Axion mass and coupling constants at zero and finite temperatures

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Outline

QCD and the topological term

The QCD Lagrangian with a topological term

$$\mathcal{L}_{\rm QCD} = \bar{q}[i\gamma_\mu D^\mu - M]q - \frac{1}{4}G^c_{\mu\nu}G^{c,\mu\nu} + \mathcal{L}_{\theta_0}$$

with

$$\mathcal{L}_{\theta_0} = -\theta_0 \frac{g^2}{32\pi^2}{\rm Tr}[G^c_{\mu\nu}\tilde{G}^{c,\mu\nu}] \qquad (\tilde{G}^c_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{c,\rho\sigma})$$

Theory

- $\theta = \theta_0 + \arg\, \det M$
- Violate the CP symmetry
- Neutron electric dipole moment: $d_n \approx m_q \theta e / M_N^2$ with $m_q = m_u m_d/(m_u + m_d)$.

Solution

Experiment+Lattice

- $|d_n| < 2.9 \times 10^{-26}\ e\ \mathrm{cm}$ (90% C.L.) $\Rightarrow \theta \lesssim 10^{-10}$

C. A. Baker, D. D. Doyle et al, PRL 97, 131801 (2006); F. K. Guo, R. Horsley et al, PRL 115, 62001 (2015)

- Why θ is so small? (Strong CP problem)

Add a new term to the QCD Lagrangian

$$\mathcal{L}_{\mathrm{eff}} = -\theta \frac{g_s^2}{32\pi^2} G^c_{\mu\nu} \tilde{G}^{c,\mu\nu} + \frac{a}{f_a} \frac{g_s^2}{32\pi^2} G^c_{\mu\nu} \tilde{G}^{c,\mu\nu} + ...$$

- An additional global chiral $U(1)$ symmetry: $U(1)_{PQ}$
- $\Theta \equiv \theta - a/f_a$
- If $\langle\Theta\rangle$=0 $\quad\Rightarrow\quad$ QCD conserves CP symmetry
- $\Rightarrow$ *Solution*: Axion S. Weinberg, PRL 40, 223 (1978); F. Wilczek, PRL 40, 279 (1978); M. Dine, W. Fischler, and M. Srednicki, PLB 104, 199 (1981)

Chiral Lagrangian

The lowest order of SU(3) chiral Lagrangian in the θ vacuum

$$\mathcal{L}^{(2)} = \frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger\rangle + \frac{F^2}{4}\langle \chi_\theta U^+ + \chi_\theta^+ U\rangle$$

- $\chi_\theta = 2B\mathcal{M}\exp(i\theta/3)$
- $U \equiv U_0\tilde{U}(x)$, where U_0 is the vacuum of the theory and can be parameterized as $U_0 = \mathrm{diag}\{e^{i\varphi_u}, e^{i\varphi_d}, e^{i\varphi_s}\}$
- Constraint: $\varphi_u + \varphi_d + \varphi_s = 0$

The stationary solution for U_0

Y. Y. Mao and T. W. Chiu, PRD 80, 34502 (2009)

$$\boxed{\phi_f = \frac{\bar{m}}{m_f}\theta + \left[\left(\frac{\bar{m}}{m_f}\right)^3 - \frac{\bar{m}^4}{m_f \bar{m}^{[3]}}\right]\frac{\theta^3}{6} + \mathcal{O}(\theta^5)}$$

- with $\frac{1}{\bar{m}} = \sum_{i=1}^N \frac{1}{m_i}$, $\frac{1}{\bar{m}^{[3]}} = \sum_{i=1}^N \frac{1}{m_i^3}$
- $\varphi_f = \frac{\theta}{3} - \phi_f$.

