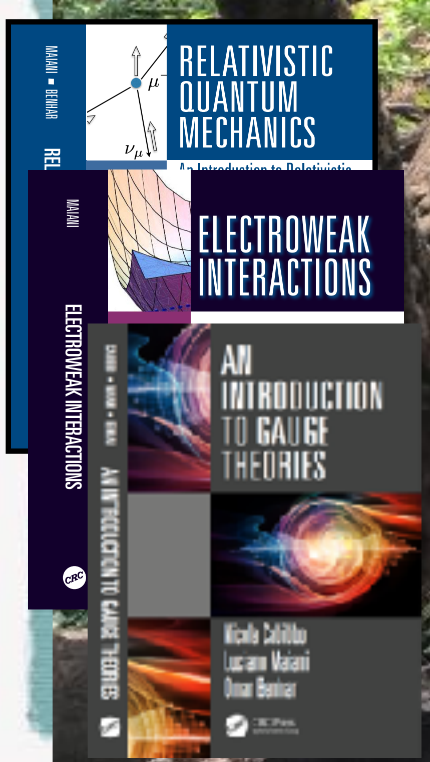

Six lectures in the Standard Theory of Elementary Particle Physics

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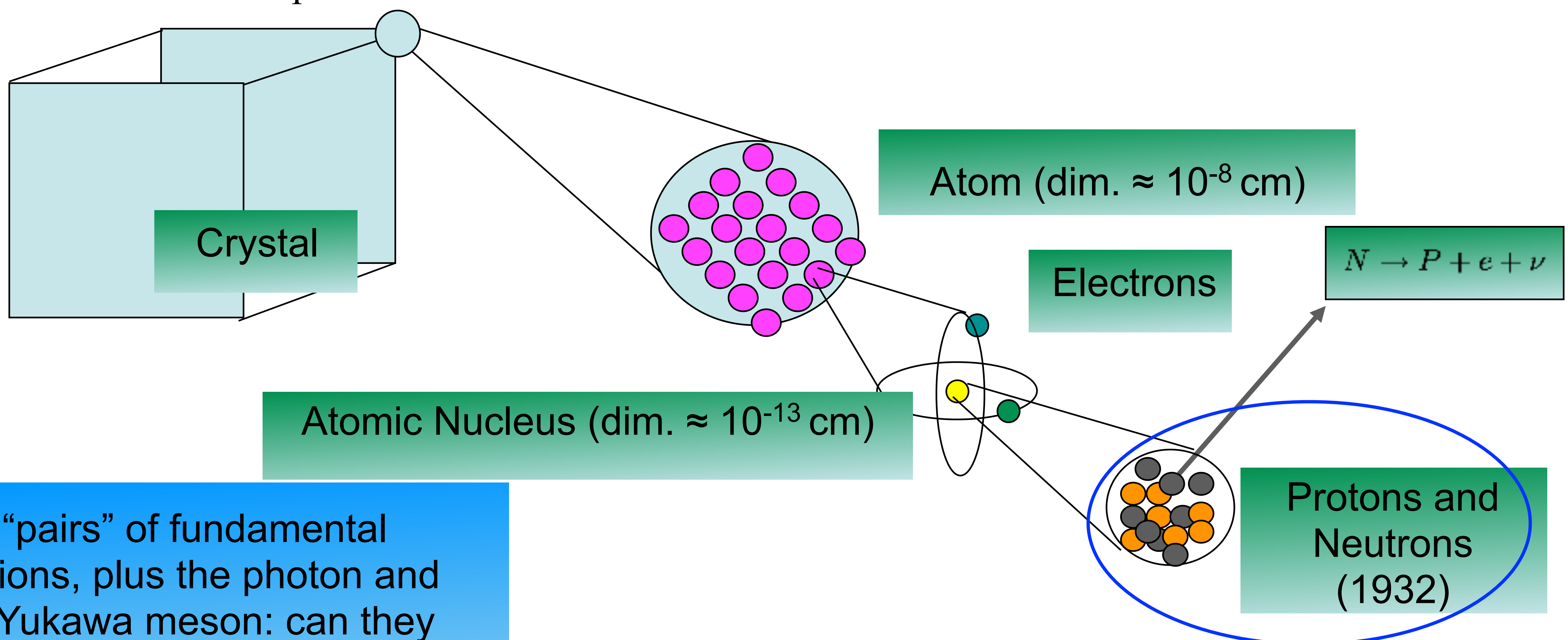
Shanghai, 6-12 July 2018

Lecture 3
Quarks 1

1. Matter constituents and interactions

After neutron's discovery (1932)

- What we are told at school;
- quite adequate, even today, for a first orientation;
- three fundamental forces plus gravity: electromagnetic, strong (nuclear), weak (beta decay);
- very few fundamental particles

