A new method to study number of colors in final state interactions

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Based on: PLB783(2018)294; arxiv:1808.05057[hep-ph];

手征2018，长春

Outlines

1. Introduction
2. Scattering amplitudes
3. Poles in large Nc
4. Semi-local duality
5. Summary

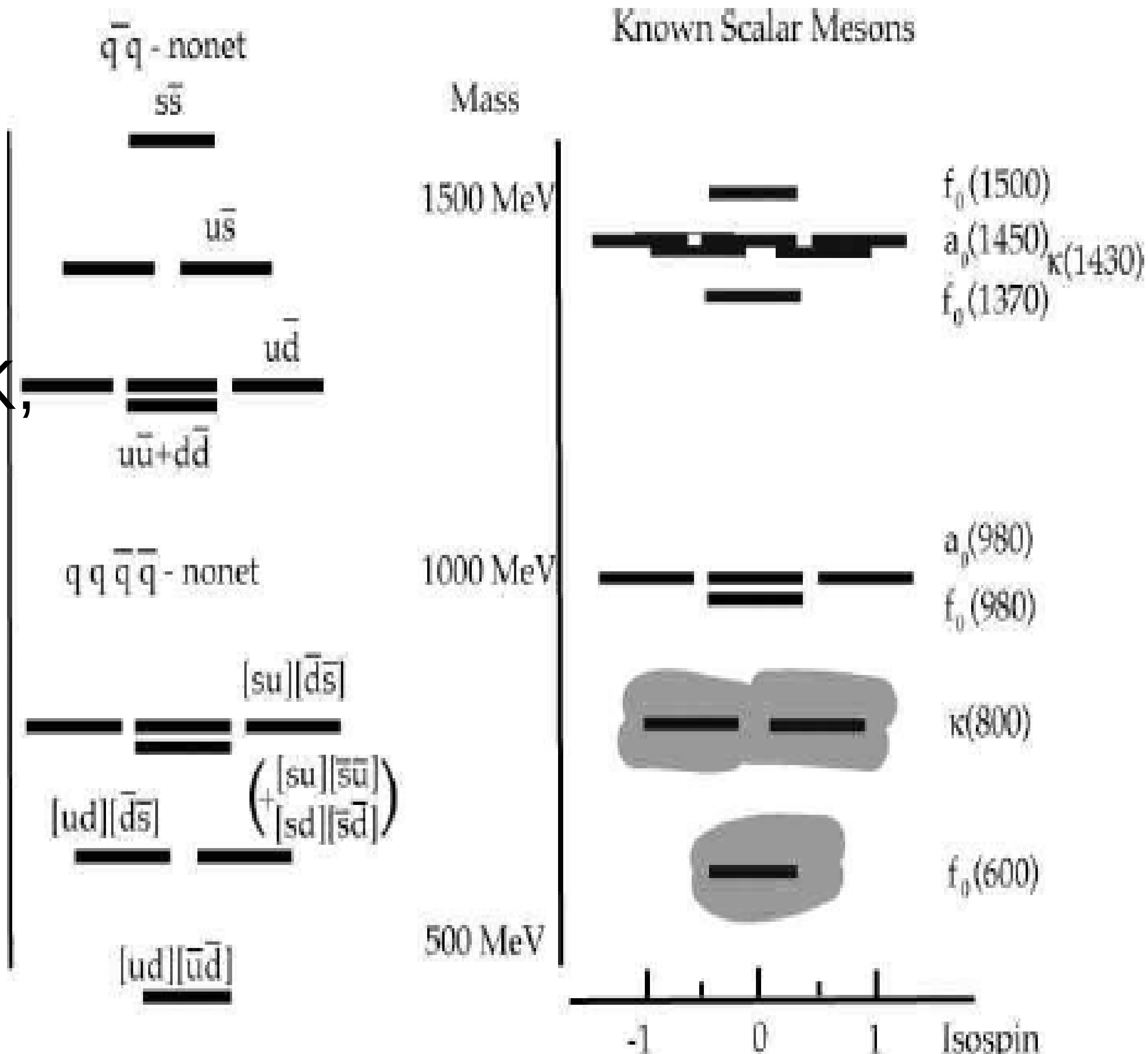
1. Introduction

- The resonances appear in processes including final states, e.g. ππ, KK, are rather interesting.