Goldstone boson fields and meson masses

- The unitary matrix collecting the Goldstone boson fields: $\tilde{U} = e^{i\Phi/f_\pi}$ with Φ given by

$$\Phi = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$

$\Rightarrow$ The θ-dependent meson mass

- $M^2_{\pi^\pm} = B\sum_{u,d} m_f \cos\phi_f$, $M^2_{K^\pm} = B\sum_{u,s} m_f \cos\phi_f$
- $M^2_{K^0} = M^2_{\bar{K}^0} = B\sum_{d,s} m_f \cos\phi_f$
- $M^2_{\pi^0} = B\Big(\sum_{u,d} m_f \cos\phi_f + \epsilon_a\Big)$
- $M^2_\eta = \frac{B}{3}\Big(\sum_{u,d} m_f \cos\phi_f + 4m_s\cos\phi_s - 3\epsilon_a\Big)$ with ϵ_a given by

$$\epsilon_a = (m_u\cos\phi_u + m_d\cos\phi_d - 2m_s\cos\phi_s)\frac{2\sin^2\epsilon}{3\cos 2\epsilon}$$

pion-eta mixing angle ϵ: $\tan 2\epsilon = \frac{\sqrt{3}(m_u\cos\phi_u - m_d\cos\phi_d)}{m_u\cos\phi_u + m_d\cos\phi_d - 2m_s\cos\phi_s}$

Tree-level Lagrangian at NLO

$$\begin{aligned}\mathscr{L}_4 = {} & L_1\left\{\mathrm{Tr}[D_\mu U(D^\mu U)^\dagger]\right\}^2 + L_2\mathrm{Tr}\left[D_\mu U(D_\nu U)^\dagger\right]\mathrm{Tr}\left[D^\mu U(D^\nu U)^\dagger\right] \\ & + L_3\mathrm{Tr}\left[D_\mu U(D^\mu U)^\dagger D_\nu U(D^\nu U)^\dagger\right] + L_4\mathrm{Tr}\left[D_\mu U(D^\mu U)^\dagger\right]\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger) \\ & + L_5\mathrm{Tr}\left[D_\mu U(D^\mu U)^\dagger(\chi U^\dagger + U\chi^\dagger)\right] + L_6\left[\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)\right]^2 \\ & + L_7\left[\mathrm{Tr}(\chi U^\dagger - U\chi^\dagger)\right]^2 + L_8\mathrm{Tr}(U\chi^\dagger U\chi^\dagger + \chi U^\dagger \chi U^\dagger) \\ & - iL_9\mathrm{Tr}\left[f^R_{\mu\nu}D^\mu U(D^\nu U)^\dagger + f^L_{\mu\nu}(D^\mu U)^\dagger D^\nu U\right] + L_{10}\mathrm{Tr}\left(Uf^L_{\mu\nu}U^\dagger f_R^{\mu\nu}\right) \\ & + H_1\mathrm{Tr}\left(f^R_{\mu\nu}f_R^{\mu\nu} + f^L_{\mu\nu}f_L^{\mu\nu}\right) + H_2\mathrm{Tr}(\chi\chi^\dagger)\end{aligned}$$

Incoperate the θ parameter or axion field $\chi \to \chi_\theta$

$$\begin{aligned}\mathcal{L}^{(4,\mathrm{tree})} = {} & L_6\langle\chi_\theta U^+ + \chi_\theta^+ U\rangle^2 \\ & + L_7\langle\chi_\theta U^+ - \chi_\theta^+ U\rangle^2 + H_2\langle\chi_\theta^+\chi_\theta\rangle \\ & + L_8\langle\chi_\theta^+ U\chi_\theta^+ U + \chi_\theta U^+\chi_\theta U^+\rangle\end{aligned}$$

One-loop generating functional

Originally, the one-loop generational functional $Z_{\mathrm{one-loop}}$ is expanded around the functional for free fields at $\theta = 0$ J. Gasser and H. Leutwyler, NPB 250, 465 (1985)

$$Z_{\mathrm{one-loop}} = \frac{i}{2} \ln \ \det D^0 + \frac{i}{4} \mathrm{Tr}[(D^0)^{-1} \delta] + \ldots$$