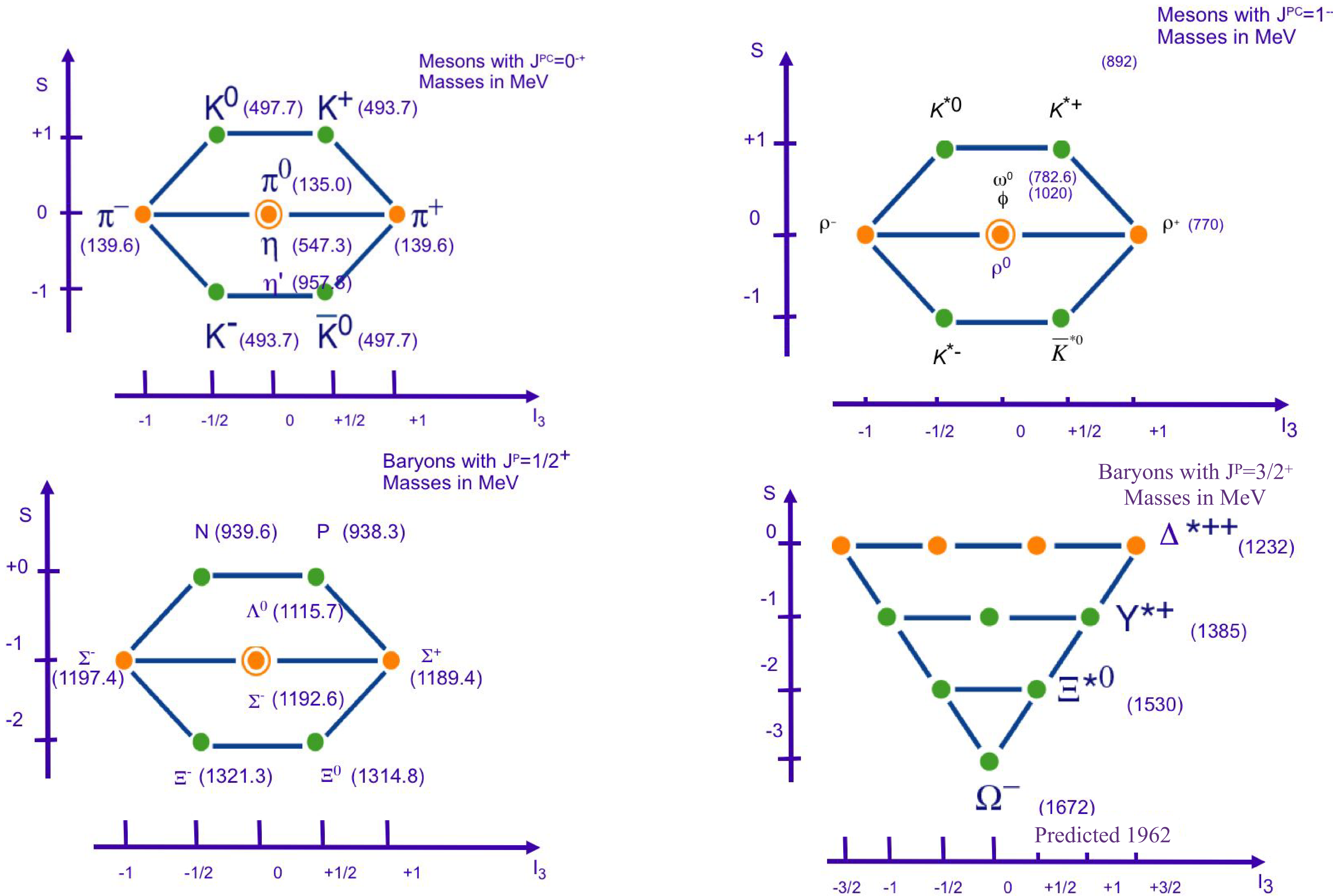


Two "pairs" of fundamental fermions, plus the photon and the Yukawa meson: can they "explain" all we see in the World?

$$\begin{pmatrix} P \\ N \end{pmatrix} \quad \begin{pmatrix} \nu \\ e \end{pmatrix} \quad \gamma \quad \pi \ ?$$

H. Yukawa (1934). Nuclear forces are transmitted by a particle with mass ≈ 200 times the electron's mass: the π meson

The *particle revolution*: lowest mass Baryons and Mesons (hadrons)



Three interactions are operative at particle level

– **Distinguished by strength and by selection rules**

1. ***Strong interactions***, O(1): act on hadrons, conserve
   – Parity, Charge Conjugation, Time reversal,
   – I (isospin), S (strangeness), B (baryon number)
   – $-I \leqq I_3 \leqq +I$, $n(I)=2I+1$
   – typical lifetimes $\sim 10^{-23}$ sec ($\Gamma \sim 100$ MeV)

2. ***Electromagnetic interactions***, O(1/137): act on hadrons and charged leptons (e, μ) ,
   - conserve P (parity), C (charge conjugation), T (time reversal)
   - Q (electric charge) and S;
   - L (lepton numbers) and B
   - typical lifetimes $\sim 10^{-18}$ secs

Gell-Mann, Nishijima formula:

$$Q = I_3 + \frac{S+B}{2} = I_3 + \frac{Y}{2}$$

Y= hypercharge

3. ***Weak Interactions*** (Fermi, 1932), O(10^{-5}): act on all particles, including ν's,
   - conserve CPT
   - violate: P, CP and T; CPT conserved
   - conserve B, violate: S, L_e and L_μ and maybe $L= L_e + L_\mu$
   - typical lifetimes 10^{-12} secs or longer

Selection rules in weak and electromagnetic decays: mesons

- Decays of light pseudoscalar mesons. Note the difference in lifetimes between weak and electromagnetic decays (e.g. π^+ vs π^0 lifetimes).

	S	dominant decay	ΔI	$\lvert\Delta S\rvert$	τ(s)	Interaction
$\pi^\pm$	0	$\mu\ \nu_\mu$	1	0	$2.6 \cdot 10^{-8}$	weak
π^0	0	$\gamma\ \gamma$	1	0	$8.4 \cdot 10^{-17}$	electromagnetic
K$^\pm$	± 1	$\pi^\pm\ \pi^0$ $\mu\ \nu_\mu$ $\pi\ l\ \nu_l$	3/2 1/2 1/2	1	$1.2 \cdot 10^{-8}$	weak
K$_L$	± 1	3π $\pi\ l\ \nu_l$	1/2, 3/2 1/2	1	$5.2 \cdot 10^{-8}$	weak
K$_S$	± 1	2π	1/2	1	$0.89 \cdot 10^{-10}$	weak
η	0	3π $\gamma\ \gamma$	≥ 1	0	$0.55 \cdot 10^{-18}$	electromagnetic

Selection rules in weak and electromagnetic decays: baryons

- Decays of baryons stable for strong interactions. For each decay mode, the branching fraction is indicated in parenthesis.

	S	Dominant decay	ΔI	ΔS	τ(s)	Interaction
p	0	not observed			$\geq 10^{31-33}$ years	??
n	0	$p\ e^-\ \bar{\nu}_e$	1	0	885.7 ± 0.8	weak
Σ^+	-1	$p\ \pi^0\ (0.52)$ $n\ \pi^+(0.48)$ $\Lambda\ e^+\nu_e\ (2.0 \pm 0.5\ \cdot 10^{-5})$	1/2 1/2 1	1 1 0	$0.802 \pm 0.003 \cdot 10^{-10}$	weak
Σ^-	-1	$n\ \pi^-\ (0.998)$ $n\ e^+\bar{\nu}_e\ (1.017 \pm 0.034\ \cdot 10^{-3})$ $\Lambda\ e^-\bar{\nu}_e\ (5.73 \pm 0.27\ \cdot 10^{-3})$	1/2 1/2 1	1 1 0	$1.479 \pm 0.011 \cdot 10^{-10}$	weak
Σ^0	-1	$\Lambda\ \gamma\ (1.00)$	1	0	$7.4 \pm 0.7 \cdot 10^{-20}$	electromagnetic
Λ	-1	$p\ \pi^-\ (0.639)$ $n\ \pi^0\ (0.358)$ $p\ e^-\bar{\nu}_e\ (8.32 \pm 0.14\ \cdot 10^{-4})$	1/2	1	$2.632 \pm 0.020 \cdot 10^{-10}$	weak
Ξ^0	-2	$\Lambda\ \pi^0\ (0.99522 \pm 0.00032)$ $\Sigma^+e^-\bar{\nu}_e\ (2.7 \pm 0.4\ \cdot 10^{-4})$	1/2	1	$2.90 \pm 0.009 \cdot 10^{-10}$	weak
Ξ^-	-2	$\Lambda\ \pi^-\ (0.99887 \pm 0.00035)$ $\Sigma^0e^-\bar{\nu}_e\ (8.7 \pm 1.7\ \cdot 10^{-5})$ $\Lambda\ e^-\bar{n}u_e\ (5.63 \pm 0.3\ \cdot 10^{-4})$	1/2	1	$1.639 \pm 0.015 \cdot 10^{-10}$	weak

Decays of meson and baryon resonances

- Decays and widths of the lowest hadron resonances.
- Conservation of strangeness is obeyed in all decays except for the last one, for which lifetime indicates a weak decay.

