EWPT, EWBG AND STOCHASTIC GRAVITATIONAL WAVES

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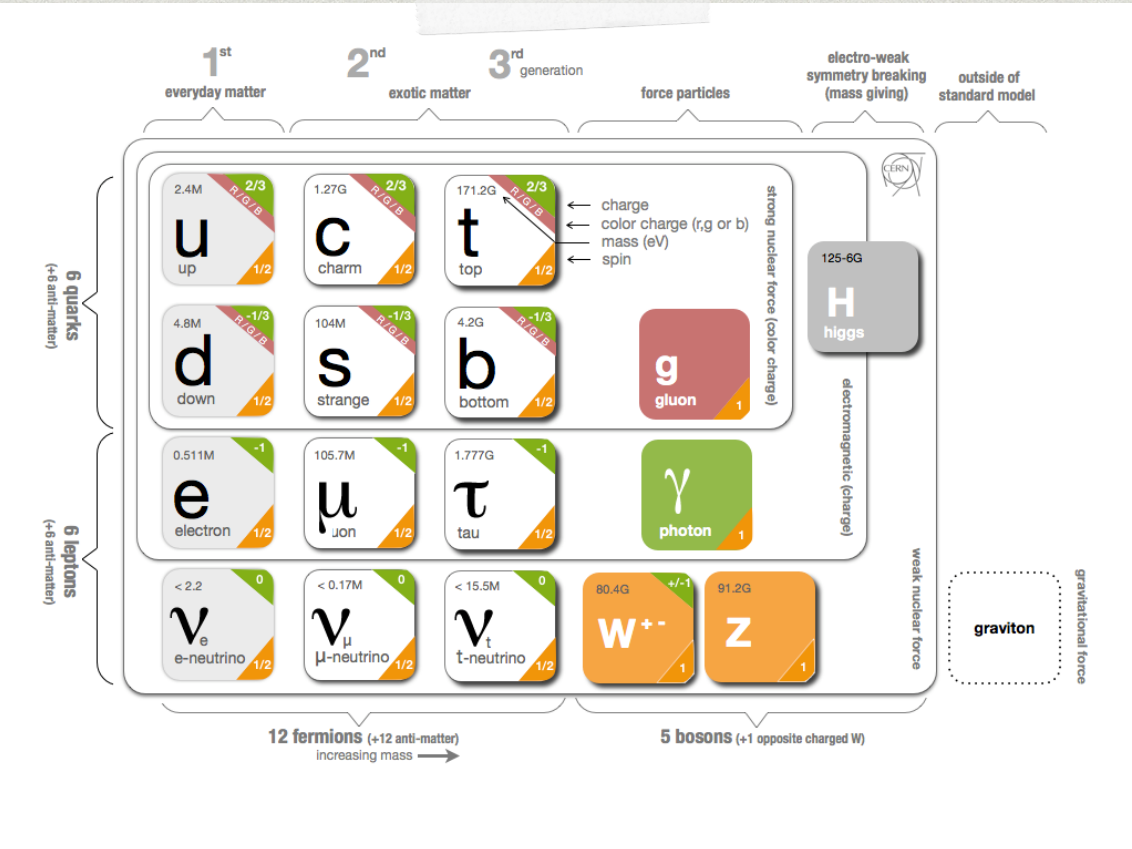
Outline&preview

- Brief overview of first order PT
- Stochastic gravitational waves from PT
- EWBG from spontaneous CPV

Preview

- *Give a quick look at the physics relevant to the electroweak phase transition.*
- *Show you stochastic gravitational wave as an indirect detection of the new gauge symmetry breaking.*
- *Showing you how to generate the BAU with a spontaneous CP phase and a two-step EWPT.*

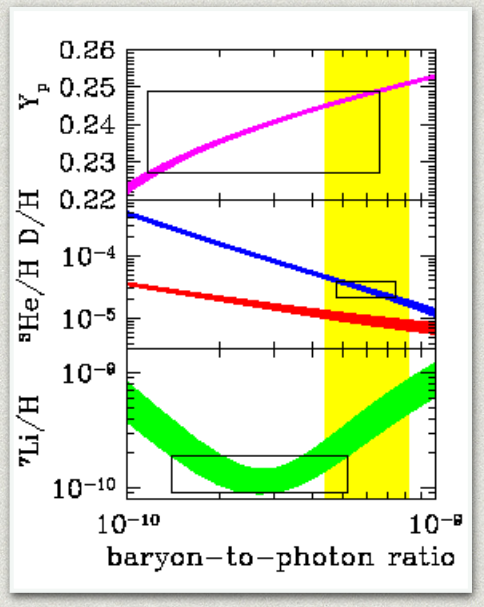
Particle Zoo

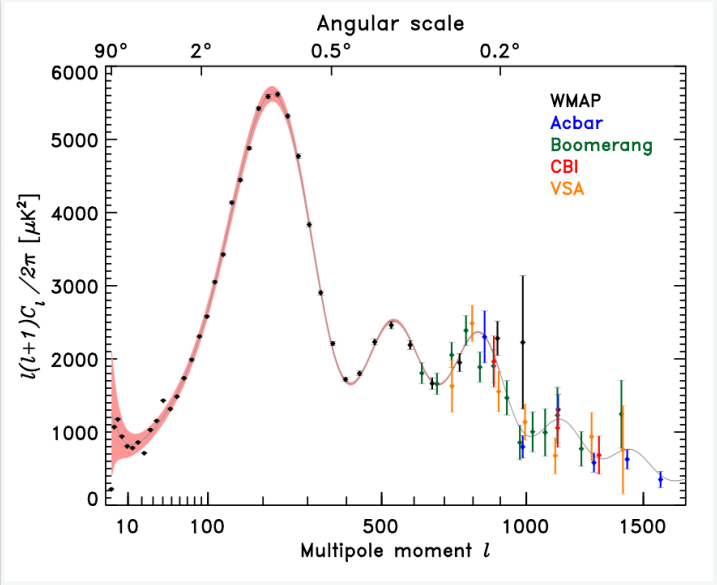


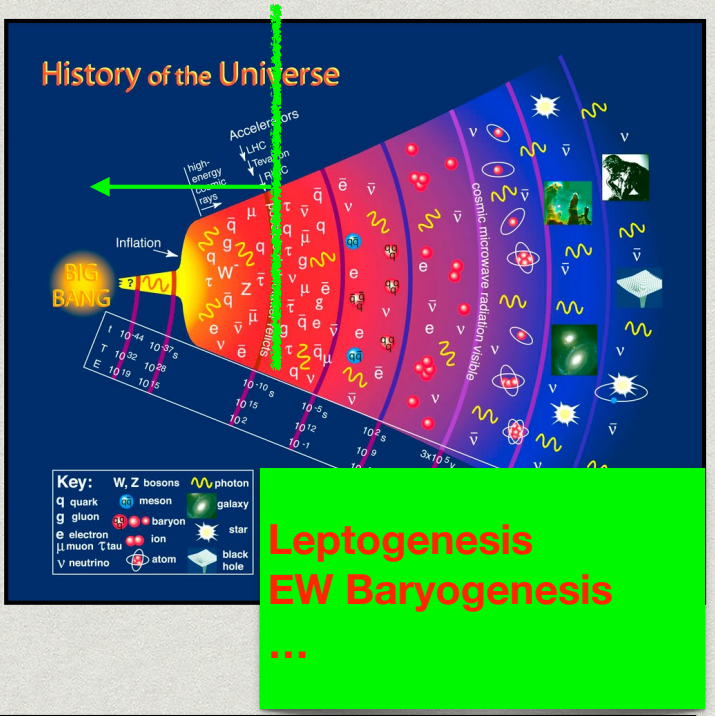
- Neutrino masses
- Dark matter
- Baryon asymmetry

Baryon asymmetry

- No anti-galaxy was observed
- The abundance of the primordial elements and the height of the CMB power spectrum depend on the ratio of of baryon to photons







Leptogenesis
EW Baryogenesis
...

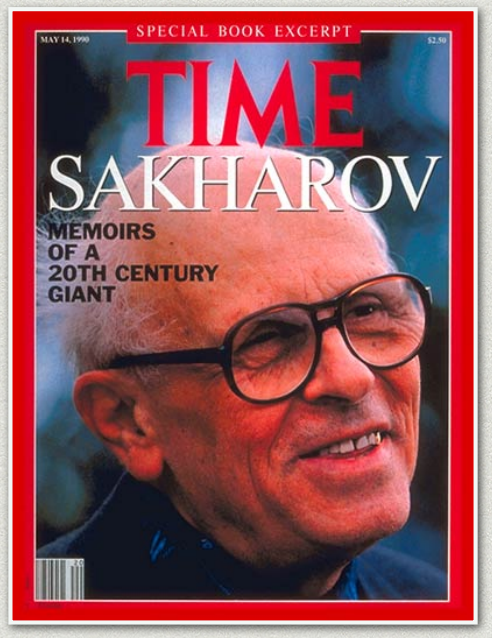
Baryon asymmetry: $$Y_B = \frac{\rho_B}{s} = (8.59 \pm 0.11) \times 10^{-11}$$ (Planck 2015)

Outline

- **Brief overview of first order PT**
- Stochastic gravitational waves from PT
- EWBG from the spontaneous CPV

