Nuclear matter from skyrmion crystal approach in magnetic field

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collaborator :
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Shinya Matsuzaki (Jilin University, Nagoya University)

arXiv:1804.09015 [nucl-th].

Outline

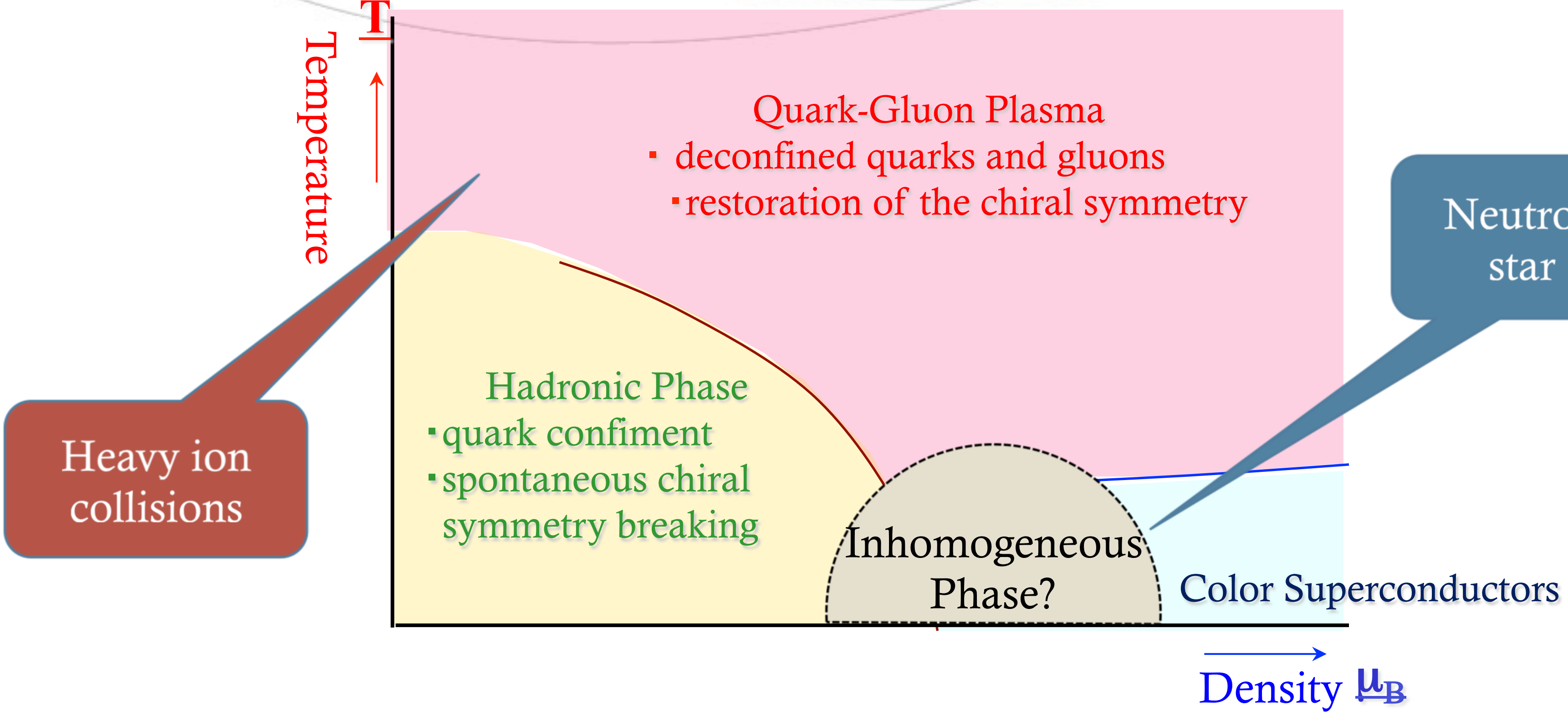
- Introduction
- Our work

  - Short review of skyrmoin and skyrmion crystal

  - Skyrmion crystal in a magnetic field
- Summary

1. Introduction

QCD phase structure

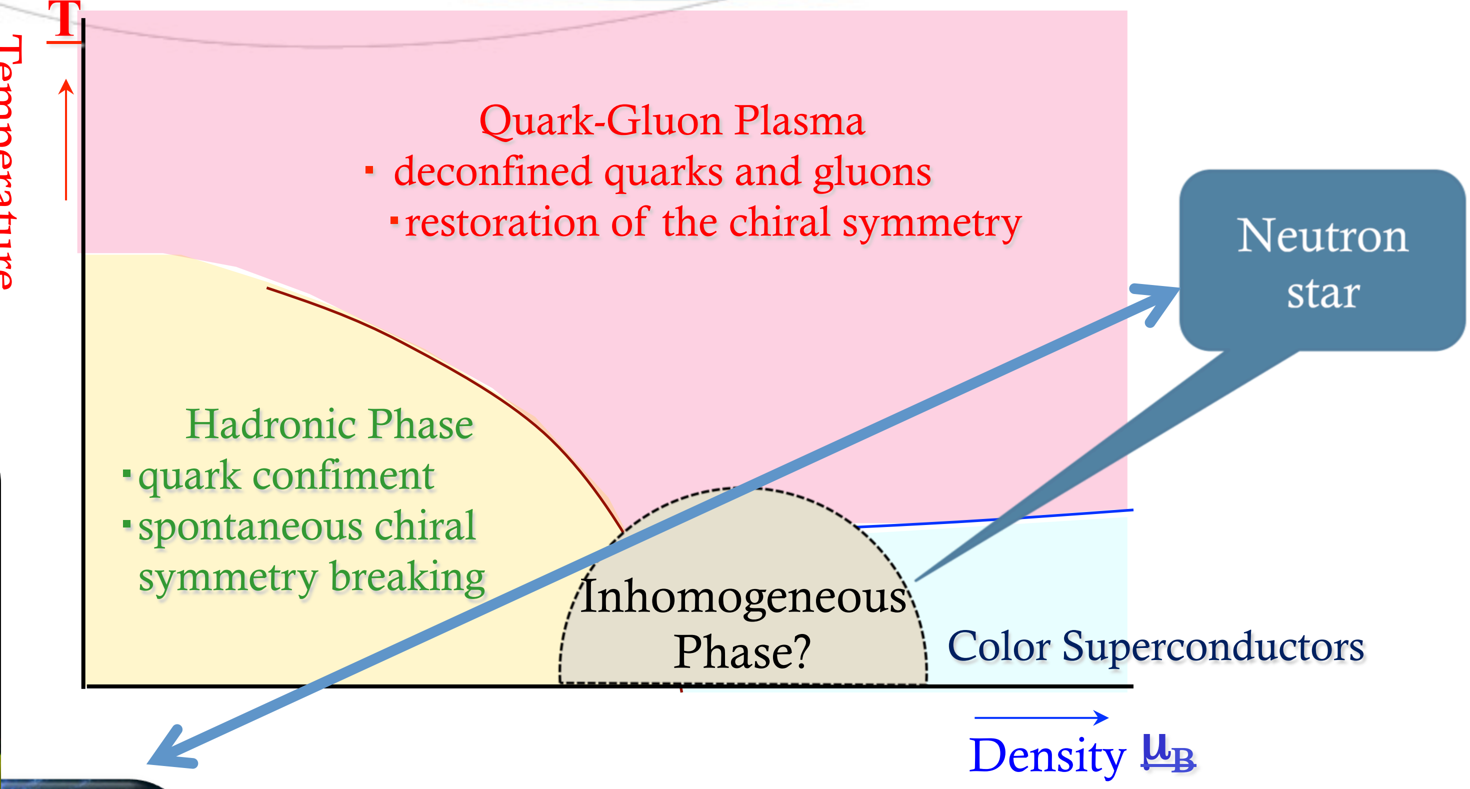


- QCD phase structure has not completely been understood yet.

 (e.g. the mass generation mechanism in terms of the chiral symmetry breaking)

- Does phase diagram have any other axis ?

QCD phase structure



T

Temperature

Quark-Gluon Plasma

- deconfined quarks and gluons
- restoration of the chiral symmetry

Hadronic Phase

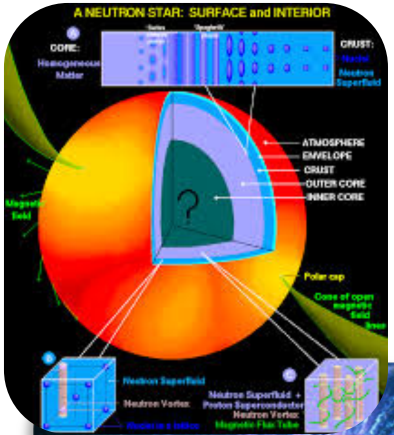
- quark confiment
- spontaneous chiral symmetry breaking

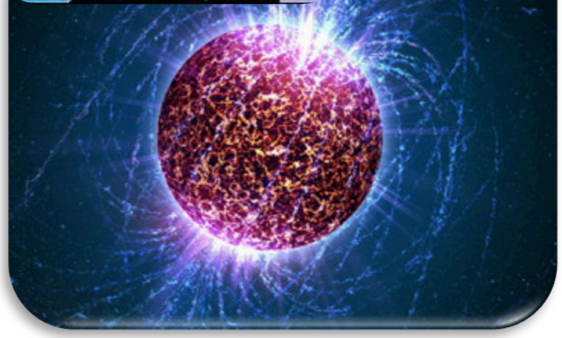
Inhomogeneous Phase?

Color Superconductors

Density μ_B

Neutron star





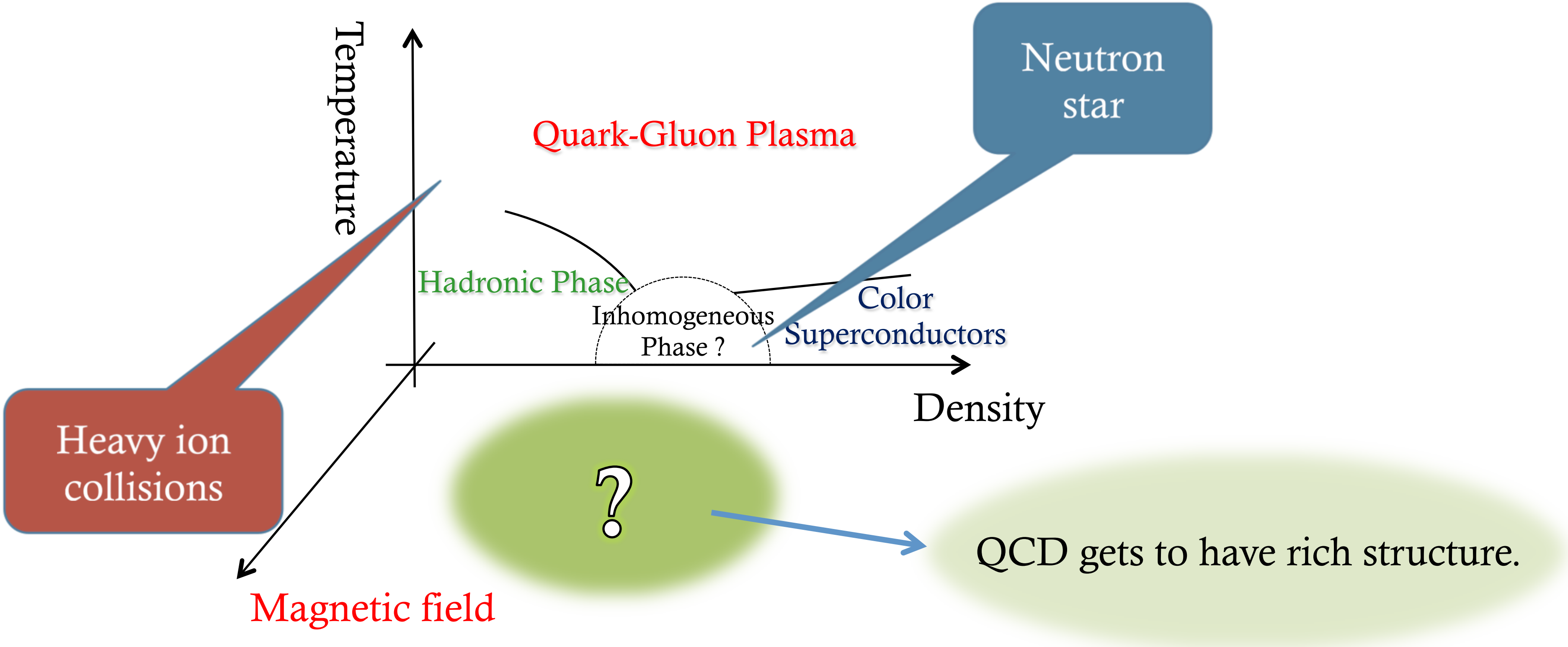
Magnetar has an extremely strong magnetic field.

Magnetar (surface)	10^{15}Gauss
Magnetar (inner core)	10^{18}Gauss

It seems that the hadron properties are changed dramatically in a magnetic field.

Need eB-axis for QCD phase structure

QCD phase diagram includes only temperature and density. ⇨ It would be important to add an axis along the magnetic field to QCD phase diagram.



Purpose of my study is to get the new insight for understanding the phase structure of QCD through such an extreme condition.

High density region and Strong magnetic field

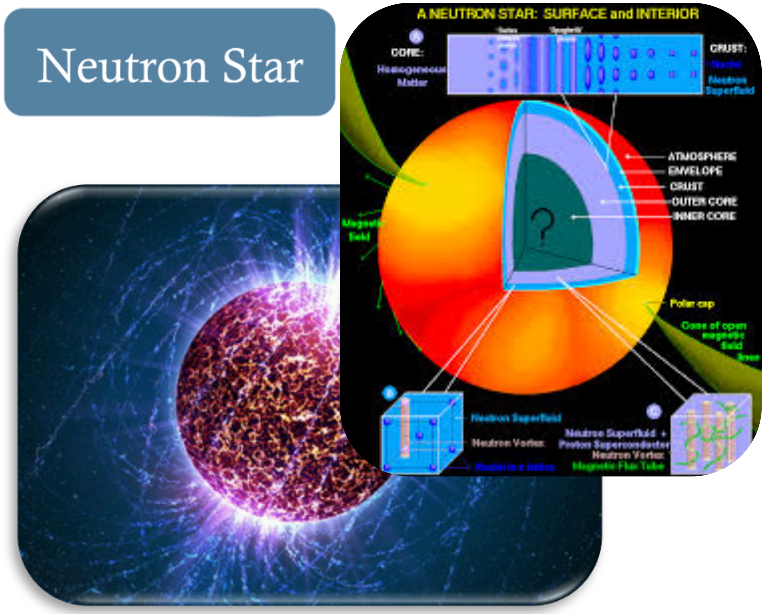
How to tackle to QCD phase structure

Summarize the above…

The purpose of my research is to extract the new aspect of QCD phase structure through extreme conditions, i.e. high density region with a magnetic field.

To accomplish my purpose, I focus on a baryonic matter with a strong magnetic field.

Neutron Star



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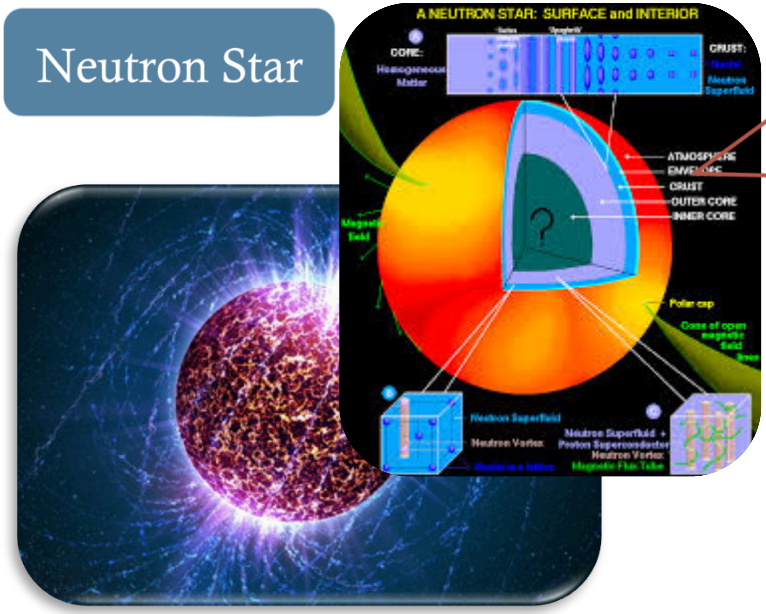
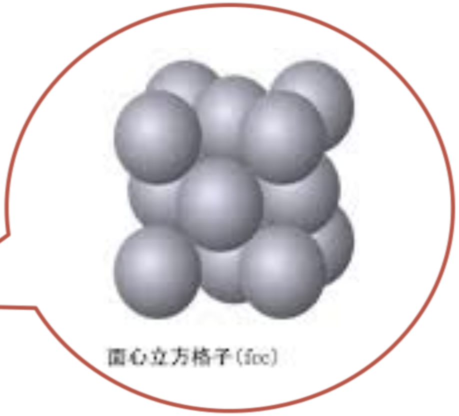
Assume that the nuclear matter consists of crystals of baryon.

Neutron Star





Face centered cubic

How to tackle to QCD phase structure

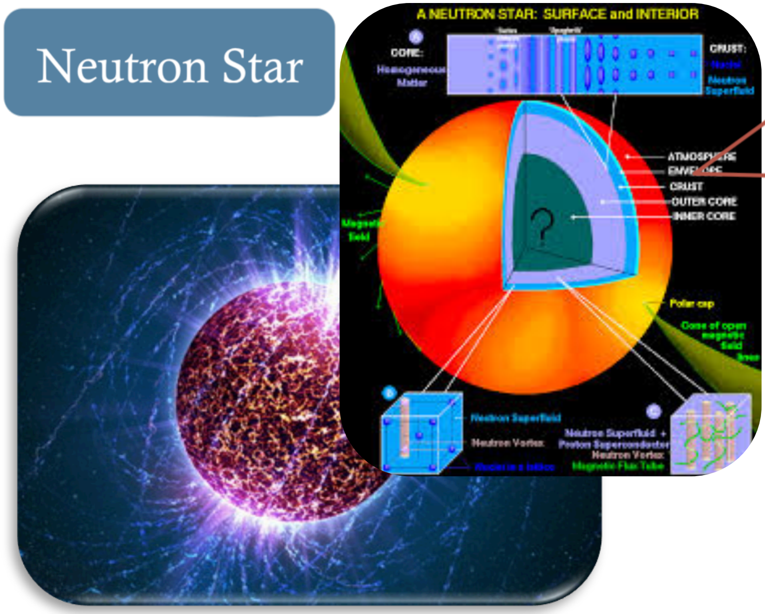
Summarize the above…

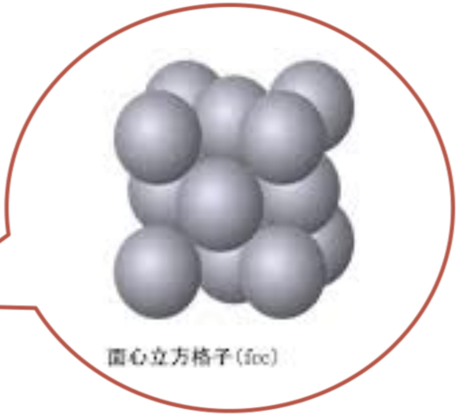
The purpose of my research is to extract the new aspect of QCD phase structure through extreme conditions, i.e. high density region with a magnetic field.