	Dominant decay	Γ (MeV)	Interaction
ρ	2π	149	strong
K^*	$K\ \pi$	51	strong
ω	3π	8.4	strong
ϕ	$K\ \bar{K}$	4.3	strong
Δ	$N\ \pi$	120	strong
Y^*	$\Lambda\ \pi$ $\Sigma\ \pi$	3.6	strong
Ξ^*	$\Xi\ \pi$	9	strong
Ω	$\Lambda\ K$ $\Xi\ \pi$	$0.8 \cdot 10^{-10}$ sec	weak

2 .Elementary…what ?

THE
PHYSICAL REVIEW
A journal of experimental and theoretical physics established by E. L. Nichols in 1893
SECOND SERIES, VOL. 76, NO. 12 DECEMBER 15, 1949

Are Mesons Elementary Particles?

E. FERMI AND C. N. YANG*
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
(Received August 24, 1949)

IN recent years several new particles have been discovered which are currently assumed to be "elementary," that is, essentially, structureless. The probability that all such particles should be really elementary becomes less and less as their number increases.

muon
strange particles
Δ^{++}
.....

composite by more elementary "constituents" ?

Fermi&Yang's proposal:

$$\pi^+ = p\bar{n}$$

related by a large symmetry?
possibly including spin ?

ALTERNATIVE: particles are all on an equal footing
poles in S-matrix, solutions of self-consistency equations
S-matrix, bootstrap, nuclear democracy?

Fermi & Yang, 1949

$$\text{mesons} = F\bar{F}$$

- Fermi&Yang: only F=(p, n) are elementary,
- in Dirac's theory:

$$\text{Parity} = \gamma^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

particle

anti particle

- particle and antiparticle have opposite parity
- one clear prediction: π^0 must have negative parity !
- same applies to all S wave, F Fbar mesons, e.g. pion, rho, omega etc.
- p and n are quasi degenerate → isospin symmetry:

$$\begin{bmatrix} p \\ n \end{bmatrix} \to U \begin{bmatrix} p \\ n \end{bmatrix}$$

- U=2x2 complex unitary matrix, can be taken with det U=1, since a phase corresponds to baryon number conservation ⇒ ***all possible U make an SU(2) group***
- ***SU(2), isospin, is a symmetry of the strong interactions***.

The Sakata Model (1956)

- Sakata: one new constituent to account for strange particles:

$$\mathrm{S} = \begin{bmatrix} p \\ n \\ \Lambda \end{bmatrix}$$

$$\text{mesons} = \mathrm{S}\bar{\mathrm{S}}; \quad \text{baryons} = \mathrm{SS}\bar{\mathrm{S}}$$

- one clear predictions: there must exist baryons with strangeness S=+1.
- ***Unfortunately it is a wrong prediction! no such resonance appears in K+p scattering (S= +1) while many appear in K-p scattering (S= -1)***.
- Basic symmetry of Sakata model: SU(3), unitary transformations of the Sakata triplet:

$$\begin{bmatrix} p \\ n \\ \Lambda \end{bmatrix} \rightarrow U \begin{bmatrix} p \\ n \\ \Lambda \end{bmatrix}$$

- U=Unitary complex matrix, 3x3, det U=1.

Nuclear Democracy

$$\pi^+ = p\bar{n} \;\rightarrow\; ?? \;\rightarrow\; n = \bar{p}\pi^+$$

which is which?

- in the presence of very strong interactions (unitarity saturated) there is no clear distinction between composites and constituents:
- for this reason, in the sixties, ***nuclear democracy*** (G. Chew and S. Frautschi) was considered the most promising approach;

XVI. ARE ALL STRONGLY INTERACTING PARTICLES COMPOSITE?

In the usual picture of atomic or nuclear physics, a very large number of composite atoms and nuclei are made up of electrons, neutrons, and protons. The electron, neutron, and proton are treated as elementary because most phenomena involve energies too low to excite their internal structure. In high-energy physics, on the other hand, the range of energies easily allows excitation and breakup of any particle. This circumstance motivated Chew and Frautschi[72)] to conjecture that they should all be treated on the same basis.

there is hope that their coupling constants and mass ratios can be determined from unitarity and maximal analyticity requirements. The way towards fulfilling this hope is believed to lie in the further development of the self-consistent or "bootstrap" method of calculation which was described in Chapter 7.

3. Symmetry

- It became clear in the 50s that the weak point of Fermi&Yang and Sakata models is that proton, neutron and Λ do not have any special role in the panorama of hadrons. M. Gell-Mann and independently, Y. Ne'eman, choose the road of ***Symmetry***
- Let us imagine that there exists a group G of transformations of the fundamental degrees of freedom of the strong interactions (whatever they are) which leave the Hamiltonian invariant.
- Given a transformation $g \in G$, the effect of g on p and n states, for the example G=SU(2), would be of the form:

$$\begin{bmatrix} p \\ n \end{bmatrix} \to U(g) \begin{bmatrix} p \\ n \end{bmatrix}$$

- with U a function of g.
- If we carry out two successive transformations, first g_1 and then g_2, the effect will be to obtain the product transformation of the two: $g=g_2\, g_1$. Correspondingly, we expect:

$$U(g_2) \cdot U(g_1) = U(g_2 \cdot g_1)$$

- The matrices U(g) provide a ***representation*** of the group G, in the mathematical sense of the term: the law of multiplication of the group is reproduced by the product of the matrices U (this is well known in quantum mechanics for space rotations or for the Lorentz group).

Symmetry (cont'd)

- Continuous groups are defined by the generators of infinitesimal transformations (there is a fine point here on topology, but I'll ignore it considering only simply connected groups like SU(2) or SU(3)).
- In the limit of exact symmetry the Hamiltonian commutes with the generators, hence (this is called Schur's lemma)
  - H is block diagonal on irreducible representations, R_i
  - on each R_i , H is a multiple of the unit matrix
- in physicists language: in the limit of exact symmetry
  - particles with same spin, baryon number, etc. fall in multiplets, each determined by one irreducible representations
  - particles inside a multiplet have all the same mass
- Examples from SU(2): 2I+1 particles in a multiplet of isospin I
  - pion (triplet), nucleon(doublet), Sigma baryon (triplet), Lambda baryon (singlet)…
- ***An important lesson***: knowing the symmetry is not enough to tell to which multiplet a given particle belongs.

Symmetries and Broken Symmetries

συμ, 'with', μετροσ, 'measure'

- Symmetry implies ***predictability***
- We can predict the hidden part of an object if we know its symmetry,
- similarly, we can predict the existence of yet undiscovered particles if we know the symmetry that links them to the known particles
- We do not know ***why*** symmetries appear in physics, but predictability was the key of their success.
- When symmetry is broken, we can still make predictions if we know the transformation properties of the term of the Hamiltonian responsible for symmetry breaking
- a key example: a constant magnetic field, B, breaks symmetry under rotations, but we can still predict the way the lines of the spectrum split if the atom is in B, since we know how B transforms under rotations.

