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# Relativistic nucleon-nucleon contact Lagrangian


**Yang Xiao（肖杨）**

School of Physics and Nuclear Energy Engineering,

Beihang University


In collaboration with Xiu-Lei Ren and Li-Sheng Geng

北京航空航天大學 BEIHANG UNIVERSITY

吉林大学

# Contents

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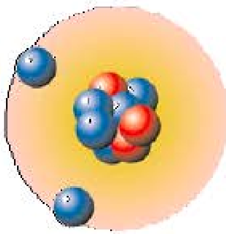
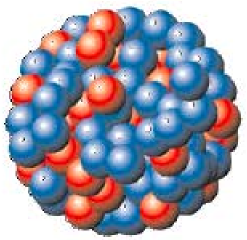
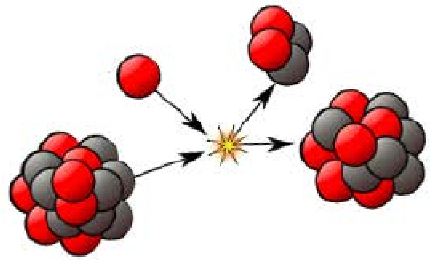
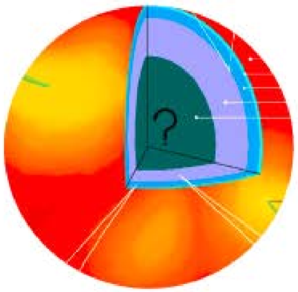
- **Why relativistic nucleon-nucleon Lagrangian**
- Generalities & strategy
- Results
- Non-relativistic reduction
- Summary and outlook

# Why nuclear force

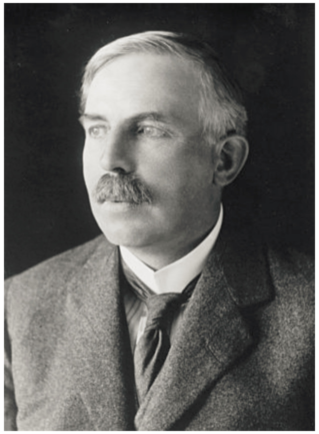
- **Microscopic nature** of the material world

- **What binds the nucleons together ?**
- One of **the most important inputs** in **nuclear physics**

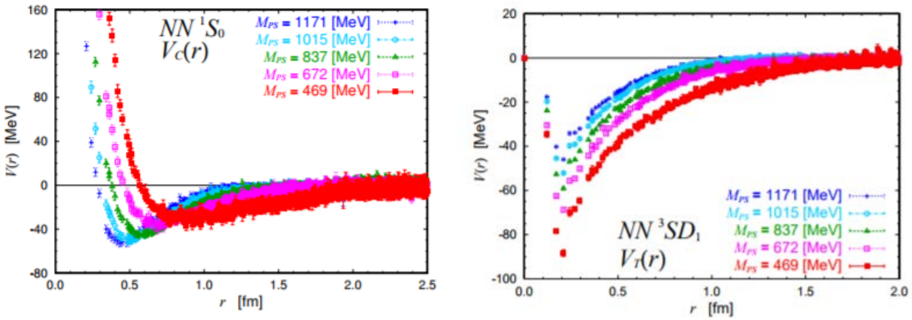

John Dalton


Thomson Joseph John


Ernest Rutherford


James Chadwick

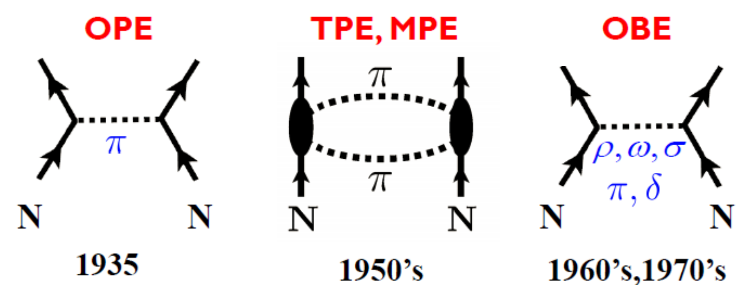
# Main methods for nuclear force study




*HAL QCD PRL(2007), arXiv: 1511.04871*

## ☐ Lattice QCD

- ✓ **numerical method of QCD**
- ✓ ***Ab initio***
- – Far away from **physical region**