In this study, we employ the skyrmion crystal model.

Skyrmion is identified as baryon while respecting the chiral symmetry.

Neutron Star





Face centered cubic

=

Skyrmions

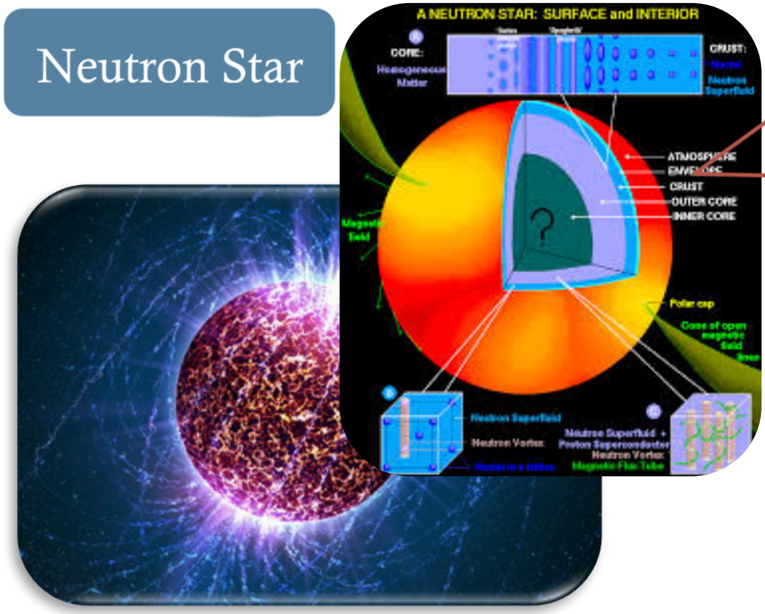
How to tackle to QCD phase structure

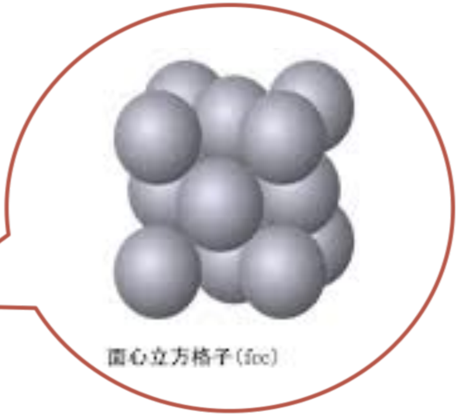
Summarize the above…

The purpose of my research is to extract the new aspect of QCD phase structure through extreme conditions, i.e. high density region with a magnetic field.

By applying a magnetic field, we study the nuclear matter properties to get the new insight for understanding QCD .

Neutron Star





Face centered cubic

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Magnetic field

Skyrmions

To get new insight for understanding QCD

Our work

(Short review of skyrmion)

Short review of skyrmion

T. H. R. Skyrme, Proc. Roy. Soc. Lond. A260 (1961) 127;
Nucl. Phys. 31 (1962) 556;
I. Zahed and G. E. Brown, Phys. Rept., 142 (1986) 1.

Skyrme model Lagrangian based on the chiral symmetry $U = \exp[i\pi^a \tau^a / F_\pi]$

$$\mathcal{L}_{\mathrm{Skyr}} = \frac{f_\pi^2}{4}\mathrm{tr}[\partial_\mu U \partial^\mu U^\dagger] + \frac{1}{32e^2}\mathrm{tr}\Big\{[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U][U^\dagger \partial^\mu U, U^\dagger \partial^\nu U]\Big\}$$

- Invariant under chiral transformation $U \to g_L U g_R^\dagger$

To describe baryon-physics, we give the hedgehog ansatz, $U = \exp[i\hat{x}^i \tau^i F(r)]$.

In topology

The ansatz is denoted as the nontrivial map $U(x) \ : \ R^3 \to S^3$

This maps constitute the third homotopy group $\pi_3(S^3) = Z.$

boundary condition

Winding number (baryon number) :$B = \int d^3x j_B^0 = 1$

$F(0) = \pi, \ F(\infty) = 0$

Baryon current : $j_B^\mu = \frac{1}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\Big[(\partial_\nu U \cdot U^\dagger)(\partial_\rho U \cdot U^\dagger)(\partial_\sigma U \cdot U^\dagger)\Big]$

The hedgehog ansats is characterized by the winding number (baryon number).
→Skyrme model describes “baryon”

Short review of skyrmion

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- Invariant under chiral transformation $U \to g_L U g_R^\dagger$

To describe baryon-physics, we give the hedgehog ansatz, $U = \exp[i\hat{x}^i\tau^i F(r)]$.

through the numerical calculation….

Baryon properties

- Energy of skyrmion (baryon)

$$M_{\rm Skyrm} = -\int d^3x \mathcal{L}_{\rm Skyrm}$$

$$M_{\rm Skyr} \sim 1150[{\rm MeV}]$$

- Isoscalar charge radius of a nucleon

$$r_0 = 0.66\,{\rm fm}$$

$$(\ r_0^{\rm (exp)} = 0.877 \pm 0.005\,{\rm fm}\ )$$

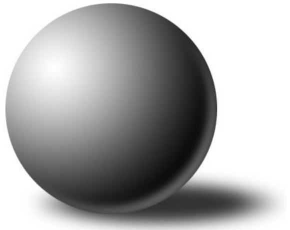
*Observables are acceptable at the leading $\mathcal{O}(N_c)$.

Input parameter

$f_\pi = 93{\rm MeV}$ (experimenta value)

$e \sim 6$ (determined from $\rho \to \pi\pi$)

Skyrmion (nucleon) is the finite size particle.



Our work

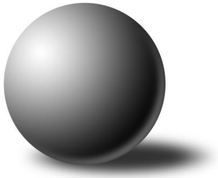
So far, I just showed the “isolated skyrmoin (= baryon)”.
Let’s move on “skyrmoins (=baryonic matter)”

(Short review of skyrmion crystal)

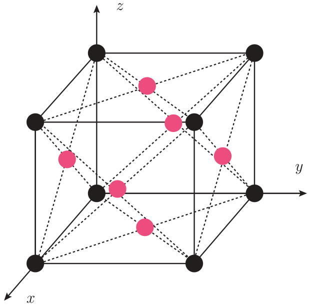
Short review of skyrmion crystal

To investigate the baryonic matter properties, we put skyrmions onto crystal lattice

Skyrmion



Put skyrmions onto crystal lattice



Skyrmion crystal

面心立方格子(fcc)

I. Klebanov, Nucl. Phys. B262(1985) 133-143

H. J. Lee, B. Y. Park, D. P. Min, M. Rho and
V. Vento, Nucl. Phys. A bf 723, 427 (2003)

Short review of skyrmion crystal

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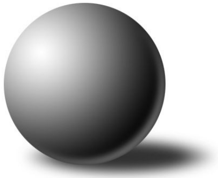
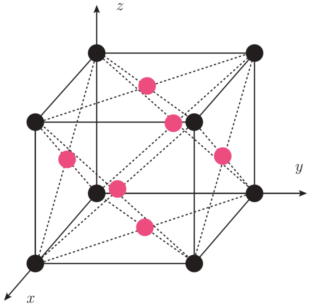
Skyrmion



Put skyrmions onto crystal lattice



Skyrmion crystal

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Identify skyrmion crystal as baryonic matter.

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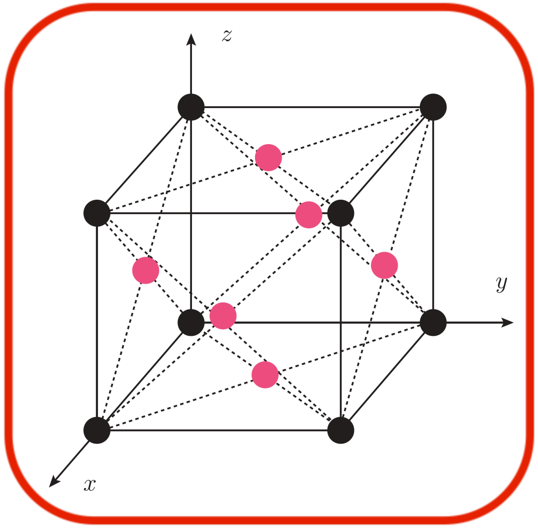
Specifically choose the face centered cubic in our work.

- Put skyrmions onto the face centered cubic(FCC) crystal
- A single FCC crystal has the volume size $(2L)^3$ and contains 4 skyrmions.



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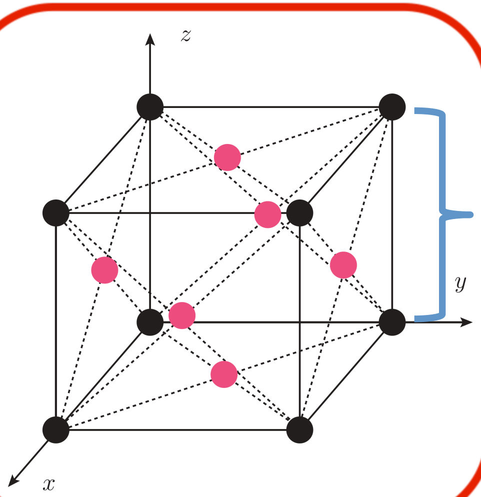

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Lattice size

- Baryonic matter density: $\rho = 4/(2L)^3$

Short review of skyrmion crystal

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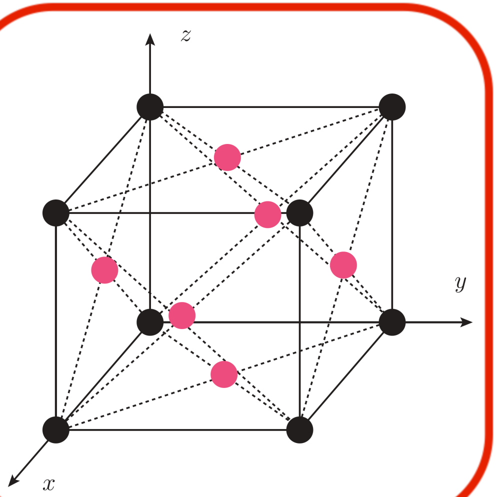
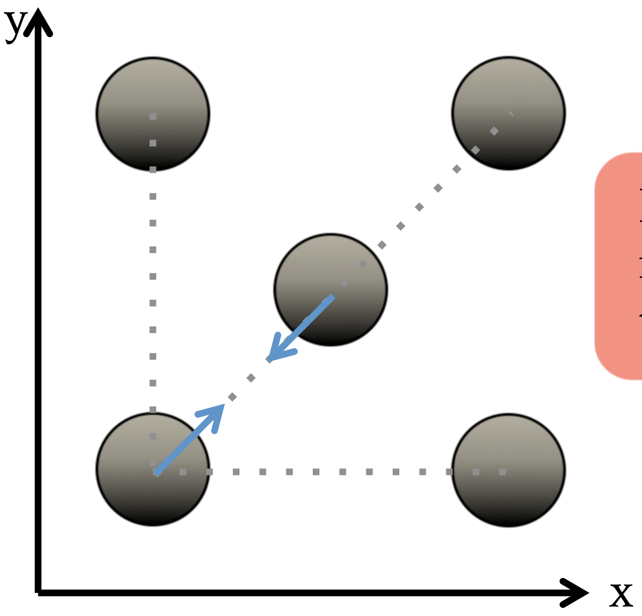


How is the skyrmion-skyrmion interaction going?



Focus on the x-y plane

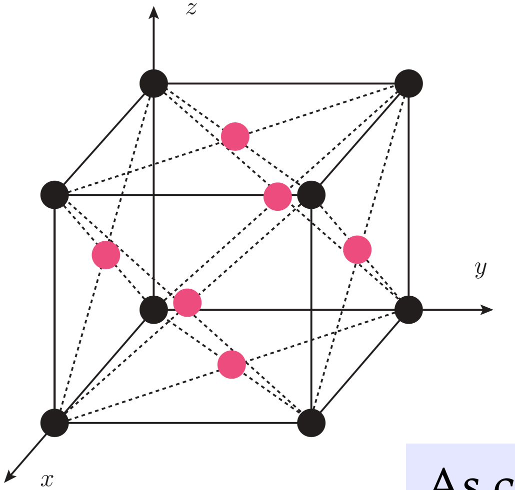
In skyrmion crystal approaches,
nearest skyrmions get
the strongest attractive interaction

(for more on this, please see “arXiv:1604.04850”)

Short review of skyrmion crystal

The skyrmion approach has a characteristic phenomena.

On the premise of this work
skyrmions are put onto a FCC crystal.



As crystal size is changed to be small,
interesting phenomena happens.
*Baryonic matter density: $\rho = 4/(2L)^3$

Short review of skyrmion crystal

The skyrmion approach has a characteristic phenomena which is the topological phase transition between the skyrmion and the half-skyrmion phase.

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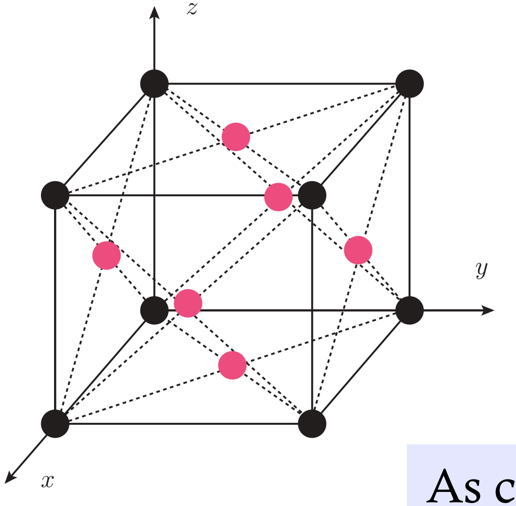

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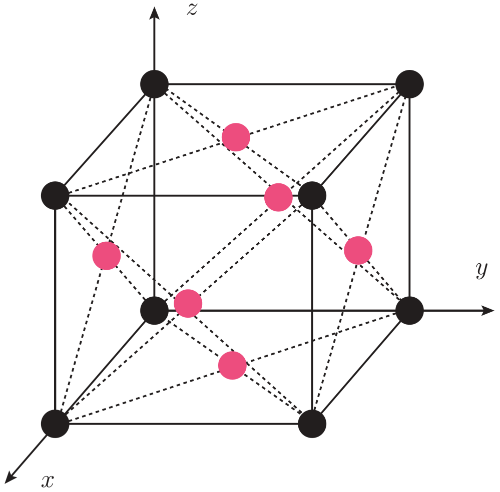
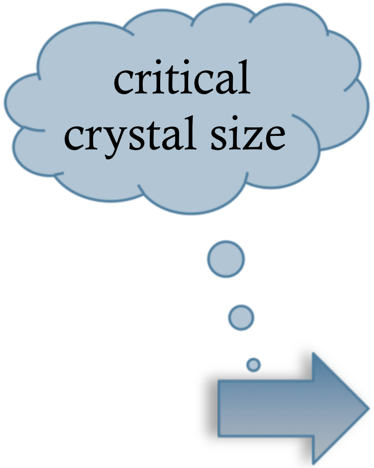
Short review of skyrmion crystal

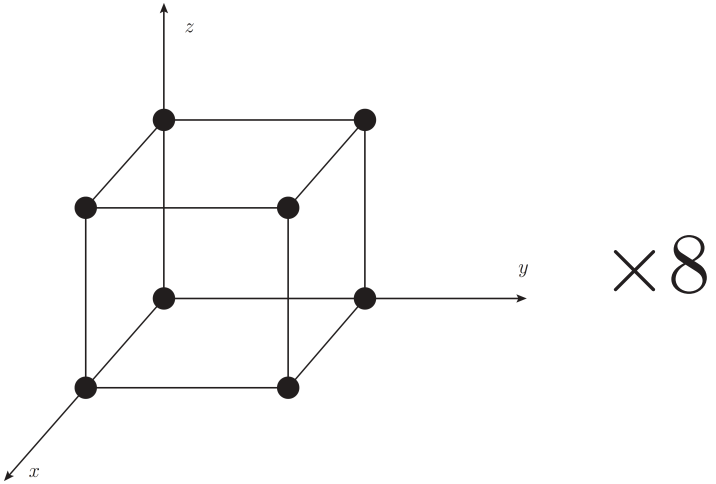
The skyrmion approach has a characteristic phenomena which is the topological phase transition between the skyrmion and the half-skyrmion phase.

- A FCC crystal with volume size $(2L)^3$ contains 4 skyrmions

- A crystal lattice with volume size $(2L)^3$ has 8 cubic-centered (CC) crystals.
- A single CC contains 1 skyrmion.




critical crystal size



* Baryon number is conserved even if this system undergoes the topological phase transition.

Short review of skyrmion crystal

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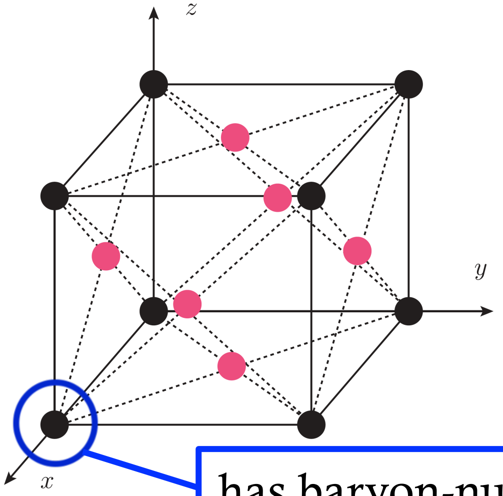

has baryon-number 1

↔ Skyrmion phase

critical crystal size

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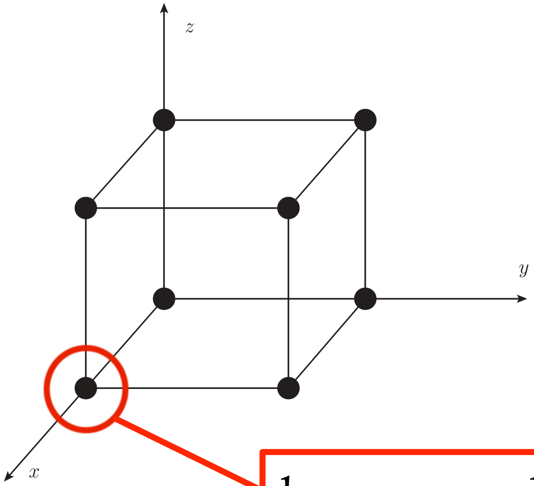
- A single CC contains 1 skyrmion.



×8

baryon-number 1/2

↔ Half-skyrmion phase

* Baryon number is conserved even if this system undergoes the topological phase transition.

