^2)}{2\sqrt{2}\ \bar{n}\cdot p} \frac{\alpha_s C_F}{2\pi} \cdot \epsilon^\nu(p') \cdot \langle O_{2,\mu\nu}(x, x') \rangle^{(0)}$$

$$* \left\{ \frac{1}{x'\bar{x}'} \left[-\left(\ln \bar{x}' + \frac{x'}{2}\right)\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{Q^2}\right) + \frac{1}{2}\ln \bar{x}' \left(\ln \bar{x}' - 2 - \bar{x}'\right) - 2x' \right] + \cdots \right\} \qquad (31)$$

convolution.

proportional to $\langle O_{\perp,\mu\nu}(x, x') \rangle^{(0)}$, $\langle O_{E,\mu\nu}(x, x') \rangle^{(0)}$

• Along the similar lines, we have

$$T^{(1)}_{\mu} = \frac{i\, g^2_{em}\, (Q_u^2 - Q_d^2)}{2\sqrt{2}\ \bar{n}\cdot p}\ \epsilon^{\nu}(p') \cdot \langle O_{2,\mu\nu}(x,x') \rangle^{(0)} * A^{(1)}_{2,\text{hard}}(x') + \cdots,$$

proportional to $\langle O_{1,\mu\nu}(x,x')\rangle^{(0)}$, and $\langle O_{E,\mu\nu}(x,x')\rangle^{(0)}$.

(32)

where the amplitude $A^{(1)}_{2,\text{hard}}$ is given by

from IR region, since vector current is renormalization invariant.

$$A^{(1)}_{2,\text{hard}}(x') = \frac{\alpha_s C_F}{4\pi} \cdot \left\{ \frac{1}{\bar{x}'} \left[-(2\ln \bar{x}' + 3)\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{Q^2}\right) + \ln^2 \bar{x}' + 7\,\frac{\bar{x}'\ln \bar{x}'}{x'} - 9 \right] + (x' \leftrightarrow \bar{x}') \right\}$$

(33)

Comments:

(1) The amplitude $A^{(1)}_{2,\text{hard}}(x')$ is independent of the γ_5-scheme.

Since the OPE of two vector currents are independent of γ_5 treatment.

• Now expanding the matching equation (25) to the one-loop order.

QCD amplitude !

$$\left[\frac{i\, g^2_{em}\, (Q_u^2 - Q_d^2)}{2\sqrt{2}\ \bar{n}\cdot p}\ \epsilon^{\nu}(p') \right] \sum_i A^{(1)}_i(x') * \langle O_{i,\mu\nu}(x,x') \rangle^{(0)}$$

$$= \left[\frac{i\, g^2_{em}\, (Q_u^2 - Q_d^2)}{2\sqrt{2}\ \bar{n}\cdot p}\ \epsilon^{\nu}(p') \right] \sum_i \left[T^{(1)}_i(x') * \langle O_{i,\mu\nu}(x,x') \rangle^{(0)} + T^{(0)}_i(x') * \langle O_{i,\mu\nu}(x,x') \rangle^{(1)} \right]$$

(34)

The next step is to perform both the U.V. renormalization & the IR. subtraction. To this end, we first consider the 1-loop U.V. renormalized SCET matrix element.

$$\langle O_{i,\mu\nu} \rangle^{(1)} = \sum_j [\underbrace{M^{(1)\,R}_{ij,\,bare}}_{\text{dependent on the I.R. regularization scheme } R} + Z^{(1)}_{ij}] * \langle O_{j,\mu\nu} \rangle^{(0)} \qquad (35)$$

Applying both the dim. reg. for both the U.V. and I.R. divergences, the bare matrix element $M^{(1),R}_{ij,\,bare}$ vanishes due to scaleless integrals entering the relevant one-loop computation.

Combining eqs. (34) and (35) yields:

$$T_2^{(1)} = A_2^{(1)} - \underbrace{\sum_{i=2,E} T_i^{(0)} * Z^{(1)}_{i2}}_{= \underbrace{T_2^{(0)} * Z^{(1)}_{22}}_{\text{remove the "1/}\epsilon\text{" term of eq. (33)}} + T_E^{(0)} * Z^{(1)}_{E2}} \qquad (36)$$

It's clear that the 1st term will only remove the divergent term of eq. (33). For the matrix element of the evanescent operator, we apply the condition that

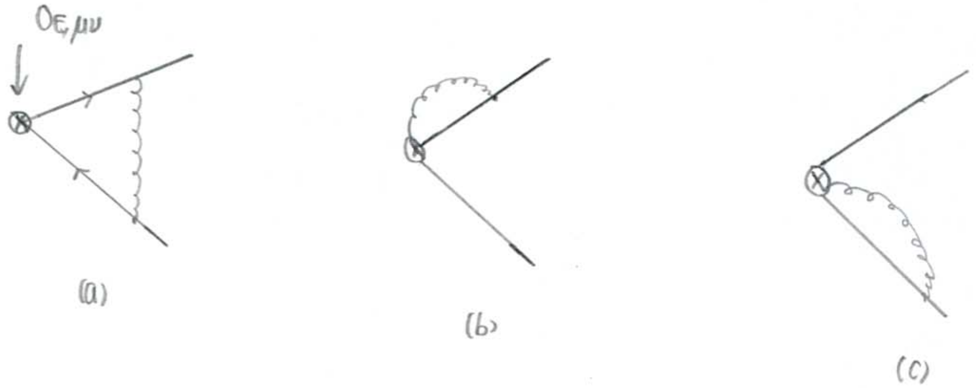
" the IR finite matrix element of the evanescent operator $\langle O_{E,\mu\nu} \rangle$ vanishes with dim. reg. applied only to the evanescent operator and with the I.R. singularities regularized by any parameter other than the dimensions of space time. Using eq. (35) gives

$$Z_{EZ}^{(1)} = - M_{EZ}^{(1)\,\mathrm{off}}. \qquad (37)$$

Inserting eqs. (36), (37) into eq. (34) yields

$$T_Z^{(1)} = \underbrace{A_{Z,\,\mathrm{ren.}}^{(1)}}_{\text{"UV" term removed.}} + T_E^{(0)} * M_{EZ}^{(1)\,\mathrm{off}} \qquad (38)$$

- The infrared subtraction term " $T_E^{(0)} * M_{EZ}^{(1)\,\mathrm{off}}$ " can be obtained by evaluating the following diagrams. The key point is that this term is dependent on the " γ_5" scheme now, as it must be the case [due to the scheme dependence of g_Z].



① NDR scheme

spin-dependent term of Brodsky-Lepage kernel.

$$T_E^{(0)} * M_{EZ}^{(1)\,\mathrm{off}} = \frac{\alpha_s C_F}{2\pi} \int_0^1 dy \left(\frac{1}{y} + \frac{1}{\bar{y}}\right) 4 \left[\frac{\bar{y}}{\bar{x}'}\, \theta(y - x') + \frac{y}{x'}\, \theta(x' - y) \right]$$

$$= \frac{\alpha_s C_F}{2\pi} \cdot (-4) \cdot \left[\frac{\ln \bar{x}'}{x'} + \frac{\ln x'}{\bar{x}'} \right] \qquad (39)$$

$+ \ (x' \leftrightarrow \bar{x}')$

② HV- scheme

$$T_E^{(0)} * M_{EZ}^{(1)\,\text{off}} = 0. \qquad (40)$$

- Finally, inserting eqs. (39), (40) into (38) yields.

$$T_2^{(1)}(x', \mu) = \frac{\alpha_s C_F}{4\pi} \cdot \left\{ \frac{1}{\bar{x}'} \left[-(2\ln\bar{x}' + 3)\ln\frac{\mu^2}{Q^2} + \ln^2\bar{x}' + \delta \cdot \frac{\bar{x}'\ln\bar{x}'}{x'} - 9 \right] + (x' \leftrightarrow \bar{x}') \right\} \qquad (41)$$

"γ_5"– scheme dependent

with $\delta = \begin{cases} -1, & \text{(NDR scheme)} \\ +7, & \text{(HV scheme)} \end{cases}$ (42)

- One-loop factorization formula.

renormalization scheme "Δ"!

$$F_{\gamma^*\gamma\to\pi^0}^{(1)}(Q^2) = \frac{\sqrt{2}\,(Q_u^2 - Q_d^2)\cdot f_\pi}{Q^2} \int_0^1 dx \left[T_2^{(0)}(x) + T_2^{(1),\Delta}(x) \right] \phi_\pi^\Delta(x, \mu) + O(\alpha_s^2). \qquad (43)$$

The scheme dependence must be cancelled between $T_2^\Delta(x)$ and $\phi_\pi^\Delta(x, \mu)$.

The relation of the twist-2 pion DA between the NDR and HV schemes

$$\phi_\pi^{HV}(x, \mu) = \int_0^1 dy\; Z_{HV}^{-1}(x, y, \mu)\; \phi_\pi^{NDR}(y, \mu) \qquad (44)$$

with the finite kernel Z_{HV}^{-1} (arxiv: hep-ph/0107295).

$$Z_{HV}^{-1}(x, y, \mu) = \delta(x-y) + \frac{\alpha_s C_F}{2\pi} \cdot 4 \left[\frac{x}{y} \theta(y-x) + \frac{\bar{x}}{\bar{y}} \theta(x-y) \right] + O(\alpha_s^2). \qquad (45)$$

One can readily show that [Exercise]

$$\int_0^1 dx \; T_2^{(0)}(x) \left[\phi_\pi^{HV}(x,\mu) - \phi_\pi^{NDR}(x,\mu) \right]$$

$$= \frac{\alpha_s C_F}{2\pi} (-4) \int_0^1 dy \; \underbrace{\left(\frac{\ln \bar{y}}{y} + \frac{\ln y}{\bar{y}} \right)}_{} \phi_\pi^{NDR}(x,\mu) + O(\alpha_s^2), \qquad (46)$$

compare with the "δ" term in Eq. (41).

which cancels against the renormalization scheme dependence of the NLO hard kernel $T_2^{(1),0}$ in Eq. (43) exactly.

- <u>Factorization-scale independence of $F_{\gamma^*\gamma\to\pi}(Q^2)$.</u>

Making use of the RGE of the pion DA: [Exercise].

$$\mu^2 \frac{d}{d\mu^2} \phi_\pi(x,\mu) = \int_0^1 dy \; V(x,y) \; \phi_\pi(y,\mu) \qquad (47)$$

with

plus function.

$$V(x,y) = \frac{\alpha_s C_F}{2\pi} \left[\frac{\bar{x}}{\bar{y}} \left(1 + \frac{1}{x-y}\right) \theta(x-y) + \frac{x}{y} \left(1 + \frac{1}{y-x}\right) \theta(y-x) \right]_+ , \qquad (48)$$

$$[f(x,y)]_+ = f(x,y) - \delta(x-y) \int_0^1 dt \; f(t,y) \qquad (49)$$

It's then easy to derive that. [Exercise]

using eq. (43).

$$\frac{d}{d\ln\mu} F_{\gamma^*\gamma\to\pi^0}(Q^2) = O(\alpha_s^2), \tag{50}$$

- <u>NLL resummation of $F_{\gamma^*\gamma\to\pi}(Q^2)$</u> (arXiv: 1706.05680).