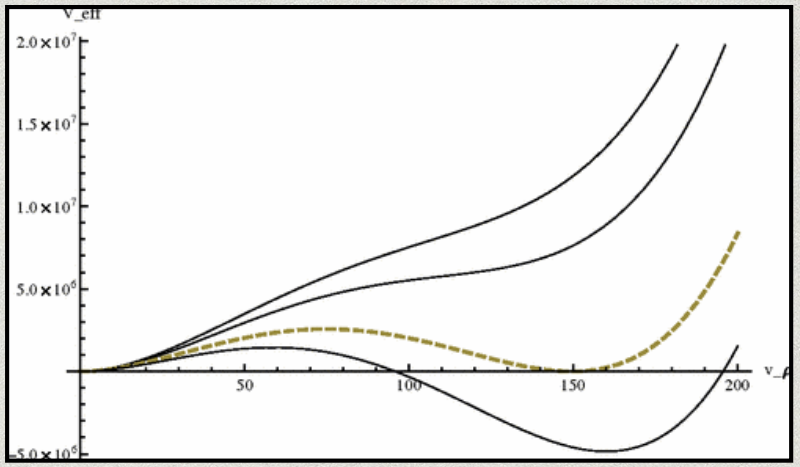
Why First order EWPT

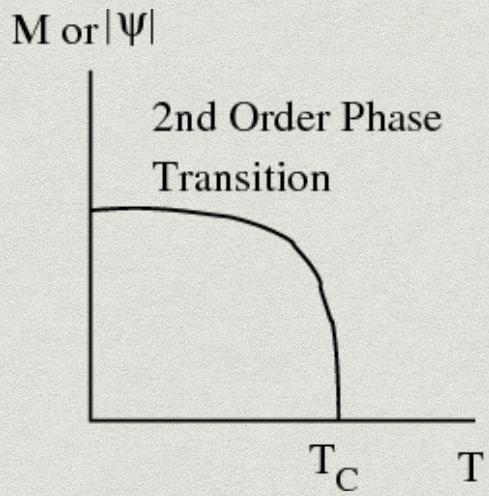
Because we believe that the BAU is generated from the EWBG

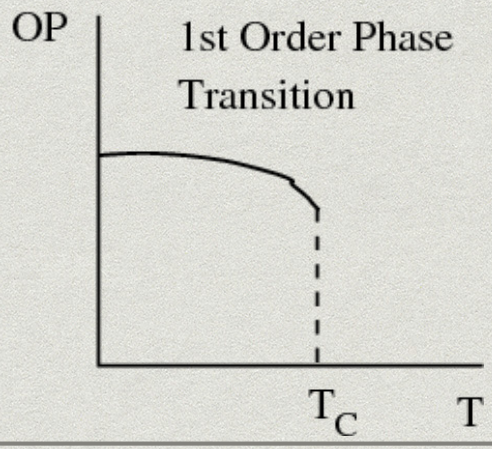


- ★Baryon number violation
- ★C&CP violation
- ★Departure from thermal equilibrium

→ First order electroweak phase transition if baryon asymmetry is generated during the EWPT without CPT violation.

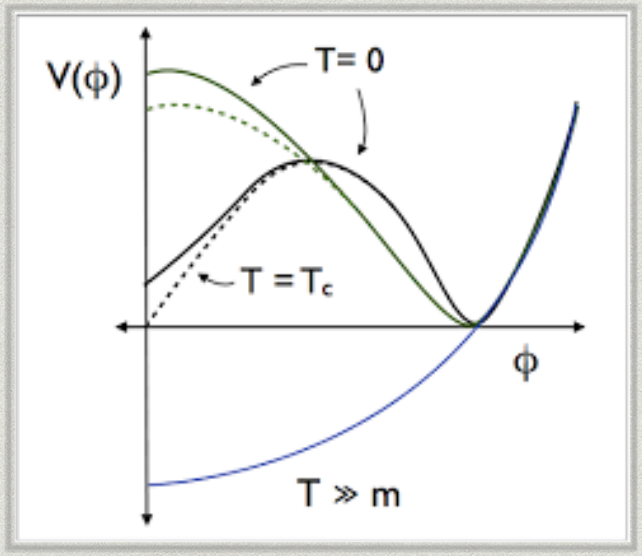


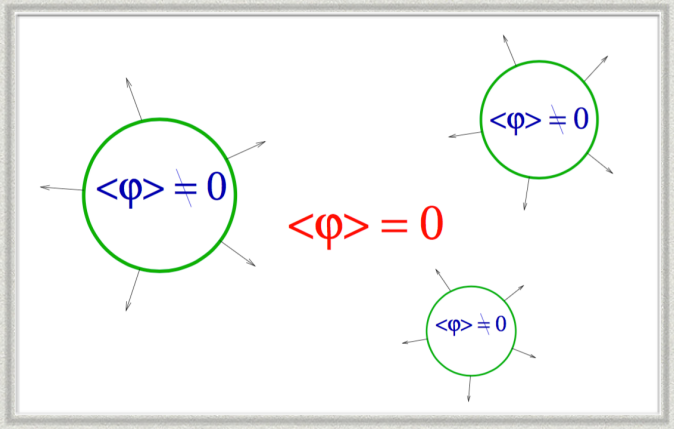




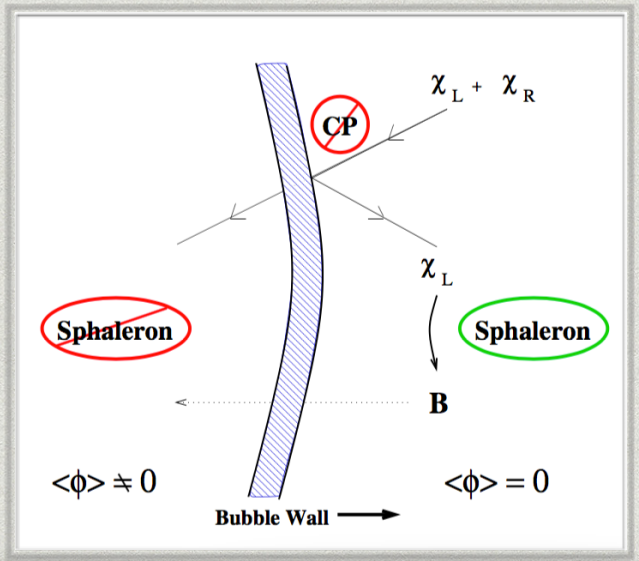
Electroweak Baryogenesis

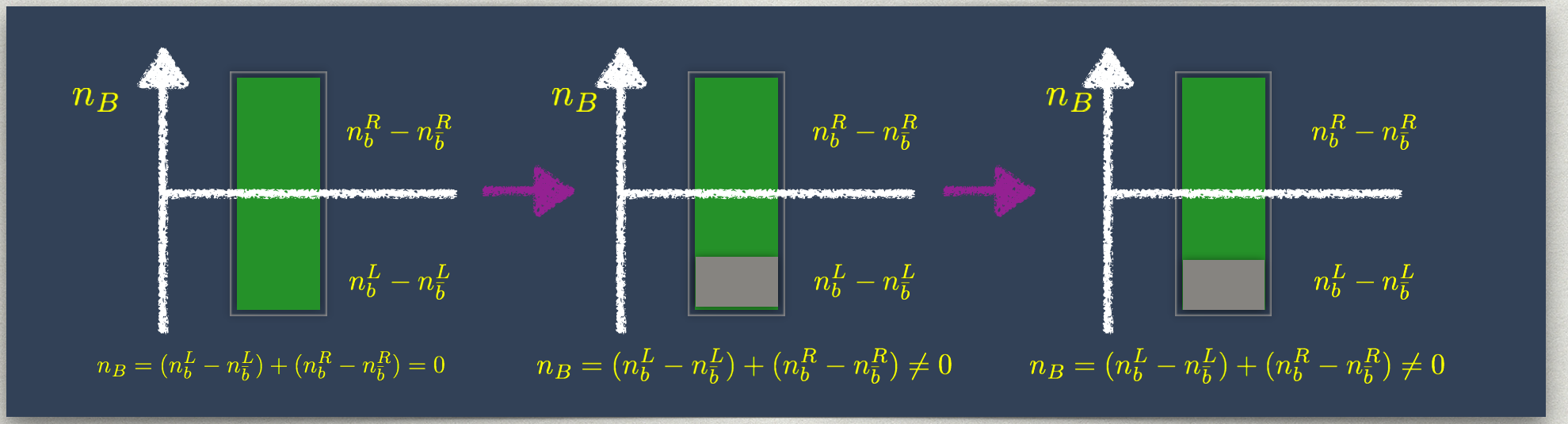
- **Generate BAU during the electroweak phase transition**





Basic description





The effective action

Generating functional (vacuum-to-vacuum amplitude):

$$Z[j] = \langle 0_{\rm out}|0_{\rm in}\rangle = \int d\phi \exp\{i(S[\phi] + \phi j)\}$$

Connected generating functional:

$$Z[j] \equiv \exp\{iW[j]\}$$

The effective action(The Legendre transformation of *W*):

$$\Gamma = W[j] - \int d^4x \frac{\delta W[j]}{\delta j(x)} j(x)$$

$$\Gamma[\phi] = S[\phi] + \frac{i}{2}\mathrm{Tr}\ln[G_0^{-1}(\phi)]$$

The standard 1pI *effective action*

$G_0(\phi)$: The green function in term of the background field!

$\Gamma[\phi]$ *is gauge dependent!*

The reason is that the generate functional is gauge dependent!

The effective potential in the SM

$$J_{B(F)}(x) = \int_0^\infty dt t^2 \ln\left(1 \mp \exp\{-\sqrt{t^2+x}\}\right)$$

$$V_T = \frac{T^4}{2\pi^2}\left\{\sum_{i\in B} n_i J_B\left[\frac{m_i^2(h,s,\xi)}{T^2}\right] - \sum_{j\in F} n_j J_F\left[\frac{m_j^2(h)}{T^2}\right] - \sum_{k\in G} n_k J_B\left[\frac{m_k^2(h,s,\xi)}{T^2}\right]\right\}$$

- **V_0: The tree-level potential**
- **V_T: Finite temperature contribution**

$$V_{\rm eff} = V_0 + V_{\rm CW} + V_T + V_{\rm Daisy}$$

- **V_{CW}: Coleman-Weinberg term**
- **V_{ring}: The ring contribution**

$$V_{\rm CW} = \frac{1}{64\pi^2}\sum_i (-1)^{2s_i} n_i m_i^4(h,s,\xi)\left[\log\frac{m_i^2(h,s,\xi)}{\mu^2} - C_i\right]$$

$$V_T^{\rm ring} = \frac{T}{12\pi}\sum_i n_i\left\{(m_i^2(h,s))^{3/2} - (M_i^2(h,s,T))^{3/2}\right\}$$