Short review of skyrmion crystal

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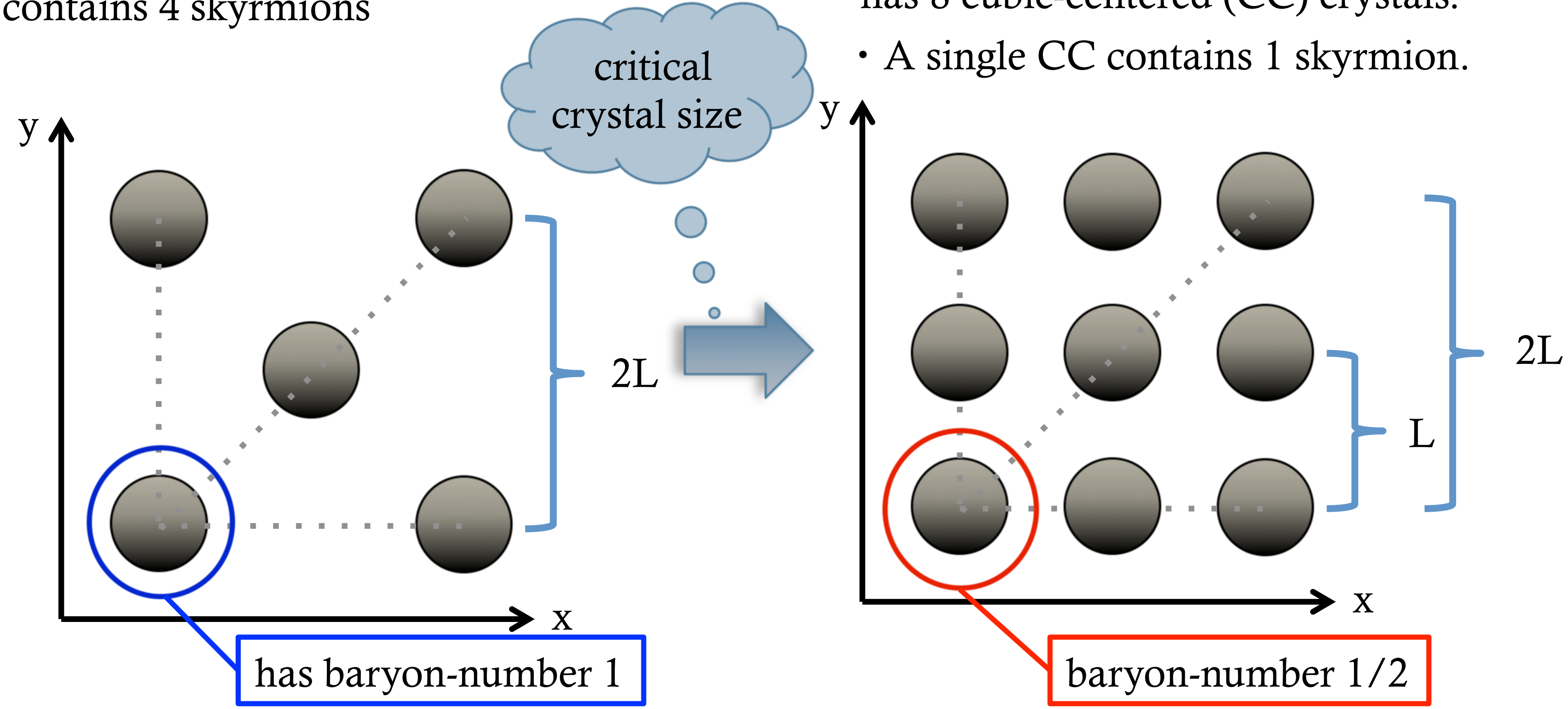
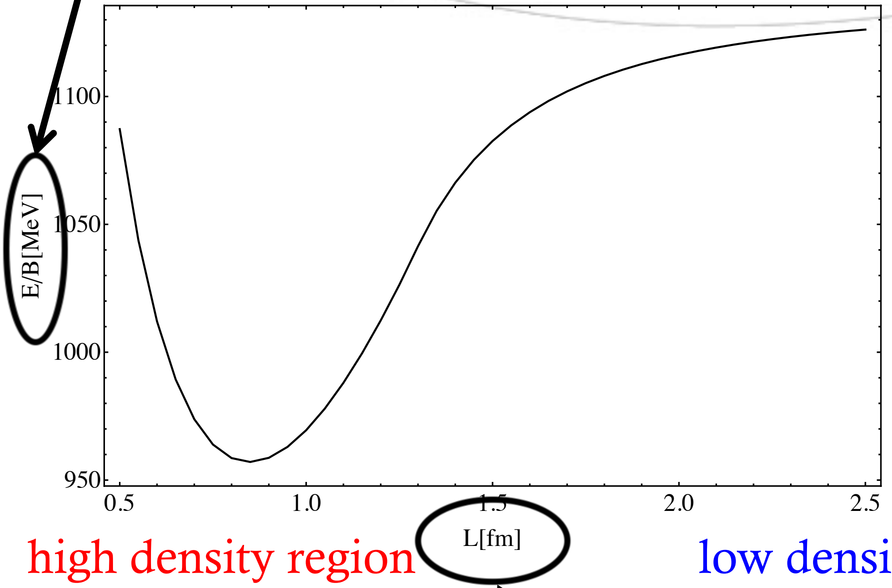


Let’s check skyrmion crystal properties through the numerical calculation

Short review of skyrmion crystal

Baryon energy per skyrmion



high density region

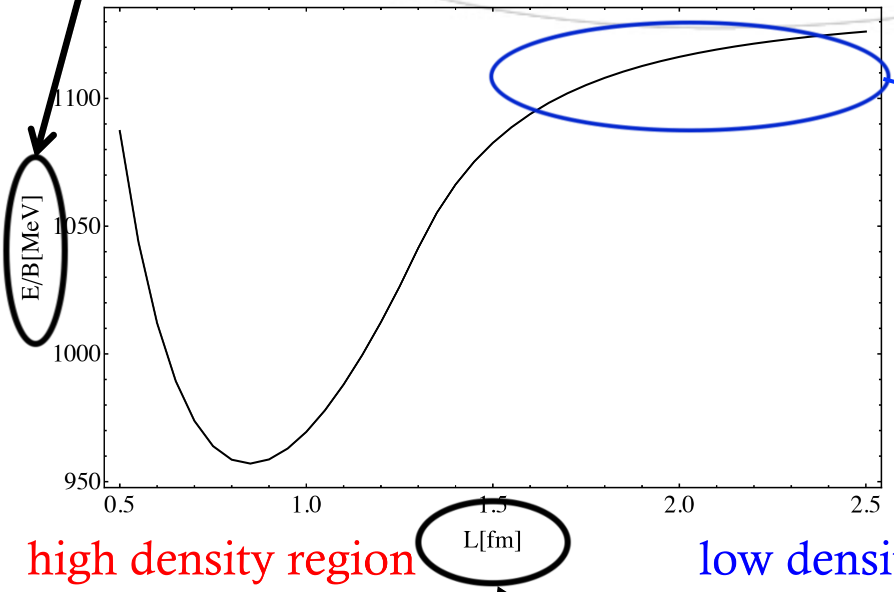
low density region

Lattice size, L[fm]

Baryonic matter density: $\rho = 4/(2L)^3$

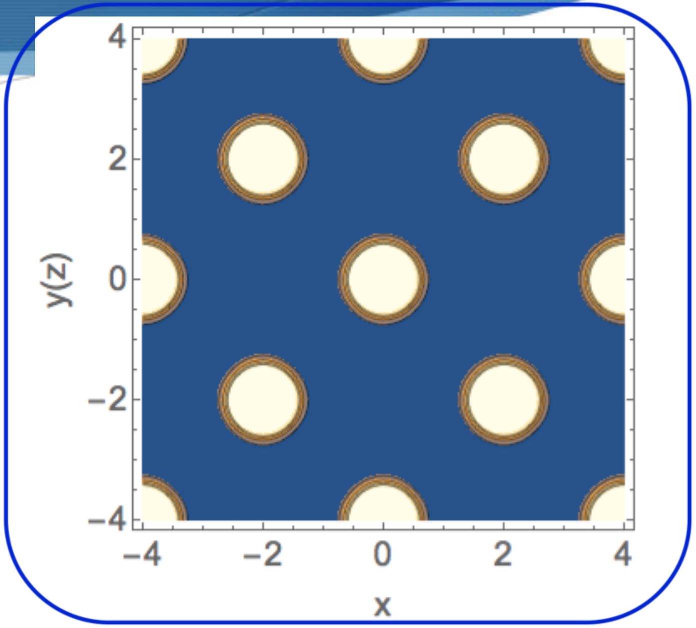
Short review of skyrmion crystal

Baryon energy per skyrmion



high density region

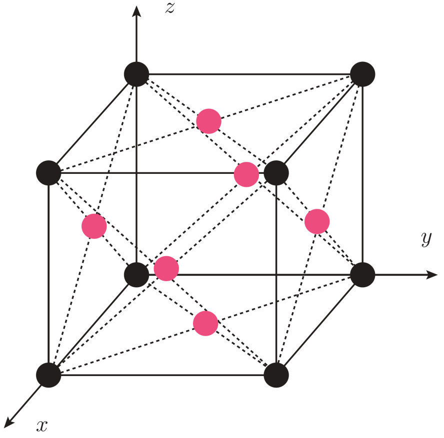
low density region



Lattice size, L[fm]

Baryonic matter density: $\rho = 4/(2L)^3$

=



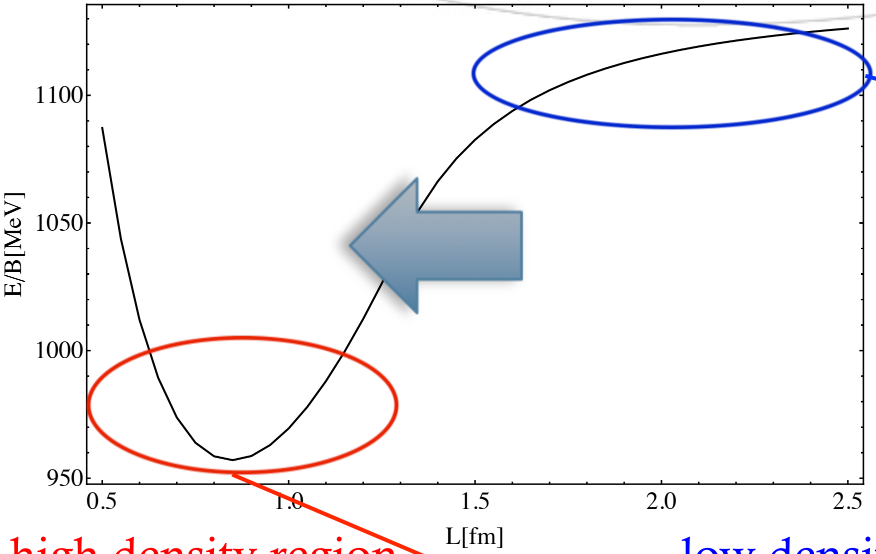
Winding number(Baryon number density)

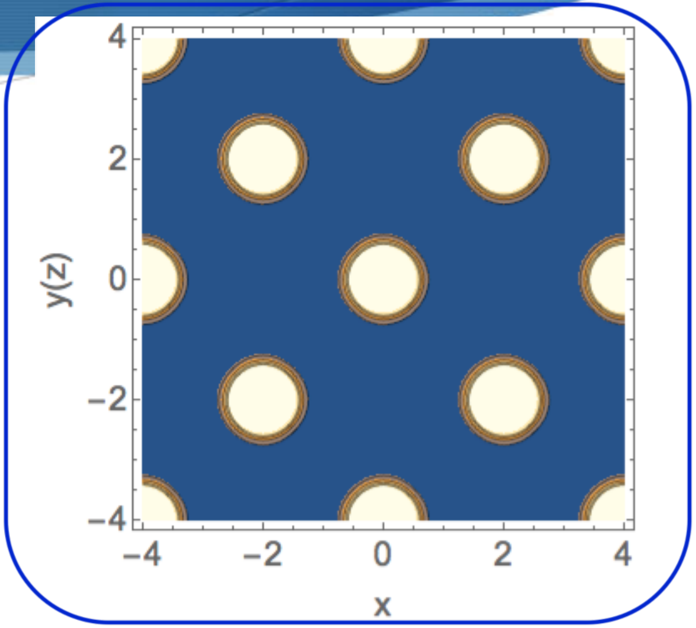
$$\rho_B = \frac{1}{24\pi^2}\epsilon^{0\nu\rho\sigma}\mathrm{tr}\left[(\partial_\nu U\cdot U^\dagger)(\partial_\rho U\cdot U^\dagger)(\partial_\sigma U\cdot U^\dagger)\right]$$

It is able to reproduce FCC crystal numerically.

Short review of skyrmion crystal

Baryon energy per skyrmion





high density region

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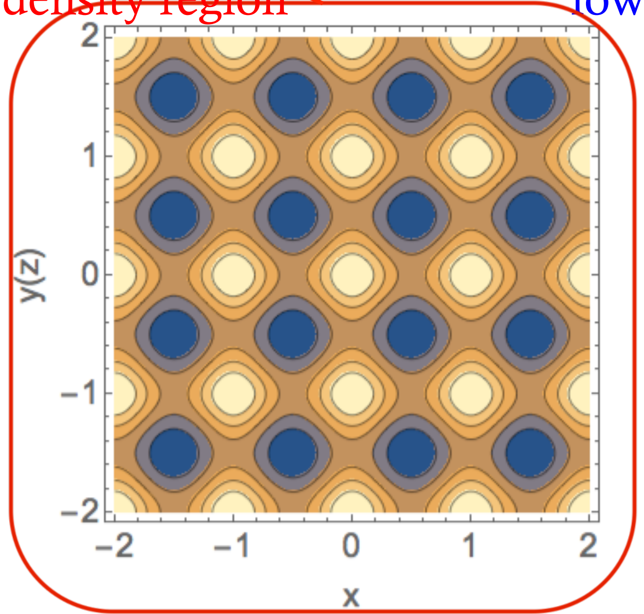

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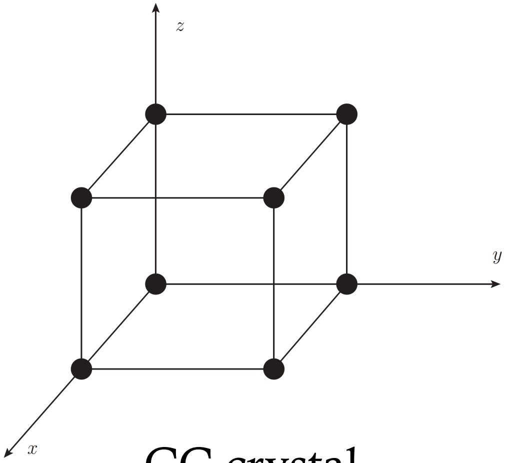
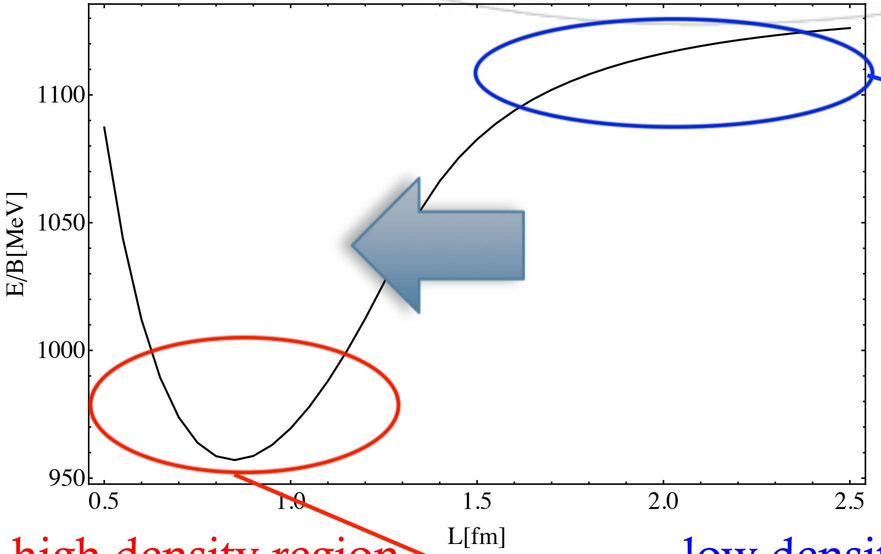


CC crystal

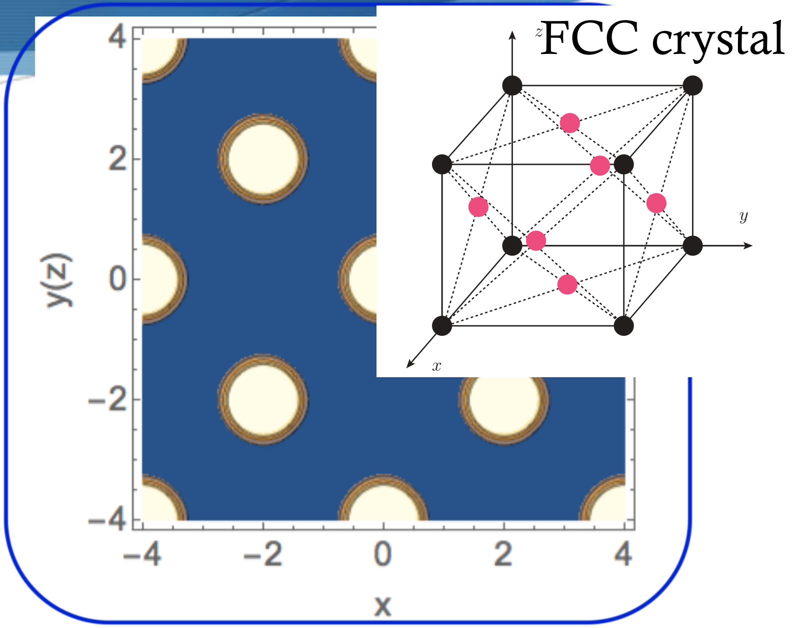
Short review of skyrmion crystal

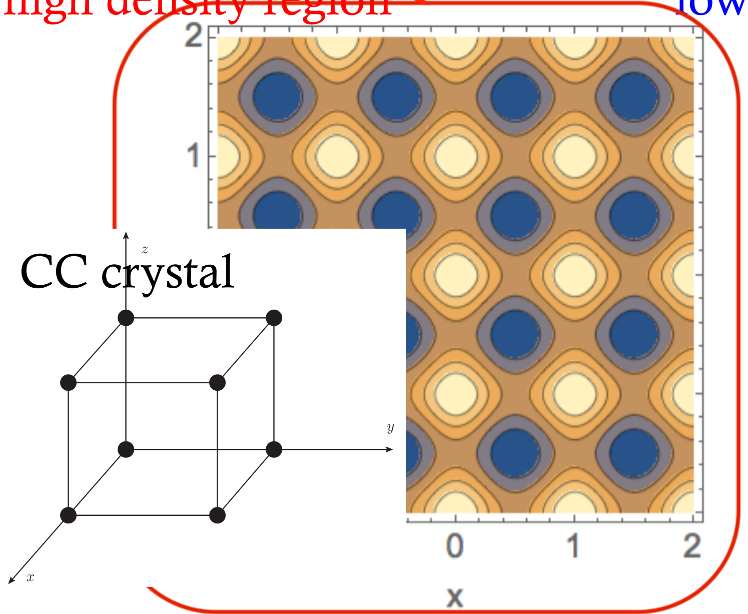
Baryon energy per skyrmion



high density region

low density region





Topological transition occurs.