Symmetry Makes Predictions

… in the real picture,
….. symmetry is broken

Piero della Francesca: Polittico della Misericordia

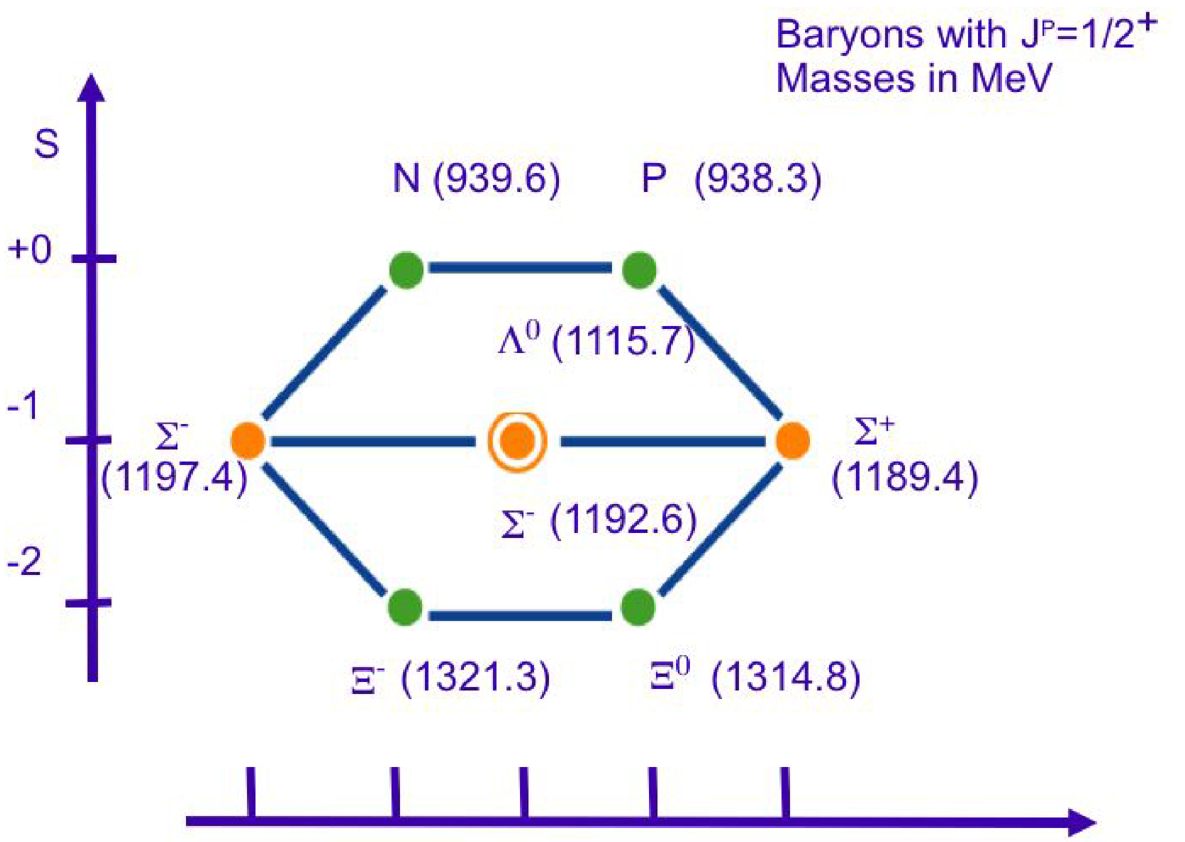
The Eightfold Way (Gell-Mann, Ne'eman, 1962)

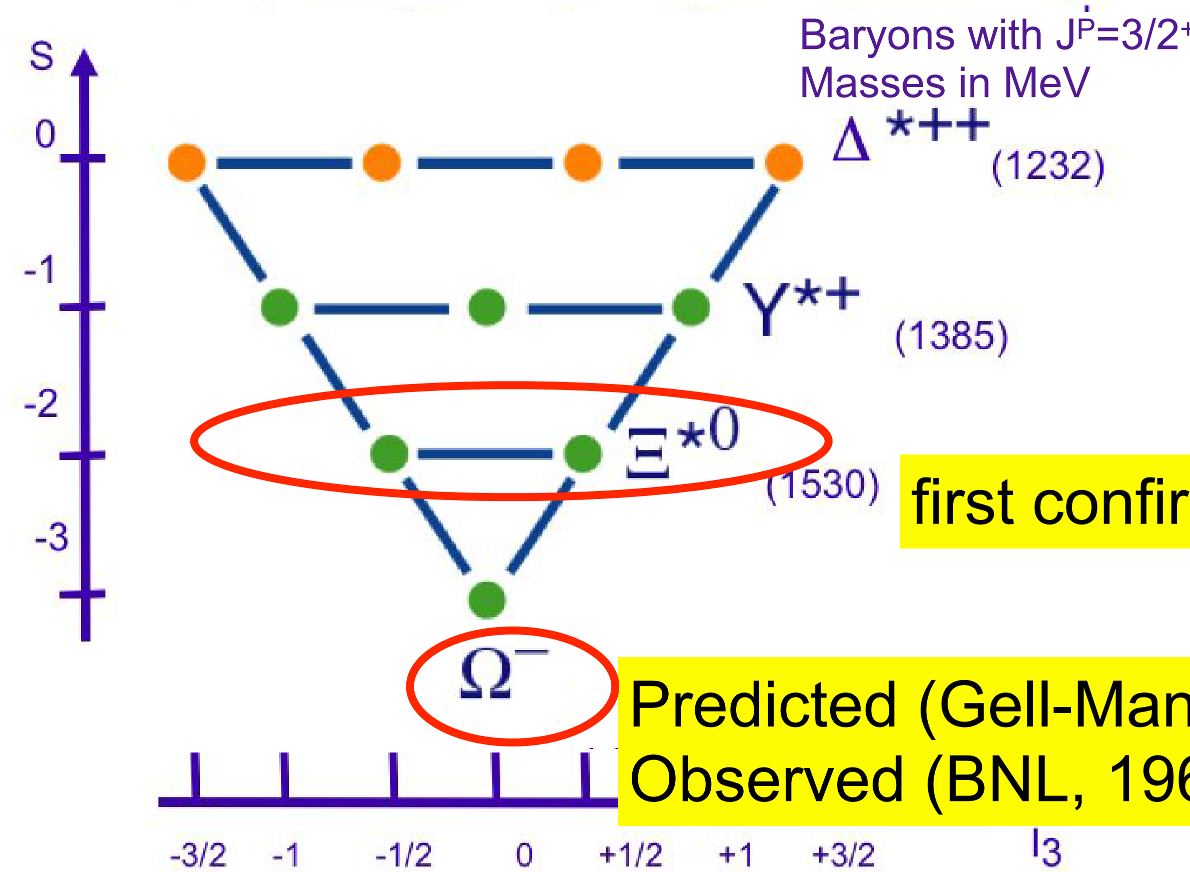
- Symmetry: SU(3), taken from Sakata model
- Mesons in octet, as in Sakata model
- Baryons in octet and decuplet, forget Sakata!
- assuming SU(3) broken by octet interaction, Gell-Mann and Okubo derived mass-formulae for octet and decuplet
  - octet baryons were known, formula is very well obeyed:

$$\frac{N+\Xi}{2}\ (1128\ \mathrm{MeV}) = \frac{3\Lambda+\Sigma}{4}\ (1136\ \mathrm{MeV})$$