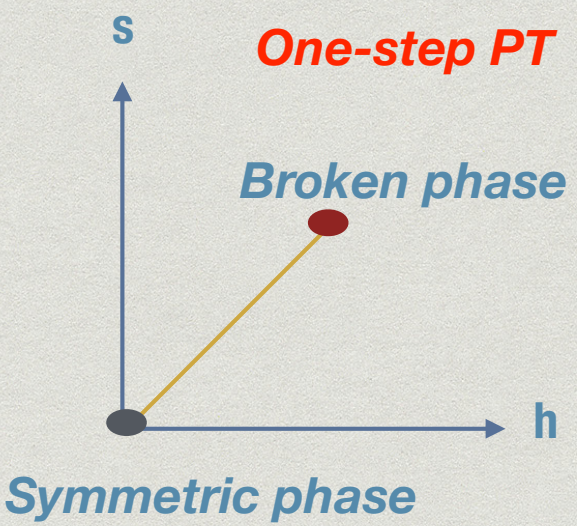
V_{eff} → Sphaleron rate ↔ ✦ Source of the BAU

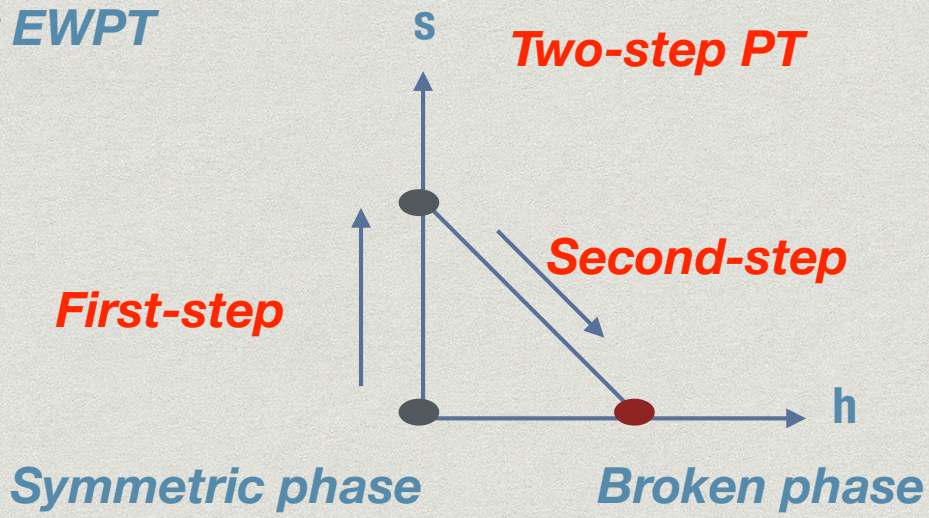
V_{eff} → EoM for Higgs → ✦ Strength of PT
✦ Bubble nucleation rate
✦ Bubble velocity
✦ Bubble wall width

Bubble dynamics

1. Patterns of PT.

SM Higgs is too heavy to saturate first order EWPT





The barrier between the symmetric and the broken phase usually comes from radiative corrections

$$V_{\rm eff}(\phi, T) = \mathcal{A}(T)\phi^2 + \mathcal{B}(T)\phi^3 + \mathcal{C}(T)\phi^4 + \cdots$$

The barrier exists at the tree-level

Merits:

1. No mixing with the SM Higgs

2. Correlated with the dark matter

Bubble dynamics

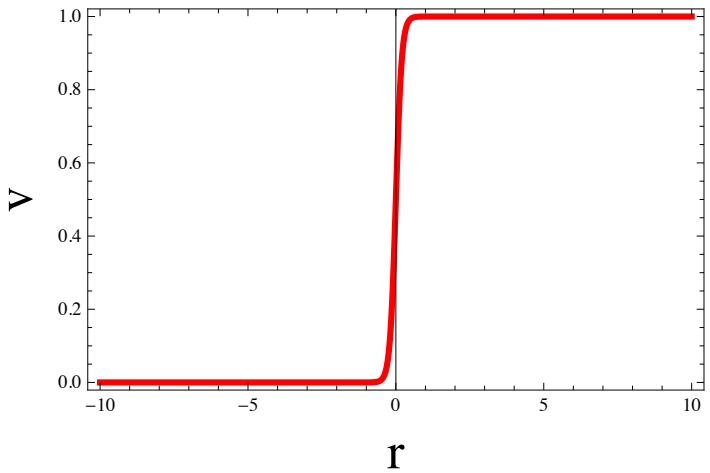
2. Typical temperatures

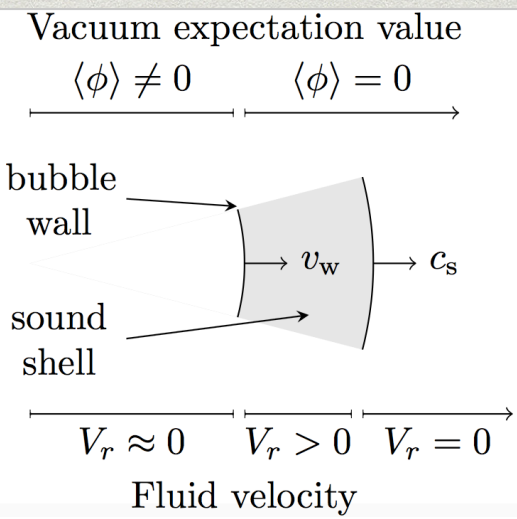
Critical temperature T_C:	$V_{\rm eff}(\phi_{\rm symmetric}, T)\|_{T_C} = V_{\rm eff}(\phi_{\rm broken}, T)\|_{T_C}$
Bubble nucleation Temperature T_n:	$\int_0^{t_n} \Gamma V_H(t) dt = \int_{T_n}^{\infty} \frac{dT}{T} \left(\frac{2\zeta M_{\rm pl}}{T}\right)^4 e^{-S_3/T} = \mathcal{O}(1),$
	Γ **Bubble nucleation rate** $V_H(t)$ **One-horizon volume**
PT completed Temperature T_d:	$f(T_d) = \frac{4\pi}{3} \int_{T_d}^{T_c} \frac{dT}{T} \frac{\Gamma(T)}{H(T)^4} v_w^3 \left(1 - \frac{T_d}{T}\right)^3 \equiv 1$
★**Relationships**	$H(T)$ **Hubble constant** v_w **Bubble wall velocity** $f(T)$ **Friction of the universe covered by the broken phase**
$T_c > T_n > T_d$	

Bubble dynamics

3. Bubble nucleation

Euclidean equation of motion	$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} - V''(\phi) = 0$
Euclidean action for the solution of EoM	$S_3 = 4\pi \int r^2 dr \left[\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + V(\phi)\right]$
Bubble nucleation rate per unit time per unit volume	$\Gamma_n(T) \approx T^4 \left(\frac{S_3(T)}{2\pi T}\right)^{3/2} \exp\left[-\frac{S_3(T)}{T}\right]$
Bounce solution to the EoM	$V(z) = \frac{1}{2}v(T)\left[1 + \tanh\left(3\frac{z}{L_w}\right)\right]$





Bubble dynamics

4. Physical parameters relating to PT

v_w	*Bubble wall velocity*	*calculated numerically*
l_w	*Bubble wall width*	*calculated numerically*
α	*Released energy to radiation energy*	$\alpha = \Lambda/\rho_{\rm rad}$
κ	*The efficiency factor*	$\kappa = \frac{3}{\varepsilon v_w^3}\int w(\xi)v^2\gamma^2\xi^2 d\xi$
Λ	*Latent heat*	$\Lambda = \Delta\left(V - \frac{dV}{dt}T\right)$

v_w
l_w

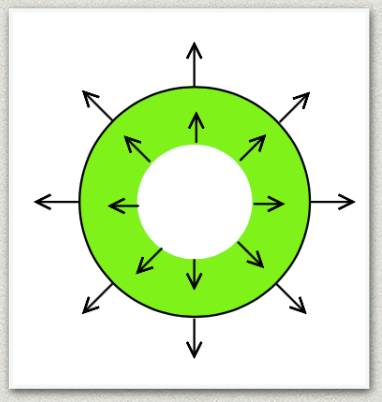
Relevant to the calculation of baryon number density generated during the EWPT

α
κ
Λ

Relevant to the calculation of stochastic gravitational wave spectrum emitted during the EWPT

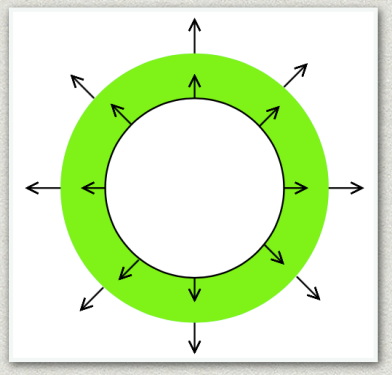
Bubble dynamics

5. Types of bubble



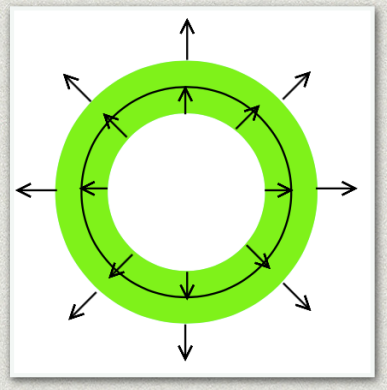
Supersonic

Fluid at rest in front of the wall



Subsonic

Fluid at rest behind the wall



Hybrid

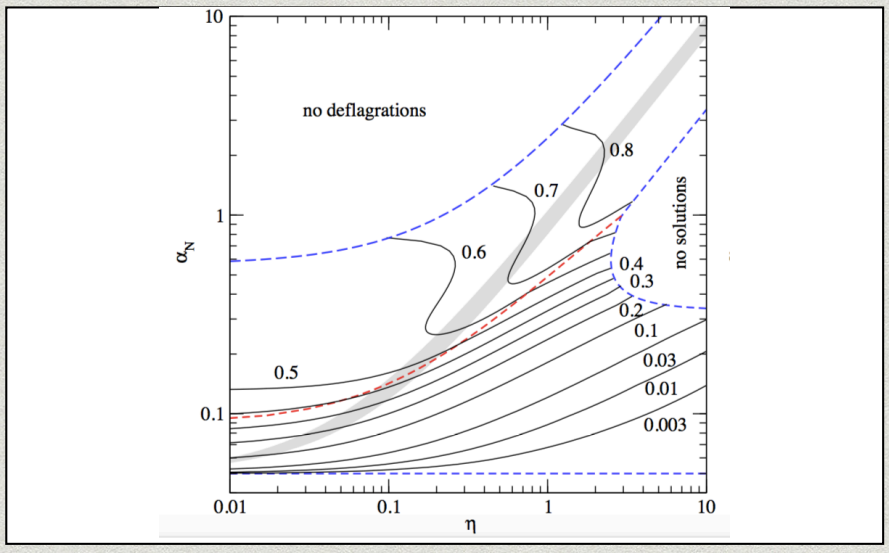
$v_w > c_s = v_- > v_+$

6.Friction

EoM of the Higgs field:

$$\Box\phi + \frac{dV}{d\phi} + \sum_n \frac{dm_n^2}{d\phi} \int \frac{d^3p}{2\pi^3} \frac{1}{2E} [f_n(p,x) + \underbrace{\delta f_n(p,x)}_{\text{Friction term}}] = 0$$

Comments: hard to calculate, but it is important for both EWBG and GW studies.