The Gegenbauer expansion of the pion DA:

$$\phi_\pi(x,\mu) = 6 x \bar{x} \sum_{n=0}^{\infty} a_n(\mu)\, C_n^{3/2}(2x-1). \tag{51}$$

At 2 loops, the Gegenbauer moments can mix with the lower moments.

$$a_n(\mu) = E_{V,n}^{NLO}(\mu,\mu_0)\, a_n(\mu_0) + \frac{\alpha_s(\mu)}{4\pi} \boxed{\sum_{k=0}^{n-2}} E_{V,n}^{LO}(\mu,\mu_0)\, d_{V,n}^{k}(\mu,\mu_0)\, a_k(\mu_0). \tag{52}$$

The resummation improved factorization formula reads.

both in NDR scheme!

$$F^{(LP)}_{\gamma^*\gamma\to\pi^0}(Q^2) = \frac{3\sqrt{2}\,(Q_u^2 - Q_d^2)}{Q^2}\, f_\pi \sum_{n=0}^{\infty} a_n(\mu)\, C_n(Q^2,\mu) + O(\alpha_s^2). \tag{53}$$

with

$H_n = \sum_{k=1}^{n} \frac{1}{k}$.

$$C_n(Q^2,\mu) = 1 + \frac{\alpha_s(\mu)\, C_F}{4\pi} \Big\{ \Big[4 H_{n+1} - \frac{3n(n+3)+8}{(n+1)(n+2)} \Big] \ln\frac{\mu^2}{Q^2} + 4 H_{n+1}^2$$
$$- 4\frac{H_{n+1}+1}{(n+1)(n+2)} + 2\Big[\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2}\Big] + 3\Big[\frac{1}{n+1} - \frac{1}{n+2}\Big]$$
$$- 9 \Big\} \tag{54}$$

Lecture III. QCDF for heavy-to-light FFs at large recoil

Aug. 16. 2017

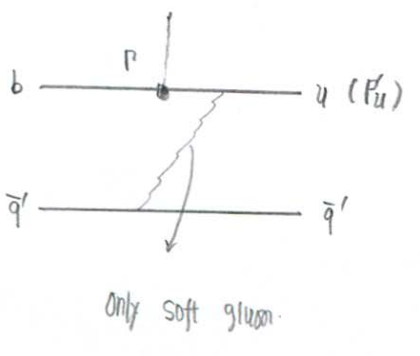
1) Definition of heavy-to-light form factors: (hep-ph/0008255).

$$\langle P(P')|\bar q \gamma_\mu b|\bar B(P)\rangle = f_+(q^2)\left[P_\mu + P'_\mu - \frac{M^2-m_P^2}{q^2} q_\mu\right] + f_0(q^2)\frac{M^2-m_P^2}{q^2} q_\mu \qquad (55)$$

$$\langle P(P')|\bar q \sigma_{\mu\nu} q^\nu b|\bar B(P)\rangle = \frac{i f_T(q^2)}{M+m_P}\left[q^2(P_\mu+P'_\mu) - (M^2-m_P^2) q_\mu\right] \qquad (56)$$

"3" B→P form factors ⊕ "7" B→V form factors.

2) Large-recoil symmetry for the form factors (only soft interactions ⇒ LEET).



only soft gluon.

(a)

$|k'| \ll n\cdot P'_u$

$$P'^{\mu}_u = \frac{n\cdot P'_u}{2}\bar n^\mu + k'^\mu, \qquad (57)$$

small residual momentum

At leading power, we obtain

only soft interaction

$$\mathcal{L}_{QCD}\to \mathcal{L}_{eik} = \bar q_n \frac{\not n}{2}(i\,\bar n\cdot D_s)\, q_n + O(\Lambda/n\cdot P') \qquad (58)$$

$q_n(x) = e^{i\, n\cdot P'_u\, \bar n\cdot x/2}\,\frac{\not{\bar n}\not n}{4}\, q(x)$ → collinear quark field.

Soft interaction with the heavy quark is given by

$$\mathcal{L}_{QCD} \to \mathcal{L}_{HQET} = \bar{Q}_v (i v \cdot D_s) Q_v + O(1/m_Q) \qquad (59)$$

$$Q_v(x) = e^{i m_Q v\cdot x} \frac{1+\not{v}}{2} Q(x)$$

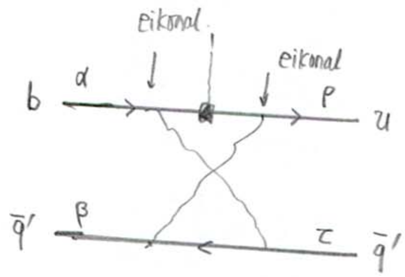
Key point: Soft gluons coupled with BOTH the heavy quark & the collinear quark are described by the eikonized Lagrangian at LP.

- Neglecting again the hard / hard-collinear interactions,

hard interaction will generate hard functions.

$$[\bar{q} \Gamma b]_{QCD} \to [\bar{q}_n \Gamma b_v]_{eff}. \qquad (59)$$

Applying the trace formula,



$$\langle L(\tfrac{n\cdot p'}{2} \bar{n}) \,|\, \bar{q}_n \Gamma b_v \,|\, B(Mv)\rangle$$

$$= \mathrm{Tr}[A_L(n\cdot p') \bar{M}_L \Gamma M_B] \qquad (60)$$

$$LH \sim (\bar{u}_\rho \Gamma^{\rho\alpha} b^\alpha)(\bar{q}'_\beta [A_L(n\cdot p')]^{\beta\tau} q'_\tau)$$

$$\sim \underbrace{(\bar{q}'_\beta b^\alpha)}_{(M_B)^{\alpha\beta}} \underbrace{(\bar{u}_\rho q'_\tau)}_{(\bar{M}_L)^{\tau\rho}} \Gamma^{\rho\alpha} [A_L(n\cdot p')]^{\beta\tau}$$

(projectors)

$$\sim \mathrm{Tr}[A_L(n\cdot p') \bar{M}_L \Gamma M_B] \;!$$

The explicit expressions of the hadronic projector are given by.

$$\bar{M}_L = \left\{ \begin{matrix} (-\gamma_5) \\ \not{\epsilon}^* \end{matrix} \right\} \frac{\not{n}\,\not{\bar{n}}}{4}, \quad \begin{matrix} L=P \\ L=V, \end{matrix} \qquad M_B = \frac{1+\not{v}}{2}(-\gamma_5). \quad \text{———} (61)$$

The function $A_L(n\cdot p')$ depends on the long-distance physics, but independent of the Dirac structure of the weak current. The general form of $A_L(n\cdot p')$ is given by.

$$A_L(n\cdot p') = a_{1L}(n\cdot p') + \underset{\substack{\swarrow \\ M_B \not{v} = M_B\,!}}{a_{2L}(n\cdot p')\not{v}} + \underbrace{a_{3L}(n\cdot p')\not{\bar{n}} + a_{4L}(n\cdot p')\not{\bar{n}}\not{v}}_{\text{Do not contribute to } B\to P \text{ FF !}}, \quad \text{———} (62)$$

Hence, only one soft form factor relevant for $B\to P$ transitions at LP.

In general, we have

$$A_L(n\cdot p') = \begin{cases} (n\cdot p')\ \xi_P(n\cdot p'), & \text{for } L=P, \\ \frac{n\cdot p'}{2}\not{\bar{n}}\left[\xi_\perp(n\cdot p') - \frac{\not{v}}{2}\xi_\parallel(n\cdot p')\right] & \text{for } L=V, \end{cases}$$

——— (63)

Inserting eq. (63). into eq. (60) gives.

$$\langle P(p')|\bar{q}\gamma_\mu b|\bar{B}(p)\rangle = (n\cdot p')\ \xi_P(n\cdot p')\ \bar{n}_\mu,$$

——— (64)

$$\langle P(p')|\bar{q}\sigma_{\mu\nu}q^\nu b|\bar{B}(p)\rangle = i(n\cdot p')\ \xi_P(n\cdot p')\left[\left(M-\frac{n\cdot p'}{2}\right)\bar{n}_\mu - M\gamma_\mu\right],$$

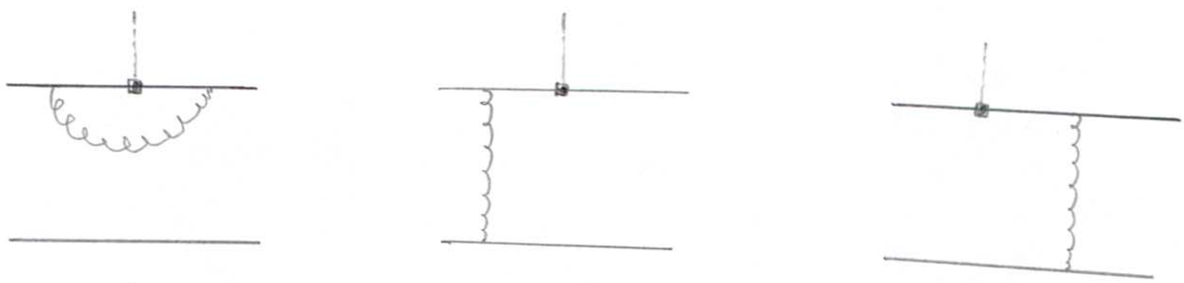
Comparing eq. (55) with eq. (64) leads to .

$$f_t(q^2) = \frac{M}{n\cdot p'} f_0(q^2) = \frac{M}{M+m_p} f_T(q^2) = \xi_p(n\cdot p') . \qquad (65)$$

Comments: ① These relations only hod for the soft contribution to QCD form factors.

② There will be corrections of order $1/m_b$ and α_s .

3) Symmetry-breaking corrections: general discussions.



(b)

(c)

(d)

hard-vertex correction

hard spectator interaction

The soft gluon contribution only projects out the asymmetric configuration of the energetic final-state meson, and we must examine the effect from the typical configuration due to the hard spectator interaction. Applying the hard-scattering approach, we can readily find that (hep-ph/0006124).

$$f_{+,0,T;\,hard}(q^2\simeq 0) \sim \alpha_s(\sqrt{\Lambda\cdot M})\cdot\left(\frac{\Lambda}{M}\right)^{3/2}, \qquad (66)$$

For the soft contribution, we can use the hard-scattering approach in the end-point region naively, and interpret α_s times the logarithmic divergence as a constant of order "1", then.

$$f_{+,0,T;\,soft}(q^2\simeq 0) \sim \xi_P(n\cdot p'\simeq M) \sim \sqrt{\frac{M}{n\cdot p'}}\left(\frac{\Lambda}{n\cdot p'}\right)^{3/2} \sim \left(\frac{\Lambda}{n\cdot p'}\right)^{3/2} \qquad (67).$$

Here, we need to assume that $\phi_\pi(u,\mu) \xrightarrow{u\to 1} \bar{u}$.

<u>Comments:</u> ① It's clear that the hard and soft contributions to the heavy-to-light form factor have the same scaling behaviour in the heavy quark limit.

② The factorization formula for $B\to P$ form factors is given by.

$$f_i(q^2) = C_i\,\xi_P(n\cdot p') + \phi_B * T_i * \phi_P. \qquad (68)$$

soft form factor NOT ONLY contains the soft gluon contribution, but also includes the divergent pieces of the hard vertex and hard spectator corrections.

4) QCD calculation of symmetry-breaking effect

A) vertex corrections (diagram b)

maintain the original Dirac structure.

$$\bar{u}(p')\,\Gamma(p',p)\,u(p) = \frac{\alpha_s C_F}{4\pi}\,\bar{u}(p')\Big\{ A_1(q^2)\cdot\Gamma + A_2(q^2)\,\gamma^\alpha\gamma^\beta\,\Gamma\,\gamma_\beta\gamma_\alpha$$

$$+ A_3(q^2)\,\gamma^\alpha \not{p}\,\Gamma\,\not{p}'\gamma_\alpha + A_4(q^2)\,m_b\,\gamma^\alpha\not{p}\,\Gamma\,\gamma_\alpha + A_5(q^2)\,m_b\,\Gamma\,\not{p}'\Big\}\,u(p) \qquad (69)$$

gluon mass

Comments: ① Only $A_1(q^2)$ contains the IR divergences: $p_n^2\,\frac{\lambda^2 m_b^2}{(m_b^2-q^2)^2}$, $\quad p_n\,\frac{\lambda^2 m_b^2}{(m_b^2-q^2)^2}$.

and such divergence cannot be cancelled by the one-loop correction in HQET/eikonal EFT, since the effective theory does not reproduce the hard-collinear IR divergence.