Jaffe
Phys. Rept. 409 (2005) 1

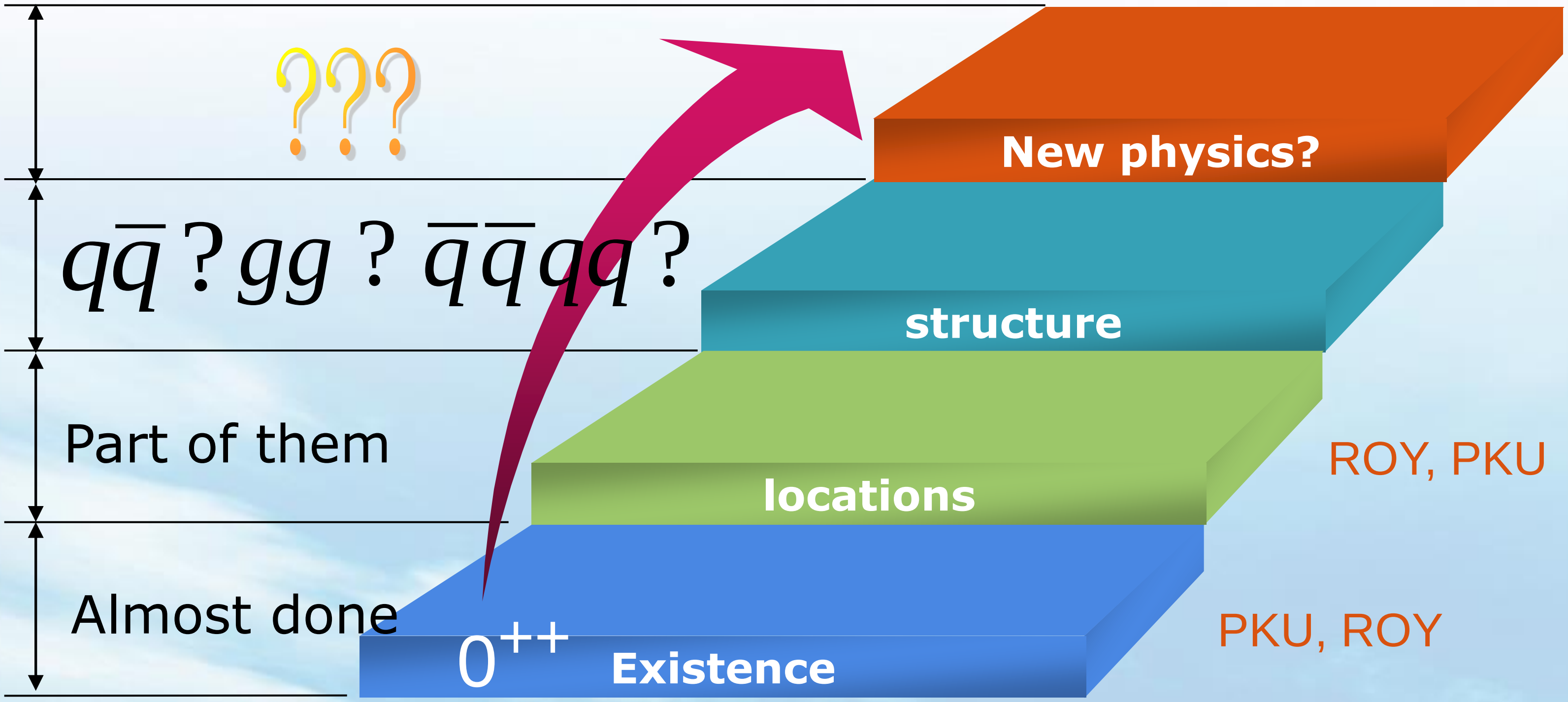
$\Lambda_{QCD} \sim 1$ GeV

χPT ≤ 0.5GeV



Scalars

- What is scalar? The same quantum number with QCD vacuum.



Structure of scalars

Mao, *et al*., PRD79 (2009) 116008;

Garcia-Martin, Moussaullam, EPJC70 (2010) 55

Dai, Pennington, PLB736(2014)11;
PRD90 (2014) 036004;

- Two photon couplings have clean background and can be well determined. However, they are related to the $\pi\pi$, KK system rather than the inner core.
- dispersive approach does not have inner information, pole counting helps to distinguish molecular.

composition	prediction (keV)	author(s)
$(\bar{u}u + \bar{d}d)/\sqrt{2}$	4.0	Babcock & Rosner [65]
	$< 1^{\dagger}$	Giacosa *et al.* [66]
$\bar{s}s$	0.2	Barnes [67]
	0.062	Giacosa *et al.* [68]
$\overline{[ns]}[ns]$	0.27	Achasov *et al.* [69]
$\overline{K}K$	0.6	Barnes [70]
	0.22	Hanhart *et al.* [72]
gg	0.2–0.6	Narison [73]

Theories fairly help

Large Nc

- 1/Nc could be an explicit model to study the inner structure.
- UχPT serves as a guide to the property of resonances. Unitarity is restored and Nc is introduced by χPT.

Sun *et al.* MPLA22 (2006) 711, σ?