Outline

- Brief overview of EWPT
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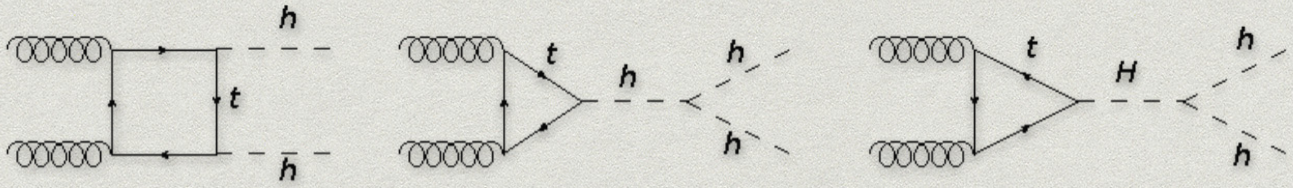
Why carry out this investigation?

QUESTION: What is the complete interaction relating to Higgs and how to test these new interactions?

- Di-Higgs…,@LHC



- Precision measurements of Higgs couplings, S,T,U,…@Precision Measurement
- …

What if the energy scale of new physics is too high to be accessible by colliders?

A Possible way out: taking stochastic gravitational wave arising from PT as an indirect detection Higgs interactions.

GW from the PT

Basics of gravitational wave from EWPT	
Gravitational waves are described by a transverse-traceless gauge invariant perturbation, h_{ij}, in a FRW metric,	$ds^2 = a^2(\tau)[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$
Einstein eq for transverse-traceless part	$h''_{ij} - \Delta h_{ij} = 16\pi G[(e+p)\gamma^2 v_i v_j + \partial_i\phi\partial_j\phi]$
Gravitational wave energy density	$\rho_{gw}(t) = \frac{\langle \dot{h}_{ij}(t,\vec{x})\dot{h}_{ij}(t,\vec{x})\rangle}{8\pi G}$
Energy spectrum	$h^2\Omega_{\rm GW}(f) = \frac{h^2}{\rho_c}\frac{d\rho_{\rm gw}}{d\log f}$

GW from the PT

Sources of GW from EWPT:

- ✦*Collisions of bubble wall and shocks in the plasma*
- ✦*Sound wave after the collision but before the expansion has dissipated the kinetic energy.*
- ✦*Magnetohydrodynamic turbulence : percolation can also induce MHD turbulence since the plasma is fully ionized.*

Fitted results of GW spectrum

Bubble collision	$h^2\Omega_{\rm coll}(f) = 1.67\times 10^{-5}\left(\frac{H_n}{\beta}\right)^2\left(\frac{\kappa\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\times\left(\frac{0.11v_w^3}{0.42+v_w^2}\right)\left[\frac{3.8(f/f_{\rm coll})^{2.8}}{1+2.8(f/f_{\rm coll})^{3.8}}\right],$
Sound wave	$h^2\Omega_{\rm sw}(f) = 2.65\times 10^{-6}\left(\frac{H_n}{\beta}\right)\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\times v_w\left(\frac{f}{f_{\rm sw}}\right)^3\left[\frac{7}{4+3(f/f_{\rm sw})^2}\right]^{7/2}$
MHD turbulence	$h^2\Omega_{\rm turb}(f) = 3.35\times 10^{-4}\left(\frac{H_n}{\beta}\right)\left(\frac{\kappa_{\rm tu}\alpha}{1+\alpha}\right)^{3/2}\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\times v_w\frac{(f/f_{\rm tu})^3}{(1+f/f_{\rm tu})^{11/3}(1+8\pi f/h_n)}$

Total energy spectrum:

$$h^2\Omega_{\rm GW} \approx h^2\Omega_{\rm coll} + h^2\Omega_{\rm sw} + h^2\Omega_{\rm turb}$$

Stochastic gravitational wave from NP

Many new physics models beyond the SM inspired by GUT theories.

Strategy: bottom-up!
Taking B-L as a prototype for the study of its gravitational wave signals , because it is one of the most simple extensions to the SM!

Type of the potential:

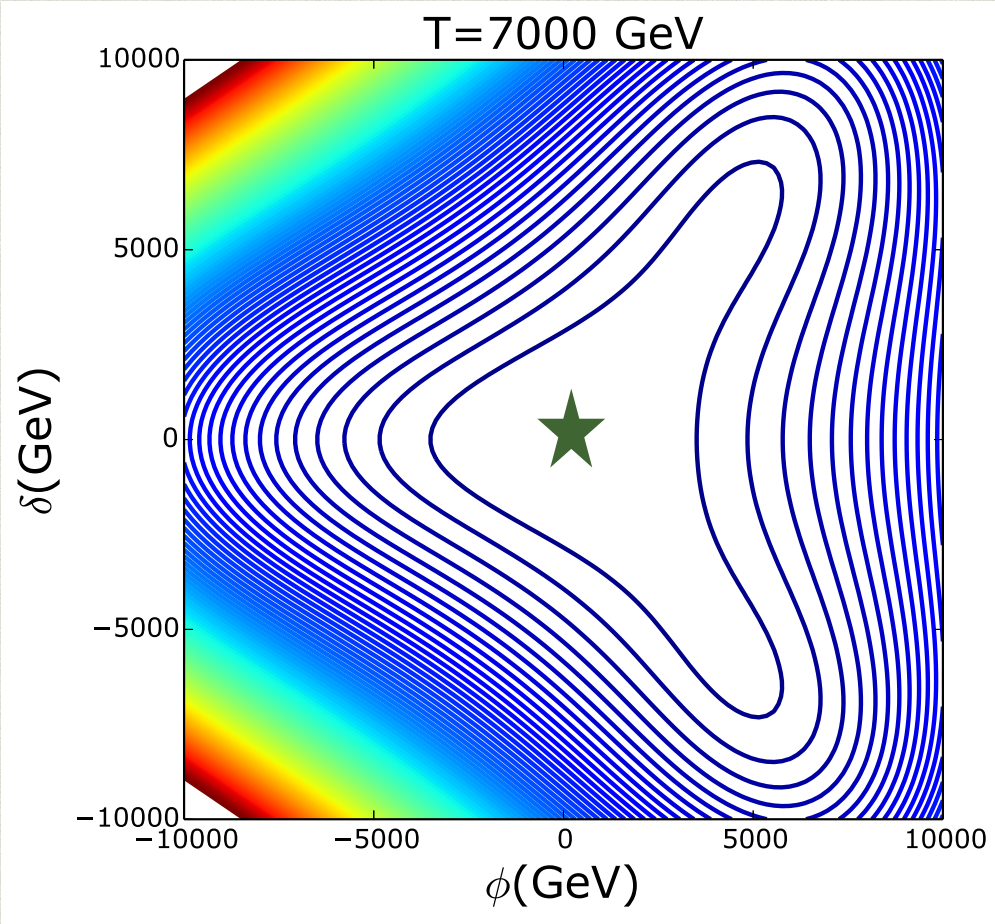
$$V_0^{(a)} = -\mu_\Phi^2 \Phi^\dagger \Phi + \kappa (\Phi^\dagger \Phi)^2,$$

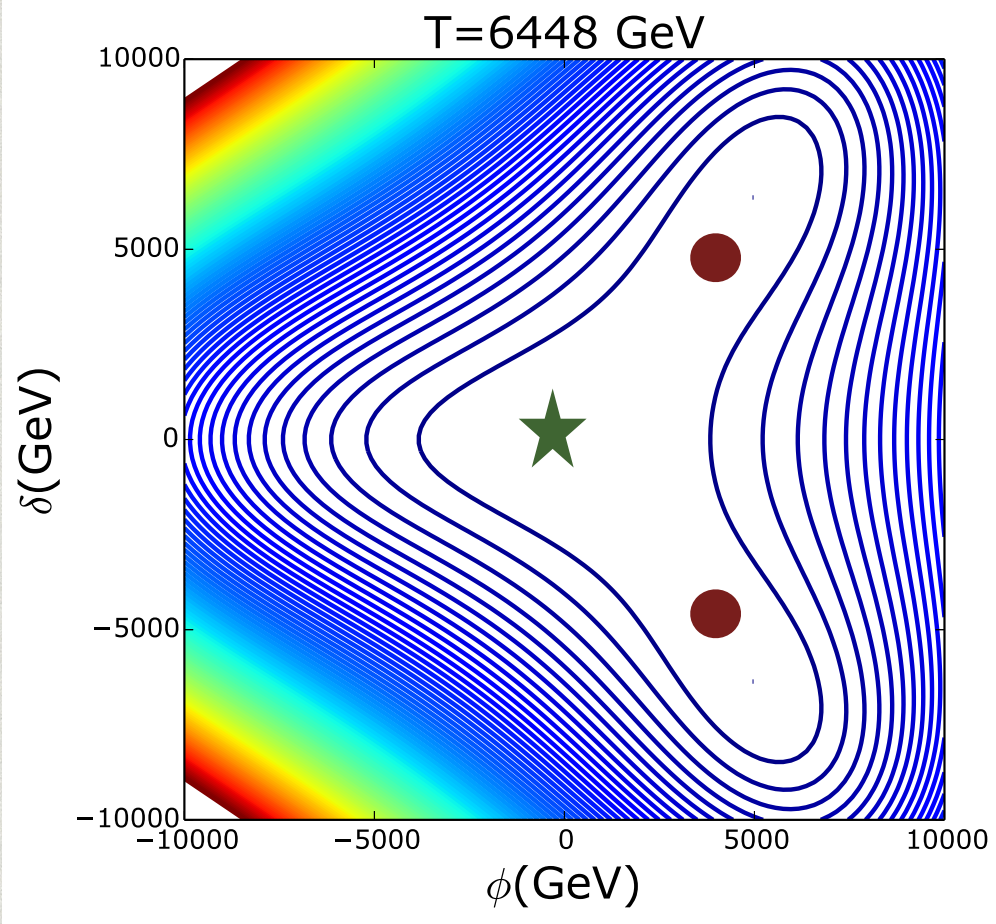
$$V_0^{(b)} = -\mu_\Phi^2 \Phi^\dagger \Phi + \kappa (\Phi^\dagger \Phi)^2 - \mu_\Delta^2 \Delta^\dagger \Delta + \kappa_1 (\Delta^\dagger \Delta)^2 + \kappa_2 (\Phi^\dagger \Phi)(\Delta^\dagger \Delta) + \{\Lambda \Delta^2 \Phi^\dagger + \text{h.c.}\},$$

scenario	Abelian symmetries	Q_L	ℓ_L	U_R	D_R	E_R	N_R	H	Φ	Δ
(a)	$\mathbf{B-L}$	1/3	-1	1/3	1/3	-1	-1	0	2	
(b)	$\mathbf{B-L}$	1/3	-1	1/3	1/3	-1	-1	0	2	1