② since the IR divergence is independent of the original Dirac structure "Γ", it therefore can be absorbed into the soft function $S_p(n\cdot p')$; irrespective of its dynamical origin.

③ We will adopt the "Physical" form factor scheme (similar to the DIS scheme by defining the quark PDF to be the structure function F_2):

$$f_t = \xi_P, \qquad V = \frac{M + m_V}{M}\,\xi_\perp, \qquad A_0 = \frac{E}{m_V}\,\xi_\parallel . \tag{70}$$

Collecting everything together, we obtain

from hard spectator interaction

$$f_0 = \frac{n\cdot p'}{M}\,\xi_P\left[1 + \frac{\alpha_s C_F}{2\pi}(1 - L)\right] + \frac{\alpha_s C_F}{4\pi}\,\Delta f_0 ,$$

from hard spectator effect.

$$f_T = \frac{M + m_P}{M}\,\xi_P\left[1 + \frac{\alpha_s C_F}{4\pi}\left(\ln\frac{m_b^2}{\mu^2} + 2L\right)\right] + \frac{\alpha_s C_F}{4\pi}\,\Delta f_T . \tag{71}$$

$$L = -\frac{n\cdot p'}{M - n\cdot p'}\ln\frac{n\cdot p'}{M} .$$

B) Hard spectator interactions (diagrams c, d).

Taking the form factor f_+ as an example,

hard-scattering approach

convergent!

asymptotically.

$$f_+^{(HSA)} = \frac{\alpha_s C_F}{4\pi}\,\frac{\pi^2 f_B f_P M}{N_c E^2}\cdot\int_0^1 du\int_0^\infty dw\,\Bigg\{\frac{2n\cdot p' - M}{M}\cdot\frac{\phi(u)\,\phi_B^+(w)}{\bar u\, w} = \frac{1}{\bar u}\frac{\phi_\sigma(u)}{3} = 2u!$$

$$+\frac{(1+\bar u)\,\phi(u)\,\phi_B^-(w)}{\bar u^2\, w} + \frac{\mu_P}{n\cdot p'}\left[\frac{\left(\phi_p(u) - \frac{\phi_\sigma'(u)}{6}\right)\phi_B^+(w)}{\bar u^2\, w}\right.$$

- both integral divergent
- involving $\phi_B^-(w)$

- divergent twist-3 effect
- LP contribution

$\phi_p(u) = 1.$

$$\left.+\frac{2n\cdot p'\,\phi_p(u)\,\phi_B^+(w)}{\bar u\, w^2}\right]\Bigg\} \tag{72}$$

- both divergent.

• <u>Comments</u>

① The IR divergent terms are universal for all the three $B \to P$ FFs, hence they can be absorbed into ξ_P.

② Only the 1st term is perturbatively calculable, and this term differs for different FFs.

③ There's no correspondence between the power expansion and the twist expansion, whenever there's LP soft contribution.

Finally, we have

$$\Delta f_0 = \frac{M - n\cdot p'}{n\cdot p'}\,\Delta F_P, \qquad \Delta f_T = -\frac{M + m_P}{n\cdot p'}\,\Delta F_P. \qquad (73)$$

$$\Delta F_P = \frac{8\pi^2 f_B f_P}{N_c M}\,\langle \omega^{-1}\rangle\,\langle \bar{u}^{-1}\rangle_P. \qquad (74)$$

$$\equiv \int d\omega\, \frac{\phi_B^+(\omega)}{\omega} = \lambda_B^{-1}(\mu). \quad (\sim 1/\Lambda).$$

$$\equiv \int_0^1 du\, \frac{\phi(u)}{\bar{u}}$$

The same as the moment entering the factorization formula of $\gamma^*\gamma \to \pi$ FF.

5) SCET for heavy-to-light form factors at large recoil

- The Lagrangian $\mathcal{L}_{eik} + \mathcal{L}_{HQET}$ is insufficient to describe the $b \to u$ transition at large recoil.

Instead, we need to apply the SCET Lagrangian.

$$= i\bar{n}\cdot D_c(x) + g_s\,\bar{n}\cdot A_s\Big(\underbrace{\tfrac{n\cdot x}{2}}_{\equiv x_-}\bar{n}\Big)$$

collinear quark field

$$\mathcal{L}_{eff}^{SCET} = \underbrace{\sum_{Q=b,c} \bar{h}_{v_Q}\, i v_Q\cdot D_s\, h_{v_Q}}_{①} + \sum_{q,\,light} \bar{\xi}_q \Big(\underbrace{i\bar{n}\cdot D}_{③} + \underbrace{i\not{D}_{\perp c}\frac{1}{i n\cdot D_c}\, i\not{D}_{\perp c}}_{②} \Big) \frac{\not{n}}{2}\xi_q$$

HQET lagrangian

$$+ \sum_{q\,light} \bar{q}\, i\not{D}_s\, q \;+\; \mathcal{L}_{pure\ gluon}\,. \qquad (75)$$

soft quark field $\qquad$ c & s.

Compare eq. (75) with (58), (59), we find that new terms appear in SCET Lagrangian

<u>Comments</u>: ① $\bar{h}_{v_Q}\, i v\cdot D_s\, h_{v_Q}$, $\Rightarrow$ no interaction of heavy quarks with collinear fields,

$$i D_s = i\partial + g_s A_s$$

since $(p_Q + p_c)^2 - m_Q^2 \sim O(m_Q^2)$.

already integrated out.

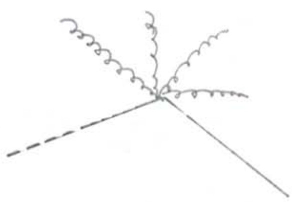
② $\bar{\xi}_q\, i\not{D}_{\perp c}\frac{1}{i n\cdot D_c}\, i\not{D}_{\perp c}\frac{\not{n}}{2}\xi_q$ $\Rightarrow$ Collinear interactions are non-local.

vertices with any number of $n\cdot A_c$ fields.

$$\frac{1}{i n\cdot D + i\epsilon} = W\frac{1}{i n\cdot\partial + i\epsilon}W^\dagger$$

$$\frac{1}{i n\cdot\partial + i\epsilon}\phi(x)$$

$$= -i\int_{-\infty}^{0} ds\ \phi(x + s n)$$

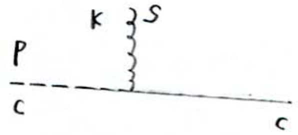


any number of $n\cdot A_c$, up to two $A_{\perp c}$.

③ $\bar{\xi}(x)\,[\,i\bar{n}\cdot D_c(x) + g_s\, \bar{n}\cdot A_s(x_-)\,]\,\frac{\not{n}}{2}\,\xi$

$$\uparrow$$

$$x_-^\mu = \frac{n\cdot x}{2}\,\bar{n}^\mu ;$$



$P + \frac{\bar{n}\cdot k}{2}\, n,$

since $n\cdot k \ll n\cdot P,$

$k_\perp \ll P_\perp$

Collinear: $(n\cdot k,\ \bar{n}\cdot k,\ k_\perp)$

$\sim m_b\,(1,\ \lambda,\ \lambda^{1/2}).$

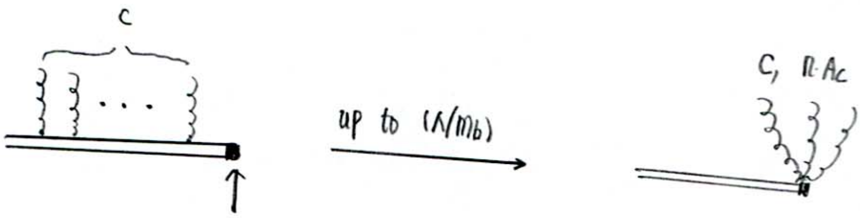
Soft: $(n\cdot k,\ \bar{n}\cdot k,\ k_\perp)$

$\sim m_b\,(\lambda,\ \lambda,\ \lambda).$

<u>Note</u>: since the soft fields vary slowly than the collinear fields in the "$\perp$" and "n" directions, we expand the position arguments of the soft fields:

$$\phi_s(x) = \phi_s(x_-) + (x_\perp \partial \phi_s)(x_-) + \frac{\bar{n}\cdot x}{2}\,[n\cdot\partial\,\phi_s](x_-) + \cdots,$$

<u>Question:</u> Where do collinear interactions with heavy quarks in full QCD go?



line ends on weak vertex

$b \to W_c\, h_v$

$\uparrow$

collinear Wilson line

$$W_c(x) = P\,\mathrm{Exp}\left[\,i g_s \int_{-\infty}^{0} ds\ n\cdot A_c(x+sn)\,\right], \qquad (76)$$

<u>Properties</u>

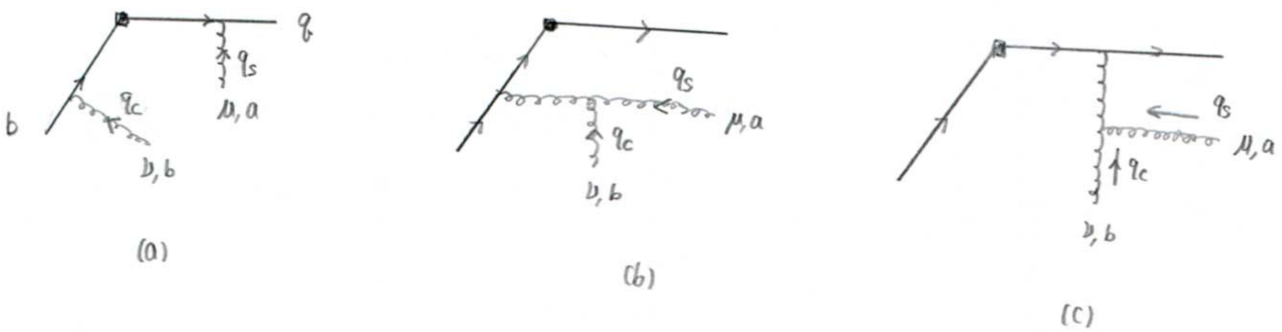
① $W_c W_c^\dagger = 1.$

② $W_c^\dagger\, f(in\cdot D_c)\, W_c = f(in\cdot\partial).$

However, the correct form of the effective current is more complicated.

$$J_A = \bar{\xi} W_c \Gamma \underset{\uparrow}{Y_s^\dagger} h_v \qquad \text{——— (77)}$$

$$Y_s^\dagger(x) = P \exp\left[i g_s \int_0^\infty dt \ \bar{n}\cdot A_s(x + t\bar{n}) \right].$$



This must be the case due to the collinear / soft gauge invariance of SCET lagrangian.

For QCD transformation.

$$U(x) = \exp\left[i \alpha^A(x) \ T^A \right]. \qquad \text{——— (78)}$$

Collinear: $U_c(x)$: $\quad i\partial^\mu U_c(x) \sim m_b (1, \lambda, \lambda^{1/2}) \ U_c(x)$ ——— (79)

Soft: $U_s(x)$: $\quad i\partial^\mu U_s(x) \sim m_b(\lambda, \lambda, \lambda) \ U_c(x)$. ——— (80)

The transformation properties are. summarized as follows.