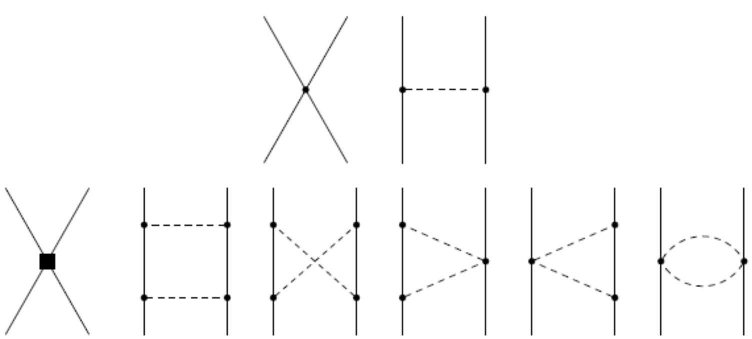



## ☐ Phenomenological model

- ✓ **High precision**
- – Not constructed from **fundamental level**

**LO:**

**NLO:**



## ☐ ChEFT

- ✓**Low energy QCD**
- ✓**Model independently**
- ✓**Improve calculations systematically**
- ✓**Estimate theoretical uncertainties**
- ✓**Consistent many body interactions**

# Chiral NN potential is of high precision

| | Phenomenological forces | | | NR Chiral nuclear force | | | |
|---|---|---|---|---|---|---|---|
| | Reid93 | CD-Bonn | LO | NLO | N2LO | N3LO | **N4LO** |
| No. of para. | 50 | 38 | 2 | 9 | 9 | 24 | **24** |
| $\chi^2$/datum | 1.03 | 1.02 | 94 | 36.7 | 5.28 | 1.27 | **1.10** |

*D.Entem, et al., PRC96(2017)024004*

# Current status for chiral nuclear force (NF)

## ◻NN interaction

- Non-relativistic
  - Up to NLO *U. van Kolck et al., PRL, PRC1992-94; N. Kaiser, NPA1997*
  - Up to NNLO *U. van Kolck et al., PRC1994; E. Epelbaum, et al.,NPA2000*
  - Up to N3LO *R. Machleidt et al., PRC2003; E. Epelbaum et al., NPA2005*
  - Up to N4LO *E. Epelbaum et al., PRL2015, D.R. Entem, et al., PRC2015*
- **Relativisitic**
  - Up to LO *Xiu-Lei Ren, Kai-Wen Li, Li-Sheng Geng, Bing-Wei Long, Peter Ring, Jie Meng., CPC 2018*

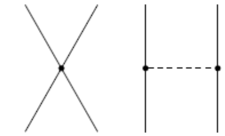
## ◻Why covariant

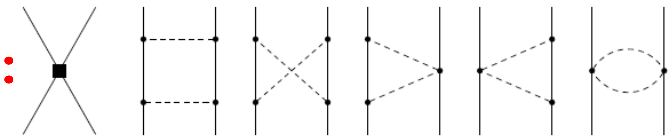
- **Lorentz invariance** is one of the most important symmetries in nature.
- Quantum mechanics & **Special relativity**: two pillars of modern physics
- **Relativistic approaches successful**
  - **Nuclear system:** spin-orbit splitting, pseudospin symmetry
  - **One-baryon sector**： magnetic moments、masses、sigma terms

## ▪Relativistic chiral nuclear force is promising !

# Current status for covariant contact Lagrangians

- **Nucleon-nucleon sector**
  - Up to LO: **5** terms *CPC 42 (2018) no.1, 014103*
  - Up to **NLO**: **36** terms *Phys.Rev. C81 (2010) 034005*

LO: 

NLO: 

- **Baryon-baryon sector**
  - Up to LO: **15** terms *CPC 42 (2018) no.1, 014105*
  - Up to **NLO**: **75** terms *Nucl. Phys., A916 (2013) 1*

- **Inadequacy**
  - Not all of them are **linear independently** (*for those up to NLO*)
  - **Too many Low energy constants** up to NLO, especially for baryon-baryon sector, compared with experiment data.