What is the signal of topological transition?

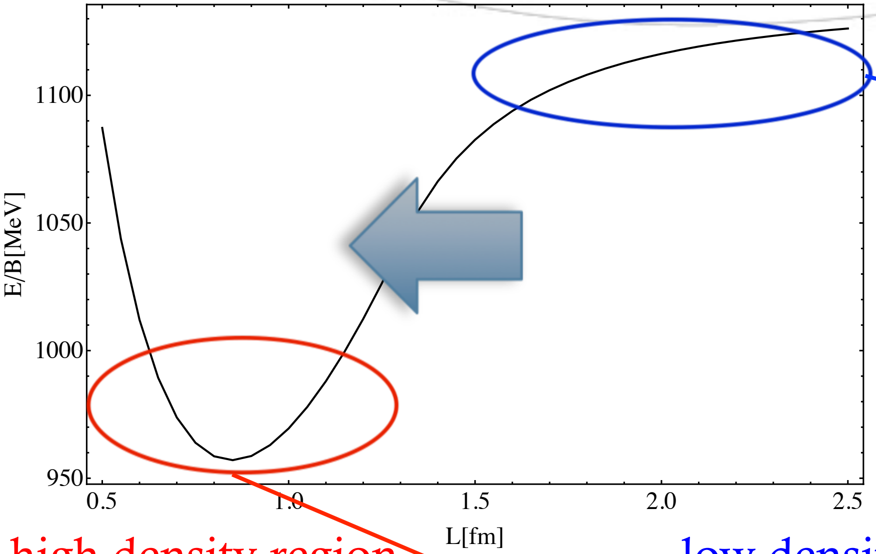
$\rightarrow$Look at the chiral field $U = \boxed{\phi_0} + i\tau_i\phi_i$.

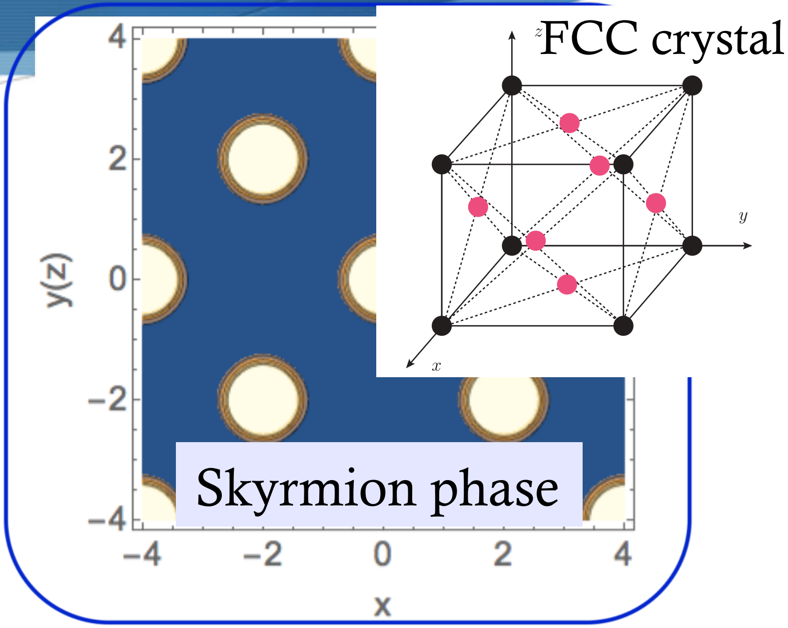
Pick out!

Space-averaged value: $\langle\phi_0\rangle = \frac{1}{(2L)^3}\int_{-L}^{L} d^3x\, \phi_0$

Short review of skyrmion crystal

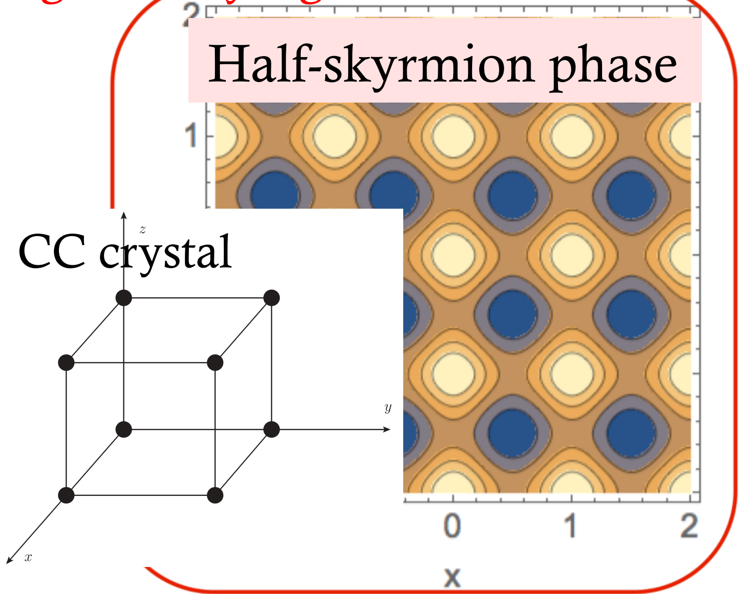
Baryon energy per skyrmion

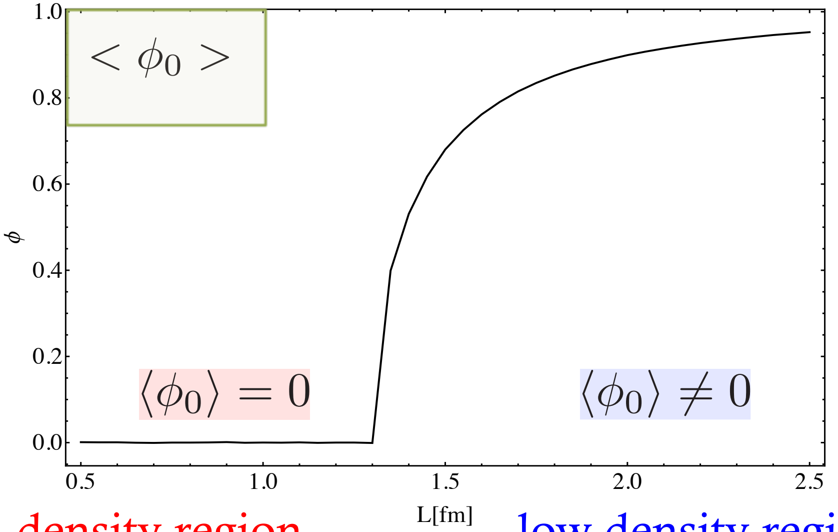




high density region

low density region



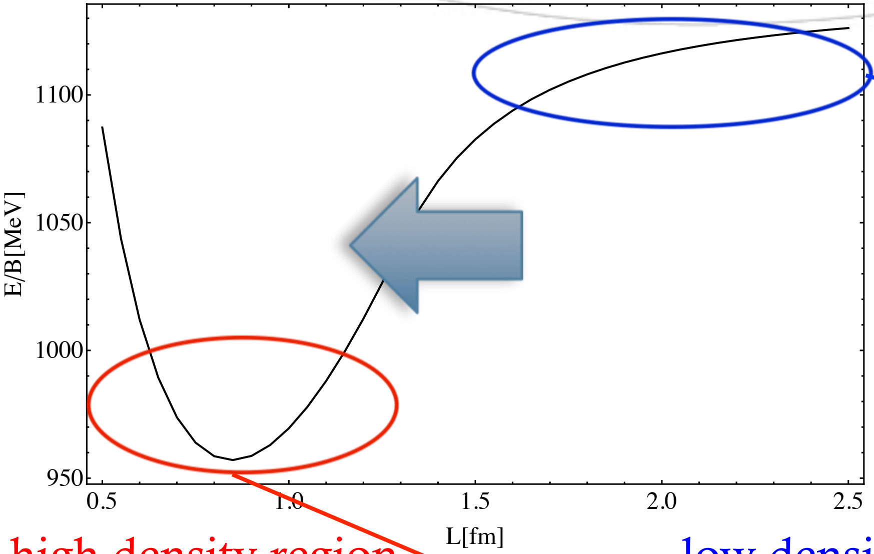


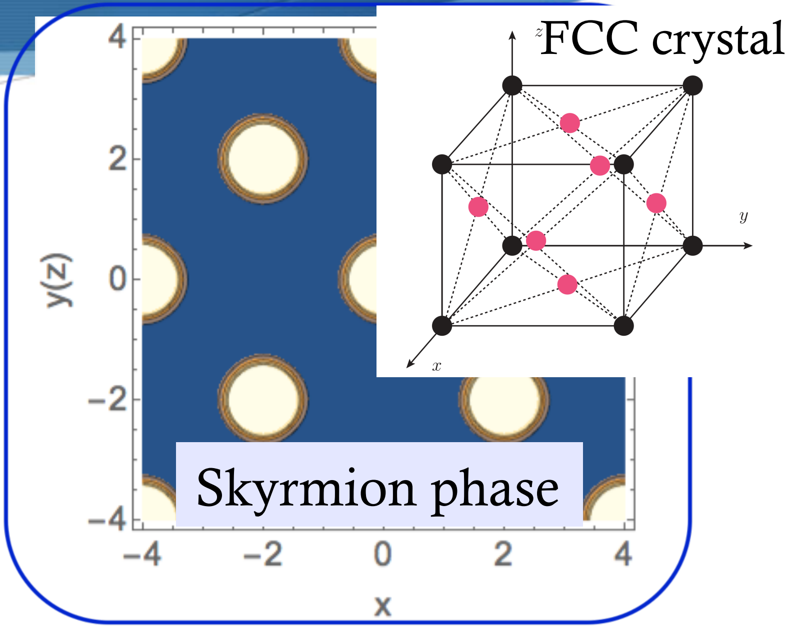
high density region

low density region

Short review of skyrmion crystal

Baryon energy per skyrmion

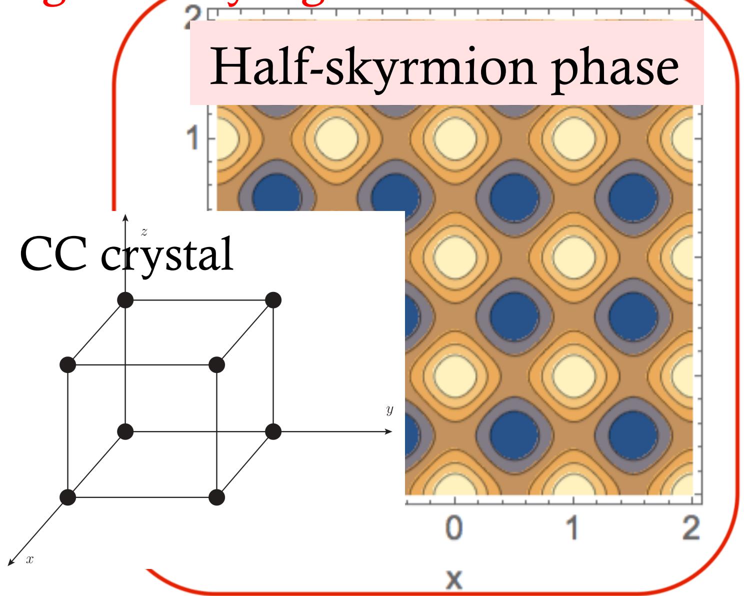




Skyrmion phase

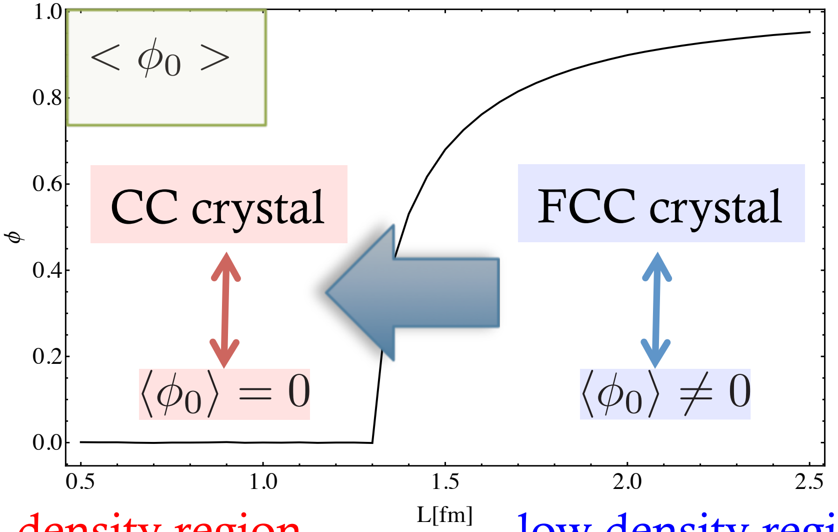
high density region

low density region



Half-skyrmion phase

CC crystal



$< \phi_0 >$

CC crystal

$\langle \phi_0 \rangle = 0$

FCC crystal

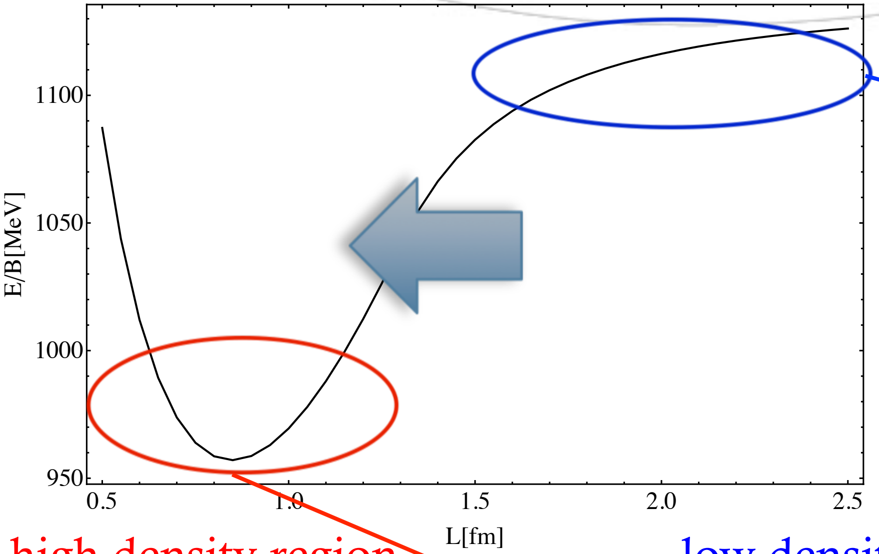
$\langle \phi_0 \rangle \neq 0$

high density region

low density region

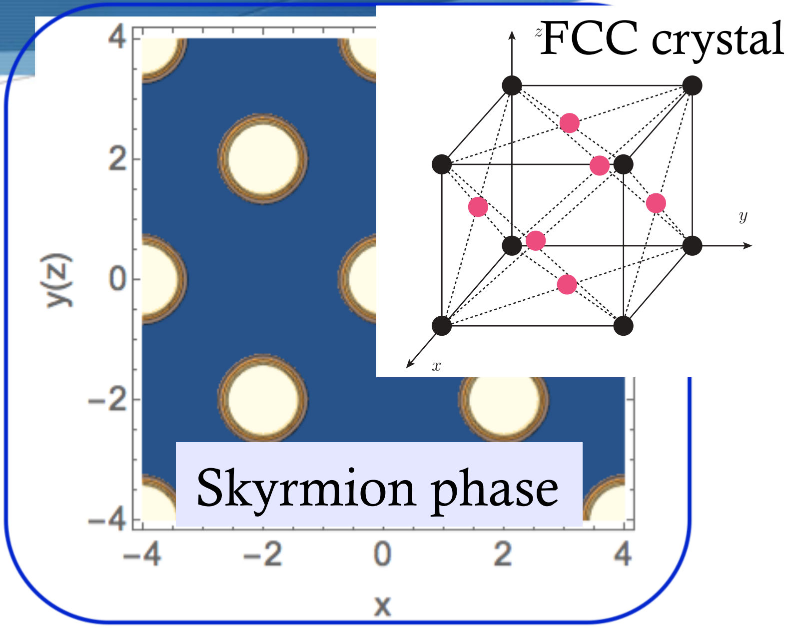
Short review of skyrmion crystal

Baryon energy per skyrmion



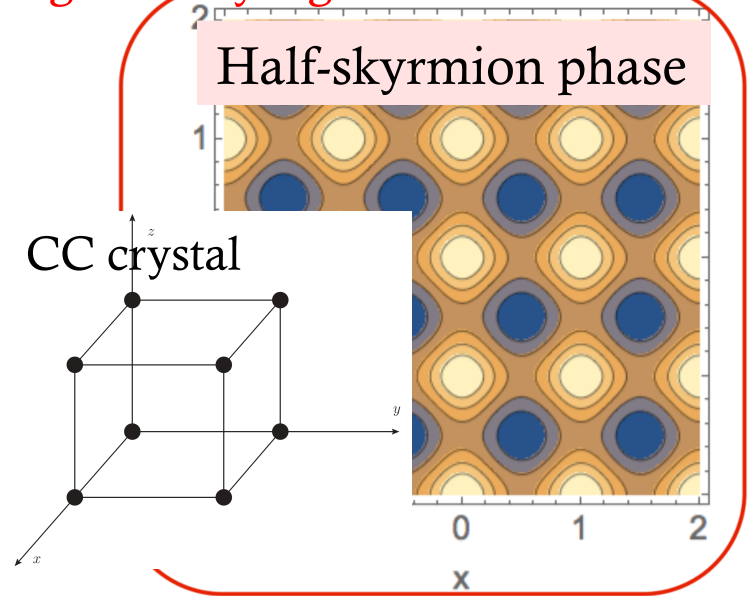
high density region

low density region



FCC crystal

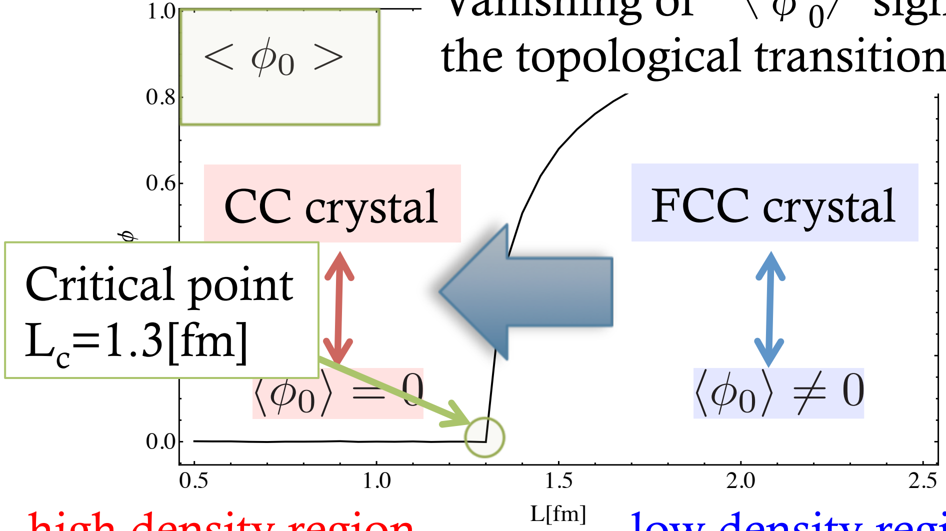
Skyrmion phase



Half-skyrmion phase

CC crystal

Vanishing of ⟨ ϕ_0 ⟩ signals the topological transition.