<u>Collinear</u>:

$A_c \to U_c A_c U_c^\dagger, \qquad \xi \to U_c \xi,$

$A_s \to A_s, \qquad q \to q,$

——— (81)

<u>Soft</u>:

$A_c \to U_s A_c U_s^\dagger, \qquad \xi \to U_s \xi,$

$A_s \to U_s A_s U_s^\dagger + \frac{1}{g_s} U_s [\partial, U_s^\dagger], \qquad q \to U_s q.$

- In general, we define the following gauge invariant building blocks. (arxiv: 0211018).

$$\mathcal{H} = S^\dagger h_v, \qquad \mathcal{Q}_s = S^\dagger q_s, \qquad \chi = W_c^\dagger \xi,$$

$$i\hat{D}_c^\mu = W_c^\dagger \, iD_c^\mu \, W_c = i\partial^\mu + \hat{A}_c^\mu,$$

gauge invariant gluon fields.

$$i\hat{D}_s^\mu = S^\dagger \, iD_s^\mu \, S = i\partial^\mu + \hat{A}_s^\mu, \qquad (82)$$

$$\hat{A}_c^\mu(x) = [W_c^\dagger \, iD_c^\mu \, W_c](x) = \int_{-\infty}^0 ds \, n_\alpha [W_c^\dagger g_s G_c^{\alpha\mu} W_c](x+sn)$$

$$\hat{A}_s^\mu(x) = [S^\dagger \, iD_s^\mu \, S](x) = \int_{-\infty}^0 ds \, \bar{n}_\alpha [S^\dagger g_s G_s^{\alpha\mu} S](x+s\bar{n}).$$

The same as Y_s, defined after eq. (77).

- Soft-gluon decoupling

Making use of the field redefinition

$$\xi(x) = Y_s(x)\,\xi^{(0)}(x), \qquad A_c(x) = Y_s(x)\,A_c^{(0)}(x)\,Y_s^\dagger(x) \qquad (83)$$

then

$$Y_s^\dagger \, i\bar{n}\cdot D_s \, Y_s = i\bar{n}\cdot\partial,$$

$$W_c = Y_s \, W_c^{(0)} \, Y_s^\dagger, \qquad (84)$$

we find that

$$\mathcal{L}_{eff}^{SCET} = \cdots + \sum_{q,\,light} \bar{\xi}_q^{(0)} \left(i\bar{n}\cdot D_c + i\not{p}_{\perp c} \frac{1}{i\bar{n}\cdot D_c} i\not{p}_{\perp c} \right) \frac{\not{n}}{2} \xi_q^{(0)} + \cdots \qquad (85)$$

- **Matching from QCD → SCET$_I$ → SCET$_{II}$.** (hep-ph/0408344.)

QCD:

$$J_{QCD} = \bar{q}\,\Gamma_i\, b$$

$\mu^2 \sim m_b^2$

A-type ↙ B-type ↘

SCET$_I$

$$\hat{s}_i = \frac{m_b}{\bar{n}\cdot v}\, s_i$$

$$= \int d\hat{s}\; \tilde{C}_i^{(A0)}(\hat{s})\; O^{(A0)}(s;0)$$

$$O^{(A0)}(s,x) = (\bar{\xi} W_c)(x + s\bar{n})\, h_v(x_-)$$

$$\equiv (\bar{\xi} W_c)_s\, h_v$$

(two-body, no "⊥" gluon, LP eff. operator)

$$= \int d\hat{s}_1\, d\hat{s}_2\; \tilde{C}_{ij}^{(B1)}(\hat{s}_1, \hat{s}_2)\; O_j^{(B1)}(s_1, s_2; 0)$$

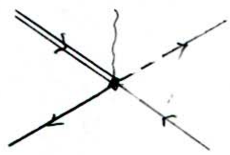
$$O_j^{(B1)}(s_1, s_2; x) = \frac{1}{m_b}\, (\bar{\xi} W_c)_{s_1}\, (W_c^\dagger\, i D_{\perp c\mu}\, W_c)_{s_2}\; \tilde{\Gamma}_j^{\mu}\, h_v$$

(three-body, with 1 "⊥" gluon, Subleading eff. operator)

SCET$_{II}$

$+ \cdots$

$O^{(A0)}$ still contains hard-collinear dynamics, Naive factorization of $O^{(A0)}$ in SCET$_{II}$ fails, due to the end-point divergence.



$+ \cdots$

$$2E \int \frac{dr}{2\pi}\, e^{-2i\tau E\cdot r}\; (\bar{\xi} W_c)(0)\; (W_c^\dagger\, i \slashed{D}_{c\perp}\, W_c)(r\bar{n})\; h_v(0)$$

$$= \int d\omega\, dv\; J(\tau; v, \ln(\tfrac{E\omega}{\mu^2}))$$

$$\left[(\bar{\xi} W_c)(s\bar{n})\, \frac{\slashed{n}}{2}\, \gamma_5\, (W_c^\dagger \xi)(0) \right]_{FT}$$

$$\left[(\bar{q}_s S)(t\bar{n})\, \frac{\slashed{\bar{n}}}{2}\, \gamma_5\, (S^\dagger h_v)(0) \right]_{FT}$$

$+ \cdots$, (only for $B \to P$)

Comments: ① Effective operator:

$$Q(v) = [(\bar{\xi} W_c)(sn)\, \frac{\not{\bar{n}}}{2}\gamma_5 (W_c^\dagger \xi)(0)]_{FT}$$

$$= \frac{n\cdot p'}{2\pi}\int ds\; e^{-isv\,n\cdot p'}\; (\bar{\xi} W_c)(sn)\, \frac{\not{\bar{n}}}{2}\gamma_5 (W_c^\dagger \xi)(0), \qquad (86)$$

$$P(\omega) = [(\bar{q}_s Y_s)(t\bar{n})\, \frac{\not{\bar{n}}}{2}\gamma_5 (Y_s^\dagger h_v)(0)]_{FT}$$

$$= \frac{1}{2\pi}\int dt\; e^{it\omega}\; (\bar{q}_s Y_s)(t\bar{n})\, \frac{\not{\bar{n}}}{2}\gamma_5 (Y_s^\dagger h_v)(0). \qquad (87)$$

② One of the "s" integrations can be removed upon taking the matrix elements by using translation invariance.

Formulation: • Step I: $SCET_I$ matching

$$(\bar{\Psi}\Gamma_i Q)(0) = \int d\hat{s}\; \tilde{C}_i^{(A0)}(\hat{s})\; O^{(A0)}(s;0) + \int d\hat{s}_1\, d\hat{s}_2\; \tilde{C}_{ij}^{(B1)}(\hat{s}_1, \hat{s}_2)\; \hat{O}_j^{(B1)}(s_1, s_2; 0)$$

sum over the index "j".

$$+ \cdots, \qquad (88)$$

Defining two LP $SCET_I$ pion form factors

$$\langle \pi(p')|\,(\bar{\xi} W_c)\, h_v\,|\bar{B}_v\rangle = 2E\, \zeta_{B\pi}(E), \qquad (89)$$

$$\langle \pi(p')|\,\frac{1}{m_b}(\bar{\xi} W_c)(W_c^\dagger\, i\not{D}_{c\perp} W_c)(rn)\, h_v\,|\bar{B}_v\rangle = 2E\int d\tau\; e^{i2\tau E r}\; \Xi_{B\pi}(\tau, E),$$

At this step, the factorization formula for $B \to \pi$ FFs reads:

$$F_i^{B\pi}(n\cdot p') = C_i(n\cdot p')\ \xi_{B\pi}(n\cdot p') + \int d\tau\ C_i^{(B1)}(E,\tau)\ \Xi_{B\pi}(\tau, E), \qquad i = +, 0, T \tag{90}$$

- Step II: $SCET_{II}$ matching.

Using the matching condition shown in Page 36 and applying the following definitions.

$$\langle \pi(p') |\ Q(v)\ | 0 \rangle = -i f_\pi \frac{n\cdot p'}{2}\ \phi_\pi(v,\mu),$$

$$\langle 0 |\ P(\omega)\ | \bar{B}_v \rangle = i \frac{\hat{f}_B\, m_B}{2}\ \phi_B^+(\omega,\mu) \tag{91}$$

We find that

$$\Xi_{B\pi}(\tau, n\cdot p') = \frac{m_B}{4 m_b}\int_0^\infty d\omega \int_0^1 dv\ J(\tau; \underset{\substack{\uparrow \\ \text{hard-collinear momentum fraction.}}}{v}, \ln(\frac{n\cdot p'\,\omega}{\mu^2}))\ \hat{f}_B\ \phi_B^+(\omega,\mu)\ f_\pi\ \phi_\pi(v,\mu). \tag{92}$$

Comments. ① The SCET-II matching is complicated by the appearance of evanescent operators and by the breakdown of Fierz transformation.

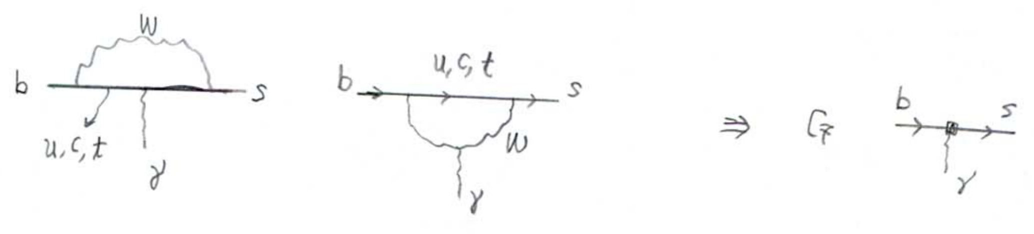
$$O_1' = (\bar{\xi}\ \gamma_{\rho\perp}\gamma_{\lambda\perp}\gamma_{\mu\perp}\ h_v)\ (\bar{q}_s\ \gamma^{\mu\perp}\gamma^{\lambda\perp}\gamma^{\rho\perp}\ \xi)$$

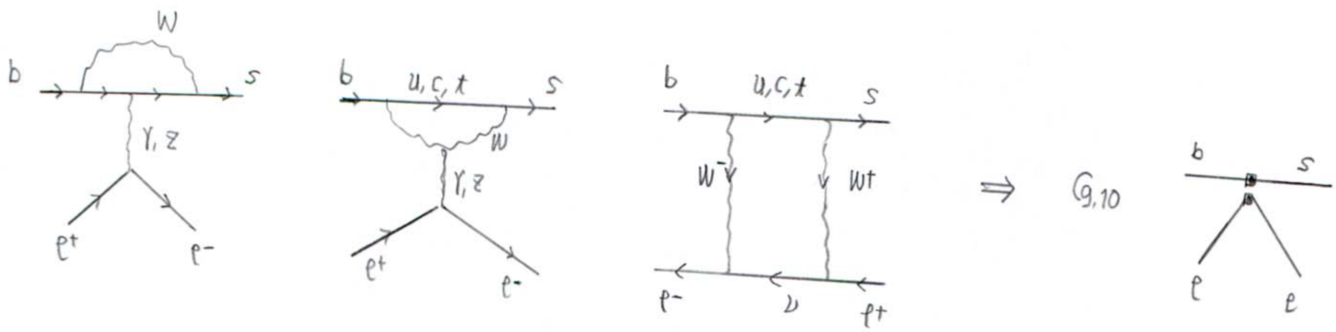
$$= f(\epsilon)\cdot O_1 + E, \qquad [\text{with } f(\epsilon=0) = 4] \tag{93}$$

$$O_1 = (\bar{\xi}\ \gamma_{\mu\perp}\ h_v)\ (\bar{q}_s\ \gamma^{\mu\perp}\ \xi).$$