  - decuplet masses equally spaced: from Δ and Σ^* masses one could predict Ξ^* and Ω masses
  - the discovery of two Ξ particles was presented at the Ginevra Conference, 1962, and Gell-Mann observed there that their mass checked
  - Ω discovered in 1964 with the expected mass
- first mass and quantum number predictions in particle physics !
- SU(3) ***symmetry*** was established.

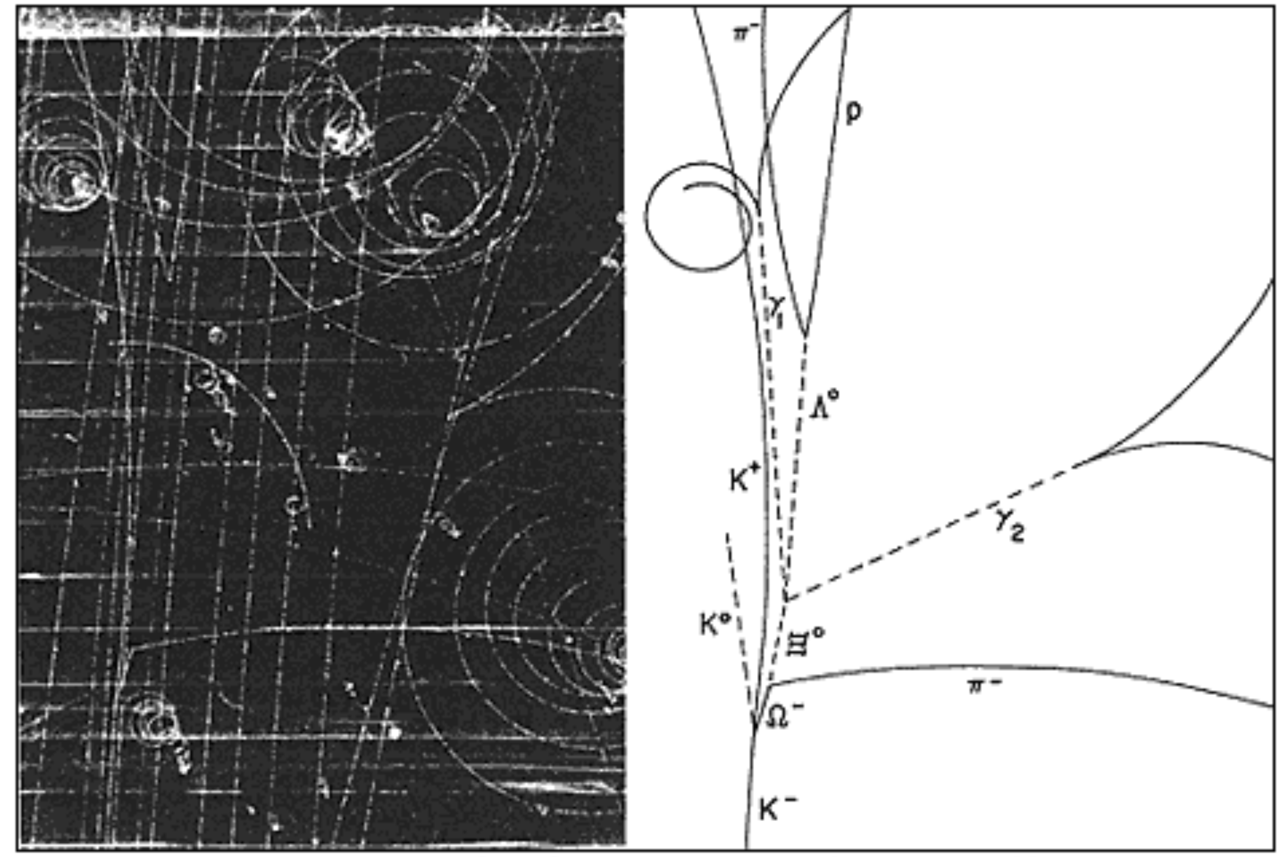
The Ω^-





first confirmation

Predicted (Gell-Mann, 1962) M=1679
Observed (BNL, 1964) M= 1672



The bubble chamber picture of the first Omega-minus (N. Samios and coworkers)

$$K^- + p \to K^+ + K^0 + \Omega^-,\ S(\Omega) = -3\ !!$$
$$\Omega^- \to \Xi^0\ \pi^-,\ S(\Xi) = -2,\ \text{(weak decay)}$$
$$\Xi^0 \to \Lambda^0\ \pi^0,\ S(\Lambda) = -1\ \text{(w.d.)}$$
$$\Lambda^0 \to p\ \pi^-,\ S(p) = 0\ \text{(w.d.)}$$

4. Quarks !!!

Baryons can now be constructed from quarks by using the combinations $(q\,q\,q)$, $(q\,q\,q\,q\,\bar{q})$, etc., while mesons are made out of $(q\,\bar{q})$, $(q\,q\,\bar{q}\,\bar{q})$, etc. It is assuming that the lowest

M. Gell-Mann, *A Schematic Model of Baryons and Mesons,* PL **8**, 214, 1964

- SU(3) representations and symmetry breaking can be studied by pure group theory
- but quarks are much simpler to handle!
- Quarks are hypothetical spin 1/2 particles in the basic SU(3) triplet, the ***first fundamental*** representation, **3**
- **we indicate SU(3) representations by their dimensionality**
- antiquarks: anti-triplet, the ***second fundamental*** representation, $\bar{\mathbf{3}}$
  - with quarks and antiquarks of spin 1/2, we should be able to construct all hadrons (forget Fermi statistics for a while, we'll come back!). How do we make mesons and baryons ?