$< \phi_0 >$

CC crystal

FCC crystal

Critical point
L_c=1.3[fm]

$\langle \phi_0 \rangle = 0$

$\langle \phi_0 \rangle \neq 0$

high density region

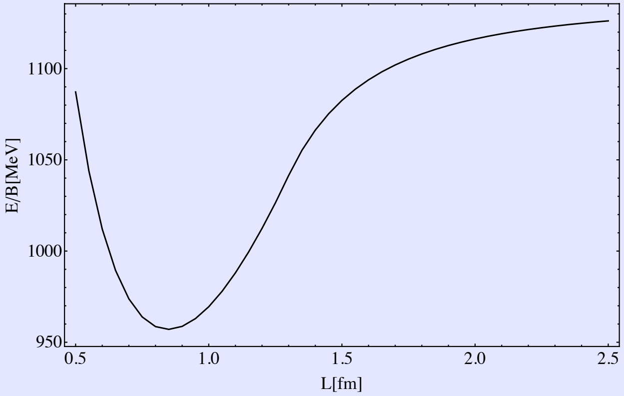
low density region

What happens in magnetic field?

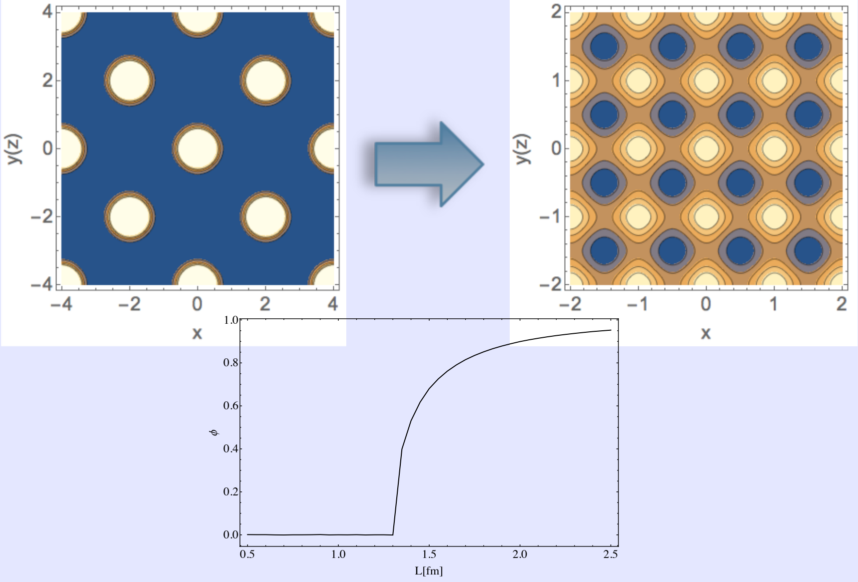
Magnetic field

Deformation of skyrmion configuration

Baryon energy per skyrmion



Topological transition



By applying magnetic field,
what changes in crystal properties?

Skyrmion crystal in a magnetic field

M. K., Y. L. Ma and S. Matsuzaki,
``Magnetic field effect on nuclear matter from skyrmion crystal model,''
arXiv:1804.09015 [nucl-th].

Skyrmion crystal in a magnetic field

Replace the derivative operator with the gauge covariant one

*Constant magnetic field along z-axis

$$\partial_\mu U \to D_\mu U = \partial_\mu U - i\mathcal{L}_\mu U + iU\mathcal{R}_\mu \qquad \mathcal{L}_\mu = \mathcal{R}_\mu = eQ_{\mathrm{em}}A_\mu$$

$$\mathcal{L}_{\mathrm{Skyr}} = \frac{f_\pi^2}{4}\mathrm{tr}[\partial_\mu U \partial^\mu U^\dagger] + \frac{1}{32e^2}\mathrm{tr}\left\{[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U][U^\dagger\partial^\mu U, U^\dagger\partial^\nu U]\right\}$$

Skyrmion crystal in a magnetic field

Replace the derivative operator with the gauge covariant one

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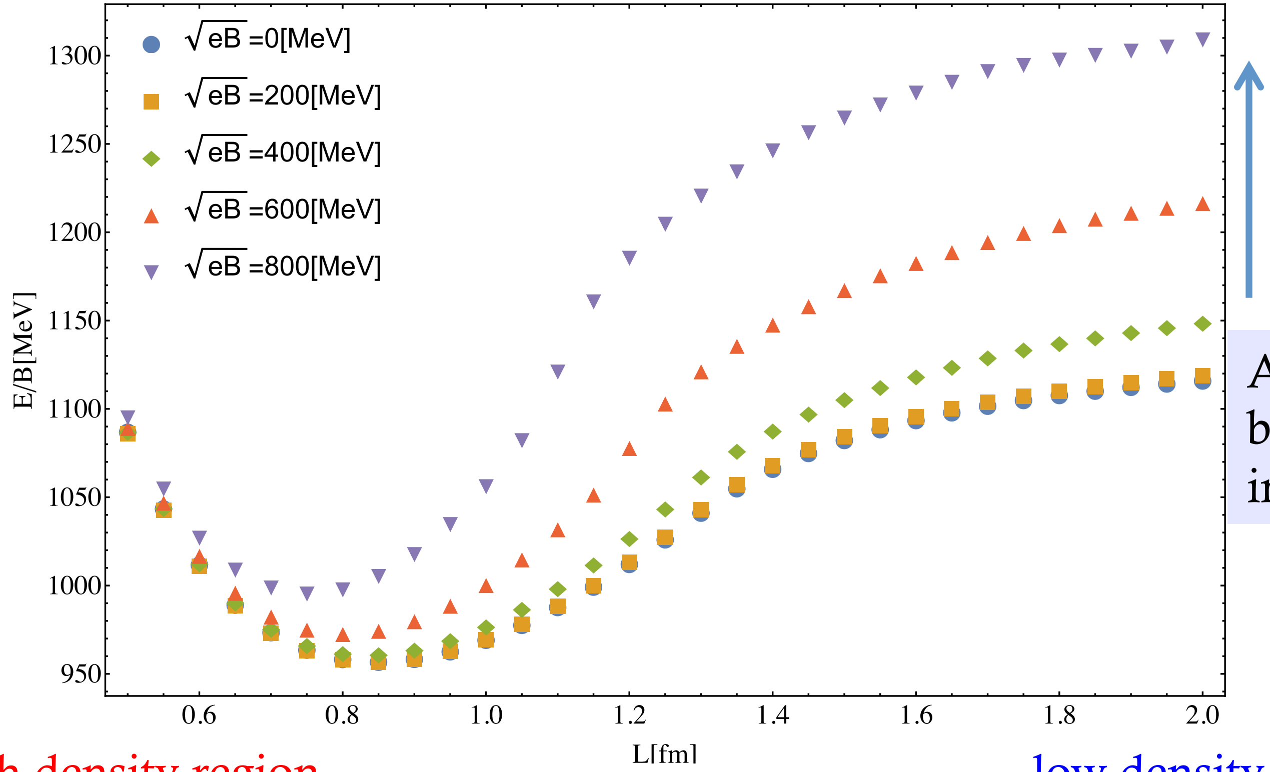
Baryon energy per skyrmion



Magnetic field eB

As magnetic field increases, baryon (skyrmion) energy increases.

high density region

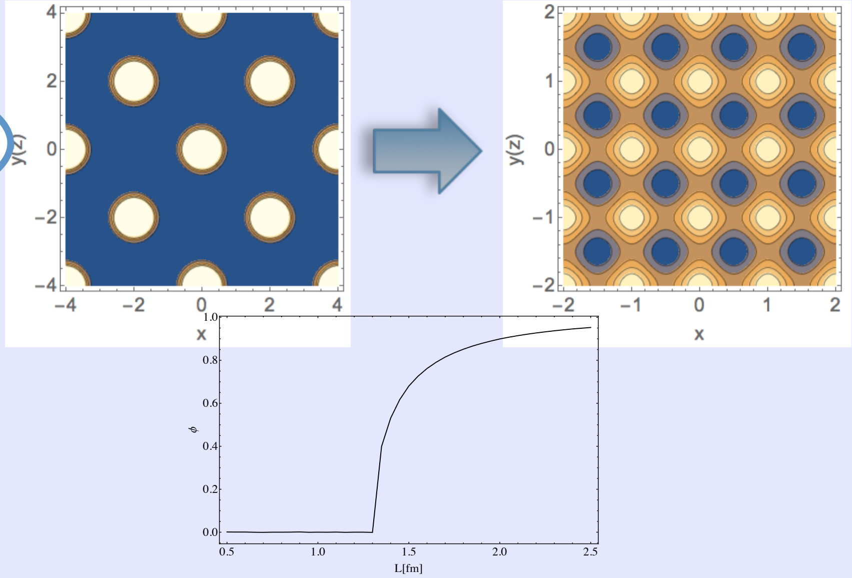
low density region

$\langle \phi_0 \rangle$ in a magnetic field

Magnetic effect on $< \phi_0 >$

Vanishing of $\langle \phi_0 \rangle$ signals the topological transition.

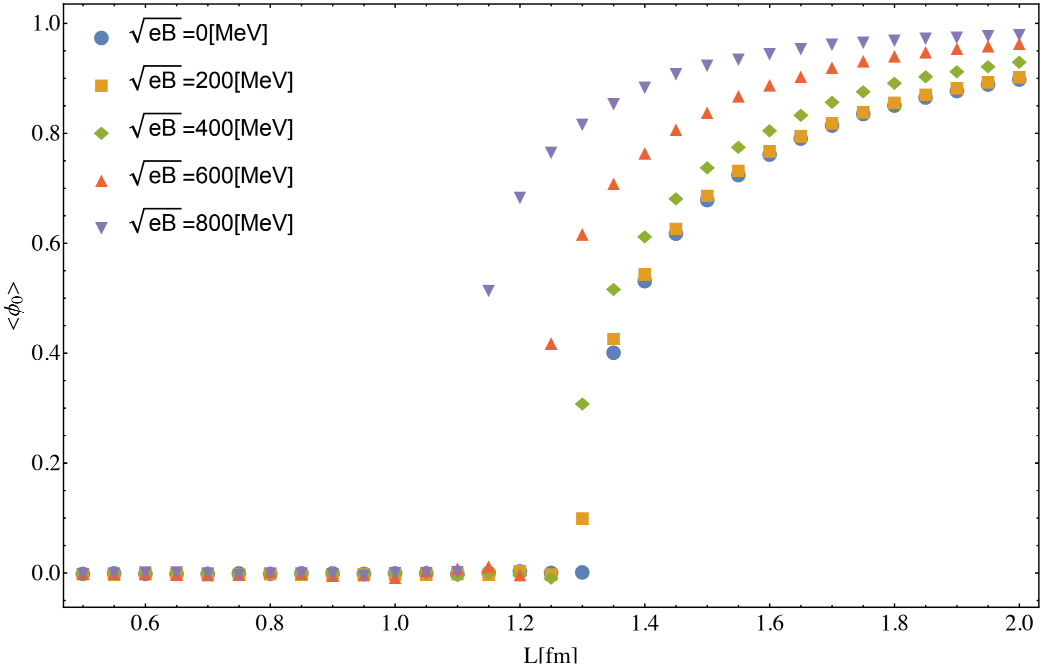
Topological transition



$\langle \phi_0 \rangle$ in a magnetic field

Magnetic effect on $< \phi_0 >$

Magnetic field eB ←



high density region

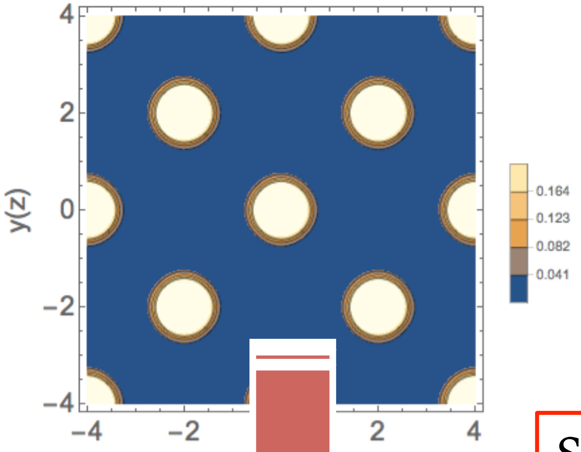
low density region

As the magnetic field increases, the topological transition point is shifted to a high density region and the value of $\langle \phi_0 \rangle$ gets larger.

Deformation of the skyrmion configuration

Skyrmion phase

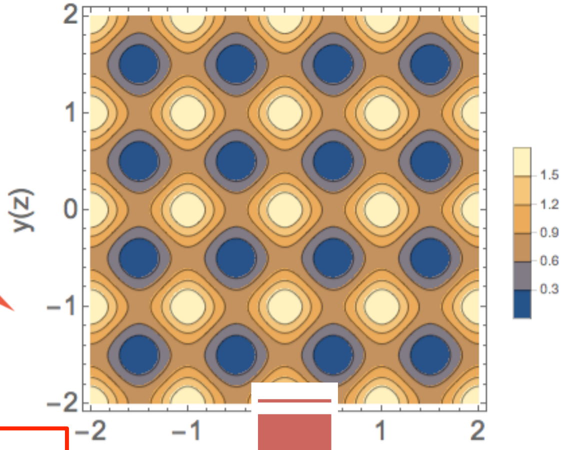
$L = 2.0[\mathrm{fm}]$



Magnetic field

Half-skyrmion phase

$L = 1.0[\mathrm{fm}]$



Skyrmion configuration
is deformed by a magnetic field.

?

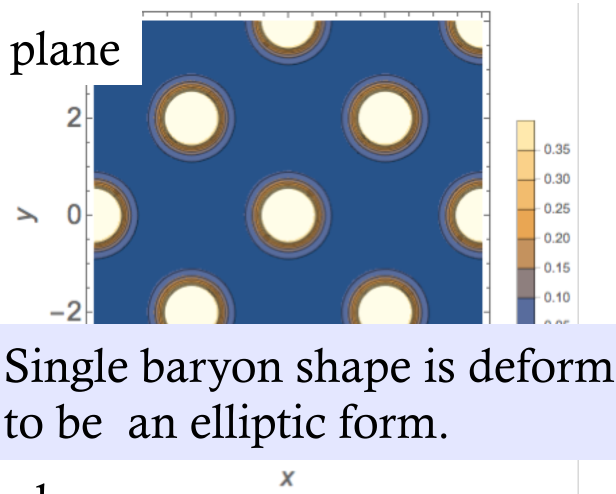
?

Deformation of the skyrmion configuration

$\sqrt{eB} = 400[\mathrm{MeV}]$

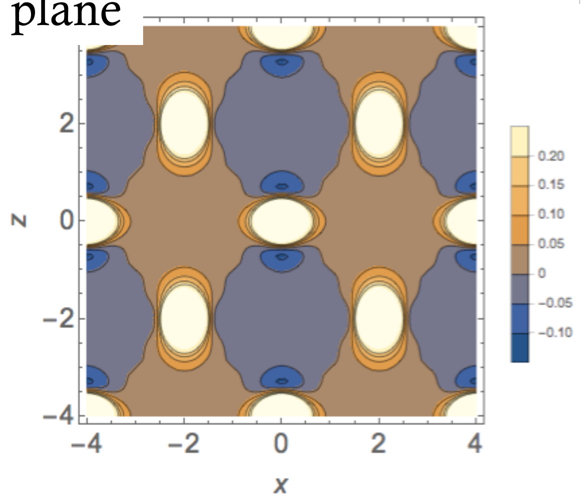
$L = 2.0[\mathrm{fm}]$ Skyrmion phase

x-y plane



Single baryon shape is deformed to be an elliptic form.

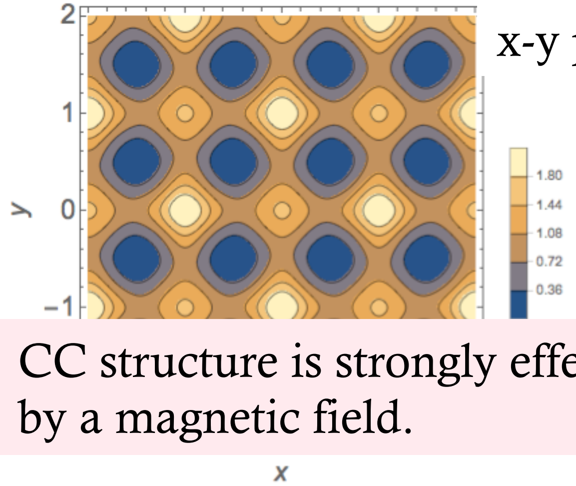
x-z plane



L_c=1.25[fm]

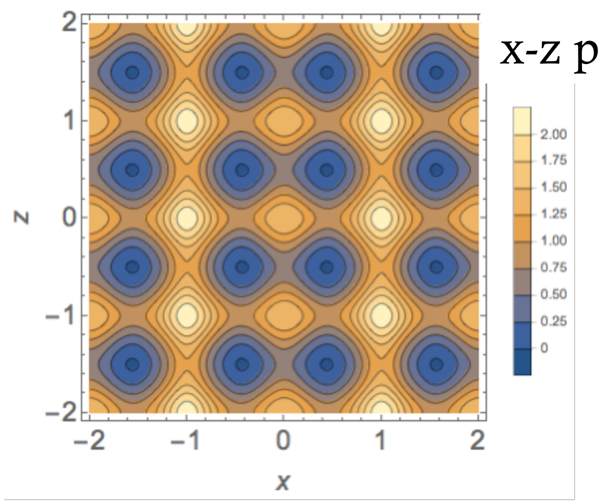
$L = 1.0[\mathrm{fm}]$ Half skyrmion phase

x-y plane



CC structure is strongly effected by a magnetic field.