- $\delta = \bar{\sigma}_{PQ} + \ldots = \sigma^{\triangle}_{PQ} + \sigma^{\chi}_{PQ} + \ldots$
- $\sigma^{\triangle}_{PQ} = \frac{1}{8} \langle [\tau^a, U^\dagger D_\mu U][\tau^b, U^\dagger D^\mu U] \rangle$ (in the vacuum, this term vanishes)
- $\sigma^{\chi}_{PQ} = \frac{1}{8} \langle \{\tau^a, \tau^b\} (\chi^+ U + U^\dagger \chi) \rangle - \delta_{PQ} M_P^2(0)$

It was then applied to study the topological susceptibility. And the loops were calculated using the Goldstone boson masses at $\theta = 0$, while the θ-dependence is kept in the first part of σ^{χ}_{PQ}:

Y. Y. Mao and T. W. Chiu, PRD 80, 34502 (2009); V. Bernard, S. Descotes-Genon, and G. Toucas, JHEP 2012, 80 (2012)

$$\sigma^{\chi}_{PQ} \quad \Rightarrow \quad \sigma^{\chi}_{PQ} = \frac{1}{8} \langle \{\tau^a, \tau^b\} (\chi^+_\theta U + U^\dagger \chi_\theta) \rangle - \delta_{PQ} M_P^2(0)$$

One-loop contribution

In fact, the pseudoscalar meson masses are θ-dependent $\Rightarrow M_P^2(\theta)$. If we expand the one-loop generating functional around the one for the free fields in a θ-vacuum, we get F. K. Guo and U.-G. Meißner, PLB 749, 278 (2015)

$$Z(\theta) = \frac{i}{2} \ln \det D^0(\theta) = \frac{i}{2} \mathrm{Tr} \ln D^0(\theta)$$

In this case, σ_{PQ}^{χ} vanishes. $Z(\theta) = \frac{i}{2} \mathrm{Tr} \ln D^0(\theta)$ is the only term left in the one-loop generating functional. **The discussion is not only valid for $N = 2$ case but also for an arbitrary N case.**

- The differential operator $D_{PQ}^0(\theta)$: $D_{PQ}^0(\theta) = [\partial^\mu \partial_\mu + M_p^2(\theta)]\delta_{PQ}$

$\Rightarrow$ Loop contribution

$$\mathcal{L}_{\text{loop}} = \frac{1}{V} \frac{i}{2} \mathrm{Tr} \ln D^0(\theta) = \sum_P \frac{M_P^4(\theta)}{128\pi^2} \left[1 - 2 \ln \frac{M_P^2(\theta)}{\mu^2} - 64\pi^2 \lambda \right]$$

Divergences

The divergence at $d = 4$ dimension

$$\lambda = \frac{\mu^{d-4}}{16\pi^2}\left\{\frac{1}{d-4} - \frac{1}{2}\left[1 + \Gamma'(1) + \ln(4\pi)\right]\right\}$$

From the $\mathcal{O}(p^4)$ tree-level Lagrangian

$$\mathcal{L}^{(4,\text{tree})} \quad \Longrightarrow \quad \mathcal{L}^{(4,\infty)}_{\text{tree}} = \frac{\lambda}{2}\sum_P M_P^4(\theta)$$

- $L_6 = L_6^r + \frac{11}{144}\lambda, \quad L_8 = L_8^r + \frac{5}{48}\lambda$
- $L_7 = L_7^r, \quad H_2 = H_2^r + \frac{5}{24}\lambda$

From the loop contribution

$$\mathcal{L}^{(4,\infty)}_{\text{loop}} = -\frac{\lambda}{2}\sum_P M_P^4(\theta)$$

$\Longrightarrow$ **The finial result is finite!!!** And the θ vacuum energy density is obtained by summing up all contributions.