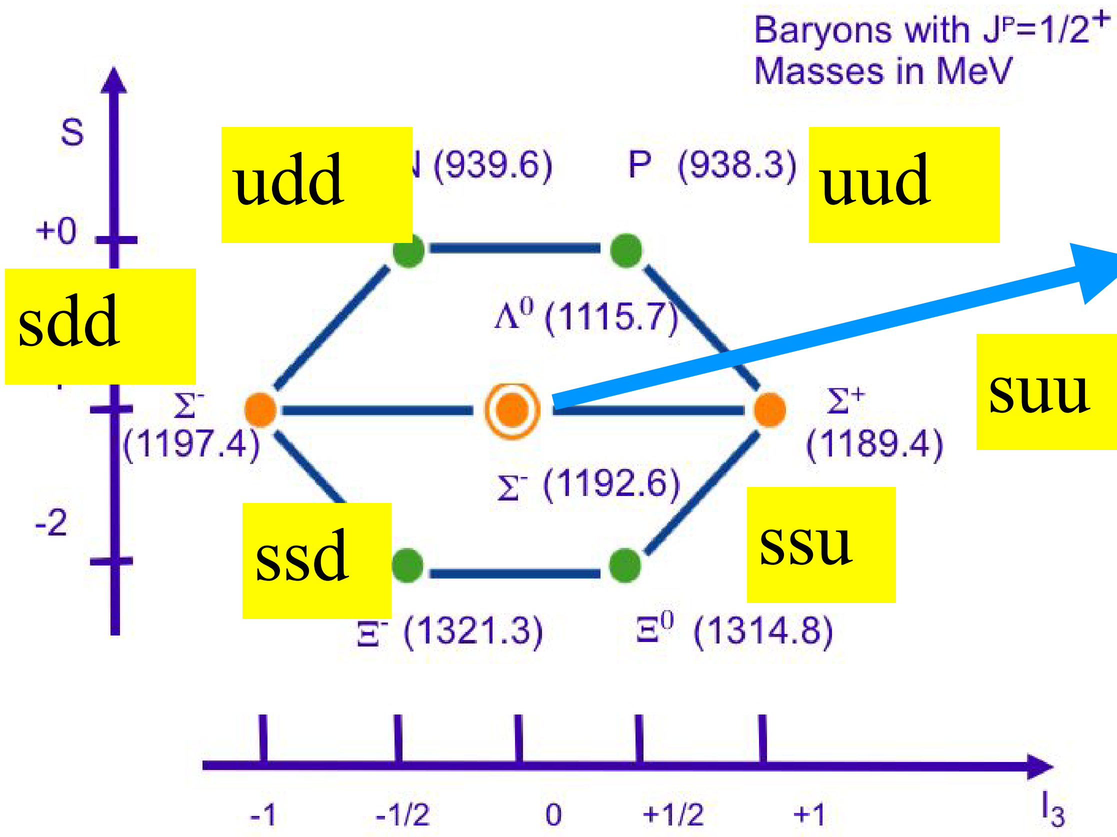
$$q = \begin{bmatrix} u \\ d \\ s \end{bmatrix}$$

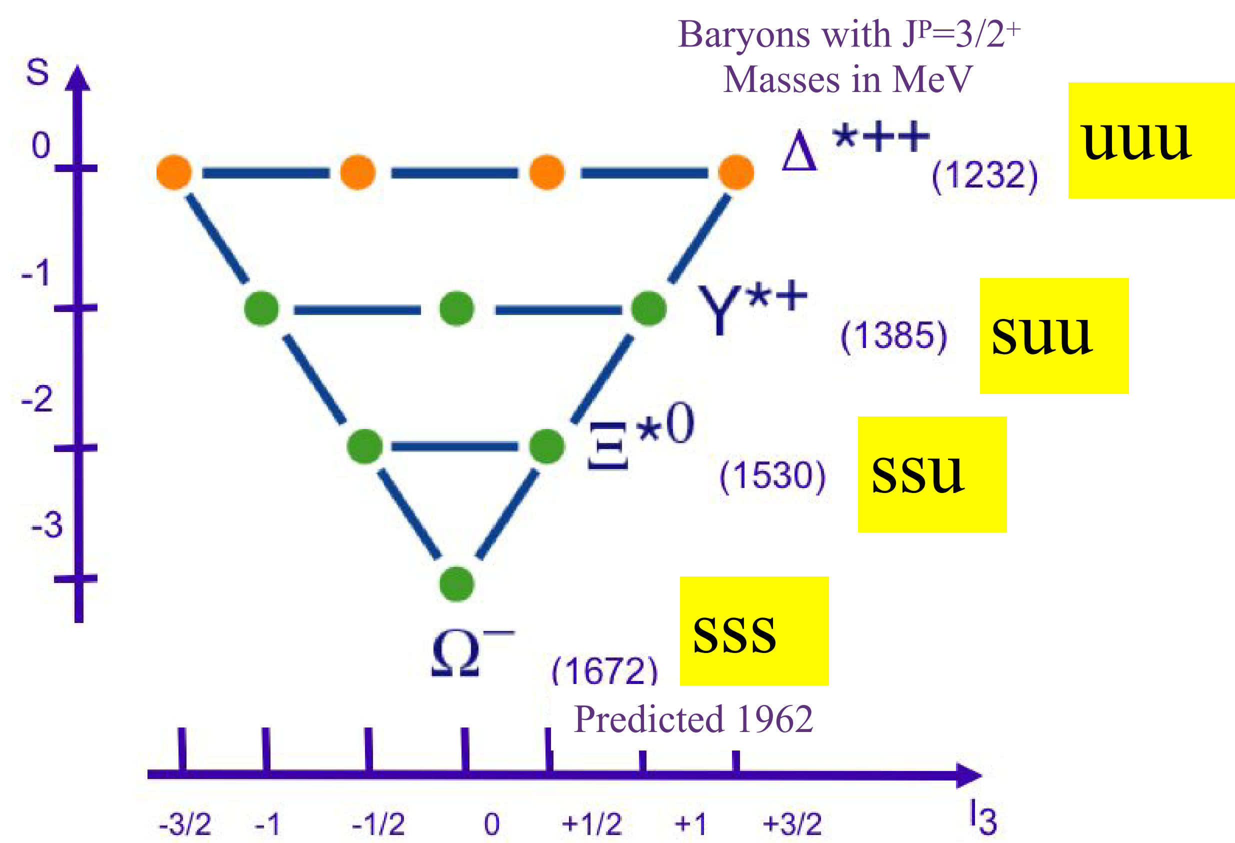
$$\bar{q} = [\bar{u}, \bar{d}, \bar{s}]$$