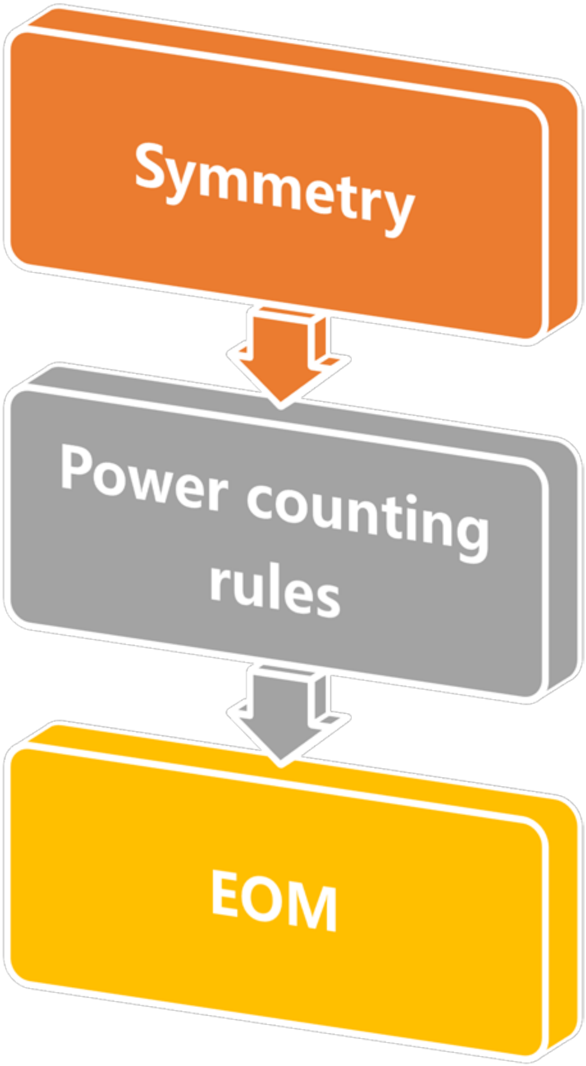
- **Our purpose**

  ※ **Construct the complete and minimal *NN (BB)* contact Lagrangians**

# Contents

# How to construct Lagrangians

- Fundamental requirement
  - **Symmetry**
    - **Lorentz**
    - **Chiral (R)**
    - Charge conjugation (C), Parity (P), Time inverse (T) and Hermitian conjugation (h.c.)
- How to raise chiral order ?
  - **Power counting rules**
- How to deal with redundant terms ?
  - **Equation of motion (EOM)**



# Symmetry requirements

- Lorentz invariant: $\alpha, \beta, \gamma$ … …
- Chiral symmetry: Automatically fulfilled. $\psi \to K\psi K^{\dagger}$
- Hermitian conjugation symmetry: No constraint.
- Parity symmetry and Charge conjugation symmetry: Important !
- Time inverse symmetry: CPT theory.

- Behavior under several transformations

| | $\mathbb{1}$ | $\gamma_5$ | $\gamma_\mu$ | $\gamma_5\gamma_\mu$ | $\sigma_{\mu\nu}$ | $\epsilon_{\mu\nu\rho\sigma}$ | $\overleftrightarrow{\partial}_\mu$ | $\partial_\mu$ |
|---|---|---|---|---|---|---|---|---|
| $\mathcal{P}$ | + | − | + | − | + | − | + | + |
| $\mathcal{C}$ | + | + | − | + | − | + | − | + |
| h.c. | + | − | + | + | + | + | − | + |
| $\mathcal{O}$ | 0 | 1 | 0 | 0 | 0 | − | 0 | 1 |

✓ $\overleftrightarrow{\partial}^\alpha = \vec{\partial}^\alpha - \overleftarrow{\partial}^\alpha$

✓ $\partial^\alpha = \partial^\alpha(\bar{\psi}\Gamma\psi)$

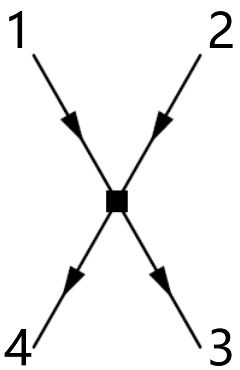
$$\frac{1}{(2m)^{N_d}}\left(\bar{\psi}i\overleftrightarrow{\partial}^\alpha i\overleftrightarrow{\partial}^\beta ...\Gamma_A\psi\right)\partial^\lambda\partial^\mu ...\left(\bar{\psi}i\overleftrightarrow{\partial}^\sigma i\overleftrightarrow{\partial}^\tau ...\Gamma_B\psi\right),$$

# Power counting rules

- **General expression:** $\frac{1}{(2m)^{N_d}}\left(\bar{\psi}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}...\Gamma_A\psi\right)\partial^{\lambda}\partial^{\mu}...\left(\bar{\psi}i\overleftrightarrow{\partial}^{\sigma}i\overleftrightarrow{\partial}^{\tau}...\Gamma_B\psi\right)$

$N_d$ is the number of four component, $\overleftrightarrow{\partial}=\vec{\partial}-\overleftarrow{\partial}$

- Nucleon field: $\psi = \binom{p}{n} \sim O(p^0)$, Nucleon mass: $m \sim O(p^0)$,




