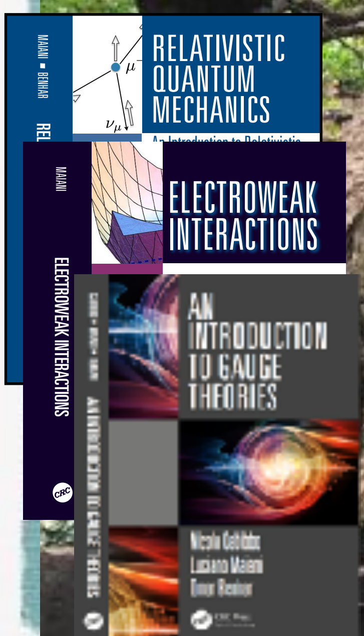


# Six lectures in the Standard Theory of Elementary Particle Physics

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## Lecture 2
## Breaking the Symmetry

# 1. A gauge theory of the Electromagnetic and Weak Interactions

- The success of the V-A theory has given momentum to the idea that Fermi interactions are mediated by an intermediate vector boson, IVB;
- Yang - Mills theory provides the conceptual framework to link the IVB to the symmetries which are associated to the weak currents;
- First proposals by J. Schwinger (in the ‘50s): he considered O(3), with gauge fields $W^{\pm}$ and A;
- S. L. Glashow (1961) extended O(3) to SU(2) ⊗U(1) and produced the first unified electroweak theory.

• Matter: $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}; \ e_R$ ↑ $T^+$ ↓ $T^-$

$$\nu_{L,R} = \frac{1 \mp \gamma_5}{2}\nu, \ \text{same for } e$$

$SU(2)_{Weak}$: $[T^+, T^-]=2T^3$

Weak Hypercharge: $Q= T^3 +1/2\, Y$

Y= -1, -2

- Y is necessary to keep Q in the gauge group
- Y commutes with $SU(2)_{Weak}$

$G= SU(2)_{Weak} \otimes U(1)_Y$

Gauge Fields: $W^{1,2,3}_\mu$, $B_\mu$

S. L. Glashow, *Partial Symmetries of Weak Interactions*, Nucl. Phys. **22** (1961) 579.

# V-A

- Lepton fields appear in the Fermi interaction always multiplied by $(1-\gamma_5)$
- We introduce fields with definite chirality, $e_L$, $\nu_L$
- $e_L$: destroys a ***left-handed electron*** or creates a ***right-handed positron***
- $e_R$: destroys a ***right-handed electron*** or creates a ***left-handed positron***
- Similarly for $\nu_L$.
- If neutrino has zero mass, we can stop here: this is the two-component neutrino theory (Landau, Lee&Yang…)
- The electron is a Dirac field: we must introduce $e_R$, which does not appear in the Fermi interaction and is thus an $SU(2)_{Weak}$ singlet.
- $Q(e_R) = -1 = Y(e_R)/2 \rightarrow Y(e_R) = -2$
- Same considerations for the $\nu_\mu - \mu$ doublet

Matter $$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}_{Y=-1/2} ;\ (e_R)_{Y=-2};\ \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}_{Y=-1/2} ;\ (\mu_R)_{Y=-2}$$

Gauge Fields (forces): $W_\mu^{1,2,3}; B_\mu$

The (Dirac) electron mass couples $e_L$ with $e_R$, so it is not invariant:

$$L_{mass} = m_e\, \bar{e}e = m_e(\bar{e}_R\, e_L + h.c.)\ :\Delta I_W = 1/2$$

**All particle are massless in the symmetry limit!**

# 2. The Abelian case

P. W. Higgs, *Spontaneous Symmetry Breakdown without Massless Bosons*, Phys. Rev. **145** (1966) 1156.
F. Englert, R. Brout, *Broken Symmetry and the Mass of Gauge Vector Mesons*, Phys. Rev. Lett. **13** (1964) 321.

- Symmetry U(1)
- Introduce scalar, complex field, 2 real components:

$$\phi(x) = \frac{1}{\sqrt{2}}[\phi_1(x) + \phi_2(x)];\ \phi(x) \to e^{i\alpha(x)}\phi(x)$$