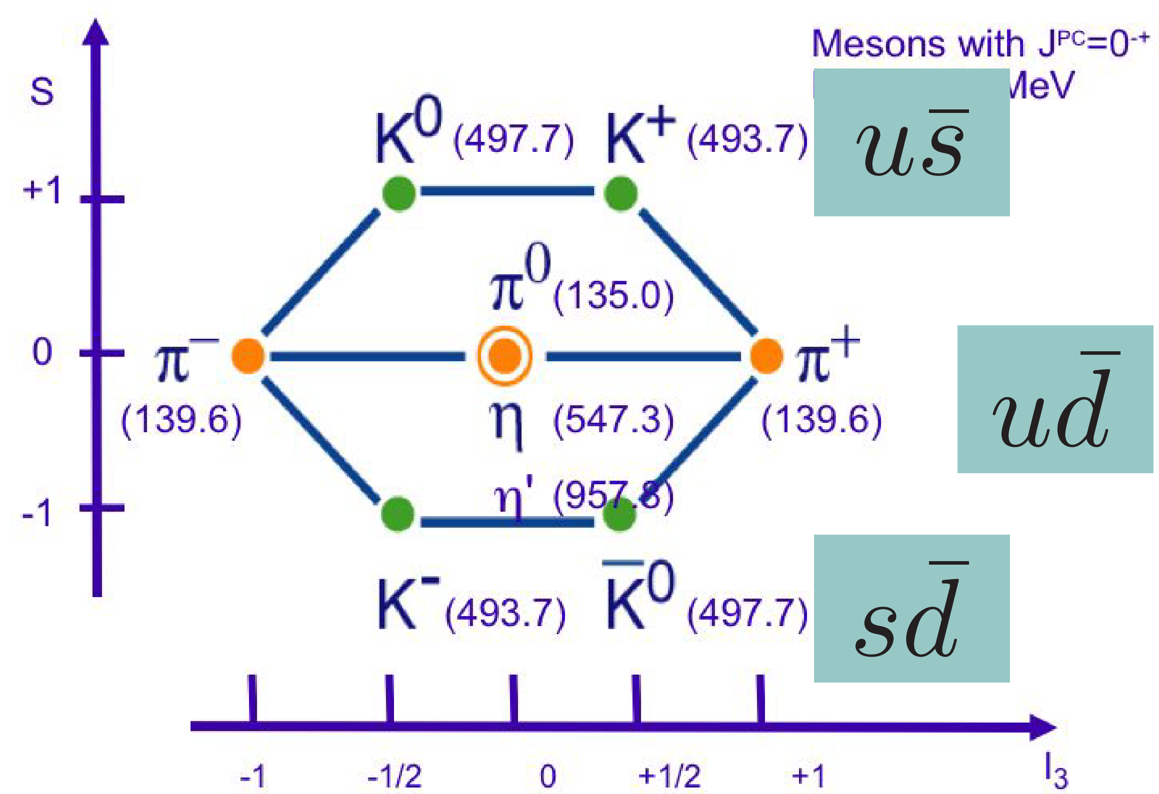
Quark quantum numbers.

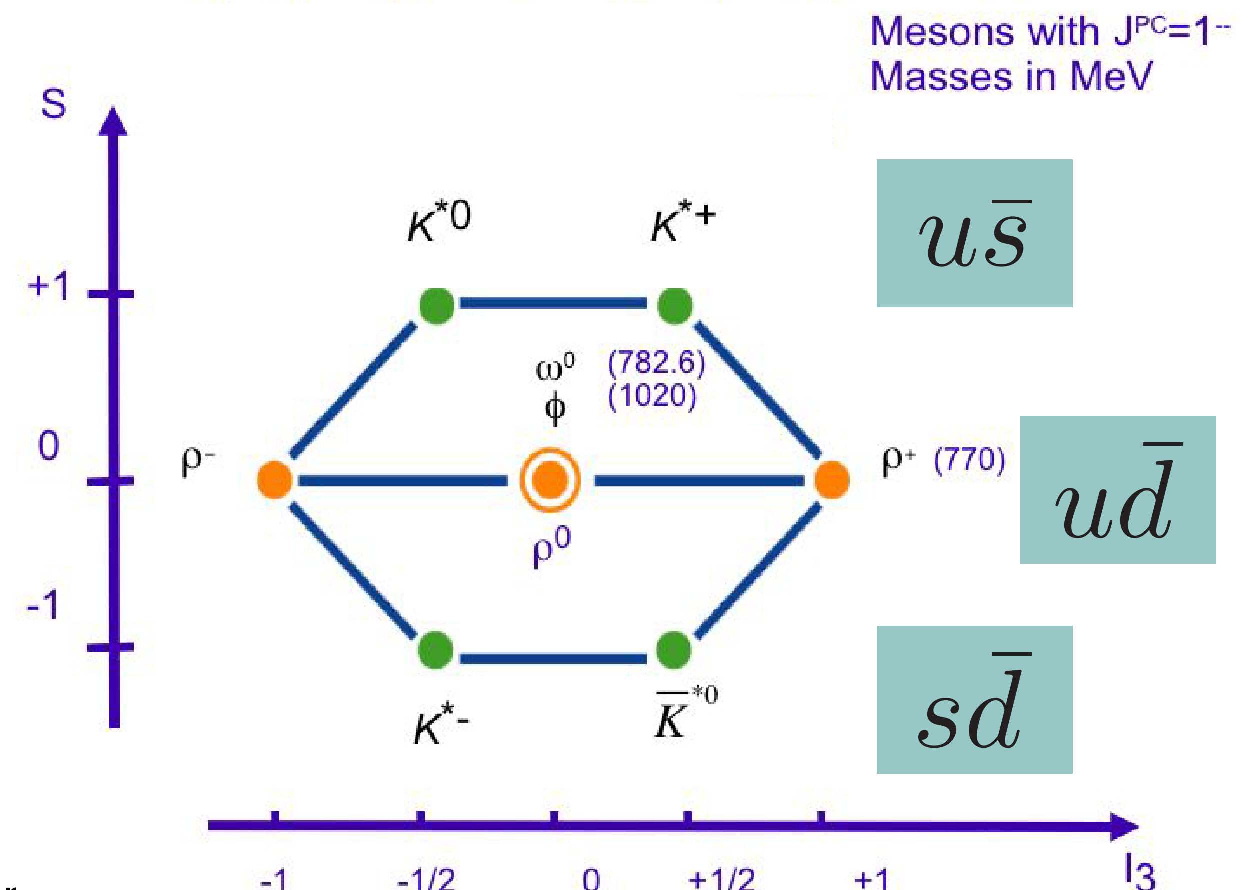
I_3	Y	(S)	Q
1/2	1/3	(0)	+2/3
-1/2	1/3	(0)	-1/3
0	-2/3	(-1)	-1/3

Quark composition of the lowest lying Baryons and Mesons









Irreducible SU(3) tensors and multi quark/antiquark constructions

- SU(3) is group with two mutually commuting operators (I_3 and Y)
- the irreducible multiplets are characterised by two integers n_1, n_2
- and are represented by standard tensors with n_1 upper and n_2 lower indices:

$$T^{a_1 a_2 \cdots a_{n_1}}_{b_1 b_2 \cdots b_{n_2}}, \ (a_1, \cdots, b_1, \cdots = 1, 2, 3)$$

- which are symmetric in the upper and in the lower indices and traceless:

$$T^{a a_2 \cdots a_{n_1}}_{a b_2 \cdots b_{n_2}} = 0$$

- quark (antiquark) =$\mathbf{q_b}$ $(\bar{\mathbf{q}}^{\mathbf{a}})$
- products of quarks and antiquarks are represented by tensors with upper (antiquark) and lower (quark) indices, but are in general not symmetric nor traceless; to express them as sums of standard tensors, we project with symmetric operations, which are
  - symmetrisation/ anti symmetrisation
  - contraction with δ^a_b (eliminates one upper and one lower indices)
  - contraction with ε^{abc} (transforms 1 (2) lower indices in 2 (1) upper indices
  - contraction with ε_{abc} (same to upper indices into lower indices).