Axion potential

Putting all the pieces together, the axion potential is ($\theta = a/f_a$)

$$\begin{aligned} V(a/f_a) &= -F^2 B \sum_i m_i \cos\phi_i - 16B^2 \Big[L_6^r \Big(\sum_i m_i \cos\phi_i \Big)^2 \\ &\quad - L_7^r \Big(\sum_i m_i \sin\phi_i \Big)^2 + L_8^r \sum_i m_i^2 \cos^2\phi_i \Big] \\ &\quad - \sum_P \frac{M_P^4(a/f_a)}{128\pi^2} \left(1 - 2\ln \frac{M_P^2(a/f_a)}{\mu^2} \right) \end{aligned}$$

Axion mass and the self-coupling constant

$$m_a = 5.87(18) \cdot \frac{10^3}{f_a}\ \mathrm{MeV}^2$$

$$\lambda_4 = -\left(\frac{58.7(23)\ \mathrm{MeV}}{f_a} \right)^4$$

Topological susceptibility and the normalised fourth cumulant

$$\chi_t^{1/4} = \sqrt{m_a f_a} = 76.60(118)\ \mathrm{MeV}$$

$$b_2 = \frac{\lambda_4 f_a^2}{12 m_a^2} = -0.0287(32)$$

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Topological susceptibility in 2+1-flavor QCD with chiral fermions

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Abstract. We compute the topological susceptibility χ_t of 2+1-flavor lattice QCD with dynamical Möbius domain-wall fermions, whose residual mass is kept at 1 MeV or smaller. In our analysis, we focus on the fluctuation of the topological charge density in a "slab" sub-volume of the simulated lattice, as proposed by Bietenholz et al. The quark mass dependence of our results agrees well with the prediction of the chiral perturbation theory, from which the chiral condensate is extracted. Combining the results for the pion mass M_π and decay constant F_π, we obtain $\chi_t = 0.227(02)(11)M_\pi^2F_\pi^2$ at the physical point, where the first error is statistical and the second is systematic.

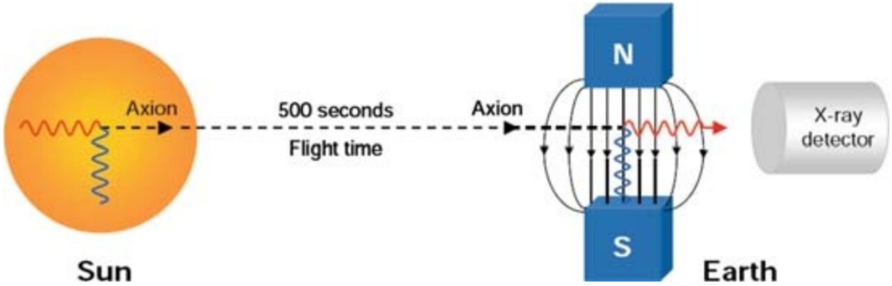
$$\Rightarrow \quad \chi_t^{1/4} \simeq 77.00 \text{ MeV}.$$

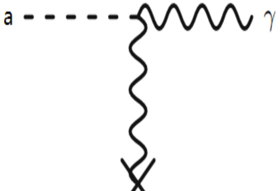
S. Borsanyi et al, Nature 539, 69-71 (2016)

$\chi(T=0) = 0.0245(24)(12)\ \text{fm}^{-4}$ in the isospin symmetric case; the first error in parentheses is statistical, the second is systematic. Isospin breaking results in a small, 12% correction, thus the physical value is $\chi(T=0) = 0.0216(21)(11)\ \text{fm}^{-4} = [75.6(1.8)(0.9)\ \text{MeV}]^4$.

Why $g_{a\gamma\gamma}$?

E.g.: Working principle of an axion helioscope





- Primakoff Effect
    - Microwave Cavity: ADMX
    - Axion Telescope: CAST
    - photon regeneration: Light shining through walls
    - . . .