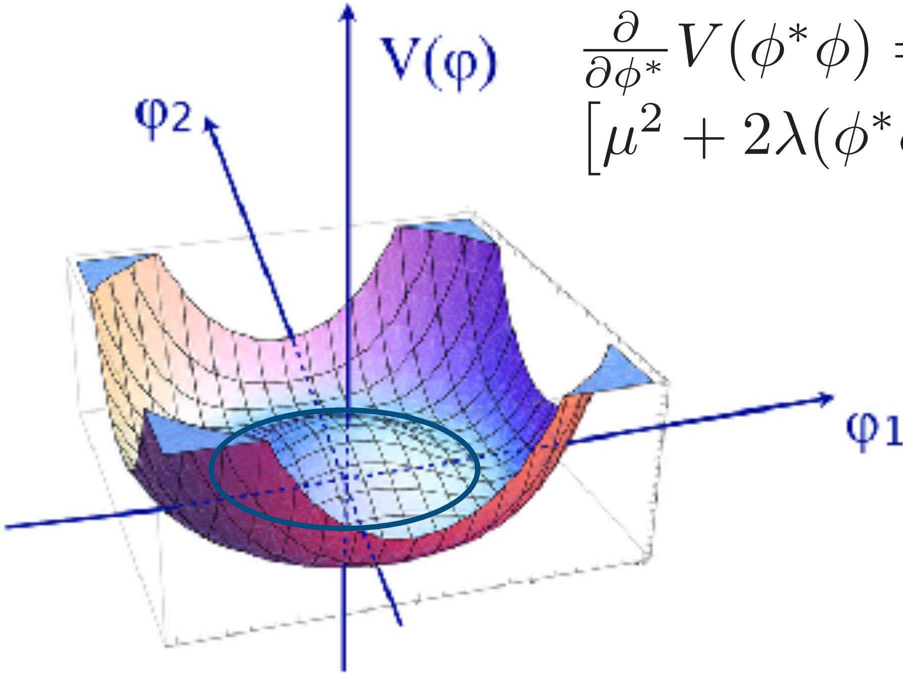
- and a lagrangian invariant under global phase transformations

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi) \qquad V(\phi) = C + \mu^2 \phi^* \phi + \lambda(\phi^* \phi)^2$$

- This invariant Lagrangian is renormalizable by power counting (can you prove it?)
- Stability: $\lambda > 0$
- Ground state (vacuum): minimum of H:

$$\mathbf{H} = \int d^3x\ H(x);\ \ H(x) = \vec{\nabla}\phi^\star \cdot \vec{\nabla}\phi + V(\phi)$$

- Translationally invariant vacuum: $\phi$=const.
- therefore $\phi$ must be the minimum of V.
- The minimum may not be invariant under the symmetry: ***spontaneous symmetry breaking***




$$\frac{\partial}{\partial \phi^*} V(\phi^* \phi) = 0$$
$$\left[\mu^2 + 2\lambda(\phi^*\phi)\right]\phi = 0$$

Solutions depend upon the sign of $\mu^2$

(a) $\mu^2 > 0$:

$\varphi = 0$

Exact symmetry, degenerate masses for $\varphi_{1,2}$ (or $\varphi^{+/-}$)

(b) $\mu^2 < 0$:

$$\varphi^+\varphi = -\frac{\mu^2}{\lambda} \qquad \eta = \sqrt{\varphi^+\varphi} = \sqrt{\frac{-\mu^2}{\lambda}}$$

$$V(0) = C$$
$$V(\phi^*\phi)|_{min.} = C - \frac{\mu^4}{4\lambda} < C$$

Broken symmetry: the stable vacuum corresponds to $\varphi \neq 0$.

What is the spectrum? Develop fields around $\eta$

***Note.*** Minima on the circle: $\varphi = \eta e^{i\alpha}$
there are infinitely many degenerate vacua, differing by a phase transformation

# A more convenient notation

$$V = \lambda(\phi^*\phi - \eta^2)^2$$

- Minimum in $\phi=\eta$;
- Coeff. of quadratic round $\phi=0$ is negative: $V^{(2)} = -2\lambda\eta^2\phi^*\phi$
- Oscillations around stable minimum parameterized by σ and ξ:

$$\phi(x) = \eta + \frac{\sigma + i\xi}{\sqrt{2}}$$
$$\phi^*\phi - \eta^2 = \sqrt{2}\eta\sigma + \sigma^2 + \xi^2$$

- Expanding the Lagrangian in powers of the fields we find:

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\xi\partial^\mu\xi\right) - \frac{1}{2}(4\lambda\eta^2)\sigma^2 + interactions$$

- "interactions" are terms at least cubic in the fields σ and ξ
- no term linear in the fields: we expanded around a minimum
- σ is a massive scalare field: $m_\sigma^2=4\lambda\eta^2$
- no term quadratic in ξ: the field ξ is a massless (Nambu-Goldstone) boson

The existence of a massless scalar particle is the sign of the spontaneous breaking of a continuous global symmetry (Y. Nambu1961, J. Goldstone 1962).