Examples

- MESONS: $\mathbf{3} \otimes \bar{\mathbf{3}}$

$$q_b \bar{q}^a = \hat{T}^a_b + \frac{1}{3}\delta^a_b(\delta^c_d T^d_c) = \hat{T}^a_b + \delta^a_b T$$

$\hat{T}^a_b$ is traceless, $\dim \hat{T} = 3 \cdot 3 - 1 = 8$

$$\rightarrow \quad \mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}$$

- TWO QUARKS: $\mathbf{3} \otimes \mathbf{3}$

$$q_a q_b = T^S_{\{ab\}} + T^A_{[ab]}, \quad T^S/T^A = \text{ symm/antisymm}$$

$$T^A_{[ab]} = \epsilon_{abc} \bar{T}^c$$

so that

$$q_a q_b = T^S_{\{ab\}} + \epsilon_{abc} \bar{T}^c,$$

$$\rightarrow \quad \mathbf{3} \otimes \mathbf{3} = \mathbf{6} \oplus \bar{\mathbf{3}}$$

- a symmetric tensor with two indices has 6 independent components

- **3** ⊗ **6** (preliminary to THREE QUARKS)

$$q_a T^S_{\{bc\}} = T^S_{\{abc\}} + \epsilon_{abd} T^d_c$$

$$T^d_c = \frac{1}{2}\epsilon^{def} q_e T^S_{\{fc\}}, \quad \rightarrow \quad T^d_d = 0$$

$$\dim\, T^S_{\{abc\}} = 10; \;\; \dim\, T^d_c = 8$$

$$\rightarrow \quad \mathbf{3} \otimes \mathbf{6} = \mathbf{8} \oplus \mathbf{10}$$

- $T^S_{\{abc\}}$ is symmetric in its three indices

- Prove that $T_{\{ab\}}$ has 6 and $T_{\{abc\}}$ has 10 components
- Decompose $T_{\{ab\}}$ and $T_{\{abc\}}$ in isospin multiplet (see App. 2)

Are quarks real?

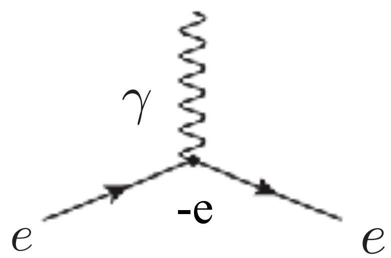
- THREE QUARKS:

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = (\bar{\mathbf{3}} \oplus \mathbf{6}) \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}$$

- This is really an extraordinary result: three quarks reproduce only the baryon multiplets that we see (octets, decuplets, singlets), with only negative or vanishing strangeness, as required by data;
- all observed baryons are treated equally (nuclear democracy !.
- Still, several puzzling features remain:
  - quarks are fractionally charged, the lightest quark must be absolutely stable
  - however, no fractionally quarks observed in matter
  - no fractionally charged stable or metastable particles observed in high energy collisions
  - Δ++=$u^{\uparrow}u^{\uparrow}u^{\uparrow}$ in S-wave: what about Fermi statistics?
- prevailing mood in the 60s was that quarks are a mathematical shorthand to summarise the solutions of the (unknown) basic equations of strong interactions (bootstrap?), in presence of SU(3) symmetry.

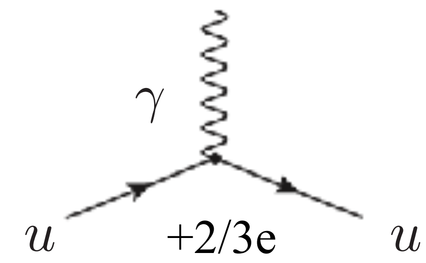
ELETTROMAGNETIC INTERACTIONS

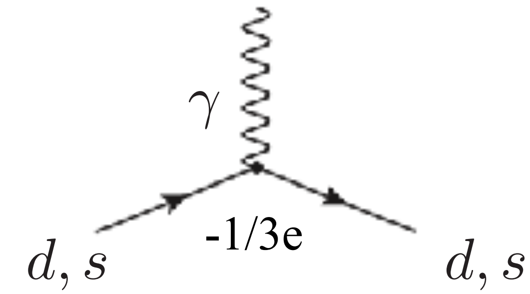
Charged Leptons



$(\mu \to \mu,\ \tau \to \tau)$

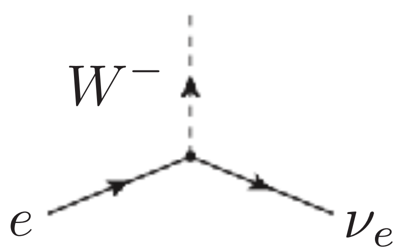
QUARKS





WEAK INTERACTIONS

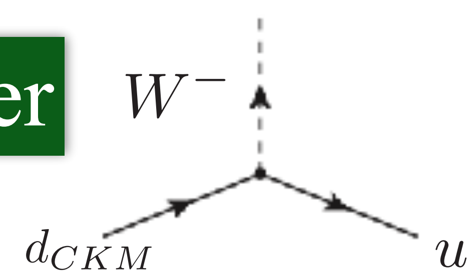
Leptons-charged currents



$(\mu \to \nu_\mu,\ \tau \to \nu_\tau)$

We'll come back later

Quark-charged currents

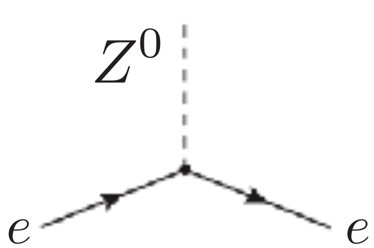


$(s_{CKM} \to c,\ b_{CKM} \to t)$

$$d_C = \cos\theta d + \sin\theta s,$$
$$\theta = \text{Cabibbo angle}$$

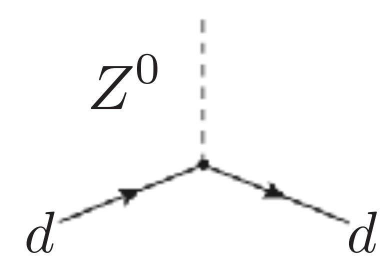
$$\begin{bmatrix} d_{CKM} \\ s_{CKM} \\ b_{CKM} \end{bmatrix} = U_{CKM} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

Leptons-neutral currents



$(\mu \to \mu,\ \tau \to \tau)$

Quark-neutral currents



$(s \to s,\ b \to b)$

with d_C only, d →s : strangeness changing neutral currents would be allowed (GIM)