Wess-Zumino-Witten (WZW) Lagrangian

- The Wess-Zumino-Witten (WZW) Lagrangian accounts for the chiral anomaly, which can be written in the following compact form as J. Wess and B. Zumino, PLB 37, 95 (1971); E. Witten, NPB 223, 422 (1983)

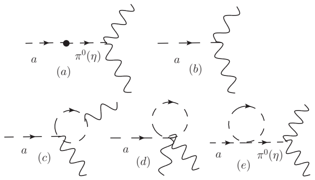
$$\begin{aligned}\mathcal{L}_{\mathrm{WZW}} &= -\frac{eN_c}{48\pi^2}\varepsilon^{\mu\nu\rho\sigma}A_\mu \mathrm{Tr}[Q\partial_\nu U U^\dagger \partial_\rho U U^\dagger \partial_\sigma U U^\dagger \\ &+QU^\dagger\partial_\nu U U^\dagger \partial_\rho U U^\dagger \partial_\sigma U] + i\frac{e^2N_c}{48\pi^2}\varepsilon^{\mu\nu\rho\sigma} \\ &\times\partial_\nu A_\rho A_\sigma \mathrm{Tr}[2Q^2(U\partial_\mu U^\dagger - U^\dagger\partial_\mu U) \\ &-QU^\dagger Q\partial_\mu U + QUQ\partial_\mu U^\dagger],\end{aligned}$$

  where $\varepsilon^{\mu\nu\rho\sigma}$ and Q denote the completely antisymmetric tensor and the usual diagonal quark charge matrix, respectively.
- The anomalous axion-photon coupling

$$\frac{1}{4}g_{a\gamma\gamma}aF^{\mu\nu}\tilde{F}_{\mu\nu}$$

Feynman diagrams

- The Feynman diagrams responsible for the correction of $g_{a\gamma\gamma}$:



The full axion-photon coupling $g_{a\gamma\gamma}$ up to NLO can be written as

$$\begin{aligned} g_{a\gamma\gamma} &= \frac{\alpha}{2\pi f_a}\Bigg[\frac{E}{C} - \frac{4\bar{m}}{m_u} - \frac{1024}{3\hat{m}}(C_7^W + 3C_8^W)\pi^2\bar{m}m_{\pi^+}^2 \\ &\quad + \frac{16 m_{\pi^+}^4 \bar{m}(3L_7^r + L_8^r)}{F_\pi^2 \hat{m}^2}\left(\frac{m_d - m_u}{m_{\pi^0}^2} + \frac{2(m_s - \hat{m})}{3m_\eta^2}\right) + g_{a\gamma\gamma}^{(e)}\Bigg] \\ &= \frac{\alpha}{2\pi f_a}\left[\frac{E}{C} - 2.674(59)\right] \end{aligned}$$

χPT at finite temperature

- The temperature effects enter χPT through the loop corrections.
- The axion potential at finite temperature:

$$\mathcal{V}(\theta,T) = \mathcal{V}_0\left[1+\frac{3}{2}\frac{T^4}{f_\pi^2 M_\theta^2}\int_0^\infty x^2\log\left(1-e^{-E_\theta}\right)\mathrm{d}x\right]$$

- Axion mass at finite temperature:

$$\frac{m_a^2(T)}{m_a^2} = 1-\frac{3T^2}{4\pi^2 f_\pi^2}\int_0^\infty \frac{x^2}{E_T}\frac{\mathrm{d}x}{e^{E_T}-1}$$

- The axion quartic self-coupling at finite temperature:

$$\frac{\lambda_a(T)}{\lambda_a} = 1-\frac{3T^2}{4\pi^2 f_\pi^2}\int_0^\infty \frac{x^2}{E_T}\frac{\mathrm{d}x}{e^{E_T}-1}+\frac{9z}{z^2-z+1} \times\frac{m_\pi^2}{8f_\pi^2}\int_0^\infty \frac{x^2}{\pi^2}\frac{e^{E_T}(E_T+1)-1}{(e^{E_T}-1)^2E_T^3}\mathrm{d}x$$

The NJL model

- The SU(2) NJL model Lagrangian with the instanton effects taken into accounts is given by

$$\mathcal{L} = \bar{q}(i\gamma^\mu \partial_\mu - m)q + \mathcal{L}_{\bar{q}q} + \mathcal{L}_{\text{det}}$$