- Clifford algebra: $\Gamma \in \{1, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu} \sim O(p^0), \gamma_5 \sim O(p^1)\}$
- **Covariant derivative:** $\partial(\bar{\psi}\Gamma\psi) \sim O(p^1)$, $\left(\bar{\psi}\overleftrightarrow{\partial}\psi\right) \sim O(p^0)$
- **Unique structure:** $(\bar{\psi}\sigma_{\mu\nu}\psi)\left(\bar{\psi}\overleftrightarrow{\partial}^{\mu}\Gamma\psi\right) \sim O(p^1)$, $(\bar{\psi}\gamma_5\gamma_\mu\psi)(\bar{\psi}\overleftrightarrow{\partial}^{\mu}\Gamma\psi) \sim O(p^1)$
- **Treatment for covariant derivative:**

$$\widetilde{O}^{(n)}_{\Gamma_A\Gamma_B} = \frac{1}{(2m)^{2n}}(\bar{\psi}\, i\overleftrightarrow{\partial}^{\mu_1}\, i\overleftrightarrow{\partial}^{\mu_2} \cdots i\overleftrightarrow{\partial}^{\mu_n}\, \Gamma_A^{\alpha}\, \psi)\,(\bar{\psi}\, i\overleftrightarrow{\partial}_{\mu_1}\, i\overleftrightarrow{\partial}_{\mu_2} \cdots i\overleftrightarrow{\partial}_{\mu_n}\, \Gamma_{B\,\alpha}\, \psi)$$

-Expansion of such structure: $1 + \frac{n}{4m^2}\left[\mathbf{p}_1^2 + \mathbf{p}_2^2 + \mathbf{p}_3^2 + \mathbf{p}_4^2 - (\mathbf{p}_1 + \mathbf{p}_3)\cdot(\mathbf{p}_2 + \mathbf{p}_4)\right]$

-Differences are of high order: $n = 0$

# A Reduction of equation of motion (EOM)

- **Equation of motion :** $\not{D}B = \gamma^\mu D_\mu B = -\mathrm{i}M_0 B + \mathcal{O}(q)$

- Beyond the obvious replacements one can bring terms not containing $\not{D}B$ into a form where they do. *Annals Phys., 283:273, (2000)*

- **Summary:**

  – $\gamma^\mu \Leftrightarrow \overleftrightarrow{\partial}^\mu$;

  – $\gamma_5\gamma^\mu \Leftrightarrow \epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}\overleftrightarrow{\partial}^\nu$;

  – $\sigma_{\mu\nu} \Leftrightarrow \epsilon_{\mu\nu\alpha\beta}\gamma_5\gamma^\alpha\overleftrightarrow{\partial}^\beta$;

  – $\epsilon_{\mu\nu\alpha\beta}\left(\bar{\psi}\overleftrightarrow{\partial}^\mu\overleftrightarrow{\partial}^\nu \ldots \Gamma\psi\right) = 0$;

| $\Gamma$ | $\Gamma'_\lambda$ | $\Gamma''_\lambda$ |
|---|---|---|
| $\mathbb{1}$ | $\gamma_\lambda$ | $0$ |
| $\gamma_\mu$ | $g_{\mu\lambda}1$ | $-i\sigma_{\mu\lambda}$ |
| $\gamma_5$ | $0$ | $\gamma_5\gamma_\lambda$ |
| $\gamma_5\gamma_\mu$ | $\frac{1}{2}\epsilon_{\mu\lambda\rho\tau}\sigma^{\rho\tau}$ | $g_{\mu\lambda}\gamma_5$ |
| $\sigma_{\mu\nu}$ | $\epsilon_{\mu\nu\lambda\tau}\gamma_5\gamma^\tau$ | $-i\left(g_{\mu\lambda}\gamma_\nu - g_{\nu\lambda}\gamma_\mu\right)$ |
| $\epsilon_{\mu\nu\rho\tau}\gamma^\tau$ | $\epsilon_{\mu\nu\rho\lambda}1$ | $g_{\mu\lambda}\gamma_5\sigma_{\nu\rho} + g_{\rho\lambda}\gamma_5\sigma_{\mu\nu} + g_{\nu\lambda}\gamma_5\sigma_{\rho\mu}$ |
| $\epsilon_{\mu\nu\rho\tau}\gamma_5\gamma^\tau$ | $g_{\mu\lambda}\sigma_{\nu\rho} + g_{\rho\lambda}\sigma_{\mu\nu} + g_{\nu\lambda}\sigma_{\rho\mu}$ | $\epsilon_{\mu\nu\rho\lambda}\gamma_5$ |
| $\epsilon_{\mu\nu\rho\alpha}\sigma^\alpha_\tau$ | $\gamma_5\gamma_\rho\left(g_{\lambda\nu}g_{\mu\tau} - g_{\lambda\mu}g_{\nu\tau}\right) + \gamma_5\gamma_\nu\left(g_{\lambda\mu}g_{\rho\tau} - g_{\lambda\rho}g_{\mu\tau}\right) + \gamma_5\gamma_\mu\left(g_{\lambda\rho}g_{\nu\tau} - g_{\lambda\nu}g_{\rho\tau}\right)$ | $ig_{\lambda\tau}\epsilon_{\mu\nu\rho\alpha}\gamma^\alpha - i\epsilon_{\mu\nu\rho\lambda}\gamma_\tau$ |
| $\frac{i}{2}\epsilon_{\mu\nu\rho\tau}\sigma^{\rho\tau} = \gamma_5\sigma_{\mu\nu}$ | $\frac{1}{i}\left(g_{\mu\lambda}\gamma_5\gamma_\nu - g_{\nu\lambda}\gamma_5\gamma_\mu\right)$ | $\epsilon_{\mu\nu\lambda\rho}\gamma^\rho$ |

$$Y(\Theta^i = \Gamma'^\lambda D^{n_i}_\lambda) \approx -\mathrm{i}M_0 Y(\Theta^i = \Gamma D^{n_i - 1})$$