Lecture IV. QCDF for $B \to K^{(*)} ee$ decays and beyond. Aug. 17. 2017

1) Weak effective Hamiltonian for $b \to s\gamma$ & $b \to see$





$$O_7 = -\frac{g_{em}\,\hat{m}_b}{8\pi^2}\,\bar{s}\,\sigma_{\mu\nu}(1+\gamma_5)\,b\,F^{\mu\nu}, \qquad O_{9,10} = \frac{\alpha_{em}}{2\pi}\,(\bar{s}\,b)_{V-A}\,(\bar{\ell}\,\ell)_{V,A}$$

$$O_8 = -\frac{g_s\,\hat{m}_b}{8\pi^2}\,\bar{s}_i\,\sigma_{\mu\nu}(1+\gamma_5)\,T^A_{ij}\,b_j\,G^A_{\mu\nu}. \tag{94}$$

More operators are relevant here:

$$H_{eff} = -\frac{G_F}{\sqrt{2}}\,V_{tb}V_{ts}^*\sum_{i=1}^{10} C_i(\mu)\,O_i(\mu). \tag{95}$$

Using the Chetyrkin–Misiak–Münz (CMM) basis,

$$O_1 = (\bar{s}_i\,T^A_{ij}\,c_j)_{V-A}\,(\bar{c}_k\,T^A_{kl}\,b_l)_{V-A}, \qquad O_2 = (\bar{s}\,c)_{V-A}\,(\bar{c}\,b)_{V-A},$$

$$O_3 = 2\,(\bar{s}\,b)_{V-A}\sum_q(\bar{q}\,q)_V, \qquad O_4 = 2\,(\bar{s}_i\,T^A_{ij}\,b_j)_{V-A}\sum_q(\bar{q}_k\,T^A_{kl}\,q_l)_V,$$

$$O_5 = 2\,(\bar{s}\,\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}(1-\gamma_5)\,b)\sum_q(\bar{q}\,\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\,q)$$

$$O_6 = 2\,(\bar{s}_i\,\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}T^A_{ij}(1-\gamma_5)\,b_j)\sum_q(\bar{q}_k\,\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\,T^A_{kl}\,q_l) \qquad (96)$$

- The advantage of the CMM basis is that Dirac traces involving γ_5 do not appear in the EFT calculations.

2) Parametrization of the hadronic matrix element:

external vertex

$$\langle \gamma^*(q,\mu)\,\bar{K}^*(p',\epsilon^*)\,|\,H_{eff}\,|\,\bar{B}(p)\rangle$$

$$= -\frac{G_F}{\sqrt{2}}V_{tb}V_{ts}^*\frac{i\,g_{em}\,m_b}{4\pi^2}\Big\{2\cdot\tilde{T}_1(q^2)\,\epsilon^{\mu\nu\rho\sigma}\,\epsilon^*_\nu\,p_\rho\,p'_\sigma$$

$$-i\,\tilde{T}_2(q^2)\cdot\left[(m_B^2-m_{K^*}^2)\,\epsilon^{*\mu} - (\epsilon^*\cdot q)(p^\mu+p'^\mu)\right]$$

$$-i\,\tilde{T}_3(q^2)(\epsilon^*\cdot q)\left[q^\mu - \frac{q^2}{m_B^2-m_{K^*}^2}(p^\mu+p'^\mu)\right]\Big\} \qquad (97)$$

- Only two independent invariant amplitudes: $\tilde{T}_\parallel(q^2)$ and $\tilde{T}_\perp(q^2)$.

$$\tilde{T}_1(q^2) = \tilde{T}_\perp(q^2), \qquad \tilde{T}_2(q^2) = \frac{2E}{m_B}\tilde{T}_\perp(q^2), \qquad E = \frac{m_B^2-q^2}{2m_B}$$

$$\tilde{T}_3(q^2) = \tilde{T}_\parallel(q^2) + \tilde{T}_\perp(q^2), \qquad (98)$$

Due to helicity conservation and the chiral weak interaction.

Comment: • $B \to K^*$ FFs are not sufficient to describe the QCD dynamics of $B \to K^* ee$ decays.

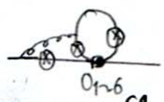
3) QCD factorization formula

$a = \parallel, \perp$

including LO factorizable quark loops.

$$\tilde{T}_a = \xi_a \left[C_a^{(0)} + \frac{\alpha_s C_F}{4\pi} C_a^{(1)} \right]$$

including the correction from expressing QCD FFs in terms of ξ_a, and the NLO quark loops (2-loop) & O_8 effect.

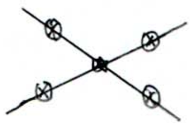


$$+ \frac{\pi^2}{N_c} \cdot \frac{f_B \cdot f_{K^*,a}}{m_B} \, \Xi_a \sum_{\pm} \int_0^\infty \frac{d\omega}{\omega} \phi_B^{\pm}(\omega) \int_0^1 du \, \phi_{K^*,a}(u) \, T_{a,\pm}(u,\omega) \qquad (99)$$

$$\Xi_a = \begin{cases} 1, & \text{for "}\perp\text{"} \\ m_{K^*}/E, & \text{for "}\parallel\text{"}. \end{cases}$$

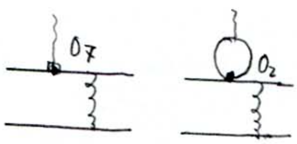
$$T_{a,\pm} = T_{a,\pm}^{(0)} + \frac{\alpha_s C_F}{4\pi} T_{a,\pm}^{(1)},$$

$T_{a,\pm}^{(0)}$: only from the weak annihilation

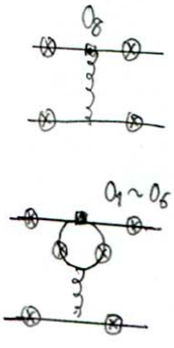


$$T_{a,\pm}^{(1)} = T_{a,\pm}^{(f)} + T_{a,\pm}^{(nf)}$$

$T_{a,\pm}^{(f)}$: from expressing QCD FFs in term of ξ_a. (Hard spectator interaction)



$T_{a,\pm}^{(nf)}$: from the below diagrams



We use the physical FF scheme:

$$\xi_\perp(0) = \frac{m_B}{m_B + m_{K^*}} V^{K^*}(0),$$

$$\xi_\parallel(0) = \frac{2 m_{K^*}}{m_B} A_0^{K^*}(0).$$

4) LO contribution.

ⓐ $$C_{\perp}^{(0)} = C_7^{\text{eff}} + \frac{q^2}{2m_b m_B} Y(q^2),$$

factorizable quark loop with all the flavours.

$$= C_7 - \frac{C_3}{3} - \frac{4}{9} C_4 - \frac{20}{3} C_5 - \frac{80}{9} C_6.$$

$$C_{\parallel}^{(0)} = -\left[C_7^{\text{eff}} + \frac{M_B}{2m_b} Y(q^2) \right], \qquad (100)$$

ⓑ weak annihilation:

$$T_{\perp,+}^{(0)}(u,\omega) = T_{\perp,-}^{(0)}(u,\omega) = 0.$$

no "transverse" contribution.

$$T_{\parallel,+}^{(0)}(u,\omega) = 0.$$ (no "$\phi_B^+(\omega)$" contribution for longitudinally polarized amplitude).

$$T_{\parallel,-}^{(0)}(u,\omega) = -e_q \underbrace{\frac{m_B\,\omega}{m_B\,\omega - q^2 - i0}} \frac{4 m_B}{m_b} \underbrace{(\bar{C}_3 + 3\bar{C}_4)}. \qquad (101)$$

e_q: electric charge of the spectator quark

End-point divergence at $q^2 = 0$!

But $T_\parallel(q^2)$ does not contribute at $q^2 = 0$!

only penguin operators contributes. for $\bar{B}^0$

$$\begin{cases} \bar{C}_3 = C_3 - \frac{C_4}{6} + 16 C_5 - \frac{8}{3} C_6, \\ \bar{C}_4 = \frac{C_4}{2} + 8 C_6. \end{cases}$$

For $B^\pm$ decays, tree operators can contribute.

$$- \frac{V_{ub} V_{us}^*}{V_{tb} V_{ts}^*} \frac{\bar{C}_1 + 3\bar{C}_2}{\bar{C}_3 + 3\bar{C}_4} \times \text{(the above result)}$$

Comments: ① The LP contribution from the photon radiation off the light-quark inside the B-meson.

② To restore the gauge invariance, one needs to sum over the contribution from all the diagrams.

Explicitly, $$A \sim g_{em}^2 \cdot f_B \cdot f_{K^*,\parallel} \left\{ \underline{\frac{2 e_q}{m_B} \int_0^\infty d\omega \frac{\phi_B^-(\omega)}{\omega - q^2/m_B} + \frac{e_q}{q^2}} - \left[\frac{Q_b - Q_s + e_q}{q^2} \right] \right\}$$

③ The weak annihilation diagrams can generate strong phase at time-like q^2.

For $b \to s$ transition, its effect is however suppressed by either the Wilson coefficients or the CKM matrix elements. (Important for $b \to d$ transition).

④ Since the LO transverse amplitude vanishes, the subleading power contribution from the weak annihilation diagrams can be sizeable. In particular, the photon radiation off the spectator quark inside the K^*-meson will provide important contribution to the isospin symmetry breaking effect. Such contribution can be computed as follows.

$$\Delta \tilde{T}_{\perp}\Big|_{\rm ann.} = (-e_q) \cdot \frac{4\pi^2}{3} \frac{f_B\, f_{K^*,\perp}}{m_b\, m_B} \left[\bar{C}_3 + \frac{4}{3}(\bar{C}_4 + 3\bar{C}_5 + 4\bar{C}_6) \right] \int_0^1 du \frac{\phi_{K^*,\perp}(u)}{\bar{u} + u q^2/m_B^2}$$

↓

due to photon radiation from the spectator quark inside the K^*

suppressed by Λ/m_B (e.g., λ_B/m_B).

$$+ e_q \cdot \frac{2\pi^2}{3} \frac{f_B\, f_{K^*,\parallel}}{m_b \cdot m_B} \frac{m_{K^*}}{(1-q^2/m_B^2)\, \lambda_{B,+}(q^2)} \left[\bar{C}_3 + \frac{4}{3}(\bar{C}_4 + 12\bar{C}_5 + 16\bar{C}_6) \right], \quad (102)$$

↓

from the photon radiation off the light-quark inside the B-meson,

↓

suppressed by m_{K^*}/m_B. (due to "longitudinal" polarized K^* meson).

<u>Key point:</u> • The above subleading power correction is calculable in QCDF. (no. end-point div.).

<u>Definition:</u> • $$\lambda_{B,\pm}(q^2) = \int_0^\infty d\omega \frac{\phi_B^{\pm}(\omega)}{\omega - q^2/m_B + i0} \quad (103)$$

<u>Note:</u> • Long-distance photon effect is not relevant here, sine $q^2 \gtrsim m_b \cdot \Lambda$.