  - the attractive part of the $\bar{q}q$ channel of the Fierz transformed color current-current interaction:

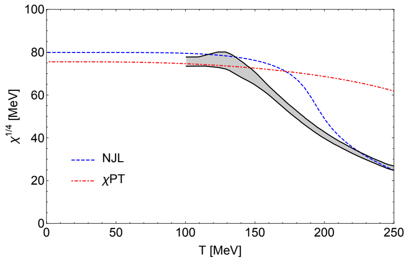
$$\mathcal{L}_{\bar{q}q} = G_1[(\bar{q}\tau_a q)^2 + (\bar{q}\tau_a i\gamma_5 q)^2]$$

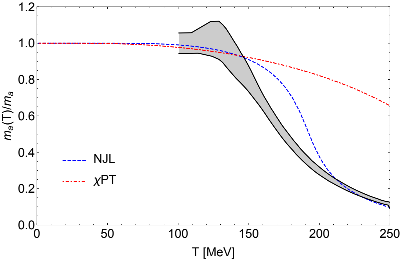
  - The 't Hooft determinant interaction, which depends on the QCD vacuum angle θ, is given by

$$\mathcal{L}_{\text{det}} = 8G_2\left[e^{i\theta}\det(q_R q_L) + \det(q_L q_R)e^{-i\theta}\right]$$

Topological susceptibility and axion mass

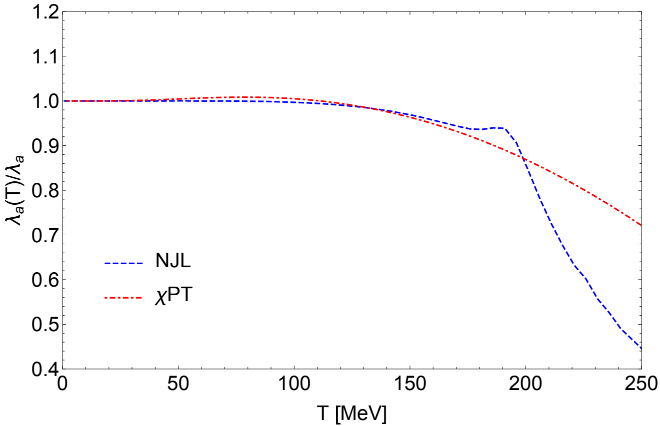
- NJL (T=0): $m_a = 6.38 \times \frac{10^3}{f_a}\ \mathrm{MeV}^2, \quad \lambda_a = -(\frac{55.64\ \mathrm{MeV}}{f_a})^4$
- χPT (T=0): $m_a = 5.70 \times \frac{10^3}{f_a}\ \mathrm{MeV}^2, \quad \lambda_a = -(\frac{57.93\ \mathrm{MeV}}{f_a})^4$





- Fourth root of the topological susceptibility obtained from several methods as a function of the temperature.

- The thermal behavior of the temperature-dependence of the axion mass from NJL model scaled by its zero temperature value.



• The thermal behavior of the axion self-coupling scaled by its zero temperature value.

Summary

- The obtained numerical QCD axion mass is $m_a = 5.87(18) \times 10^3 / f_a \ \mathrm{MeV}^2$;
- As a by product, we also obtain: $\chi_t^{1/4} = 76.60(118)\ \mathrm{MeV}$, which is **in good agreement with** the lattice results;
- We also derive the axion-photon coupling **analytically and numerically up to NLO**. Numerically, we obtain $g_{a\gamma\gamma} = \frac{\alpha}{2\pi f_a}\left[\frac{E}{C} - 2.674(59)\right]$;
- Numerically, **the inclusion of the strange quark does not change** the axion properties **dramatically** compared to the SU(2) case;
- The χPT results at finite temperature might be quantitatively reliable when $T \lesssim 140$ MeV;
- Both the axion mass and the quadratic self-coupling are not showing obvious drop. However, after the chiral transition point both of them decrease obviously as the temperature increases.

Thank You