x-z plane



Deformation of the skyrmion configuration

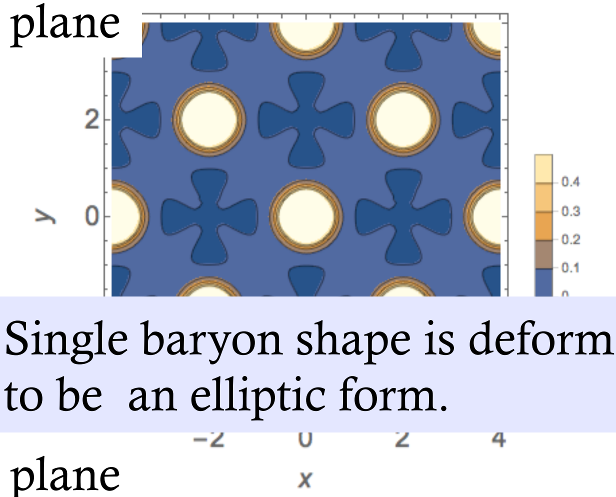
$\sqrt{eB} = 800[\mathrm{MeV}]$

$L = 2.0[\mathrm{fm}]$ Skyrmion phase

$L = 1.0[\mathrm{fm}]$ Half-skyrmion phase

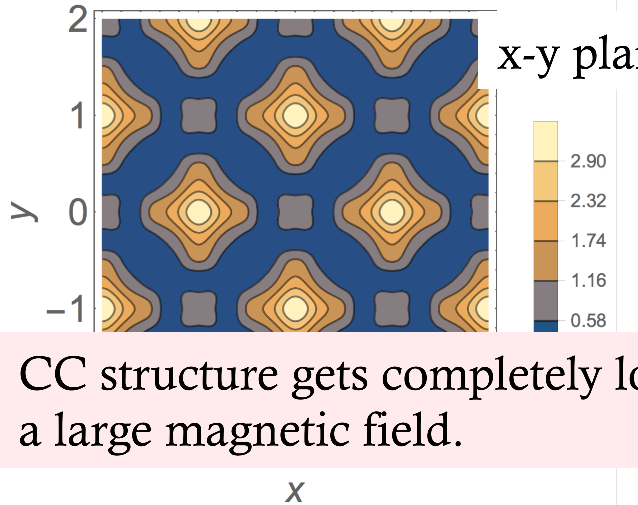
L_c=1.1[fm]

x-y plane



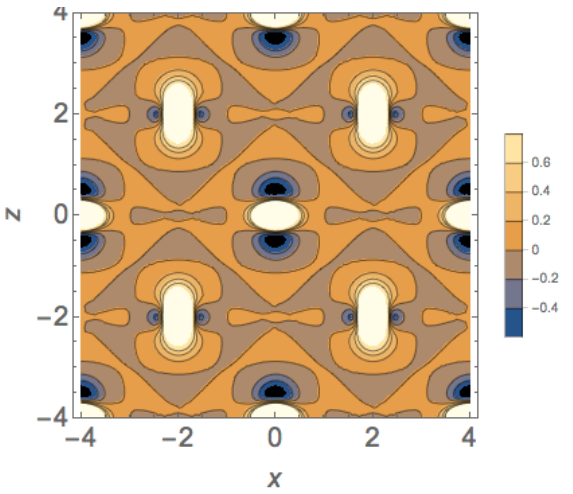
Single baryon shape is deformed to be an elliptic form.

x-y plane

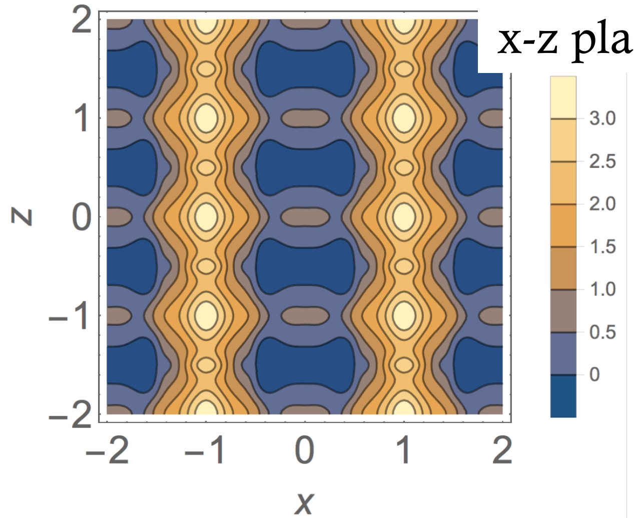


CC structure gets completely lost for a large magnetic field.

x-z plane



x-z plane



Summary

Discussed the magnetic effect on the baryonic matter based on the skyrmion crystal approach.

- As magnetic field increases, baryon (skyrmion) energy increases for any crystal size.
- As the magnetic field increases, the topological transition point is shifted to a high density region and the value of $\langle \phi_0 \rangle$ gets larger.
  → Magnetic effect plays the role of a catalyzer for the topological transition.
- Magnetic field distorts the skyrmion crystal structure.
  - Low density region : Single baryon shape is deformed to be an elliptic form.
  - Highr density region : CC structure is strongly effected by a magnetic field.
    In particularly, CC structure gets completely lost for a large magnetic field.

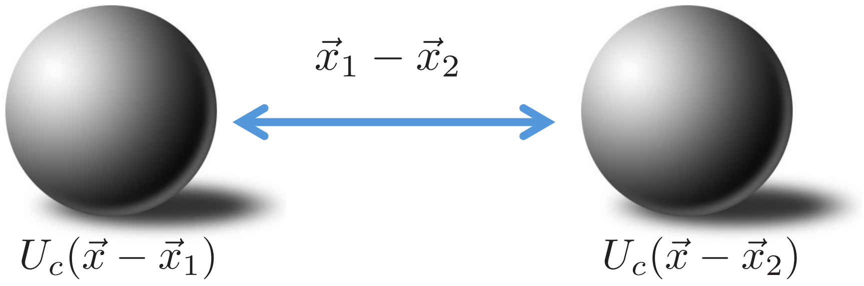
Thank you very much!

back up

スカーミオン間の相互作用

スカーミオン二つ用意 $U_c(\vec{x}-\vec{x}_1),\ U_c(\vec{x}-\vec{x}_2).$
スカーミオン間のポテンシャルについてみてみる

スカーミオン結晶を作るときの
条件が知りたい



カイラル場を用意する：　相互作用**する**二つのスカーミオンについて記述する

$$U_{cc}(\vec{x},\vec{x}_1,\vec{x}_2)=U_c(\vec{x}+\vec{x}_1)C(\vec{\alpha})U_c(\vec{x}+\vec{x}_2)C^{\dagger}(\vec{\alpha})\qquad C(\alpha)=\exp(i\vec{\alpha}\cdot\vec{\tau}/2)$$

$\vec{\alpha}$　を変えることで相互作用の強弱を調節できる
引力が強くなる条件は・・・

スカーミオン間の相互作用

スカーミオン二つ用意 $U_c(\vec{x}-\vec{x}_1)$, $U_c(\vec{x}-\vec{x}_2)$.
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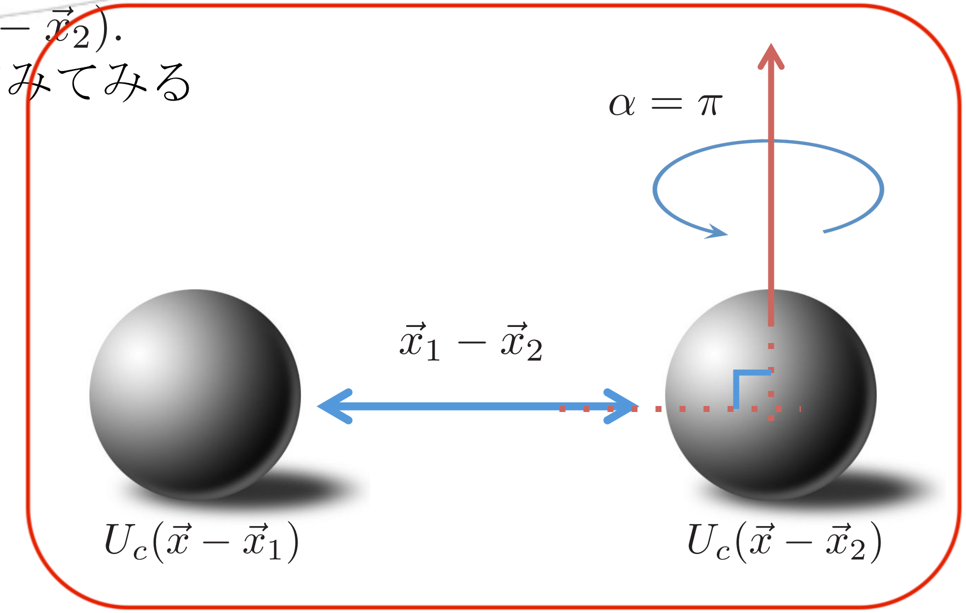

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$$U_{cc}(\vec{x},\vec{x}_1,\vec{x}_2)=U_c(\vec{x}+\vec{x}_1)C(\vec{\alpha})U_c(\vec{x}+\vec{x}_2)C^\dagger(\vec{\alpha}) \qquad C(\alpha)=\exp(i\vec{\alpha}\cdot\vec{\tau}/2)$$

引力が最も強くなる条件

$$\vec{x}_1=(0,0,0) \quad \vec{x}_2=(L,0,0)$$

$$U_{cc}(x,y,z)=U_c(x,y,z)e^{i\pi\tau_y/2}U_c(x+L,y,z)e^{-i\pi\tau_y/2}$$

- メモ
- 「Nuclear matter」の説明をどこまで真面目にやる？
- パスタ構造とか説明いる？　→いらない
- skyrmion の話をどれだけ真面目にやるか・・・？
- skyrmon crystal の説明をどれだけ真面目にやるか…
- EFT の会議だからカイラル対称性とか議論しとく？
- Scale symmetry について最後にコメントしておく？

chiral symmetry in magnetic field

Focus magnetic effect

- magnetic catalysis/inverse catalysis: chiral condensate is enhanced (suppressed) by the magnetic field.

chiral phase transition easily(difficultly) happen in magnetic field
→ magnetic field affects the chiral symmetry.
(And also, it is expected that the properties of nuclear matter is affected by magnetic field)
(nuclear matter properties : mass of baryon (nucleon) and structure of baryons)

In this work, we focus on the magnetic dependence of chiral symmetry (baryonic matter)

Why focus on a baryonic matter?

(Again) In a baryonic matter, the chiral symmetry is expected to restore.

why use the skymion approach?

- skyrmion is identified as baryon while respecting the chiral symmetry.
  → chiral restoration phenomenon is observed on this approach.

- Inner structure of baryonic matter can be visualized through baryon number density
  （which focus on “inner structure” or “deformation”?)

How to tackle to the mass generation

Summarize the above…

The purpose of my research is to extract the new aspect of QCD phase structure through extreme conditions, i.e. high density region with a magnetic field.

To accomplish my purpose, I focus on a baryonic matter with a strong magnetic field.

In this study, we employ the skyrmion crystal model.

Advantage of skyrmion apprach

- Skyrmion is identified as baryon while respecting the chiral symmetry.
  → Chiral restoration phenomenon is observed on this approach.
- Inner structure of baryonic matter can be visualized through baryon number density.

(The details will be described later.)

How to tackle to the mass generation

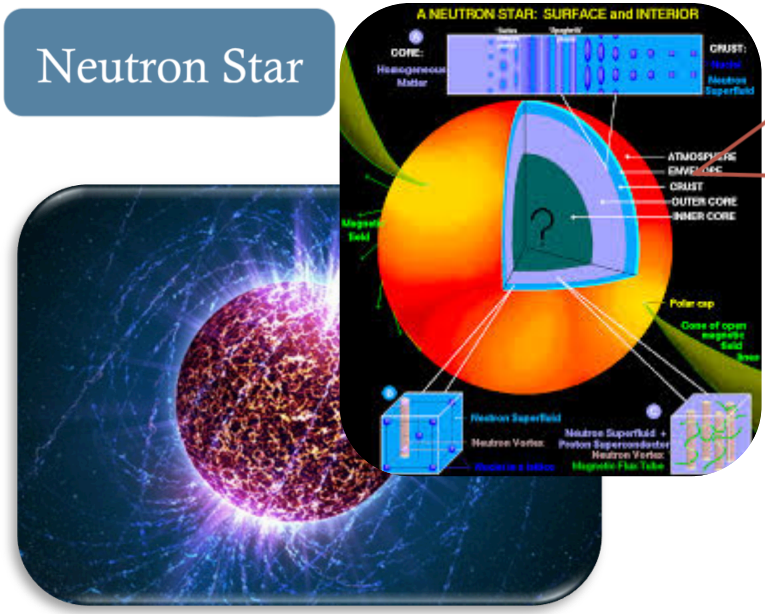
Summarize the above…

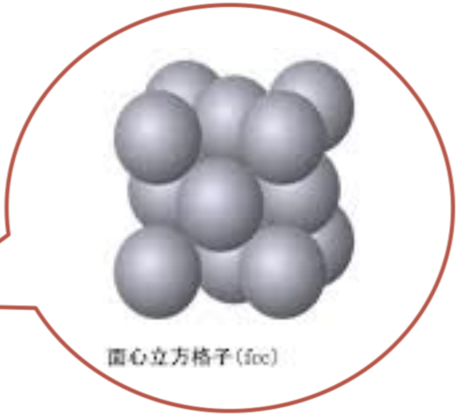
The purpose of my research is to extract the new aspect of QCD phase structure through extreme conditions, i.e. high density region with a magnetic field.

To accomplish my purpose, I focus on a baryonic matter with a strong magnetic field.

Magnetic field

Neutron Star





Face centered cubic

= Skyrmions

To get new insight for understanding QCD

Skyrmion properties

$\mathcal{L}_{\mathrm{Skyrm}}(U) \quad U = \exp[i\hat{x}^i\tau^i F(r)]$

Input parameter

$f_\pi = 93\mathrm{MeV}$ (experimenta value)

$e \sim 6$ (experimenta value determined from $\rho \to \pi\pi$)

Baryon properties

- Energy of skyrmion

$$M_{\mathrm{Skyrm}} = -\int d^3x \mathcal{L}_{\mathrm{Skyrm}}$$

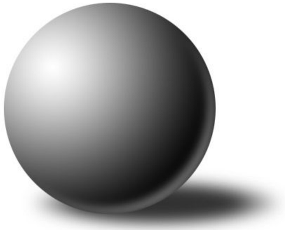
$$M_{\mathrm{Skyr}} \sim 1150[\mathrm{MeV}]$$

- Isoscalar charge radius of a nucleon

$$r_0 = 0.66\,\mathrm{fm}$$

$$(\, r_0^{(\mathrm{exp})} = 0.877 \pm 0.005\,\mathrm{fm}\,)$$

Skyrmion (nucleon) is the finite size particle.



*Observables are acceptable at the leading $\mathcal{O}(N_c)$.

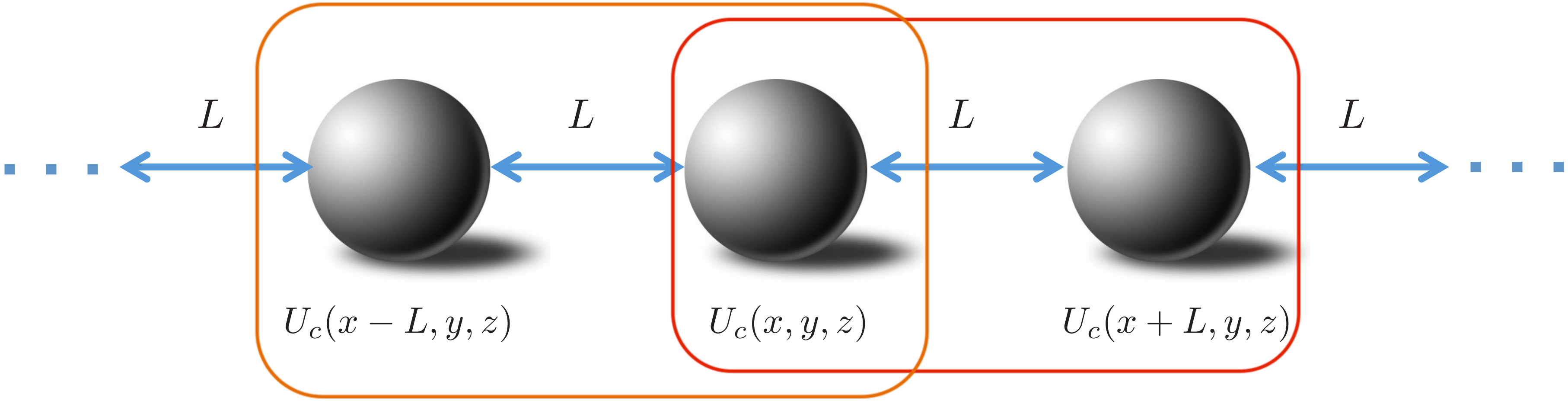
skyrmion-skyrmion interaction

スカーミオン間の相互作用

スカーミオンを間隔Lで格子状に並べたとき

引力が最も強くなる条件

$$U_{cc}(x,y,z) = U_c(x,y,z)e^{i\pi\tau_y/2}U_c(x+L,y,z)e^{-i\pi\tau_y/2}$$



引力が最も強くなる条件

$$U'_{cc}(x,y,z) = U_c(x-L,y,z)e^{i\pi\tau_y/2}U_c(x,y,z)e^{-i\pi\tau_y/2}$$

スカーミオン結晶が束縛するための条件 $U_c(x,y,z) = e^{i\pi\tau_y/2}U_c(x+L,y,z)e^{-i\pi\tau_y/2}$

磁場中でのカイラル不均一

磁場中でのカイラ不均

$$\rho_B = \left.\frac{\partial \mathcal{L}_{\rm wzw}}{\partial \mu_B}\right|_{\mu_B=0}$$

バリオン数

$$V^{\mu=0} = \bar{q}_L \gamma^0 q_L + \bar{q}_R \gamma^0 q_R$$

磁場中でのカイラ不均一

$$\rho_B = \left.\frac{\partial \mathcal{L}_{\mathrm{wzw}}}{\partial \mu_B}\right|_{\mu_B=0}$$

バリオン数

$$V^{\mu=0} = \bar{q}_L \gamma^0 q_L + \bar{q}_R \gamma^0 q_R$$

$$\rho_5 = \left.\frac{\partial \mathcal{L}_{\mathrm{wzw}}}{\partial \mu_5}\right|_{\mu_5=0}$$

右巻き・左巻き粒子数の差（カイラル不均一）

$$A^{\mu=0} = \bar{q}_L \gamma^0 q_L - \bar{q}_R \gamma^0 q_R$$

磁場中での**カイラ**不均一

$$\rho_B = \left.\frac{\partial \mathcal{L}_{\mathrm{wzw}}}{\partial \mu_B}\right|_{\mu_B=0}$$

バリオン数

$$V^{\mu=0} = \bar{q}_L \gamma^0 q_L + \bar{q}_R \gamma^0 q_R$$

$$\boxed{\rho_5 = \left.\frac{\partial \mathcal{L}_{\mathrm{wzw}}}{\partial \mu_5}\right|_{\mu_5=0}}$$

右巻き・左巻き粒子数の差（カイラル不均一）

$$A^{\mu=0} = \bar{q}_L \gamma^0 q_L - \bar{q}_R \gamma^0 q_R$$

真空（磁場がない環境）では現れない $\rho_5 := 0$

磁場中でのカイラ不均一

$$\rho_B = \left.\frac{\partial \mathcal{L}_{\rm wzw}}{\partial \mu_B}\right|_{\mu_B=0}$$