# Unitary gauge: the Higgs-Brout-Englert miracle

$$\phi(x) = \eta + \frac{\sigma + i\xi}{\sqrt{2}} \sim e^{i\frac{\xi}{\eta\sqrt{2}}}(1 + \frac{\sigma}{\sqrt{2}}) = U[\xi(x)]\phi_{real}$$

- with a change in parametrization we see that with a gauge transformation ***we can make $\phi$ real and completely independent from $\xi$***.
- In the ***Unitary Gauge***, the Goldstone boson has disappeared and the scalar spectrum consists only of a scalar massive particle, the Higgs-Brout-Englert boson
- the gauge invariant scalar lagrangian, in the unitary gauge $\phi$ =real, is

$$\mathcal{L}_{loc.symm} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + g^2\eta^2 A_\mu A^\mu(1 + \frac{\sigma}{\eta\sqrt{2}})^2 - V =$$

$$= \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + m_\sigma^2\sigma^2) + \frac{1}{2}(2g^2\eta^2)A_\mu A^\mu + interactions$$

# The Higgs-Brout-Englert miracle (cont'd)

$$\mathcal{L}_{loc.symm} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} =$$

$$= \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + m_\sigma^2\sigma^2) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(2g^2\eta^2)A_\mu A^\mu + interactions$$

- particle spectrum of the ***unbroken theory*** had two massive scalars and one massless vector
- ***spontaneous broken global symmetry***: one massles scalar (Goldstone boson) one massles vector one massive scalar
- Higgs-Brout-Englert mechanism: the ***spontaneously broken local symmetry*** has no massless particles!
- spectrum made by one massive scalar and one massive vector

| — | – | $\phi_1$ | $\phi_2$ | $A^\mu$ | no. degrees of freedom |
|---|---|---|---|---|---|
| glob.&loc. sym. | $\mu^2 > 0$ | $\mu$ | $\mu$ | 0 | 1+1+2=4 |
| global sym. | $\mu^2 < 0$ | $M_H$ | 0 | 0 | 1+1+2=4 |
| local sym. | $\mu^2 < 0$ | $M_H$ | — | $M_A$ | 1+0+3=4 |

Table 1: Mass spectrum of the particles in the Abelian model.

- the Goldstone boson provides the missing helicity=0 state to make a massive vector particle!

# 3. The Weinberg-Salam theory

S.~Weinberg, *A Model of Leptons*, Phys. Rev. Lett. **19** (1967) 1264.
A. Salam, in N. Svartholm: *Elementary Particle Theory*, Proc. Nobel Symp., Lerum Sweden (1968) 367.

- We try now with the non-abelian symmetry $SU(2)_W \otimes U(1)_Y$ introduced by Glashow, as done by S. Weinberg and A. Salam in 1967-68
- Assume scalar, complex fields, making a doublet with Y=+1 and a lagrangian invariant under global $SU(2)_W \otimes U(1)_Y$ transformations:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \qquad \begin{array}{l} \mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - V(\phi) \\ V(\phi) = C + \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \end{array}$$

$$\mathbf{H} = \int d^3x \; H(x); \quad H(x) = \vec{\nabla}\phi^+ \cdot \vec{\nabla}\phi + V(\phi)$$

- Minimum of H in $\phi=\eta$: $\phi(x) = \bar{\phi} + \xi(x); \; \bar{\phi} = \begin{pmatrix} 0 \\ \eta \end{pmatrix}$
- Oscillations above vacuum parameterized by $\xi$: $\xi = \phi - \bar{\phi} = \begin{pmatrix} \frac{\xi_1 + i\xi_2}{\sqrt{2}} \\ \frac{\sigma + i\xi_4}{\sqrt{2}} \end{pmatrix}$
- Expanding the Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \xi_1 \partial^\mu \xi_i) - \frac{1}{2}(4\lambda^2)\sigma^2 + interactions$$

- "interactions" are terms of order 3 or higher in the $\xi$s
- $\sigma$ is a massive scalare field: $m_\sigma^2 = 4\lambda\eta^2$
- the fields $\xi^i$ are the Goldstone bosons, as indicated by the lack of quadratic terms in $\xi^i$ in $\mathcal{L}$; there is one Goldstone boson for each broken generator

# Goldstone fields are eliminated by a local gauge transformation

- Given an SU(2) spinor, $\phi$, there exists a SU(2)xU(1) transformation that brings it in a standard form, with only the down component $\neq 0$ and real (prove it)
- Write the non vanishing component as $\eta+\sigma/\sqrt{2}$; then:

$$\phi = U \begin{pmatrix} 0 \\ \eta + \frac{\sigma}{\sqrt{2}} \end{pmatrix}; \quad U = e^{i\epsilon^A T^A} e^{i\epsilon Q}$$

where $T^A$, A=1, 2, 3 are the three broken generators and Q the conserved electric charge.