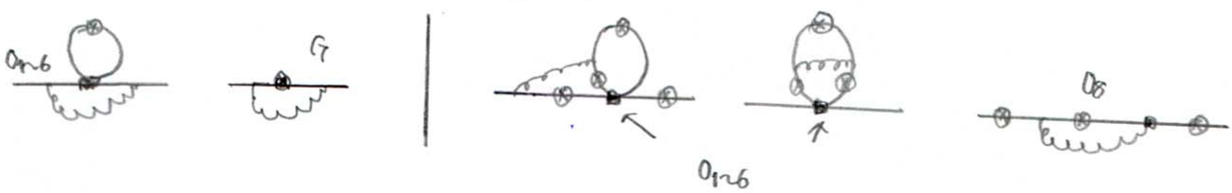
5) NLO contribution

A) Form-factor type correction.

due to expressing QCD FFs as soft FFs

Decomposition:

$$C_a^{(1)} = C_a^{(f)} + C_a^{(nf)}, \qquad (104)$$



- More specifically, we have

$$L = -\frac{m_b^2 - q^2}{q^2} \ln\left(1 - \frac{q^2}{m_b^2}\right).$$

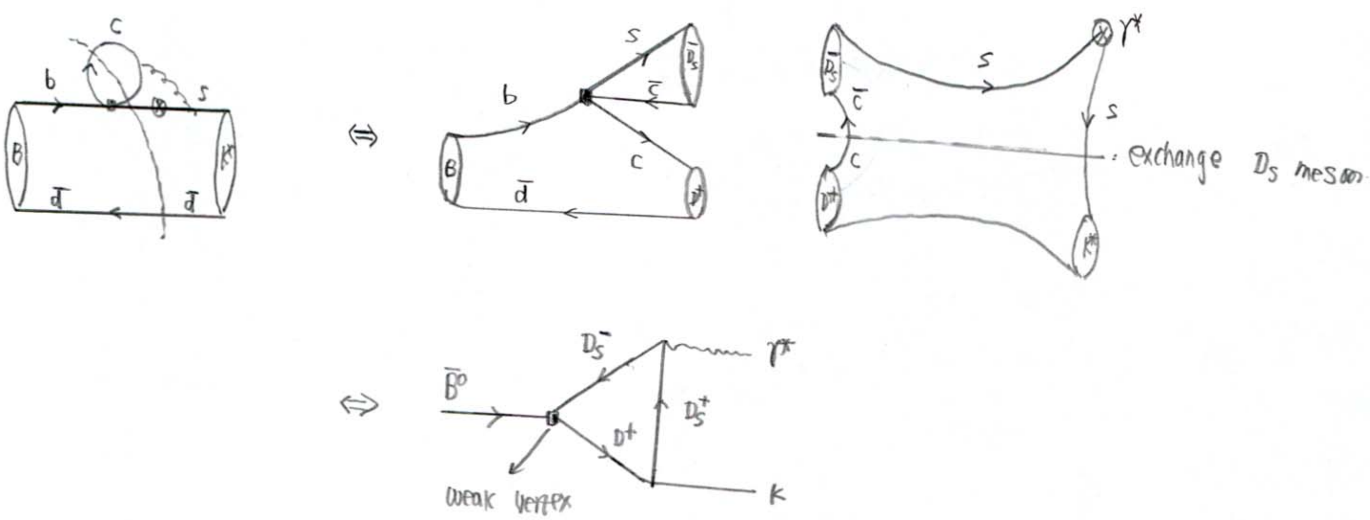
$$C_1^{(f)} = C_7^{eff} \cdot \left(4 \ln\frac{m_b^2}{\mu^2} - 4 - L\right),$$

$$C_{11}^{(f)} = -C_7^{eff}\left(4 \ln\frac{m_b^2}{\mu^2} - 6 + 4L\right) + \frac{m_B}{2m_b} Y(q^2) \cdot (2 - 2L), \qquad (105)$$

- For $C_a^{(nf)}$, only two-loop tree-operator contributions are known.

Three scales, (q^2, m_b, m_c) appear, double expansion in q^2/m_b^2 & m_c/m_b.

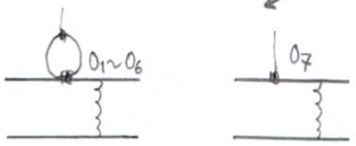
Comments: ① Generate the dominant strong phase!



B) NLO hard spectator interaction.

express QCD FFs in terms of soft FFs.

Decomposition $T^{(1)}_{a,\pm} = T^{(f)}_{a,\pm} + T^{(nf)}_{a,\pm}$, ——— (106)



- More specifically, we have

only $\phi^+_B(\omega)$ relevant, as must be the case.

$$T^{(f)}_{\perp,+}(u,\omega) = C^{\text{eff}}_7 \cdot \frac{2m_B}{\bar{u}E}, \qquad T^{(f)}_{\parallel,+}(u,\omega) = \left[C^{\text{eff}}_7 + \frac{q^2}{2m_b m_B} Y(q^2)\right]\frac{2m_B^2}{\bar{u}E^2},$$

$$T^{(f)}_{\perp,-}(u,\omega) = T^{(f)}_{\parallel,-}(u,\omega) = 0. \qquad \text{——— (107)}$$

- NLO non-factorizable contribution.

① photon radiation off the spectator quark. (QCD → SCET$_I$)

* Similar to the annihilation diagrams.

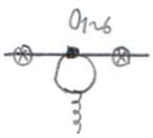
* only contribute to $T^{(nf)}_{\parallel,-}(u,\omega)$ → Longitudinal Polarization.

* End-point divergence at $q^2 = 0$

$$\int_0^\infty \frac{d\omega}{\omega} \frac{m_B\,\omega}{m_B\,\omega - q^2 - i\epsilon} \phi^-_B(\omega).$$

② photon emission from the active quarks; only contribute to $T^{(nf)}_{\perp,+}(u,\omega)$.

$$T^{(nf)}_{\perp,+}(u,\omega) \supset -\frac{4 e_d\, C^{\text{eff}}_8}{u + \bar{u}\, q^2/m_B^2}, \qquad C^{\text{eff}}_8 = C_8 + (4\bar{C}_3 - \bar{C}_5)/3. \qquad \text{——— (108)}$$



Do not contribute to $T_{a,\pm}$ $(a = \parallel, \perp)$

③ Photon emission from the internal quark loop:

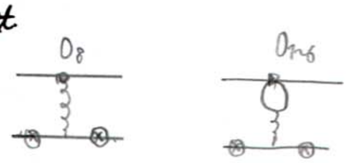
* Contribute to both $T_{\perp,+}^{(nf)}$ & $T_{\parallel,+}^{(nf)}$

q-flavour loop

$t_\perp(u, m_q)$ well defined at $q^2 = 0$, hower, $t_\parallel(u, m_q)$ develops a logarithmic singularity, which is of no consequence due to the "q^2" suppression of the longitudinal contribution relative to the transverse contribution.

* Generate large strong phase even at $q^2 < 4m_q^2$; since the typical scale is "$\bar u m_B^2 + u q^2$". True even for space-like q^2.

④ Aagain, Power suppressed correction with the photon radiation off the spectator quark is relevant to predict the isospin symmetry breaking effect



$$\Delta \tilde{T}_\perp \big|_{hsa} = e_q \frac{\alpha_s C_F}{4\pi} \cdot \frac{\pi^2 f_B}{N_c \cdot m_b \cdot m_B} \Big\{ 12 \cdot C_8^{eff} \cdot \frac{m_b}{m_B} \cdot f_{K^*,\perp} \, X_\perp(q^2)$$

suppressed by λ_B / m_B

$$+ 8 \cdot f_{K^*,\perp} \int_0^1 du \cdot \frac{\phi_\perp(u)}{\bar u + u q^2/m_B^2} \, F_V(\bar u m_B^2 + u q^2)$$

quark loop function, including all flavours & wilson coefficients.

$$- \frac{4 m_{K^*} f_{K^*,\parallel}}{(1 - q^2/m_B^2)\, \lambda_{B,+}(q^2)} \int_0^1 du \int_0^u dv \, \frac{\phi_\parallel(v)}{\bar v} \, F_V(\bar u m_B^2 + u q^2). \qquad (109)$$

"dv" integral due to "$\epsilon^* \cdot z / p \cdot z$" in the definition of the K^* DA

- The function $X_\perp(q^2)$ defined as

$$X_\perp(q^2) = \frac{1}{3} \int_0^1 du \, \frac{\phi_\perp(u)}{\bar{u} + u\, q^2/m_B^2} \left[1 + \underbrace{\frac{1}{\bar{u} + u\, q^2/m_B^2}} \right] \qquad (110)$$

this term will generate end-point divergence at $q^2=0$,

relevant to $B \to K^* \gamma$.

<u>Naive parametrization</u>:

$\phi \in [0, 2\pi)$. ≈ 0.5 GeV.

$$\int_0^1 du \;\to\; (1 + \rho\, e^{i\phi}) \int_0^{1-\Lambda_h/m_B} du \qquad (111).$$

$0 \le \rho \le 1$

- Function $F_V(s)$:

$$F_V(s) = \frac{3}{4} \Big[h(s, m_c)(\bar{C}_2 + \bar{C}_4 + \bar{C}_6) + h(s, m_b)(\bar{C}_3 + \bar{C}_4 + \bar{C}_6)$$
$$+ h(s,0)(\bar{C}_3 + 3\bar{C}_4 + 3\bar{C}_6) - \frac{8}{27}(\bar{C}_3 - \bar{C}_5 - 15\bar{C}_6) \Big] \qquad (112).$$

C) Missing parts at NLO

① NLO weak annihilation contribution

② Two-loop form-factor type contribution from the penguin operators

③ A complete NLL resummation for $\tilde{T}_a(q^2)$. $(a = \parallel, \perp)$

6) Phenomenologies

A). zero-point of A_{FB} for $B \to K^* \ell \ell$:

$$q_0^2 = 3.4 {}^{+0.6}_{-0.5} \ \mathrm{GeV}^2, \qquad \Rightarrow \qquad q_0^2 = 4.2 \pm 0.6 \ \mathrm{GeV}^2.$$

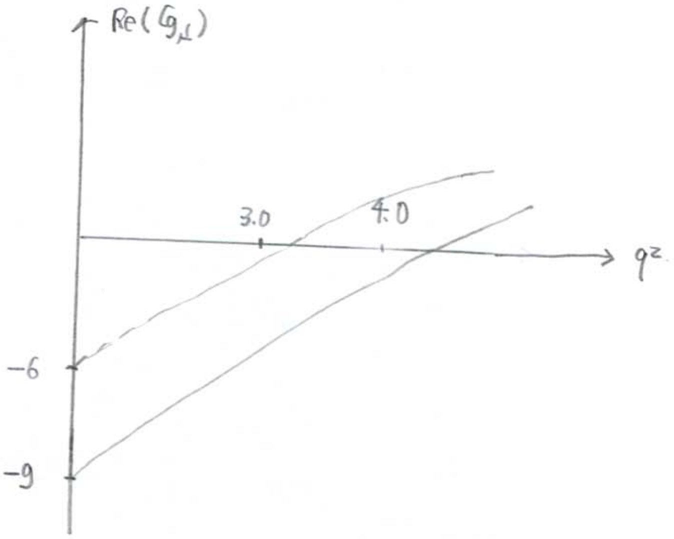
B) C_7 changes sizeably at small q^2 (≈ 0).

$$|C_7|^2_{NLO} / |C_7|^2_{LO} \approx 1.78.$$

Both form-factor type and non-form-factor type corrections are important

C) $\mathrm{Re}(C_{9\perp})$ changes drastically @ small q^2.

$$\Delta C_{9\perp} = \frac{2 m_b m_B}{q^2} \cdot \frac{\tilde{T}_\perp(q^2)}{\xi_\perp(q^2)},$$



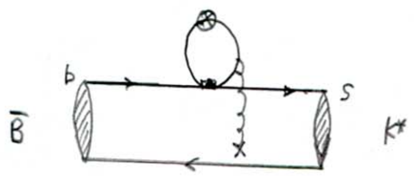
7) Beyond QCDF:

- Subleading power O_8 contribution suffers from the end-point divergence

  $\Rightarrow$ LCSR. (regularized by the threshold parameter).

- Subleading power charm-loop effect $\Rightarrow$ LCSR with B-meson and K^* DAs.

[due to "$1/q^2$" enhancement and the color enhancement]



- How to go beyond the small q^2 region ?

  Hadronic dispersion relation, heavy quark expansion at large q^2, ($\sqrt{q^2} \sim O(m_B)$).