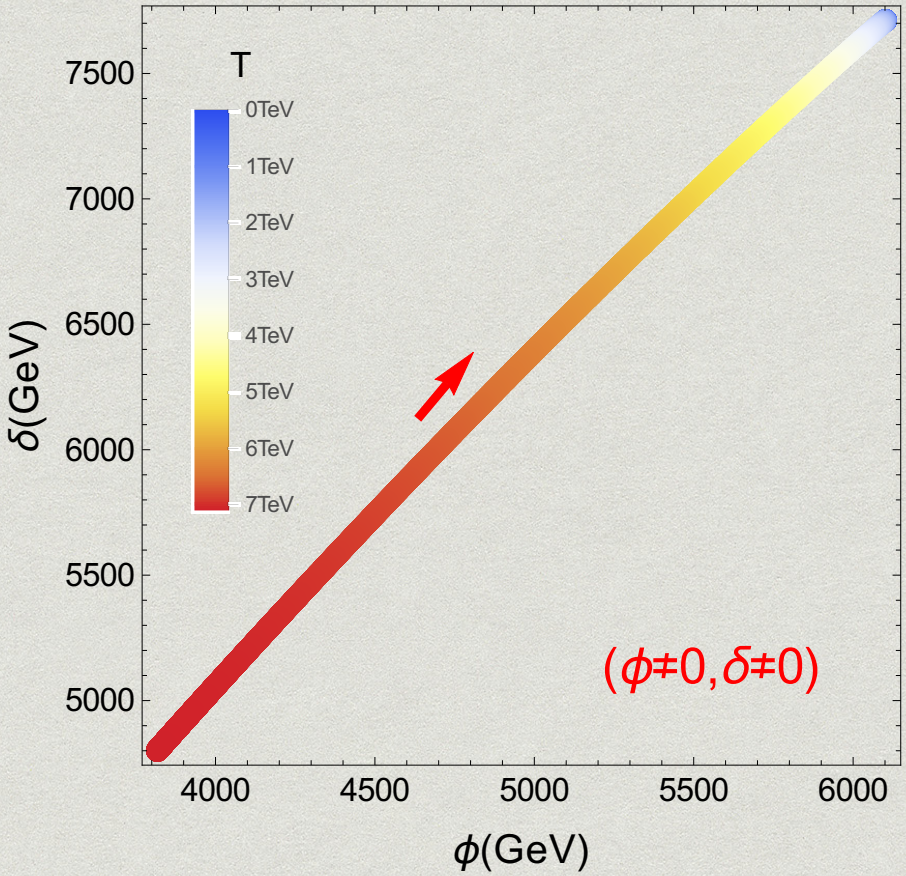
scenario (a)		sceinario (b)	
fields	masses	fields	masses
ϕ	$-\mu_\Phi^2 + 3\kappa\phi^2$	ϕ	$-\mu_\Phi^2 + 3\kappa\phi^2 + \frac{1}{2}\kappa_2\delta^2$
χ	$-\mu_\Phi^2 + \kappa\phi^2$	χ	$-\mu_\Phi^2 + \kappa\phi^2 + \frac{1}{2}\kappa_2\delta^2$
N	$y_{\mathbf{N}}^2\phi^2$	N	$y_{\mathbf{N}}^2\phi^2$
Z'	$4g_{\mathbf{B-L}}^2\phi^2$	Z'	$g_{\mathbf{B-L}}^2(4\phi^2 + \delta^2)$
		δ	$-\mu_\Delta^2 + 3\kappa_1\delta^2 + \frac{1}{2}\kappa_2\phi^2$
		χ'	$-\mu_\Delta^2 + \kappa_1\delta^2 + \frac{1}{2}\kappa_2\phi^2$