- The generic spinor is the gauge-transformed of a "standard" diagonal spinor, with only one real component, $\phi_{diag}$. Unitary gauge: $\phi = \phi_{diag}$; $\phi_{diag}(\eta) = \phi_0$

$$L = (D_\mu(A)\phi_0)^+ D_\mu(A)\phi_0 - V(\phi_0) = \frac{1}{2}\,\partial_\mu\sigma\partial^\mu\sigma - V + \frac{1}{2}\,\mu^2_{AB}(\phi_0)\,A_\mu A^{\mu B} + \ldots;$$

$$\mu^2_{AB} = g_A g_B\,(\phi_0^\dagger \{T^A, T^B\}\phi_0)$$

- ***The same miracle***. Adding the Yang Mills lagrangian, one sees that the last term gives a mass to all the gauge fields whose generators do not annihilate the vacuum state (Higgs-Brout-Englert mechanism), $W^\pm$, Z.
- Photon remains massless.
- The 3 Goldstone bosons of the broken global symmetry provide the longitudinal spin states of W and Z.

# Masses of the gauge bosons in SU(2)xU(1)

- in our case: $$g^A T^A A_\mu^A = g\frac{\tau^i}{2} W_\mu^i + \frac{1}{2} g' B_\mu$$
- in the basis of the fields $W^1$, $W^2$, $W^3$, B, one obtains:

$$(\mu^2)_{ab} = M_W^2 \delta_{ab},\ a,b = 1,2;\ M_W^2 = \frac{1}{2} g^2 \eta^2$$

$$\mu^2 = M_W^2 \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}$$

- the $W^3$, B matrix has $Det(\mu^2)=0$, corresponding to the massless photon.
- Defining the pysical Z and photon field as

$$Z_\mu = cos\theta\ W_\mu^3\ -\ sin\theta\ B_\mu;\ A_\mu = sin\theta\ W_\mu^3\ +\ cos\theta\ B_\mu$$

one finds: $$\tan\theta = \frac{g'}{g}$$

and

$$M_W^2 = \frac{g^2}{2}\eta^2;\ M_Z^2 = \frac{M_W^2}{cos^2\theta};\ M_A^2 = 0$$

# Currents and Fermi couplings

- Back to the leptons:

$$D_\mu(A)\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L=[\partial_\mu+i(g\frac{\tau^i}{2}W^i_\mu-\frac{1}{2}g'\,B_\mu)]\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L$$

$$D_\mu(A)\;e_R=[\partial_\mu+i(-g'B_\mu)]e_R$$

- from this, one finds easily the interaction lagrangian:

$$L_{int}=\frac{g}{2\sqrt{2}}[W^\lambda\,J^{weak}_\lambda+h.c.]+\frac{g}{cos\theta}Z^\lambda(J^3_\lambda-sin^2\theta J^{e.m.}_\lambda)+eJ^{e.m.}_\lambda A^\lambda$$

$$J^{weak}_\lambda=\bar{\nu}_e\gamma_\lambda(1-\gamma_5)e+(e\to\mu);$$

$$J^3_\lambda=\frac{1}{4}[\bar{\nu}_e\gamma_\lambda(1-\gamma_5)\nu_e-\bar{e}\gamma_\lambda(1-\gamma_5)e+(e\to\mu)];$$

$$J^{e.m.}_\lambda=-\bar{e}\gamma_\lambda e-\bar{\mu}\gamma_\lambda\mu$$

- and the leptonic charged current and neutral current Fermi interactions between e.g. electronic and muonic leptons:

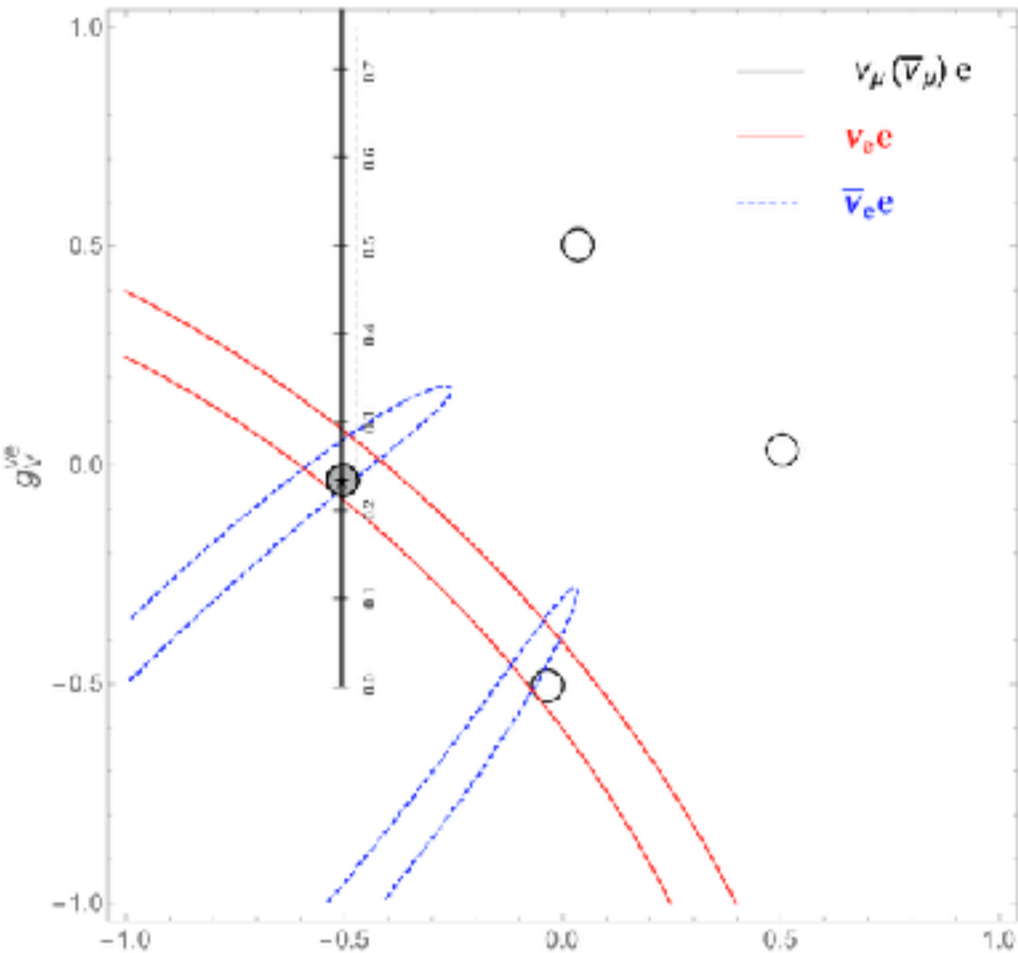
$$L_{Fermi}=\frac{G}{\sqrt{2}}[\bar{\nu}_\mu\gamma_\lambda(1-\gamma_5)\mu+...][\bar{e}\gamma^\lambda(1-\gamma_5)\nu_e+...]+$$
$$+\frac{G}{\sqrt{2}}[\bar{\nu}_\mu\gamma_\lambda(1-\gamma_5)\nu_\mu+...][\bar{e}\gamma_\lambda(g_V-g_A\gamma_5)e+\dots]$$

$$\frac{G}{\sqrt{2}}=\frac{g^2}{8M_W^2}\qquad g_V=-\frac{1}{2}+2\sin^2\theta;\;\; g_A=-\frac{1}{2}$$

**Note: relative normalization of neutral and charged currents is fixed**

# $\sin^2\theta$ from e-ν neutral current scattering

J. Erler and A. Freitas in pdgLive
M. Tanabashi *et al*. (Particle Data Group), Phys. Rev. D **98**, 030001 (2018)




Allowed contours in $g^{\nu e}$ vs. $g^{\nu e}$ from neutrino-electron scattering and the SM prediction as a function of $s^2_Z$. (The SM best fit value $s^2_Z = 0.23122$ is also indicated.) The $\nu_e$e [80] and $\bar\nu_e$e [81] constraints are at 1 σ, while each of the four equivalent $\nu_\mu(\bar\nu_\mu)$e [78–79] solutions ($g_{V,A} \to -g_{V,A}$ and $g_{V,A} \to g_{A,V}$ ) are at the 90% C.L. The global best fit region (shaded) almost exactly coincides with the corresponding $\nu_\mu(\bar\nu_\mu)$e region.

***The solution near $g_A = 0$, $g_V = -0.5$ is eliminated by $e^+e^- \to l^+l^-$ data*** under the weak additional assumption that the neutral current is dominated by the exchange of a single Z boson.

$s^2_Z = 0.23122$ → $M_W$=80.0 GeV; $M_Z$=90.0 GeV

# 4. The electron mass

- The Higgs doublet has: $I_W$=1/2, Y= -1 and it can couple the left-handed electron doublet, $e_L$ ($I_W$=1/2, Y= -1) to the singlet $e_R$ ($I_W$=1/2, Y= -2):

$$\ell_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}; \quad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

- Trilinear coupling:

$$L_e = g_e(\bar{e_R}\phi^\dagger \ell_L + \ h.c.) = g_e\left(\bar{e_R}\phi^- \nu_{eL} + \bar{e_R}\bar{\phi}^0 e_L \ + h.c.\right)$$

- In the vacuum configuration ($<\phi^+>_0=0$, $<\phi^0>_0=\eta$):

$$L_e \to g_e\eta(\bar{e_R}e_L + \bar{e_L}e_R) = g_e\eta\ \bar{e}e, \text{ i.e. } m_e = g_e\eta \neq 0$$