- How to understand the various " anomalies "?

  - "P_5" anomaly: Subleading power contribution

  - "R_K" anomaly: $\dfrac{BR(B \to K\mu\mu)}{BR(B \to K ee)}$: [miss-measurement, QED corrections].

Lecture V   Introduction to QCD Sum Rules on the Light Cone   Aug. 18. 2017

1) History of QCD sum rules:

- SVZ SR (70's): mostly for the "static" properties of hadrons, [mass, decay constant, ...]

- Main idea:

correlation function

local OPE ↙ ↘ dispersion relation

QCD representation
[including both the perturbative contribution and the condensate effect]

Hadronic representation
[including the information of ground state and of the continuum states]

↓ matching with the duality approximation

preliminary QCD sum rules

↓ Borel transformation

$$\Pi(M^2) \equiv \hat{B}_{M^2}\, \Pi(q^2) = \lim_{\substack{-q^2,\, n \to \infty \\ -q^2/n = M^2}} \frac{(-q^2)^{n+1}}{n!} \left(\frac{d}{dq^2}\right)^n \Pi(q^2)$$

Final QCD sum rules

Assumption: existence of of hadrons in nature, NOT a proof of the existence.

- An example:

$j_\mu = \bar{\psi}\gamma_\mu\psi$

$$\Pi_{\mu\nu}(q) = i\int d^4x \; e^{iq\cdot x} \; \langle 0 | T\{ j_\mu(x),\; j_\nu(0)\} | 0\rangle$$

$$= (q_\mu q_\nu - q^2 g_{\mu\nu})\; \Pi(q^2) \qquad (113)$$

① QCD representation:

$\Pi(q^2=0)=0$

$\frac{1}{\pi}\,\mathrm{Im}\,\Pi(s) = \rho^{QCD}(s) \rightarrow$ QCD spectral function.

$$\Pi(q^2) = \frac{1}{\pi}\cdot q^2 \int_{-\infty}^{+\infty} ds \; \frac{\mathrm{Im}\,\Pi(s)}{s\,(s-q^2)}, \qquad (114)$$

$v = \sqrt{1-4m_q^2/s}$

$$\mathrm{Im}\,\Pi(s) = \frac{1}{8\pi}\, v\,(3-v^2)\,\theta(s-4m_q^2)\cdot\left\{1+\alpha_s C_F\left[\frac{\pi}{2v} - \frac{v+3}{4}\left(\frac{\pi}{2}-\frac{3}{4\pi}\right)\right]\right\}$$

$$+ \cdots, \qquad (115)$$

condensate contribution

② Hadronic representation:

$\langle V(q)| j_\mu |0\rangle = f_V\, m_V\, \epsilon_\mu^{(V)*}(q)$

hadronic spectral function

$$\Pi(q^2) = q^2\left\{\frac{f_V^2}{m_V^2\,(m_V^2-q^2)} + \int_{s_0^h}^{\infty} ds\; \frac{\rho^h(s)}{s\,(s-q^2)}\right\} \qquad (116)$$

③ Duality approximation: (Semi-local) (difficult to justify this approximation).

$$\int_{s_0^h}^{\infty} ds\; \frac{\rho^h(s)}{s\,(s-q^2)} = \int_{s_0}^{\infty} ds\; \frac{\rho^{QCD}(s)}{s\,(s-q^2)} \qquad (117)$$

④ Continuum subtraction:

$$\frac{f_V^2}{m_V^2\,(m_V^2-q^2)} \cong \int_{4m_q^2}^{s_0} ds\; \frac{\rho^{QCD}(s)}{s\,(s-q^2)} \qquad (118)$$

⑤ Borel transformation:

$$B_{M^2}\left(\frac{1}{(m^2-q^2)^k}\right) = \frac{1}{(k-1)!}\,\frac{1}{M^{2(k-1)}}\,\exp\left(-\frac{m^2}{M^2}\right) \qquad (119)$$

$$\frac{f_V^2}{m_V^2}\, e^{-m_V^2/M^2} = \int_{4m_q^2}^{s_0} \frac{ds}{s}\, e^{-s/M^2}\, \rho^{QCD}(s) \qquad (120)$$

$\Leftrightarrow$

$$f_V^2\, e^{-m_V^2/M^2} = \int_{4m_q^2}^{s_0} ds\, e^{-s/M^2}\, \rho^{QCD}(s) \qquad (121)$$

[(121) can be obtained from (120) by taking the deriative " $-\frac{d}{d(1/M^2)}$ " on both sides].

- Eq. (121) gives the final form of the QCD SR for f_V.

<u>Comments:</u> ① QCD SR cannot be treated as mathematically formulae naively, due to many approximations employed in the construction (perturbative correction, power correction, subleading power contribution).

② Not all the non-perturbative quantities (objects) can be evaluated from SVZ sum rules, in particular for exotic hadrons.

③ In many cases, SVZ-SR fail to compute the "dynamical" objects with more complicated strong interaction dynamics. (e.g., hadronic form factors with large momentum transfer). $\Rightarrow$ light-cone sum rules.

2) Why do we need light-cone sum rules?

① OPE (short-distance expansion in condensates) upsets power counting in the large momentum/mass

Ex. ⓐ The pion e.m. form factor with large Q^2.

$$F_\pi(Q^2) \sim \# \frac{1}{Q^2} + \# \frac{\langle g_s^2 G^2 \rangle}{M^4} + \# Q^2 \cdot \frac{\langle \bar{q}q \rangle^2}{M^8} + \cdots, \qquad (122)$$

$M^2 \sim O(1\,\mathrm{GeV}^2)$, Borel mass.

The sum rule result for $F_\pi(Q^2)$ starts to rise at $Q^2 > 3 \sim 5\ \mathrm{GeV}^2$. Such behaviour is clearly unphysical and indicates the breakdown of the local OPE.

Ex. ⓑ The $B \to \rho e \nu$ form factor $A_1(q^2=0)$ from the 3-point QCD SR:

$$A_1(q^2=0) \sim \# \frac{1}{m_b^{3/2}} + \# m_b^{1/2} \langle \bar{q}q \rangle + \# m_b^{3/2} \langle \bar{q} g_s \sigma G q \rangle + \cdots \qquad (123)$$

The rise of the form factor $A_1(q^2=0)$ from the 3-point QCDSR is due to the following problem: expansion in slowly varying (vacuum) fields is inadequate if a short-distance subprocess is involved.

Note: For $B \to \pi e \nu$, the 3-point QCDSR works, since (accidently) the quark condensate contribution is only $\sim m_b^{-1/2}$, and the problem is numerically less important.

② • Contamination of the sum rule by "non diagonal" transitions of the ground states to excited states, (when applying the single dispersion relation for the 3-point SR).

• Introducing the double dispersion relation to get rid of the non-diagonal transitions by using the double dispersion relation & the double Borel transformation.

$$\int dx\, dy\; e^{-ip_1 x + ip_2 y} \quad \langle 0 | T\{ H(y),\ J(0),\ H(x) \} | 0 \rangle$$

$$\sim \langle 0 | H | h \rangle \frac{1}{m_h^2 - p_1^2} \langle h | J | h \rangle \underbrace{\frac{1}{m_h^2 - p_2^2}}_{\downarrow} \langle h | H | 0 \rangle + \cdots \qquad (124)$$

For single dispersion relation, setting $p_1 = p_2$, then (with zero momentum transfer).

$$\frac{1}{(m_h^2 - p^2)^2} \langle h | J | h \rangle + \underbrace{\frac{1}{(m_h^2 - p^2)(m_h^2 - m_{h'}^2)}}_{\text{not suppressed after single Borel transformation}} \langle h | J | h' \rangle + \cdots \qquad (125)$$

• The problems of double dispersion relation:

ⓐ The results highly depend on the shape of the duality region.

ⓑ There are formal problems with double dispersion relations <u>in the decay kinematics</u> in presence of Landau singularities.

ⓒ It's becoming increasingly clear that suppression of non diagonal transitions by the double transformation is more formal than real.

3) LCSR for $B \to \pi$ form factors at large recoil : General discussions

A) Different versions of LCSRs :

① LCSR with π DAs in QCD / SCET

② LCSR with B-meson DAs in QCD / SCET.

B) General strategies

① Only replace the "local OPE" for the SVZ SR by the "light-cone OPE".

② In general, "LCSR" ≈ "QCDF for the correlation function"

model independent.

+ " semi-global quark-hadron duality".

" difficult to quantify the systematical uncertainty "

③ Non-perturbative inputs are the hadronic DAs on the light cone with increasing twists. [⇒ RGEs of LCDAs ⇒ Conformal symmetry analysis].

④ Subleading power corrections to the correlation function are generally rather complicated, since various light-cone DAs are related by EOM in a nontrivial way and the operator-mixing pattern becomes involved.

4) LCSR for $B \to \pi$ form factors at large recoil: detailed analysis

A) step 1: starting with the following correlation function.

$$p_\mu = \frac{n\cdot p}{2}\bar{n}_\mu + \frac{\bar{n}\cdot p}{2} n_\mu, \quad \text{with} \begin{cases} \bar{n}\cdot p \sim O(\Lambda) \\ n\cdot p \sim O(m_B) \end{cases}$$

$$\Pi_\mu(n\cdot p, \bar{n}\cdot p) = \int d^4x\, e^{ip\cdot x} \langle 0 | T\{\bar{d}(x)\, \not{n}\, \gamma_5\, u(x),\ \bar{u}(0)\, \Gamma_\mu\, b(0)\} | \bar{B}(p+q)\rangle. \qquad = p_B = m_B \not{v}$$

$$= \Pi(n\cdot p, \bar{n}\cdot p)\, n_\mu + \tilde{\Pi}(n\cdot p, \bar{n}\cdot p)\, \bar{n}_\mu. \qquad (126)$$

$$\gamma_\mu = \frac{\not{n}}{2}\bar{n}_\mu + \frac{\not{\bar{n}}}{2} n_\mu,$$

no "transverse" component

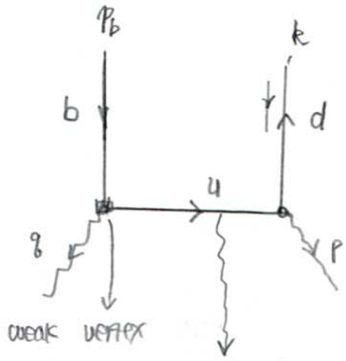
Power counting scheme:

$$\begin{cases} n\cdot p \simeq \dfrac{m_B^2 + m_\pi^2 - q^2}{m_B} = 2E_\pi \\ \bar{n}\cdot p \sim O(\Lambda_{QCD}) \end{cases} \qquad (127)$$

$\downarrow$

hard-collinear interpolating current.

B) step 2: QCDF for the correlation function at tree level.