  – **No two Lorentz indices** inside a fermion bilinear can be contracted with one another, **except for Levi-Civita tensor, $\partial^\mu$ contracting with $\sigma_{\mu\nu}$ and $\partial^2$ acting on the whole bilinear.**

# Contents

# Covariant *NN* contact Lagrangians (NLO)

| | |
|---|---|
| $\widetilde{\mathcal{O}}_1$ | $(\bar{\psi}\psi)\,(\bar{\psi}\psi)$ |
| $\widetilde{\mathcal{O}}_2$ | $(\bar{\psi}\gamma^{\mu}\psi)\,(\bar{\psi}\gamma_{\mu}\psi)$ |
| $\widetilde{\mathcal{O}}_3$ | $(\bar{\psi}\gamma_5\gamma^{\mu}\psi)\,(\bar{\psi}\gamma_5\gamma_{\mu}\psi)$ |
| $\widetilde{\mathcal{O}}_4$ | $(\bar{\psi}\sigma^{\mu\nu}\psi)\,(\bar{\psi}\sigma_{\mu\nu}\psi)$ |
| $\widetilde{\mathcal{O}}_5$ | $\frac{1}{4m^2}\,(\bar{\psi}\psi)\,\partial^2\,(\bar{\psi}\psi)$ |
| $\widetilde{\mathcal{O}}_6$ | $\frac{1}{4m^2}\,(\bar{\psi}\gamma^{\mu}\psi)\,\partial^2\,(\bar{\psi}\gamma_{\mu}\psi)$ |
| $\widetilde{\mathcal{O}}_7$ | $\frac{1}{4m^2}\,(\bar{\psi}\gamma_5\gamma^{\mu}\psi)\,\partial^2\,(\bar{\psi}\gamma_5\gamma_{\mu}\psi)$ |
| $\widetilde{\mathcal{O}}_8$ | $\frac{1}{4m^2}\,(\bar{\psi}\sigma^{\mu\nu}\psi)\,\partial^2\,(\bar{\psi}\sigma_{\mu\nu}\psi)$ |
| $\widetilde{\mathcal{O}}_9$ | $\frac{1}{4m^2}\left(\bar{\psi}\sigma^{\mu\rho}\overleftrightarrow{\partial}^{\nu}\psi\right)\left(\bar{\psi}\sigma_{\mu\nu}\overleftrightarrow{\partial}_{\rho}\psi\right)$ |
| $\widetilde{\mathcal{O}}_{10}$ | $\frac{1}{4m^2}\left(\bar{\psi}\gamma_5\gamma^{\mu}\overleftrightarrow{\partial}^{\nu}\psi\right)\left(\bar{\psi}\gamma_5\gamma_{\nu}\overleftrightarrow{\partial}_{\mu}\psi\right)$ |
| $\widetilde{\mathcal{O}}_{11}$ | $(\bar{\psi}\gamma_5\psi)\,(\bar{\psi}\gamma_5\psi)$ |
| $\widetilde{\mathcal{O}}_{12}$ | $\frac{i}{4m^2}\,(\bar{\psi}\sigma^{\mu\rho}\psi)\,\partial_{\mu}\left(\bar{\psi}\overleftrightarrow{\partial}_{\rho}\psi\right)$ |
| $\widetilde{\mathcal{O}}_{13}$ | $\frac{1}{2m}\,(\bar{\psi}\sigma^{\mu\rho}\psi)\,\partial_{\mu}\,(\bar{\psi}\gamma_{\rho}\psi)$ |



**Relativistic: 4+9**

vs.