# The Higgs boson mass

- The physical scalar field, σ, is the ***Higgs boson***;
- Its coupling to the vector bosons is fixed by their masses and by the value of η:

$$L_{\sigma VV} = \left[2\frac{\sigma}{\sqrt{2}\eta} + (\frac{\sigma}{\sqrt{2}\eta})^2\right] (M_W^2 W_\mu^\dagger W^\mu + \frac{1}{2} M_Z^2 Z_\mu Z^\mu)$$

- We know η:

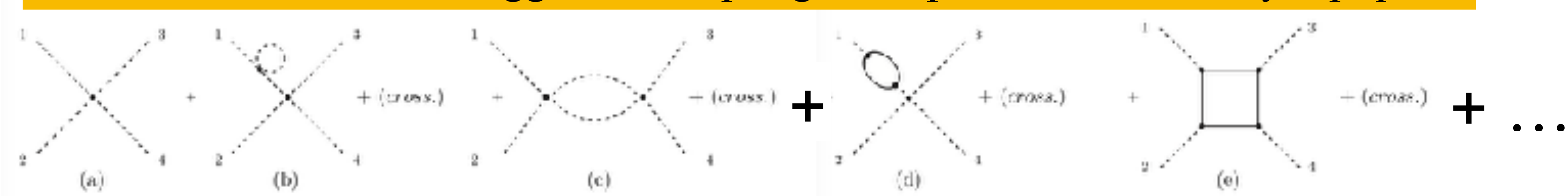
$$\frac{G}{\sqrt{2}} = \frac{g^2}{8M_W{}^2} = \frac{g^2}{8(g^2\eta^2/2)} = \frac{1}{4\eta^2} \qquad \eta = (\frac{1}{2\sqrt{2}G})^{1/2} \approx (\frac{10^5\,\mathrm{GeV}^2}{1.16\cdot 2.82})^{1/2} \approx 170\mathrm{GeV}$$

$$m_H^2 = 4\lambda\eta^2$$
$$m_H = \sqrt{\lambda}\ 340\ \mathrm{GeV}$$

- The direct ratio between σ-VV coupling and the V mass, σ-ff and f mass, and between σ-VV e σ-σ-VV are the signatures of the Higgs mechanism.
- The Higgs boson mass depends from λ, ***not predicted by theory***.

# Bounds to the Higgs boson mass

## Renormalization of the Higgs self-coupling, λ, in presence of a heavy top quark




## Renormalization of top quark-Higgs coupling, $g_t$, due to the Higgs interaction


$$m_H^2 = 4\lambda\eta^2$$
$$m_H = \sqrt{\lambda}\ 340\ \text{GeV}$$

- The mass of the Higgs boson is determined by the Higgs self coupling, λ
- stability: $\lambda > 0$;
- without Higgs boson, WW scattering violates unitarity at E > 800 GeV, so $m_H < 800$ GeV $\rightarrow\ \lambda < 1.5$
- more restrictive bounds come from the ultraviolet behavior of the running coupling λ(t), $t=\log[q^2/\mu^2]$
- The beta function of λ receives dominant contributions from the Higgs interaction itself and from the coupling to a heavy fermion, the top quark

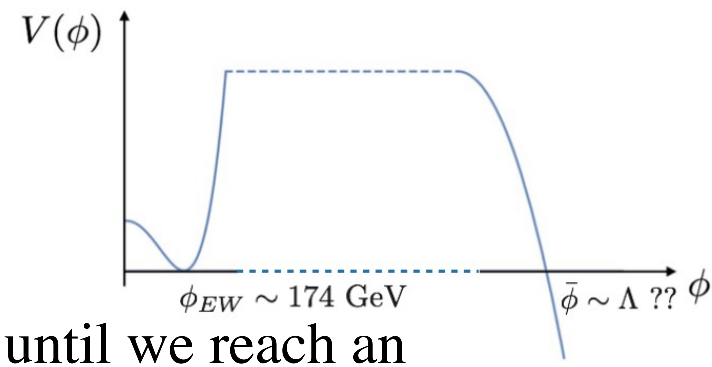
$$\beta(\lambda) = \frac{1}{16\pi^2}\ \left(4\lambda^2 - 36\, g_t^4 + 12\, g_t^2\right)\ +\ \text{smaller EW corr.s}$$

- given λ, $g_t$ can drive it to smaller values until λ becomes negative at some energy scale $\Lambda_{low}$
- alternatively, a value of λ too large can generate a Landau pole at some energy

# Bounds to the Higgs boson mass (cont'd)

$$\beta(\lambda) = \frac{1}{16\pi^2}\ \left(4\lambda^2 - 36\, g_t^4 + 12\, g_t^2\right)\ \ +\ \text{smaller EW corr.s}$$