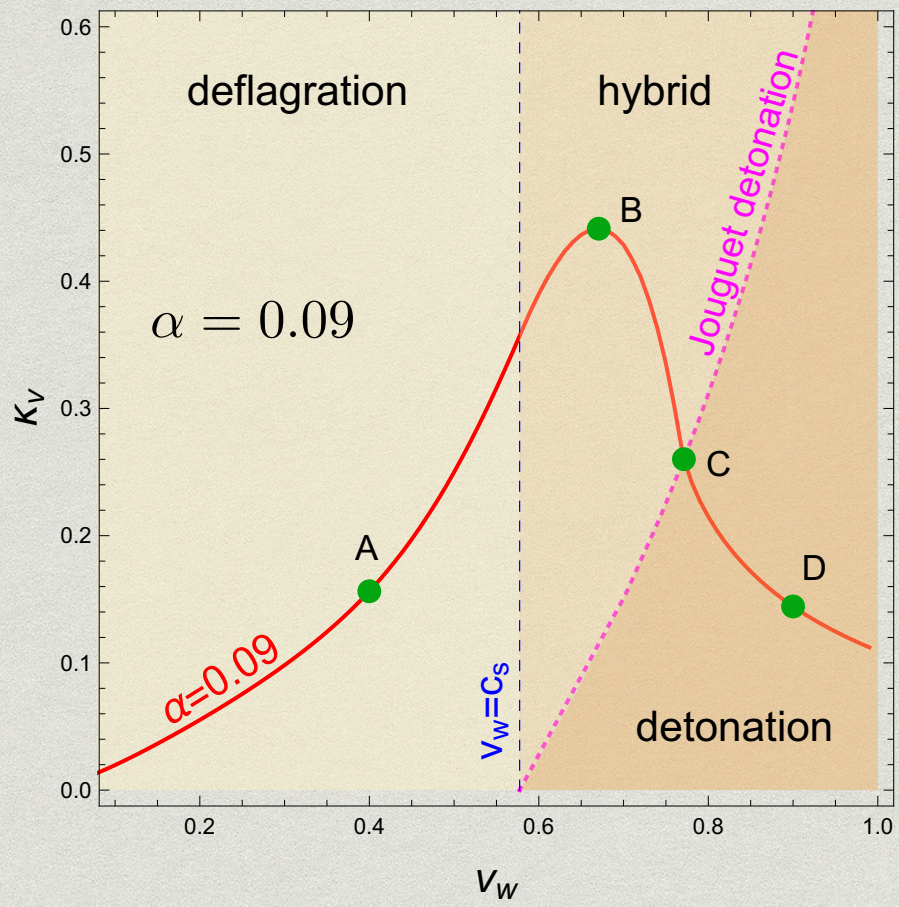
Contours of the potential at finite T



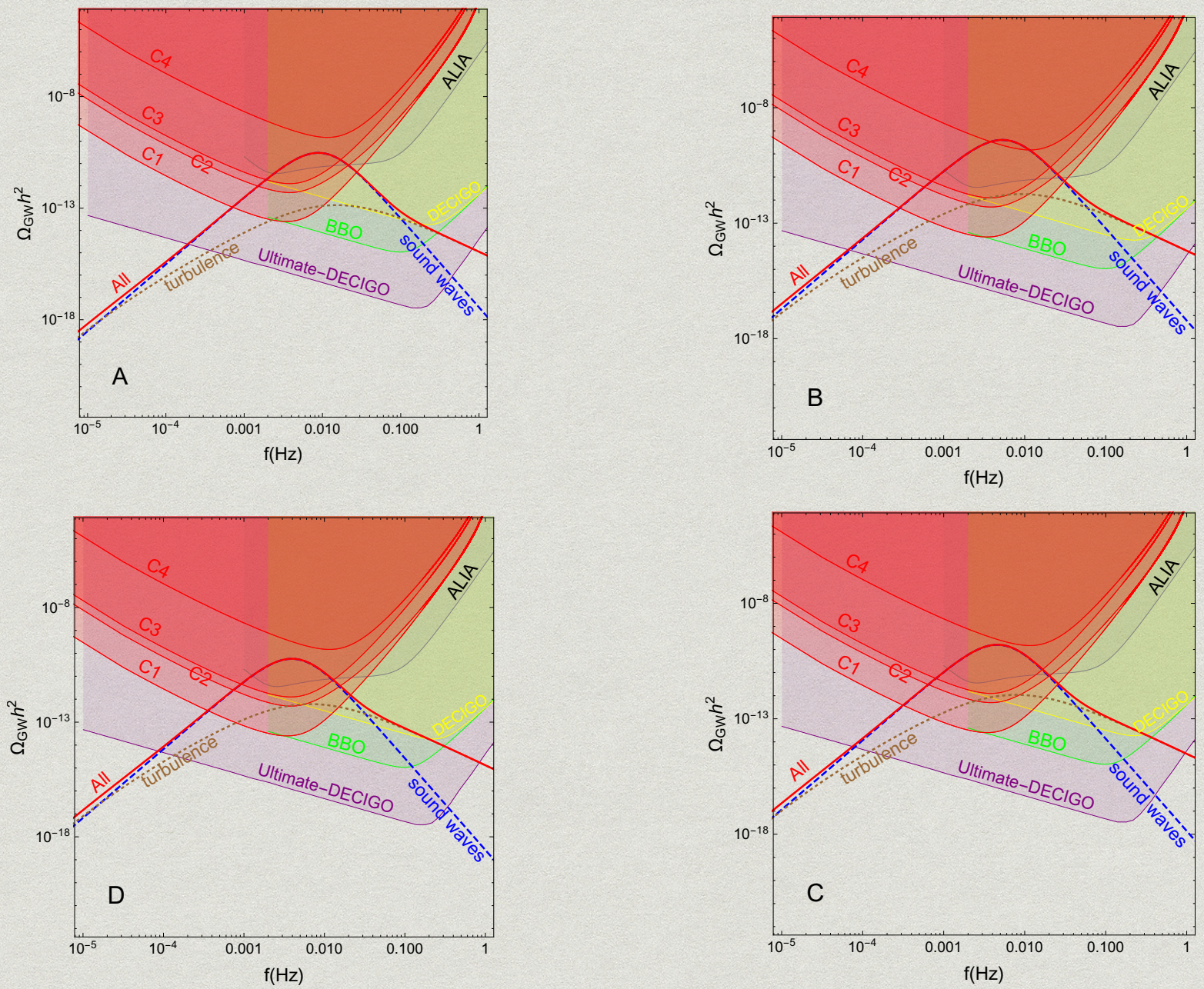


Further detail of vacuum evolution

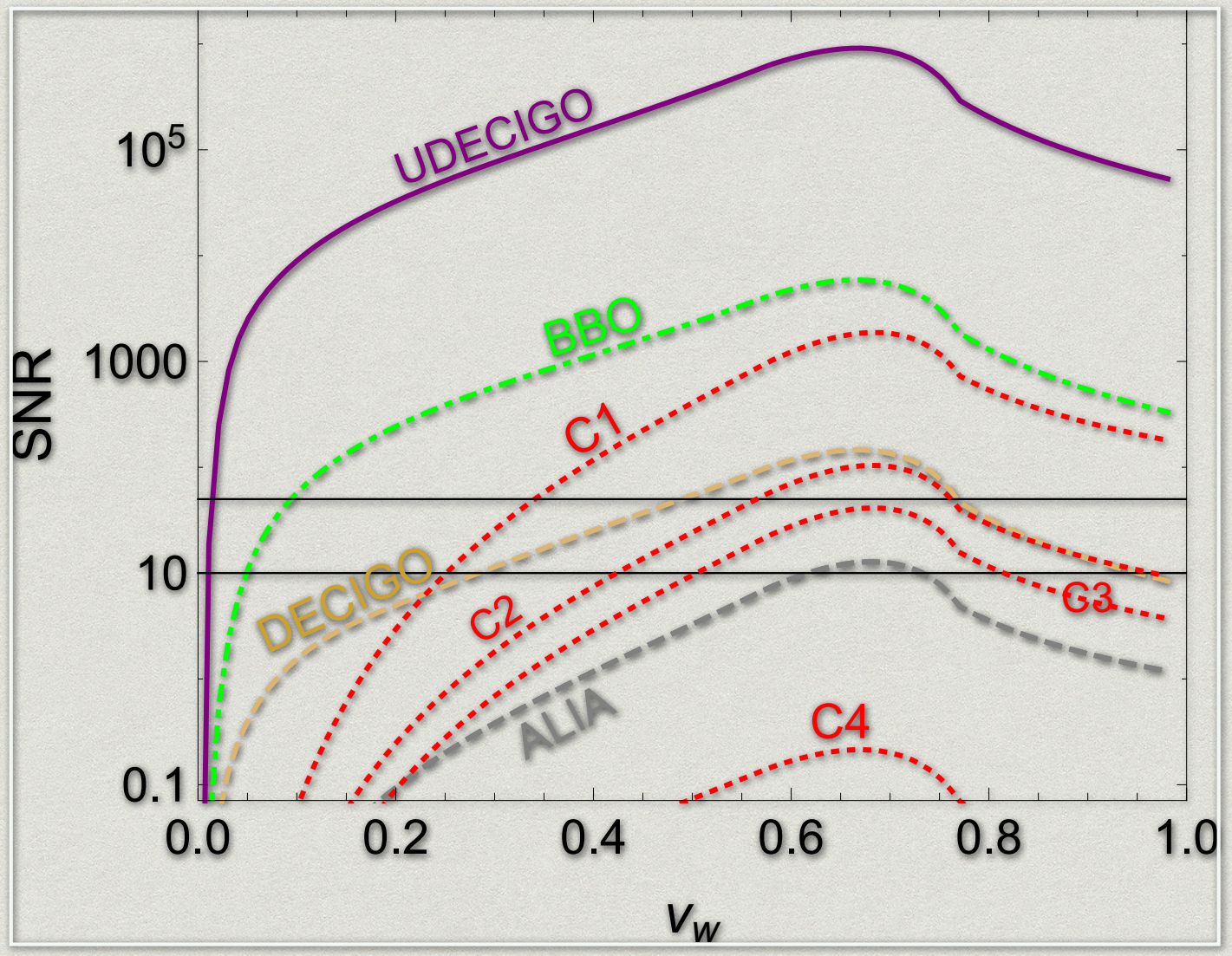




Signals


C4
C3
C1
C2
ALIA
DECIGO
BBO
Ultimate-DECIGO
sound waves
turbulence
All
A
B
C
D
$\Omega_{GW}h^2$
f(Hz)
10^{-8}
10^{-13}
10^{-18}
10^{-5}
10^{-4}
0.001
0.010
0.100
1

SNR



$$\mathrm{SNR} = \sqrt{\mathcal{T} \int_{f_{\min}}^{f_{\max}} df \left[\frac{h^2 \Omega_{\mathrm{GW}}(f)}{h^2 \Omega_{\exp}(f)} \right]^2},$$

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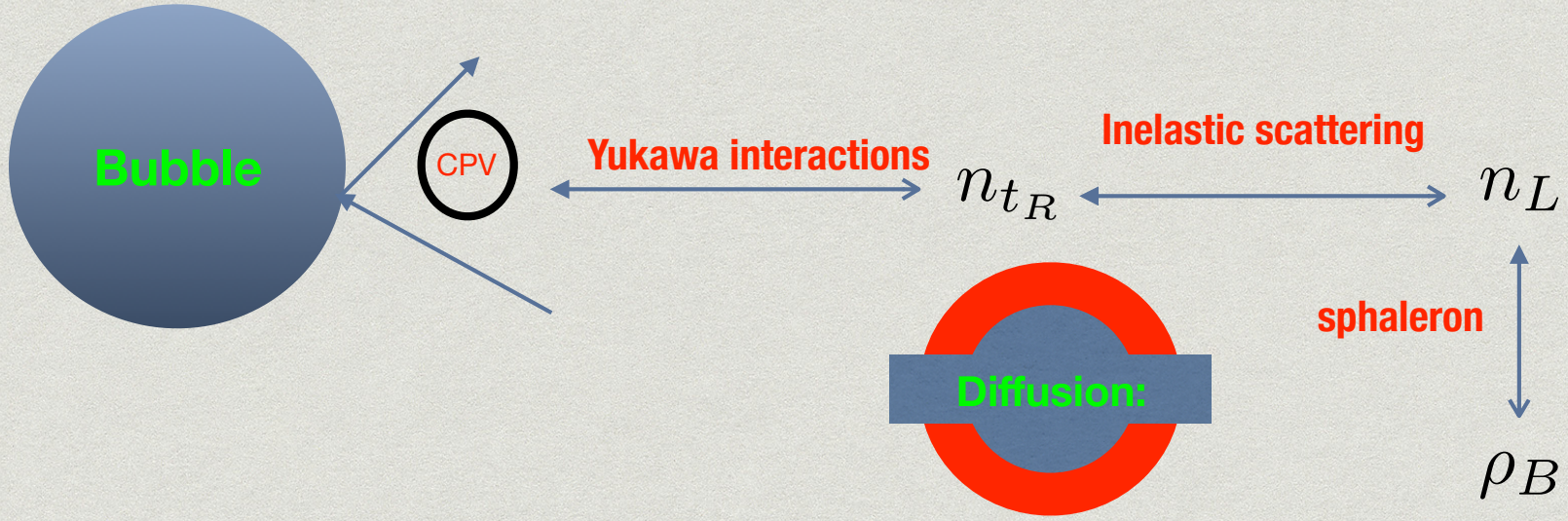
BAU during the EWPT

Basic description

Sakharov:

- $\not{B}$
- $\not{C}$&$\not{CP}$
- First order EWPT



Transport equation:

$$\partial_t \rho_B(x) - D \nabla \rho_B(x) = -\Gamma_{\rm ws} F_{\rm ws}(x)[n_L(x) - R\rho_B(x)]$$

Fate of the EWBG

Three Detectives

Conventional EWBG mechanism might be found or excluded in the near future when these three methods are combined.

Questions: Is there a mechanism of electroweak baryogenesis that can escape from (some of) hunters?