**Non-relativistic: 2+7**

# Contents

- Why relativistic nucleon-nucleon Lagrangian
- Generalities & strategy
- Results
- **Non-relativistic reduction**
- Summary and outlook

# Non-relativistic reduction

- **Why do non-relativistic reduction: Self consistent check**
- **Non-relativistic expansion:** $\psi \to N$, expand Lagrangians in terms of **1/m**
  - **Relativistic nucleon field operator:** $\psi(x) = \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{m}{E_p} \tilde{b}_s(\mathbf{p})\, u^{(s)}(\mathbf{p})\, \mathrm{e}^{-ip\cdot x},$
  - **Non-relativistic nucleon field operator:** $N(x) = \int \frac{d\mathbf{p}}{(2\pi)^3} b_s(\mathbf{p})\, \chi_s\, \mathrm{e}^{-ip\cdot x}$
  - **Expansion of field operator**

$$\psi(x) = \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{i}{2m}\begin{pmatrix} 0 \\ \boldsymbol{\sigma}\cdot\boldsymbol{\nabla} \end{pmatrix} + \frac{1}{8m^2}\begin{pmatrix} \boldsymbol{\nabla}^2 \\ 0 \end{pmatrix}\right] N(x) + \mathcal{O}(Q^3)$$

  - **Dirac matrices**

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu].$$

# Non-relativistic reduction

Non-relativistic Lagrangians $O(p^2)$

*Phys. Rev. C 53, 2086 (1996).*

| | |
|---|---|
| $O_S$ | $(N^\dagger N)(N^\dagger N)$ |
| $O_T$ | $(N^\dagger \boldsymbol{\sigma} N)\cdot(N^\dagger \boldsymbol{\sigma} N)$ |
| $O_1$ | $(N^\dagger \overrightarrow{\boldsymbol{\nabla}} N)^2 + \text{h.c.}$ |
| $O_2$ | $(N^\dagger \overrightarrow{\boldsymbol{\nabla}} N)\cdot(N^\dagger \overleftarrow{\boldsymbol{\nabla}} N)$ |
| $O_3$ | $(N^\dagger N)(N^\dagger \overrightarrow{\boldsymbol{\nabla}}^2 N) + \text{h.c.}$ |
| $O_4$ | $i\,(N^\dagger \overrightarrow{\boldsymbol{\nabla}} N)\cdot(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \times \boldsymbol{\sigma} N) + \text{h.c.}$ |
| $O_5$ | $i\,(N^\dagger N)(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \cdot \boldsymbol{\sigma} \times \overrightarrow{\boldsymbol{\nabla}} N)$ |
| $O_6$ | $i\,(N^\dagger \boldsymbol{\sigma} N)\cdot(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \times \overrightarrow{\boldsymbol{\nabla}} N)$ |
| $O_7$ | $(N^\dagger \boldsymbol{\sigma}\cdot \overrightarrow{\boldsymbol{\nabla}} N)(N^\dagger \boldsymbol{\sigma}\cdot \overrightarrow{\boldsymbol{\nabla}} N) + \text{h.c.}$ |
| $O_8$ | $(N^\dagger \sigma^j \overrightarrow{\nabla}^k N)(N^\dagger \sigma^k \overrightarrow{\nabla}^j N) + \text{h.c.}$ |
| $O_9$ | $(N^\dagger \sigma^j \overrightarrow{\nabla}^k N)(N^\dagger \sigma^j \overrightarrow{\nabla}^k N) + \text{h.c.}$ |
| $O_{10}$ | $(N^\dagger \boldsymbol{\sigma}\cdot \overrightarrow{\boldsymbol{\nabla}} N)(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \cdot \boldsymbol{\sigma} N)$ |
| $O_{11}$ | $(N^\dagger \sigma^j \overrightarrow{\nabla}^k N)(N^\dagger \overleftarrow{\nabla}^j \sigma^k N)$ |
| $O_{12}$ | $(N^\dagger \sigma^j \overrightarrow{\nabla}^k N)(N^\dagger \overleftarrow{\nabla}^k \sigma^j N)$ |
| $O_{13}$ | $(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \cdot \boldsymbol{\sigma} \overrightarrow{\nabla}^j N)(N^\dagger \sigma^j N) + \text{h.c.}$ |
| $O_{14}$ | $2\,(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \sigma^j \cdot \overrightarrow{\boldsymbol{\nabla}} N)(N^\dagger \sigma^j N)$ |