- given $\lambda$, $g_t$ can drive it to smaller values until $\lambda$ becomes negative at some energy scale $q \sim \Lambda_{low}$
- potential decreasing at value of the field $\phi \sim \Lambda_{low}$ making the electroweak minimum $\phi_{EW}$ to become unstable!
- alternatively, $\lambda$ too large can generate a Landau pole at some energy scale $\Lambda_{up}$



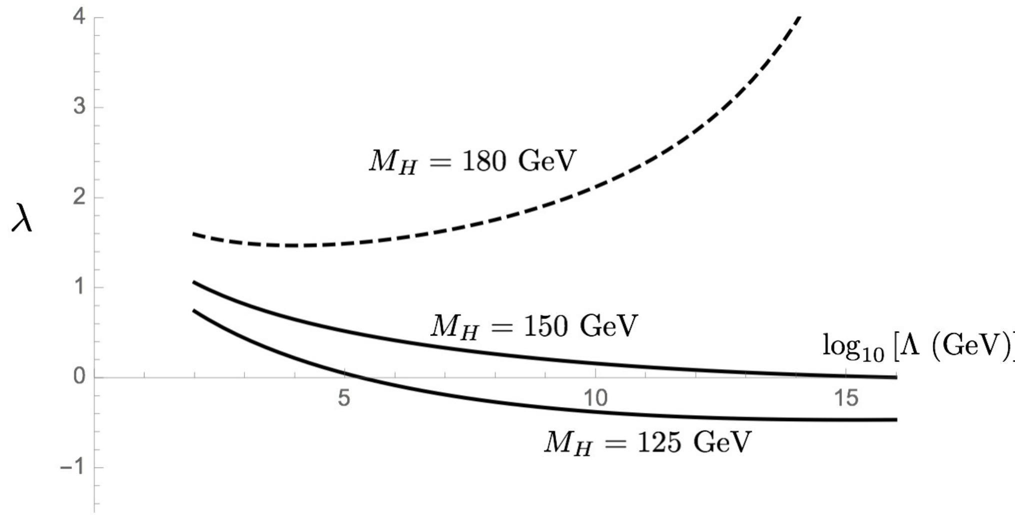

- the initial value of $\lambda$, say at $q \sim M_W$, must be between an upper and a lower bound if we want to avoid all that until we reach an energy scale, $\Lambda$, where new interactions will come in that may correct the "bad" behavior of $\lambda(t)$.

N. Cabibbo, L. Maiani, G. Parisi and R. Petronzio, Nucl. Phys. **B158** (1979) 295

- possible choices are $\Lambda = M_{Planck} \sim 10^{19}$ GeV or $\Lambda = M_{GUT} \sim 10^{14}$ GeV.
- bounds derived for $\Lambda = M_{GUT}$, to leading log approximation by Cabibbo et al.
- higher order estimates in:

G. Altarelli and G. Isidori, Phys. Lett. **B 337** (1994) 141;
M. Sher, Phys. Lett. **B 317** (1993) 159, **B 331** (1994) 448.
G. Degrassi *et al.* , JHEP **1208**} (2012) 098

$$150 \text{ GeV} < M_H < 180 \text{ GeV} ,$$
$$(\Lambda = 10^{15} \text{ GeV} , \; m_t = 174 \text{ GeV})$$