Properties of the EWPT (1)

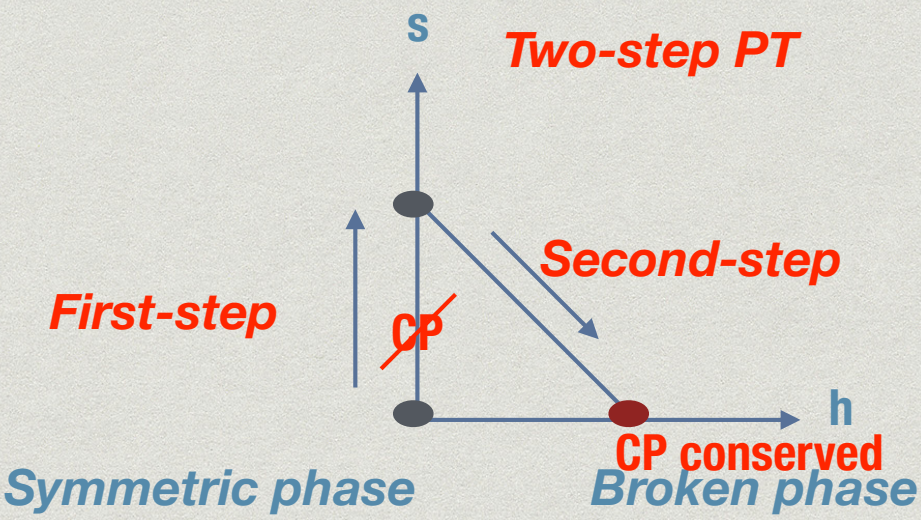
EWBG from spontaneous CP at the finite T

Lemma:
spontaneous CP violation in the theory of one complex scalar field may occur only when the related U(1) is explicitly broken by at least two spurions whose U(1) charges are different in magnitude

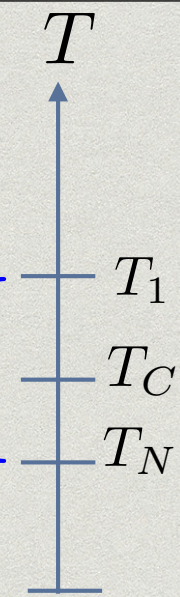
$$V = -\mu^2(H^\dagger H) + \lambda(H^\dagger H)^2 - \mu_A^2(S^\dagger S) + \lambda_1(S^\dagger S)^2 + \lambda_2(H^\dagger H)(S^\dagger S) - \frac{1}{2}\mu_B^2 S^2 + \frac{1}{2}\lambda_3 S^4 + \text{h.c.}$$

spurions spurions

How to avoid the constraint of non-observation of EDMs?



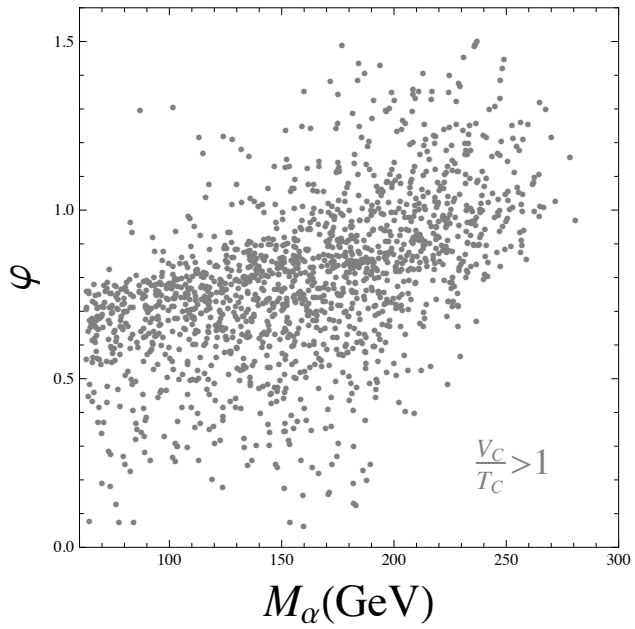
$$\varphi = \pm\frac{1}{2}\arccos\left[\frac{\lambda_1 - \lambda_3}{2\lambda_3}\frac{m_\beta^2 - m_\alpha^2}{\lambda_2 v^2 - m_\alpha^2 - m_\beta^2 + 2\Pi_\alpha}\right]$$

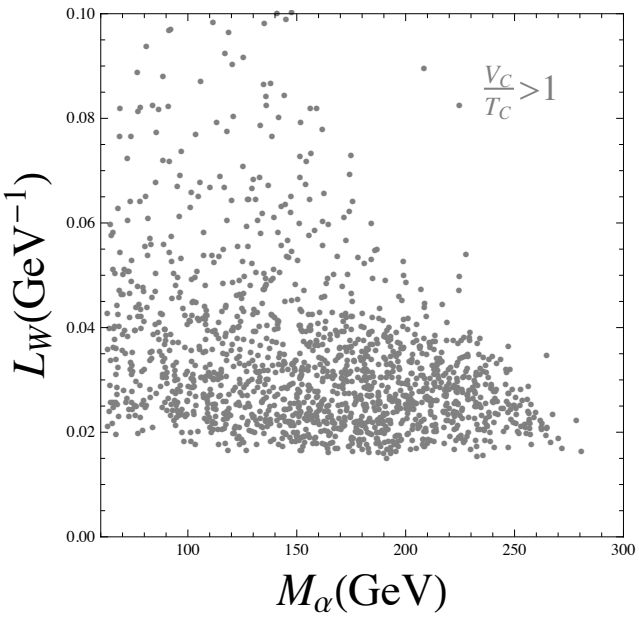


Properties of the EWPT (2)

CP phase and strength of the EWPT

EoM for three background fields:	$$\frac{d^2\phi_i}{dr^2} + \frac{2}{r}\frac{d\phi_i}{dr} = \bar{V}'(\vec{\phi})$$
Bubble wall width:	$$L_w^2 \approx 1.35 \frac{\lambda + \sqrt{\lambda\lambda_\varrho}}{(\lambda_2 - 2\sqrt{\lambda\lambda_\varrho})[\lambda v_0^2 - \Pi_h(T_C^2)]} \times \left(1 + \sqrt{\frac{\lambda_2^2}{4\lambda\lambda_\varrho}}\right)$$





Transport equations

EWBG

Transport equation

$$\frac{\partial n}{\partial t} + \nabla \cdot j(x) = -\int d^3z \int_{-\infty}^{x_0} dz^0 \mathrm{Tr}[\Sigma^>(x,z)S^<(z,x) - S^>(x,z)\Sigma^<(z,x) + S^<(x,z)\Sigma^>(z,x) - \Sigma^<(x,z)S^>(z,x)]$$

Source term:

$$S_{\mathrm{top}}^{\mathrm{CPV}} = -2\zeta^2 v_s^2 \dot{\varphi} \int \frac{k^2 dk}{\pi^2 \omega_L \omega_R} \mathrm{Im}\left\{(\varepsilon_L \varepsilon_R^* - k^2)\frac{n(\varepsilon_L) - n(\varepsilon_R^*)}{(\varepsilon_L - \varepsilon_R^*)^2} + (\varepsilon_L \varepsilon_R + k^2)\frac{n(\varepsilon_L) + n(\varepsilon_R)}{(\varepsilon_L + \varepsilon_R)^2}\right\}$$

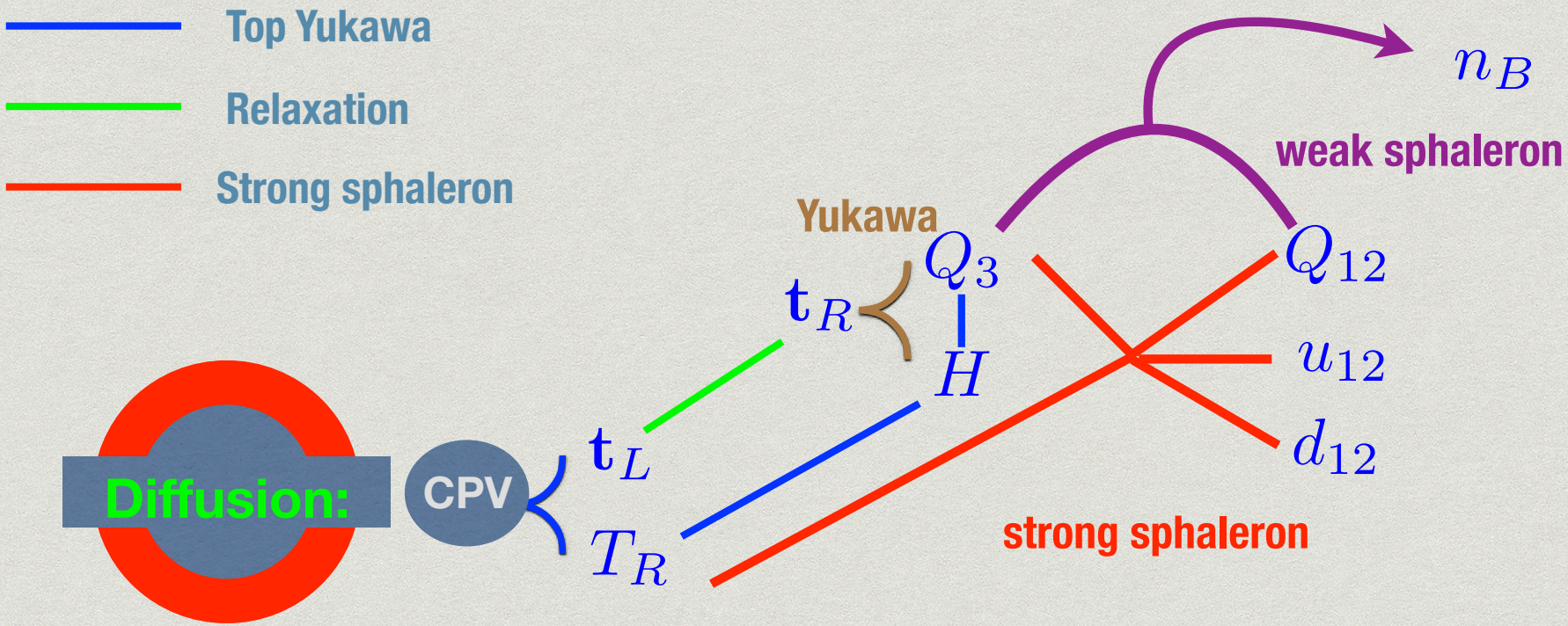
$$\zeta \overline{\mathfrak{t}_L} S t_R + (M_{\mathfrak{t}})\overline{\mathfrak{t}_L}\mathfrak{t}_R + \mathrm{h.c.} \longrightarrow \frac{y_t \zeta}{\Lambda}\overline{Q_3}\widetilde{H} S t_R$$