Non-relativistic expnasion $O(p^2)$

| | |
|---|---|
| $\tilde{\mathcal{O}}_1$ | $\mathcal{O}_S + \frac{1}{4m^2}(\mathcal{O}_1 + 2\mathcal{O}_2 + 2\mathcal{O}_3 + 2\mathcal{O}_5)$ |
| $\tilde{\mathcal{O}}_2$ | $\mathcal{O}_S + \frac{1}{4m^2}(-4\mathcal{O}_2 - 2\mathcal{O}_5 + 4\mathcal{O}_6 + \mathcal{O}_7 - \mathcal{O}_9 + 2\mathcal{O}_{10} - 2\mathcal{O}_{12})$ |
| $\tilde{\mathcal{O}}_3$ | $-\mathcal{O}_T - \frac{1}{4m^2}(-2\mathcal{O}_6 + \mathcal{O}_7 - \mathcal{O}_9 - 2\mathcal{O}_{10} - 2\mathcal{O}_{12} + 2\mathcal{O}_{13} - 2\mathcal{O}_{14})$ |
| $\tilde{\mathcal{O}}_4$ | $2\mathcal{O}_T + \frac{1}{2m^2}(-\mathcal{O}_1 - 2\mathcal{O}_2 - 4\mathcal{O}_5 + 2\mathcal{O}_6 - \mathcal{O}_8 + 2\mathcal{O}_{11} - 4\mathcal{O}_{12} - 2\mathcal{O}_{13})$ |
| $\tilde{\mathcal{O}}_5$ | $\frac{1}{4m^2}(\mathcal{O}_1 + 2\mathcal{O}_2)$ |
| $\tilde{\mathcal{O}}_6$ | $\frac{1}{4m^2}(\mathcal{O}_1 + 2\mathcal{O}_2)$ |
| $\tilde{\mathcal{O}}_7$ | $-\frac{1}{4m^2}(\mathcal{O}_9 + 2\mathcal{O}_{12})$ |
| $\tilde{\mathcal{O}}_8$ | $\frac{1}{2m^2}(\mathcal{O}_9 + 2\mathcal{O}_{12})$ |
| $\tilde{\mathcal{O}}_9$ | $-\frac{1}{4m^2}(-\mathcal{O}_1 - 2\mathcal{O}_2 - 4\mathcal{O}_5 + 4\mathcal{O}_6 - 2\mathcal{O}_7 - 2\mathcal{O}_8 + 4\mathcal{O}_9 - 4\mathcal{O}_{13} + 4\mathcal{O}_{14})$ |
| $\tilde{\mathcal{O}}_{10}$ | $\frac{1}{4m^2}(2\mathcal{O}_7 + 2\mathcal{O}_8 + 4\mathcal{O}_{13})$ |
| $\tilde{\mathcal{O}}_{11}$ | $-\frac{1}{4m^2}(\mathcal{O}_7 + 2\mathcal{O}_{10})$ |
| $\tilde{\mathcal{O}}_{12}$ | $\frac{1}{4m^2}(\mathcal{O}_1 + 2\mathcal{O}_2 - 2\mathcal{O}_4)$ |
| $\tilde{\mathcal{O}}_{13}$ | $\frac{1}{4m^2}(\mathcal{O}_1 + 2\mathcal{O}_2 - 2\mathcal{O}_4 - \mathcal{O}_7 + \mathcal{O}_9 - 2\mathcal{O}_{10} + 2\mathcal{O}_{12})$ |

# Non-relativistic reduction

Non-relativistic Lagrangians $O(p^2)$

*Phys. Rev. C 53, 2086 (1996).*

| | |
|---|---|
| $O_S$ | $(N^\dagger N)(N^\dagger N)$ |
| $O_T$ | $(N^\dagger \boldsymbol{\sigma} N)\cdot(N^\dagger \boldsymbol{\sigma} N)$ |
| $O_1$ | $(N^\dagger \overrightarrow{\boldsymbol{\nabla}} N)^2 + \text{h.c.}$ |
| $O_2$ | $(N^\dagger \overrightarrow{\boldsymbol{\nabla}} N)\cdot(N^\dagger \overleftarrow{\boldsymbol{\nabla}} N)$ |
| $O_3$ | $(N^\dagger N)(N^\dagger \overrightarrow{\boldsymbol{\nabla}}^2 N) + \text{h.c.}$ |
| $O_4$ | $i\,(N^\dagger \overrightarrow{\boldsymbol{\nabla}} N)\cdot(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \times \boldsymbol{\sigma} N) + \text{h.c.}$ |
| $O_5$ | $i\,(N^\dagger N)(N^\dagger \overleftarrow{\boldsymbol{\nabla}}\cdot\boldsymbol{\sigma}\times\overrightarrow{\boldsymbol{\nabla}} N)$ |
| $O_6$ | $i\,(N^\dagger \boldsymbol{\sigma} N)\cdot(N^\dagger \overleftarrow{\boldsymbol{\nabla}}\times\overrightarrow{\boldsymbol{\nabla}} N)$ |
| $O_7$ | $(N^\dagger \boldsymbol{\sigma}\cdot\overrightarrow{\boldsymbol{\nabla}} N)(N^\dagger \boldsymbol{\sigma}\cdot\overrightarrow{\boldsymbol{\nabla}} N) + \text{h.c.}$ |
| $O_8$ | $(N^\dagger \sigma^j \overrightarrow{\nabla}^k N)(N^\dagger \sigma^k \overrightarrow{\nabla}^j N) + \text{h.c.}$ |
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| $O_{12}$ | $(N^\dagger \sigma^j \overrightarrow{\nabla}^k N)(N^\dagger \overleftarrow{\nabla}^k \sigma^j N)$ |
| $O_{13}$ | $(N^\dagger \overleftarrow{\boldsymbol{\nabla}}\cdot\boldsymbol{\sigma}\,\overrightarrow{\nabla}^j N)(N^\dagger \sigma^j N) + \text{h.c.}$ |
| $O_{14}$ | $2\,(N^\dagger \overleftarrow{\boldsymbol{\nabla}}\sigma^j\cdot\overrightarrow{\boldsymbol{\nabla}} N)(N^\dagger \sigma^j N)$ |