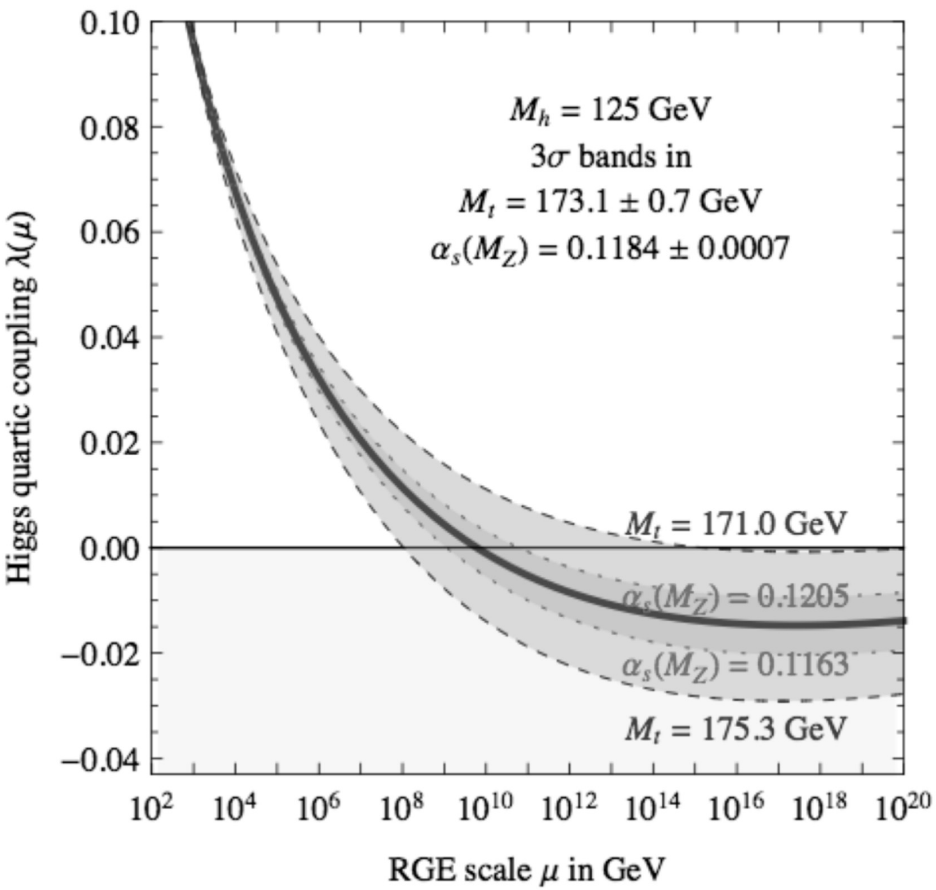

## LL approximation

N. Cabibbo, L. Maiani, G. Parisi and R. Petronzio, Nucl. Phys. **B158** (1979) 295

$$M_H \geq 135 \text{ GeV} ,$$
$$(\Lambda = 10^{15} \text{ GeV} , \; m_t = 174 \text{ GeV})$$

## NLL approximation

G. Altarelli and G. Isidori, Phys. Lett. **B 337** (1994) 141; M. Sher, Phys. Lett. **B 317** (1993) 159, **B 331** (1994) 448.

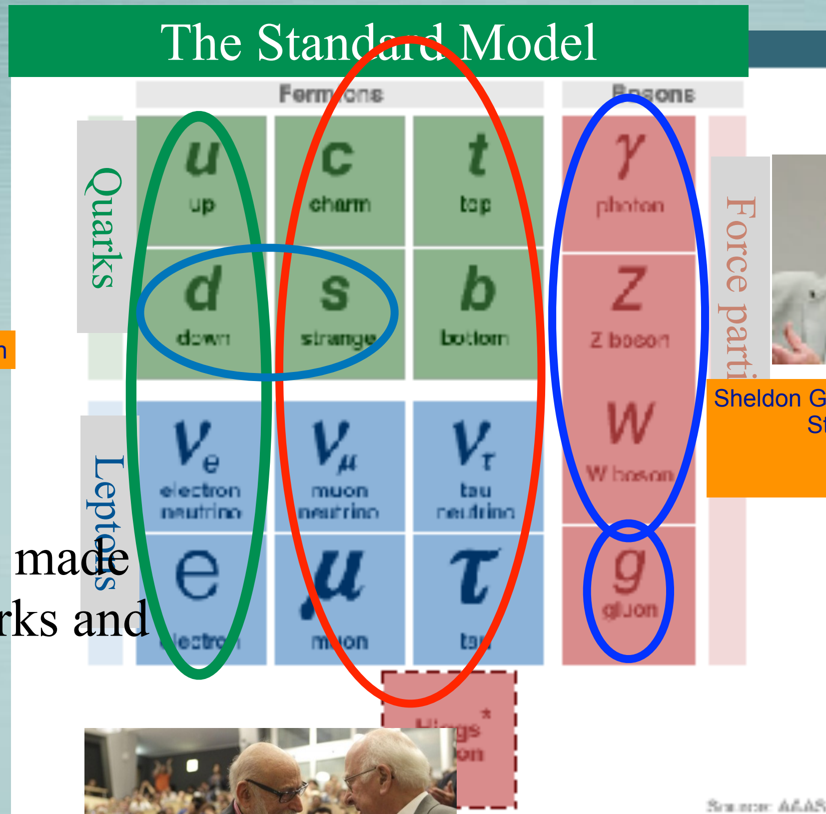



## NNLL approximation

The dependence on uncertainties of the colour constant, $\alpha_s$, and top quark mass is shown

G. Degrassi *et al.* , JHEP **1208**} (2012) 098

# Constituentsof matter and fundamental forces (circa 2016)





Murray Gell-Mann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Carlo Rubbia


Sheldon Glashow, John Iliopoulos, Luciano Maiani


Makoto Kobayashi , Toshihide Maskawa

Ordinary matter is made of the lightest quarks and leptons


Robert Englert e Peter Higgs

Heavier quarks are unstable: what is their role in the Universe?

Strong interactions between quarks are mediated by neutral vector mesons (gluons) coupled to color, and are asymptotically free
Gross&Wilczeck, Politzer (1973)