All equations

$$\begin{aligned}
\partial^\mu Q_\mu &= +\Gamma_{m_t}\mathcal{R}_T^- + \Gamma_{Y_t}\delta_t + \Gamma_{y'}\delta_{\mathfrak{t}'} + 2\Gamma_s\delta_s \\
\partial^\mu T_\mu &= -\Gamma_{m_t}\mathcal{R}_T^- - \Gamma_{Y_t}\delta_t - \Gamma_s\delta_s - \Gamma_\zeta\delta_{\mathfrak{t}} \\
&\quad +\Gamma_{\mathfrak{t}}^+\mathcal{R}_{\mathfrak{t}}^+ + \Gamma_{\mathfrak{t}}^-\mathcal{R}_{\mathfrak{t}}^- + S_{\mathrm{top}}^{\mathrm{CPV}} \\
\partial^\mu \mathfrak{t}_\mu &= +\Gamma_{m_{\mathfrak{t}}}\mathcal{R}_\Lambda^- - \Gamma_{\mathfrak{t}}^+\mathcal{R}_{\mathfrak{t}}^+ - \Gamma_{\mathfrak{t}}^-\mathcal{R}_{\mathfrak{t}}^- + \Gamma_\zeta\delta_{\mathfrak{t}} - S_{\mathrm{top}}^{\mathrm{CPV}} \\
\partial^\mu \mathfrak{t}'_\mu &= -\Gamma_{m_{\mathfrak{t}}}\mathcal{R}_\Lambda^- - \Gamma_{y'}\delta_{\mathfrak{t}'} \\
\partial^\mu S_\mu &= -\Gamma_\zeta\delta_{\mathfrak{t}} \\
\partial^\mu H_\mu &= -\Gamma_{Y_t}\delta_t - \Gamma_{y'}\delta_{\mathfrak{t}'} \qquad (13)
\end{aligned}$$

Diffusions

EWBG



Baryon number density:

$$\hat{n}_B = -\frac{3\Gamma_{ws}}{2D_Q\lambda_+}\int_{-\infty}^{-L_w/2} dz n_L(z) e^{-\lambda_- z}$$

Damping of the domain wall

EWBG

Problems

$+\varphi$ + $-\varphi$ =0

No BAU left

Solution: Adding a Z_2 breaking term to the Higgs potential: Δs+h.c.

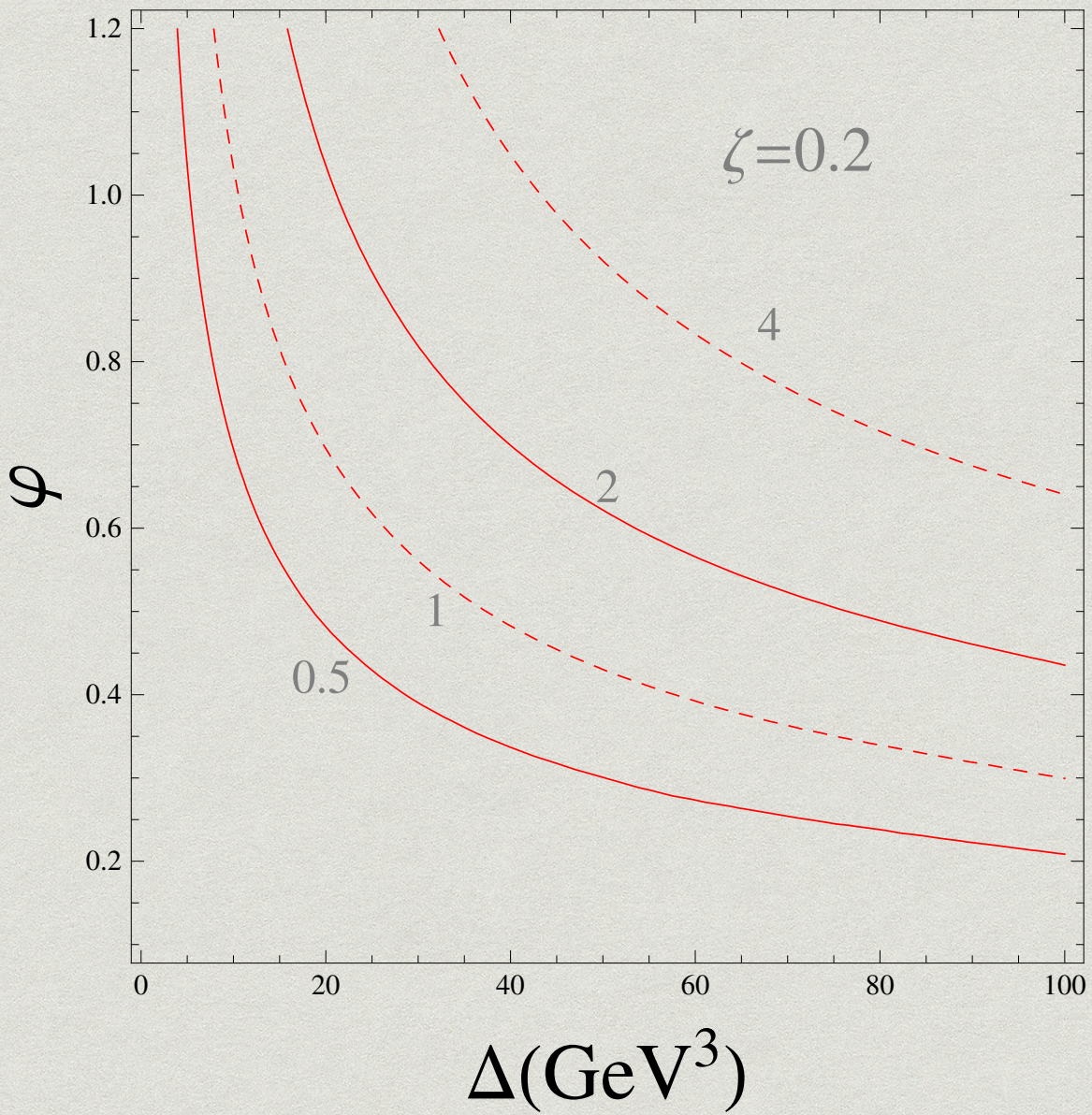
Ratio of bubbles

$$\frac{N_+}{N_-} = \exp\left(\frac{\Delta F}{T}\right)$$

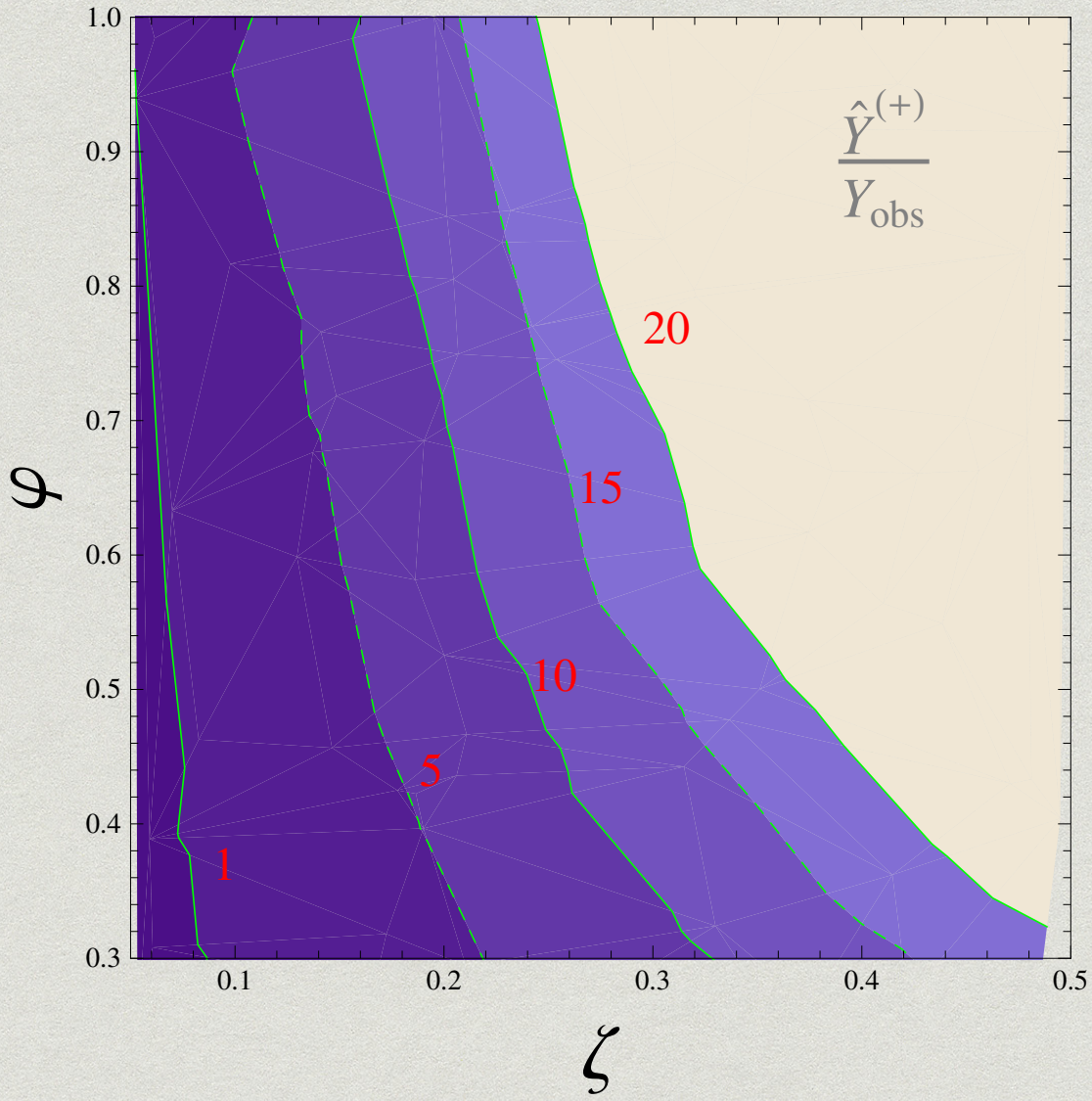
Final BAU

$$n_B = \hat{n}_B^{(+)} \frac{N_+ - N_-}{N_+ + N_-}$$

Numerical results



Numerical results



Summary

- Physics relevant to electroweak phase transition are briefly reviewed. We need to calculate many physical quantities, but many are very hard to do. Further study is needed! (Hard work is really hard.)
- Show you stochastic gravitational wave as an indirect detection of the new gauge symmetry breaking.
- The baryon asymmetry of the universe generated during the first order EWPT is discussed, especially I showed how to generate sufficient BAU with the spontaneous CP phase and a two-step EWPT.

Thank you