$(p-k)^2$

$= n\cdot p\ \bar{n}\cdot(p-k)$

$\sim O(m_b \Lambda)$

$\Rightarrow$ light-cone separation.

$$\Pi^{(0)}_{\mu,\,\text{pert.}}(n\cdot p\ \ \bar{n}\cdot p)$$

$$= \frac{i}{2}\, \frac{1}{\underbrace{\bar{n}\cdot p - \omega + i0}_{\omega = \bar{n}\cdot k}}\ \underbrace{\bar{d}(k)\, \not{n}\, \gamma_5\, \not{\bar{n}}\, \gamma_\mu\, b(p_b)}_{= 2\bar{n}_\mu\, \bar{d}(k)\, \not{n}\, \gamma_5\, b(p_b)} \qquad \gamma_\mu \Rightarrow \frac{\not{n}}{2}\bar{n}_\mu$$

$$= i\, \frac{\bar{n}_\mu}{\bar{n}\cdot p - \omega + i0}\ \underbrace{\bar{d}(k)\, \not{n}\, \gamma_5\, b(p_b)}_{\text{B-meson LCDAs}} \qquad (128)$$

Definition of B-meson LCDA:

$$[z_2, z_1] = P\exp\left[i g_s \int_{z_2}^{z_1} dz^\mu A_\mu(z)\right]$$

$$\langle 0 | \bar{q}_\beta(z)\, [z, 0]\, h_{v,\alpha}(0) | \bar{B}(v) \rangle$$

$$= -\frac{i \tilde{f}_B m_B}{4} \left[\frac{1+\not v}{2} \left\{ \tilde{\phi}_B^+(\tau) \not n + \tilde{\phi}_B^-(\tau) \not{\bar n} + \frac{\tilde{\phi}_B^-(\tau) - \tilde{\phi}_B^+(\tau)}{\tau} \not z_\perp \right\} \gamma_5 \right]_{\alpha\beta} \qquad (129)$$

$$\tau = \frac{n\cdot z}{2}$$

light-cone projector: $\{\cdots\} \to \left\{ \phi_B^+(\omega) \not n + \phi_B^-(\omega) \not{\bar n} - \frac{2\omega}{D-2} \phi_B^-(\omega) \gamma_\perp^\rho \frac{\partial}{\partial k_{\perp\rho}} \right\}$

Note: $z_\mu = \frac{n\cdot z}{2} \bar{n}_\mu + \frac{\bar{n}\cdot z}{2} n_\mu + z_{\perp,\mu}$,

with $n\cdot z \gg z_\perp \gg \bar{n}\cdot z$, $\qquad (130)$

Inserting eq. (129) into eq. (128) yields.

$$\Pi_\mu^{(0)}(n\cdot p, \bar{n}\cdot p) = \tilde{f}_B(\mu)\, m_B \int_0^\infty d\omega \frac{\phi_B^-(\omega)}{\omega - \bar{n}\cdot p - i0} \bar{n}_\mu + O(\alpha_s) \qquad (131)$$

C) Step 3: Hadronic dispersion relation.

$$\langle \pi(p) | \bar{d} \not n \gamma_5 u | 0 \rangle = -i\, n\cdot p\, f_\pi$$

$$\Pi_\mu(n\cdot p, \bar{n}\cdot p) = \frac{f_\pi\, n\cdot p\, m_B}{2(m_\pi^2 - p^2)} \left\{ \bar{n}_\mu \left[\frac{n\cdot p}{m_B} f_{B\pi}^+(q^2) + f_{B\pi}^0(q^2) \right] \right.$$

$$\left. + n_\mu \frac{m_B}{n\cdot p - m_B} \left[\frac{n\cdot p}{m_B} f_{B\pi}^+(q^2) - f_{B\pi}^0(q^2) \right] \right\}$$

$$+ \int_{\omega_s}^\infty d\omega' \frac{1}{\omega' - \bar{n}\cdot p - i0} \left[\rho^h(\omega', n\cdot p)\, n_\mu + \tilde{\rho}^h(\omega', n\cdot p)\, \bar{n}_\mu \right] \qquad (132)$$

D) step 4. Matching with the aid of the duality approximation & Borel transformation (for $\bar{n}\cdot p$).

$$f^{+}_{B\pi}(q^2) = \frac{\tilde{f}_B(\mu)\, m_B}{f_\pi\, n\cdot p} \operatorname{Exp}\left[\frac{m_\pi^2}{\bar{n}\cdot p\, \omega_M}\right] \int_0^{\omega_s} d\omega' \; e^{-\omega'/\omega_M} \; \phi_B^-(\omega') + O(\alpha_s),$$

$$f^{0}_{B\pi}(q^2) = \underbrace{\frac{n\cdot p}{m_B}}_{\text{large recoil symmetry}} f^{+}_{B\pi}(q^2) + O(\alpha_s) \qquad (133)$$

<u>Comments:</u>

① Since $\phi_B^{+}(\omega)$ does not enter the factorization formulae of $\Pi_\mu(n\cdot p, \bar{n}\cdot p)$ (see eq. (131)) at tree level, and $\phi_B^{\pm}(\omega)$ do not mix under renormalization at one loop (in the massless light-quark limit), the convolution integrals of $\phi_B^{+}(\omega)$ entering the one-loop QCD contributions must be IR finite, due to the absense of the subtraction term.

② Along the same line, only "$\tilde{\Pi}$" survives at tree level, hence the one-loop contributions to "Π" in QCD must be IR finite.

③ Power counting.

$$\omega_M = \frac{\overset{\text{Borel mass} \sim O(\Lambda^2),}{M^2}}{\bar{n}\cdot p} \sim O(\Lambda^2/m_b), \qquad \omega_s = \frac{s_0}{\bar{n}\cdot p} \sim O(\Lambda^2/m_b). \qquad (134)$$

$$\Rightarrow \quad f^{+}_{B\pi} \sim f^{0}_{B\pi} \sim (\Lambda/m_b)^{3/2} \qquad (135)$$

<u>note:</u> the scaling of the interpolating momentum has been changed from the hard-collinear to collinear mode.

E) Step 5. QCD factorization for the correlation function at $O(\alpha_s)$.

(a) (b) (c) (d)

- Evaluating the QCD diagrams with the method of regions to extract the hard & jet functions

- The resulting factorization formulae at NLO in α_s.

$$\Pi = \tilde{f}_B \cdot m_B \sum_{k=\pm} C^{(k)}(n\cdot p, \mu) \int_0^\infty \frac{d\omega}{\omega - \bar{n}\cdot p} J^{(k)}\left(\frac{\mu^2}{n\cdot p\, \omega}, \frac{\omega}{\bar{n}\cdot p}\right) \phi_B^{(k)}(\omega, \mu),$$

$$\tilde{\Pi} = \tilde{f}_B\, m_B \sum_{k=\pm} \tilde{C}^{(k)}(n\cdot p, \mu) \int_0^\infty \frac{d\omega}{\omega - \bar{n}\cdot p} \tilde{J}^{(k)}\left(\frac{\mu^2}{n\cdot p\, \omega}, \frac{\omega}{\bar{n}\cdot p}\right) \phi_B^{(k)}(\omega, \mu). \tag{136}$$

The hard functions are exactly the same as that for the heavy-to-light current.

$$\bar{q}\,\Gamma_\mu\, b \rightarrow [\, C_4\, \bar{n}_\mu + C_5\, v_\mu\,]\; \bar{\xi}_{\bar{n}}\, W_{hc}\, Y_s^\dagger\, b_v + \cdots \tag{137}$$

then, $C^{(-)} = \frac{C_5}{2}$, $\quad \tilde{C}^{(-)} = C_4 + \frac{C_5}{2}$.

$$\underbrace{C^{(+)} = \tilde{C}^{(+)}}_{\uparrow} = 1, \tag{138}$$

QCD correction does not change the Dirac structure of SCET operator.

At tree level, only $\phi_B^-(\omega)$ contributes, hence there's no contribution to $C^{(+)}$ & $\tilde{C}^{(+)}$.

Also, the hard function only comes from the diagram (a) and the b-quark w.f. renormalization.

The jet function is new, and it can be contributed from all the diagrams.

F) Step 6: QCD resummation of large logarithms.

Setting the factorization scale to be a hard-collinear scale, and ignoring the summation of $\ln^n(\mu_{hc}/\mu_0)$ from the evolution of the B-meson DA, we only need to consider the resummation of large logarithms in the hard functions.

RGE:

$$\frac{d}{d\ln\mu}\tilde{C}^{(-)}(n\cdot p,\mu) = -\left[\underbrace{\Gamma_{cusp}(\alpha_s)}_{=\frac{\alpha_s C_F}{4\pi}\left[\Gamma_{cusp}^{(0)}+\frac{\alpha_s}{4\pi}\Gamma_{cusp}^{(1)}+\cdots\right],\ \Gamma_{cusp}^{(0)}=4!}\ln\frac{\mu}{n\cdot p} + \underbrace{\gamma(\alpha_s)}_{=\frac{\alpha_s C_F}{4\pi}\left[\gamma^{(0)}+\left(\frac{\alpha_s}{4\pi}\right)\gamma^{(1)}+\cdots\right],\ \gamma^{(0)}=5}\right]\tilde{C}^{(-)}(n\cdot p,\mu). \qquad (139)$$

$\Rightarrow$

$$\tilde{C}^{(-)}(n\cdot p,\mu) = U_1(n\cdot p,\mu_{h_1},\mu)\,\tilde{C}^{(-)}(n\cdot p,\mu_{h_1}), \qquad (140)$$

Similarly,

$$\tilde{f}_B(\mu) = U_2(\mu_{h_2},\mu)\,\tilde{f}_B(\mu_{h_2}). \qquad (141)$$

G) Step 7: Resummation improved LCSRs.

$$f_\pi\, e^{-m_\pi^2/(n\cdot p\,\omega_M)}\left\{\frac{n\cdot p}{m_B}f_{B\pi}^{+}(q^2),\ f_{B\pi}^{0}(q^2)\right\} \qquad r = n\cdot p/m_b.$$

$$= \left[U_2(\mu_{h_2},\mu)\,\tilde{f}_B(\mu_{h_2})\right]\int_0^{\omega_s}d\omega'\,e^{-\omega'/\omega_M}\Big[r\cdot\phi^{+}_{B,\mathrm{eff}}(\omega',\mu) \qquad \phi^{+}_{B,\mathrm{eff}} = \frac{\alpha_s C_F}{4\pi}\int_{\omega'}^{\infty}\frac{d\omega}{\omega}\phi_B^{+}(\omega,\mu)$$

$$+\left[U_1(n\cdot p,\mu_{h_1},\mu)\,\tilde{C}^{(-)}(n\cdot p,\mu_{h_1})\right]\cdot\phi^{-}_{B,\mathrm{eff}}(\omega',\mu) \qquad \phi^{-}_{B,\mathrm{eff}} = \phi_B^{-}(\omega',\mu)+O(\alpha_s)$$

$$\pm\frac{n\cdot p-m_B}{m_B}\Big(\phi^{+}_{B,\mathrm{eff}}(\omega',\mu)+C^{(-)}(n\cdot p,\mu)\,\phi_B^{-}(\omega',\mu)\Big)\Big] \qquad (142)$$

symmetry breaking effect

$\phi^{+}_{B,\mathrm{eff}}$: from the h-c correction. $C^{(-)}$: from the hard correction.

Comments: (1) symmetry breaking effect from both the hard & hard-collinear correction

(2) $$\phi^-_{B,\text{eff}}(\omega',\mu) \supset \frac{\alpha_s C_F}{4\pi}\int_{\omega'}^{\infty} d\omega \, \underbrace{\ln\frac{\omega}{\omega'}}\, \frac{d\phi_B^-(\omega,\mu)}{d\omega} \qquad (143)$$

$\omega \sim O(\Lambda)$, due to $\phi_B^-(\omega,\mu)$

$\omega' \sim O(\omega_s) \sim O(\Lambda^2/m_b)$

$\uparrow$

from the integral in eq. (142)

$\ln\frac{\Lambda}{\Lambda^2/m_b} \sim \ln\frac{m_b}{\Lambda}$,

$\downarrow$

reproduce the structure of the end-point divergences.

H) Final comments

① QCD factorization for the correlation function can be generalized to the 3-particle case, which involves highly non-trivial operator mixing.

② A complete NLL resummation requires the two-loop RGEs of the B-meson LCDAs, which is hower not known yet.

③ LCSR can be also applied to compute the QED corrections to $B\to\pi\ell\nu$, which is however not a trivial task.