Non-relativistic expnasion $O(p^2)$

| | |
|---|---|
| $\tilde{\mathcal{O}}_1$ | $\mathcal{O}_S + \frac{1}{4m^2}(\mathcal{O}_1 + 2\mathcal{O}_2 + 2\mathcal{O}_3 + 2\mathcal{O}_5)$ |
| $\tilde{\mathcal{O}}_2$ | $\mathcal{O}_S + \frac{1}{4m^2}(-4\mathcal{O}_2 - 2\mathcal{O}_5 + 4\mathcal{O}_6 + \mathcal{O}_7 - \mathcal{O}_9 + 2\mathcal{O}_{10} - 2\mathcal{O}_{12})$ |
| $\tilde{\mathcal{O}}_3$ | $-\mathcal{O}_T - \frac{1}{4m^2}(-2\mathcal{O}_6 + \mathcal{O}_7 - \mathcal{O}_9 - 2\mathcal{O}_{10} - 2\mathcal{O}_{12} + 2\mathcal{O}_{13} - 2\mathcal{O}_{14})$ |
| $\tilde{\mathcal{O}}_4$ | $2\mathcal{O}_T + \frac{1}{2m^2}(-\mathcal{O}_1 - 2\mathcal{O}_2 - 4\mathcal{O}_5 + 2\mathcal{O}_6 - \mathcal{O}_8 + 2\mathcal{O}_{11} - 4\mathcal{O}_{12} - 2\mathcal{O}_{13})$ |
| $\tilde{\mathcal{O}}_5$ | $\frac{1}{4m^2}(\mathcal{O}_1 + 2\mathcal{O}_2)$ |
| $\tilde{\mathcal{O}}_6$ | $\frac{1}{4m^2}(\mathcal{O}_1 + 2\mathcal{O}_2)$ |
| $\tilde{\mathcal{O}}_7$ | $-\frac{1}{4m^2}(\mathcal{O}_9 + 2\mathcal{O}_{12})$ |
| $\tilde{\mathcal{O}}_8$ | $\frac{1}{2m^2}(\mathcal{O}_9 + 2\mathcal{O}_{12})$ |
| $\tilde{\mathcal{O}}_9$ | $-\frac{1}{4m^2}(-\mathcal{O}_1 - 2\mathcal{O}_2 - 4\mathcal{O}_5 + 4\mathcal{O}_6 - 2\mathcal{O}_7 - 2\mathcal{O}_8 + 4\mathcal{O}_9 - 4\mathcal{O}_{13} + 4\mathcal{O}_{14})$ |
| $\tilde{\mathcal{O}}_{10}$ | $\frac{1}{4m^2}(2\mathcal{O}_7 + 2\mathcal{O}_8 + 4\mathcal{O}_{13})$ |
| $\tilde{\mathcal{O}}_{11}$ | $-\frac{1}{4m^2}(\mathcal{O}_7 + 2\mathcal{O}_{10})$ |
| $\tilde{\mathcal{O}}_{12}$ | $\frac{1}{4m^2}(\mathcal{O}_1 + 2\mathcal{O}_2 - 2\mathcal{O}_4)$ |
| $\tilde{\mathcal{O}}_{13}$ | $\frac{1}{4m^2}(\mathcal{O}_1 + 2\mathcal{O}_2 - 2\mathcal{O}_4 - \mathcal{O}_7 + \mathcal{O}_9 - 2\mathcal{O}_{10} + 2\mathcal{O}_{12})$ |

**Reproduce the 2+7 non-relativistic structure**

# Contents

- Why relativistic nucleon-nucleon Lagrangian
- Generalities & strategy
- Results
- Non-relativistic reduction
- **Summary and outlook**

# □Summary

- We construct the covariant nucleon-nucleon contact Lagrangian up to **next-to-leding order** $O(q^2)$ in our power counting rule.
- There are in total **13** terms up to **next-to-leading order**.

# □Outlook

- Construct the covariant nucleon-nucleon contact Lagrangian up to **next-to-next-to-leading order** $O(q^4)$ and do the **non-relativistic reduction** to check our **power counting rules**.
- Extend our study to **baryon-baryon** sector.


Thank you!