バリオン数

$$V^{\mu=0} = \bar{q}_L\gamma^0 q_L + \bar{q}_R\gamma^0 q_R$$

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右巻き・左巻き粒子数の差（カイラル不均一）

$$A^{\mu=0} = \bar{q}_L\gamma^0 q_L - \bar{q}_R\gamma^0 q_R$$

バリオン数 (winding number + eBに比例する項)

$$\begin{aligned}\rho_B &= \frac{1}{24\pi^2}\epsilon^{0\nu\rho\sigma}{\rm tr}\left[(\partial_\nu U\cdot U^\dagger)(\partial_\rho U\cdot U^\dagger)(\partial_\sigma U\cdot U^\dagger)\right] \\ &\quad +\frac{1}{16\pi^2}\epsilon^{0\nu\rho\sigma}{\rm tr}\left[ie(\partial_\nu A_\rho)Q_E(\partial_\sigma U\cdot U^\dagger + U^\dagger\partial_\sigma U) + ieA_\nu Q_E(\partial_\rho U\partial_\sigma U^\dagger - \partial_\rho U^\dagger\partial_\sigma U)\right]\end{aligned}$$

カイラル不均一 (eBに比例する項のみ)

$$\rho_5 = \frac{N_c}{48\pi^2}\epsilon^{0\nu\rho\sigma}{\rm tr}\Big[-ie(\partial_\nu A_\rho)Q_E[\partial_\sigma U, U^\dagger] + ieA_\nu Q_E[\partial_\rho U, \partial_\sigma U^\dagger]\Big]$$

磁場中でのカイラル不均一

カイラル不均一

$$\rho_5 = \frac{N_c}{48\pi^2}\epsilon^{0\nu\rho\sigma}\mathrm{tr}\Big[-ie(\partial_\nu A_\rho)Q_E[\partial_\sigma U, U^\dagger] + ieA_\nu Q_E[\partial_\rho U, \partial_\sigma U^\dagger]\Big]$$

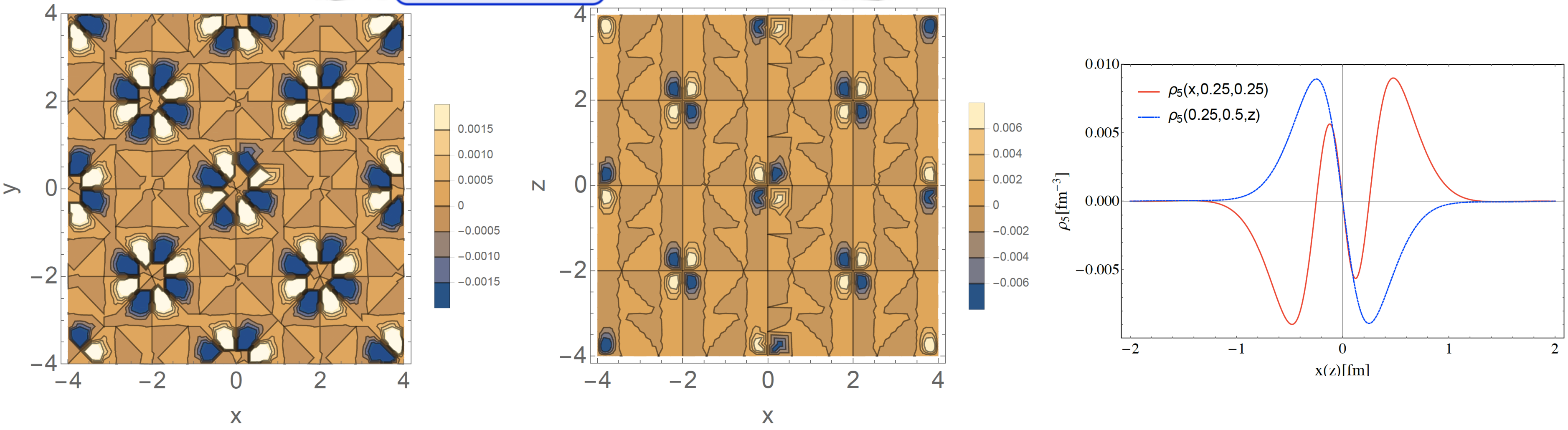
- 磁場がないとき $B = 0$ 、カイラル不均一は消える $\rho_5 = 0$

- 空間平均をでカイラル不均一はゼロになる $\int_{-L}^{L} d^3x\rho_5 = 0$

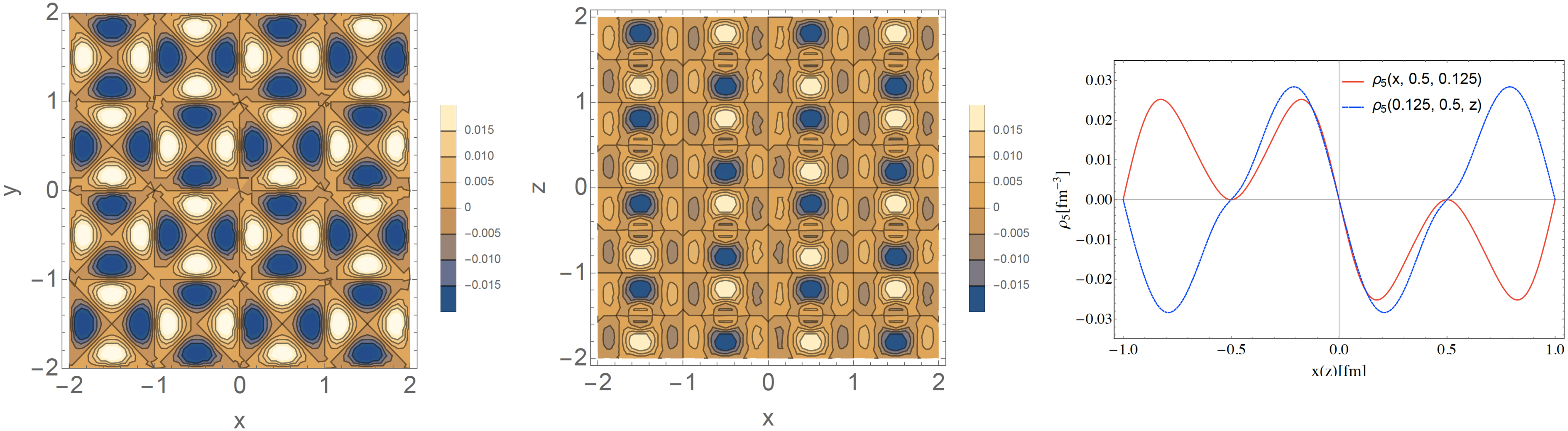
磁場中でのスカーミオン結晶は局所的にカイラル不均一な物質になっている。

磁場中でのカイラ不均一

$\sqrt{eB} = 400[\mathrm{MeV}]$ $L = 2.0[\mathrm{fm}]$

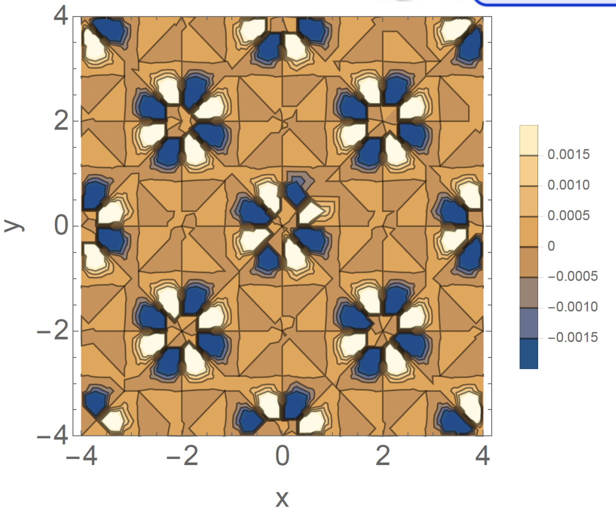


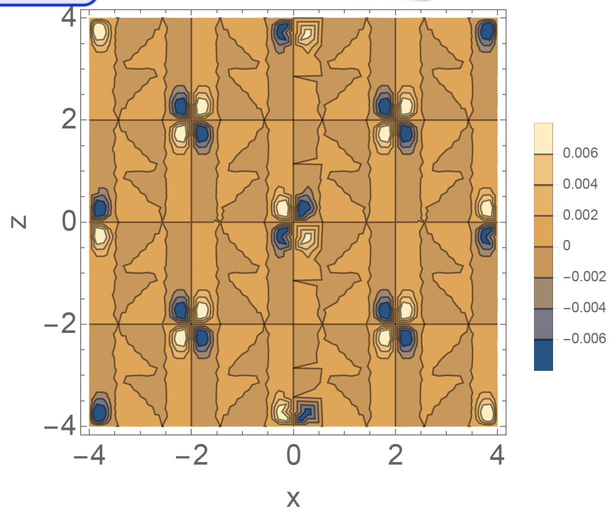
$\sqrt{eB} = 400[\mathrm{MeV}]$ $L = 1.0[\mathrm{fm}]$

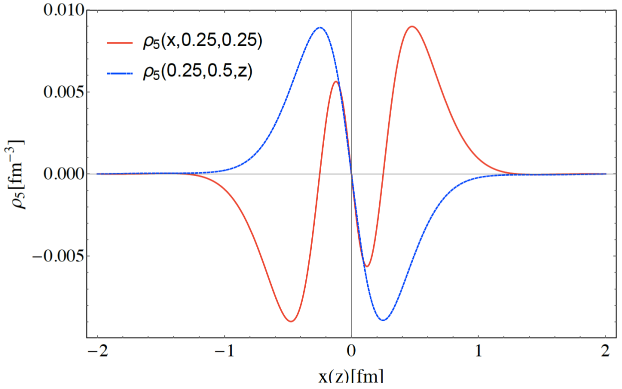


磁場中での**カイラル不均一**

$\sqrt{eB} = 400[\mathrm{MeV}]$ $L = 2.0[\mathrm{fm}]$

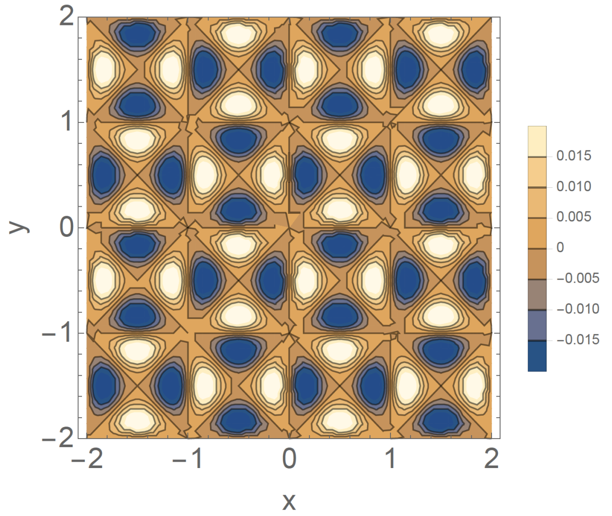


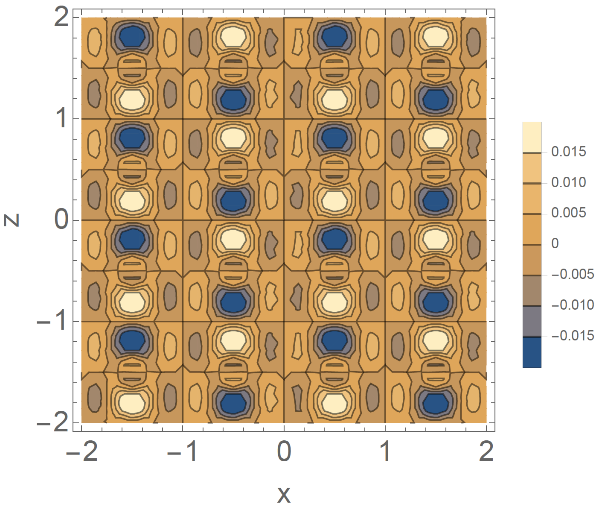


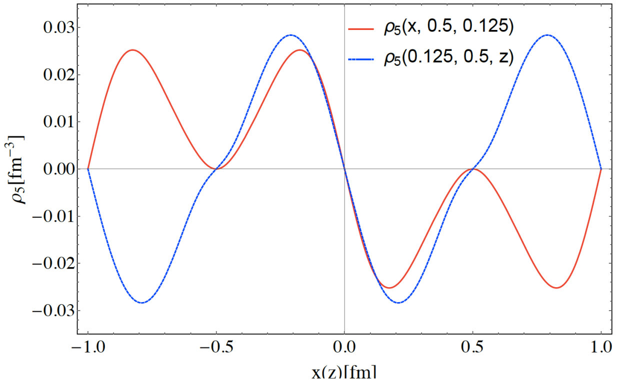


$\sqrt{eB} = 400[\mathrm{MeV}]$ $L = 1.0[\mathrm{fm}]$

高密度になると**カイラル不均一の振幅が**大きくなる



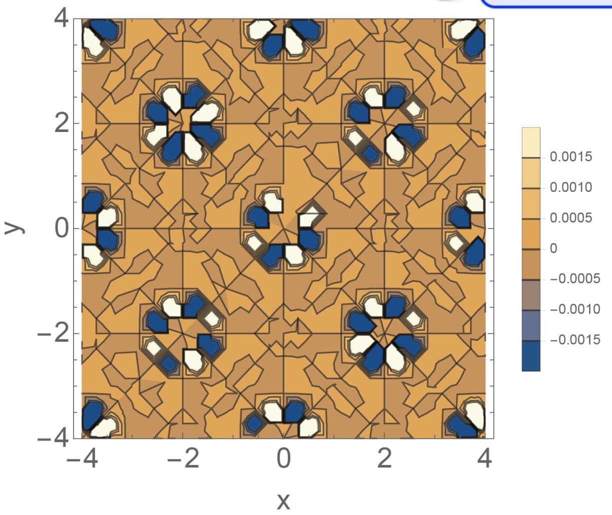


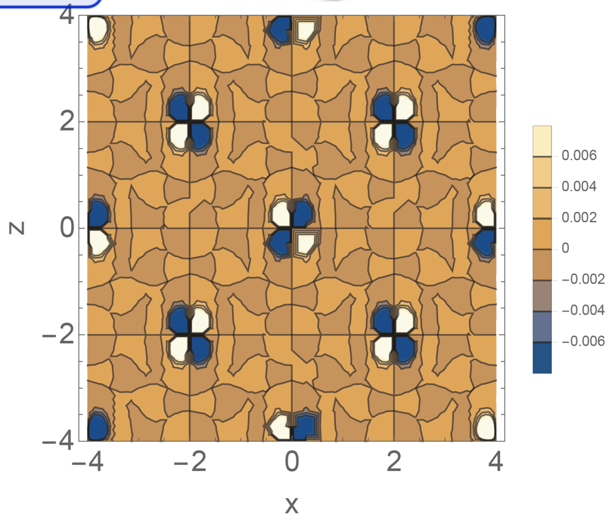


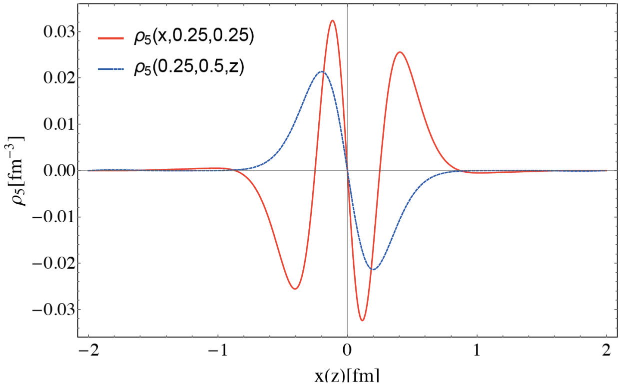
磁場中でのカイラ不均一

$\sqrt{eB} = 800[\mathrm{MeV}]$

$L = 2.0[\mathrm{fm}]$

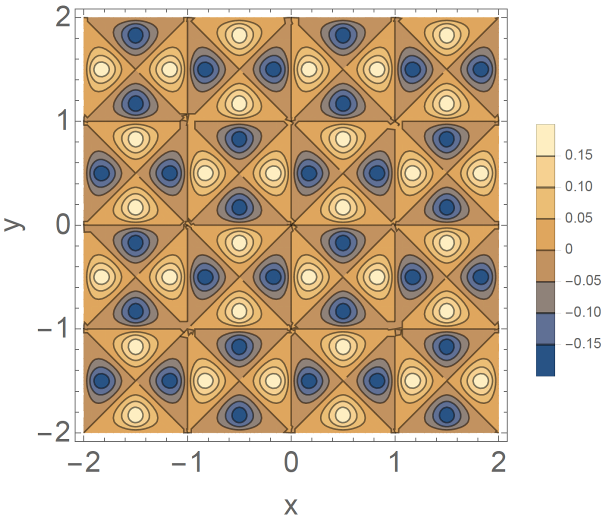


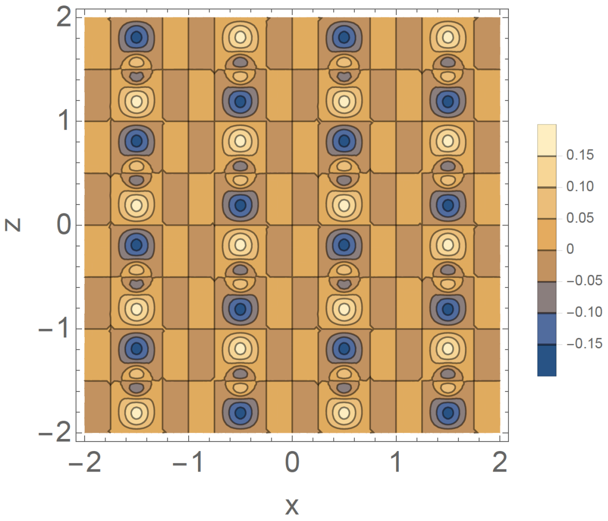


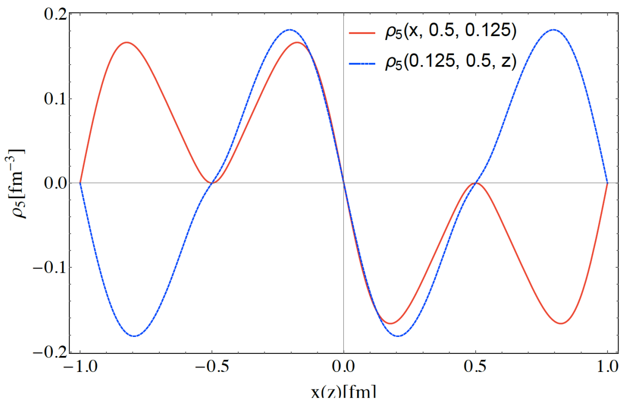


$\sqrt{eB} = 800[\mathrm{MeV}]$

$L = 1.0[\mathrm{fm}]$







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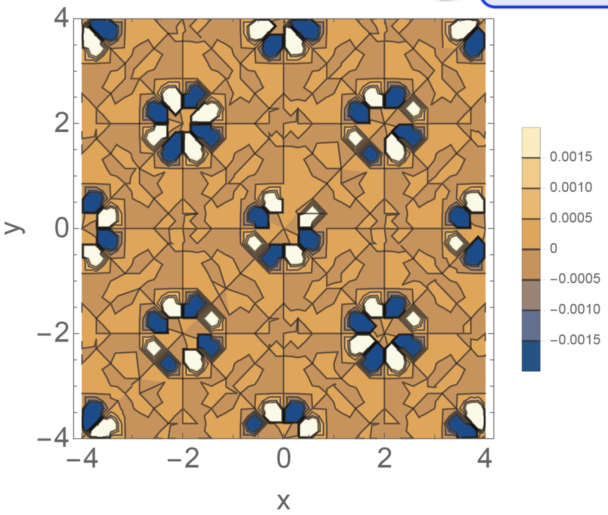
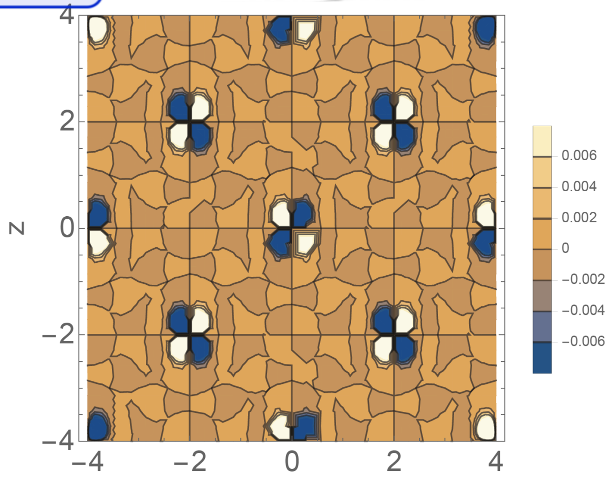
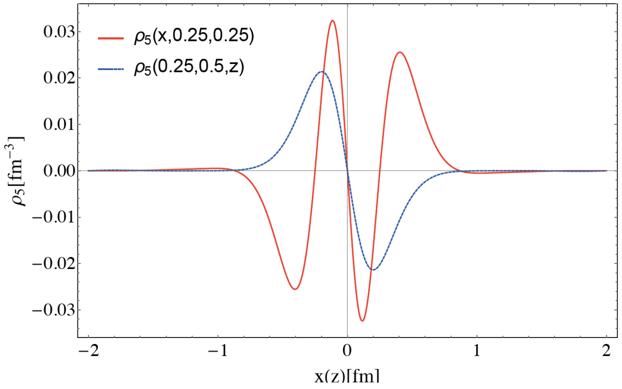
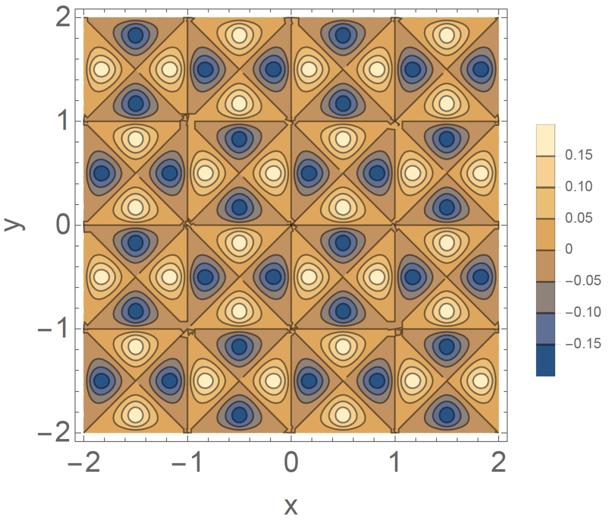


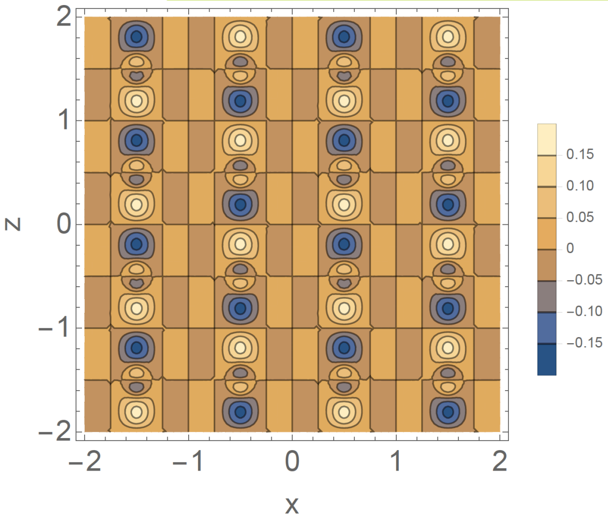


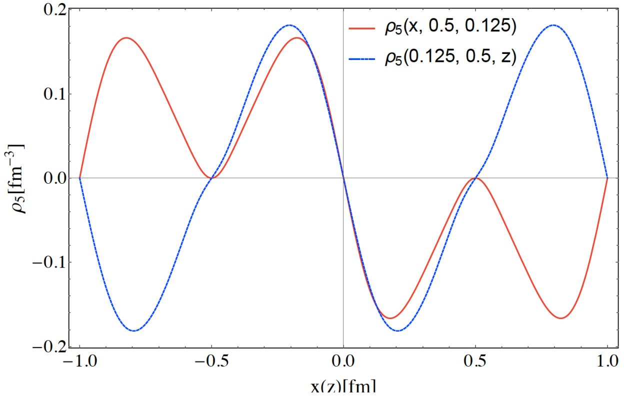


$\sqrt{eB} = 800[\mathrm{MeV}]$ $L = 1.0[\mathrm{fm}]$

強磁場・高密度領域ではカイラル不均一の振幅の大きさが一番強くなる







まと

「スカーミオンクリスタル」という手法で
バリオン物質の性質ついての研究を行なった

- 結晶構造が変化する相転移点が磁場によって高密度領域にずれる
  （half skyrmion phase への相転移が磁場によって遅くなる）

- 磁場によって結晶構造が変化する
  skyrmion phase: 円形から楕円形
  half skyrmion phase:強磁場の領域で立方格子の結晶構造ではなくなる

- 外部磁場によってスカーミオン結晶がカイラル不均一物質になっている。
  →どのような物理/物理現象と関係するの？？

スカーミオンクリスタルの構成方法

スカーミオンクリスタルでのラグランジアン

$$\mathcal{L}_{\text{Skyr}} = \frac{f_\pi^2}{4}\mathrm{tr}[\partial_\mu U \partial^\mu U^\dagger] + \frac{1}{32e^2}\mathrm{tr}\left\{[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U][U^\dagger\partial^\mu U, U^\dagger\partial^\nu U]\right\}$$

１. カイラル場の配位

$$U = \phi_0 + i\tau_i\phi_i$$

$$\phi_a = \frac{\bar{\phi}_a}{\sqrt{\bar{\phi}_b\bar{\phi}_b}}$$

$$\begin{aligned}
\bar{\phi}_0 &= \sum_{a,b,c} \bar{\beta}_{abc}\cos(a\pi x/L)\cos(b\pi y/L)\cos(c\pi z/L) \\
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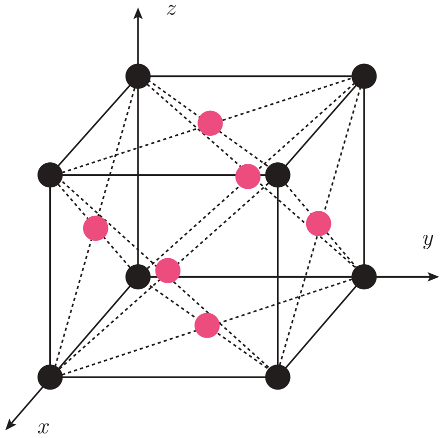
\end{aligned}$$



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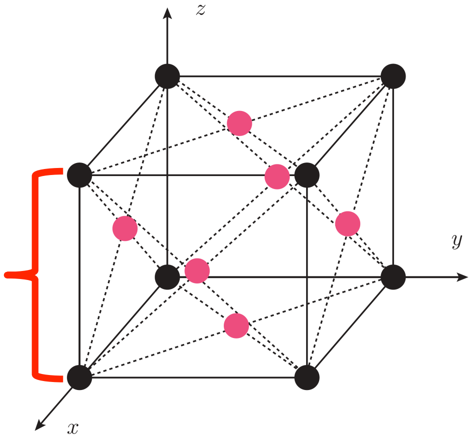
\end{aligned}$$

格子サイズ



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１. カイラル場の配位

２ .面心立方格子の
対称性**から**フーリエ係数
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\end{aligned}$$

reflection symmetry

$$(x,y,z) \to (-x,y,z): \quad (\sigma,\pi_1,\pi_2,\pi_3) \to (\sigma,-\pi_1,\pi_2,\pi_3)$$

three fold symmetry

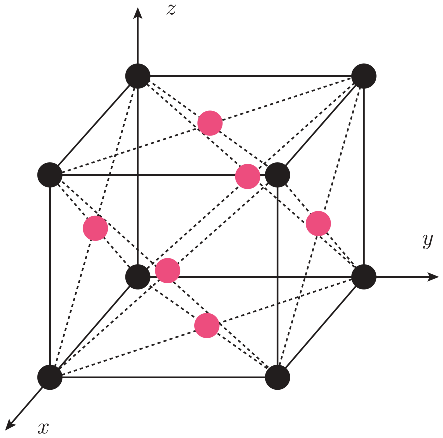
$$(x,y,z) \to (z,x,y); \quad (\sigma,\pi_1,\pi_2,\pi_3) \to (\sigma,\pi_3,\pi_1,\pi_2)$$

four fold symmetry

$$(x,y,z) \to (x,z,-y); \ (\sigma,\pi_1,\pi_2,\pi_3) \to (\sigma,\pi_1,\pi_3,-\pi_2)$$

translation symmetry

$$(x,y,z) \to (x+L,y+L,z): \quad (\sigma,\pi_1,\pi_2,\pi_3) \to (\sigma,-\pi_1,-\pi_2,\pi_3)$$



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\end{aligned}$$

２ .面心立方格子の対称性**から**フーリエ係数に制限を与え結晶を構成

reflection symmetry　(結晶が持つ対称性)

$$(x,y,z)\to(-x,y,z):\quad (\sigma,\pi_1,\pi_2,\pi_3)\to(\sigma,-\pi_1,\pi_2,\pi_3)$$

three fold symmetry　(結晶が持つ対称性)

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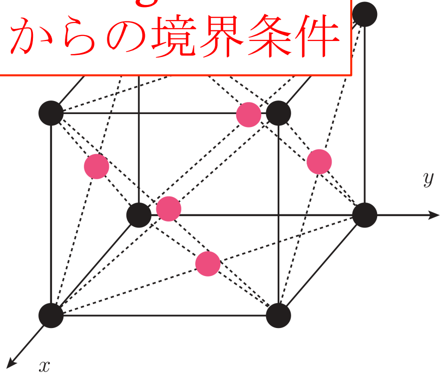
four fold symmetry　(結晶が持つ対称性)

$$(x,y,z)\to(x,z,-y);\quad (\sigma,\pi_1,\pi_2,\pi_3)\to(\sigma,\pi_1,\pi_3,-\pi_2)$$

translation symmetry　(スカーミオン同士が相互作用する条件)

$$(x,y,z)\to(x+L,y+L,z):\quad (\sigma,\pi_1,\pi_2,\pi_3)\to(\sigma,-\pi_1,-\pi_2,\pi_3)$$

＋ winding number からの境界条件



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スカーミオンクリスタルでのラグランジアン

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$$\phi_a = \frac{\bar{\phi}_a}{\sqrt{\bar{\phi}_b\bar{\phi}_b}}$$

３. スカーミオンクリスタル
のエネルギーが最小に
なるようにフーリエ係数を決定

$$\begin{aligned}
\bar{\phi}_0 &= \sum_{a,b,c}\bar{\beta}_{abc}\cos(a\pi x/L)\cos(b\pi y/L)\cos(c\pi z/L)\\
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\end{aligned}$$

$$\boxed{M_{\mathrm{Skyrm}} = -\int d^3x\,\mathcal{L}_{\mathrm{Skyrm}}}$$

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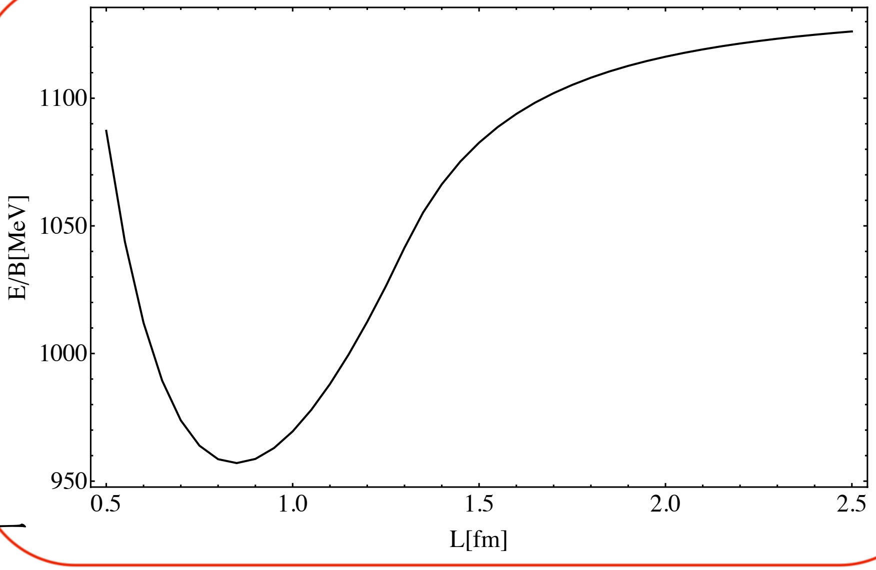
$$\boxed{M_{\mathrm{Skyrm}} = -\int d^3x\mathcal{L}_{\mathrm{Skyrm}}}$$

４. 格子サイズを変更する
ことでバリオン物質の
密度を変更

スカーミオン一つあたりのエネルギー



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\end{aligned}$$

３. スカーミオンクリスタル
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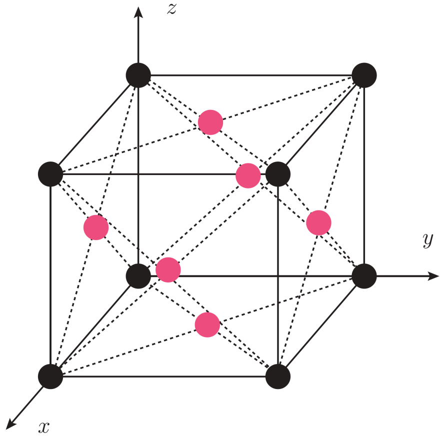
４. 格子サイズを変更する
ことでバリオン物質の
密度を変更

$$\rho_B = \frac{1}{24\pi^2}\epsilon^{0\nu\rho\sigma}\mathrm{tr}\left[(\partial_\nu U\cdot U^\dagger)(\partial_\rho U\cdot U^\dagger)(\partial_\sigma U\cdot U^\dagger)\right]$$

$$\int_{-L}^{L} d^3x\rho_B = 4$$

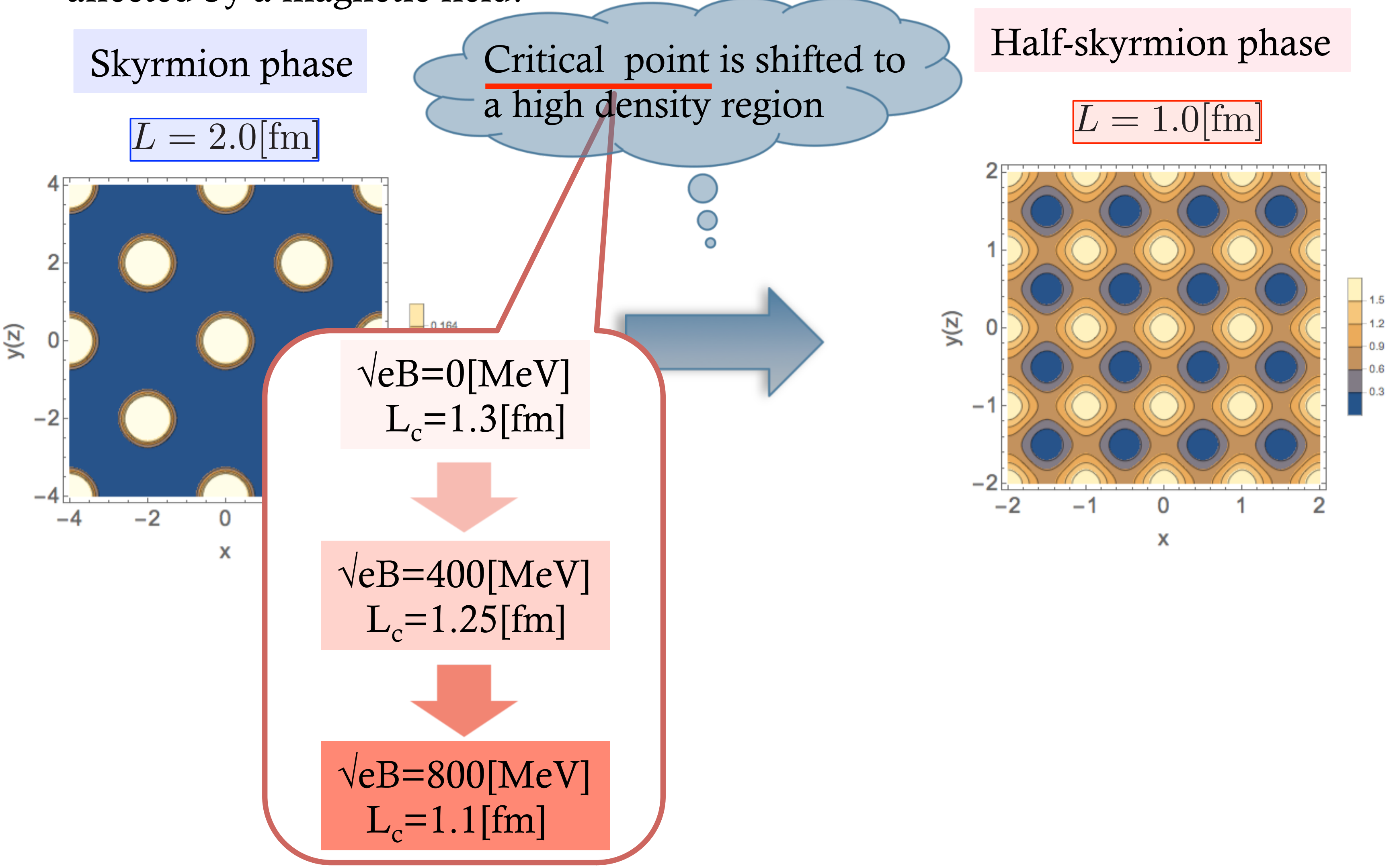
5. バリオン(スカーミオン)の
分布はwinding number密度
から記述される

面心立方格子の
スカーミオン結晶
が描けるはず



Deformation of the skyrmion configuration

It turned out that the critical point (topological transition point) is affected by a magnetic field.



Skyrmion phase

$L = 2.0[\mathrm{fm}]$

Critical point is shifted to a high density region

Half-skyrmion phase

$L = 1.0[\mathrm{fm}]$

$\sqrt{eB}=0$[MeV]
$L_c=1.3$[fm]

$\sqrt{eB}=400$[MeV]
$L_c=1.25$[fm]

$\sqrt{eB}=800$[MeV]
$L_c=1.1$[fm]