Pelaez *et al*. PRL97 (2006) 242002

Pelaez *et al*. PRD84 (2011) 096006, semi-local duality?

Dai,Wang and Zheng CTP57 (2012) 841, CTP58 (2012) 410

Breit-Wigner origin?

Guo&Oller, PRD84 (2011) 034005

- Crossing, spurious poles and cuts, Nc introduce and sensitive LECs?
- Dispersive approach+Nc?

Strategy

amplitudes → **Nc** → **Poles duality**

Use dispersive approach to get ππ - KK coupled channel scattering amplitudes

Matching with χPT to introduce Nc dependence

Nc trajectories of poles, test semi-local duality in large Nc limit

2. Scattering amplitudes

- We calculate the amplitudes by a polynomial times Omnes function of the phase.

$$T_J^I(s) = P_J^I(s)\Omega_J^I(s)$$

- The Adler zero and threshold factors are implemented, the first two terms are dtermined by the scattering lengths and slope parameters.

$$P_J^I(s) = (s - z_J^I)^{n_J} \sum_{k=1}^{n} \alpha_{J\,k}^{I}(s - 4M_\pi^2)^{k-1}$$

- We fit to the experiment data, Roy-like equations, as well as χPT amplitudes in the low energy region.

phases up to $2GeV^2$

- $\pi\pi$ - KK **scattering inputs**
  - Data on Phase shifts and inelasticities of $\pi\pi$ - KK coupled channel scattering.
  - Dispersion analysis based on symmetry and fit to data:
    Descotes *et.al* EPJC33 (2004) 409
    Pelaez *et al*. PRD83 (2011) 074004
    - ❿T-matrix of ππ scattering by CFDIV .
    - ❿$\pi\pi \to KK$ amplitudes given by Roy-Steiner Equation
  - BABAR's Dalitz plot analysis of $D_s^+ \to (\pi^+\pi^-)\pi^+$ and $D_s^+ \to (K^+K^-)\pi^+$ process. BES's $J/\Psi \to \phi\pi^+\pi^-$, $J/\Psi \to \phi K^+K^-$.

phases up to $4GeV^2$

- We make a bit trick to make the amplitudes to be limited in infinite energy region:

$$\Phi^I_J(s) = \frac{T^I_J(s)Q_0[x^I_J(s)]}{\Omega^I_J(s)Q_{nJ}[x^I_J(s)]}$$

- Where Q is the second hand Legendre function. Φ^I_J has l.h.c and r.h.c above $2GeV^2$, we write:

Dispersion relations

- Now what we need is the l.h.c (r.h.c) of Φ^I_J,

$$\Phi^I_J(s) = \Phi^I_J(4m_\pi^2) + \frac{(s-4m_\pi^2)}{\pi}\int_{-\infty}^{0}\frac{\mathrm{Im}_L\Phi^I_J(s')}{(s'-4m_\pi^2)(s'-s)}ds' + \frac{(s-4m_\pi^2)}{\pi}\int_{s_R}^{\infty}\frac{\mathrm{Im}_R\Phi^I_J(s')}{(s'-4m_\pi^2)(s'-s)}ds'$$

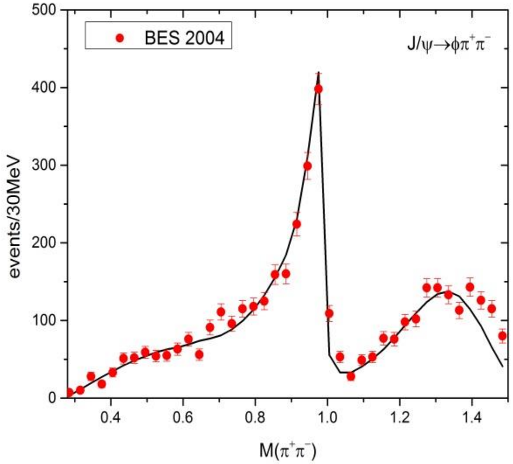
Fixed by scattering lengths

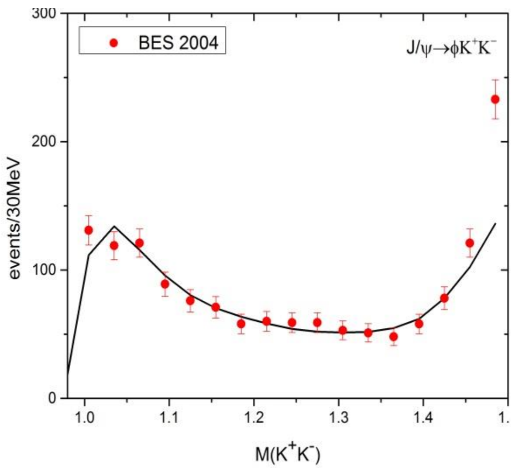
Comformal mapping

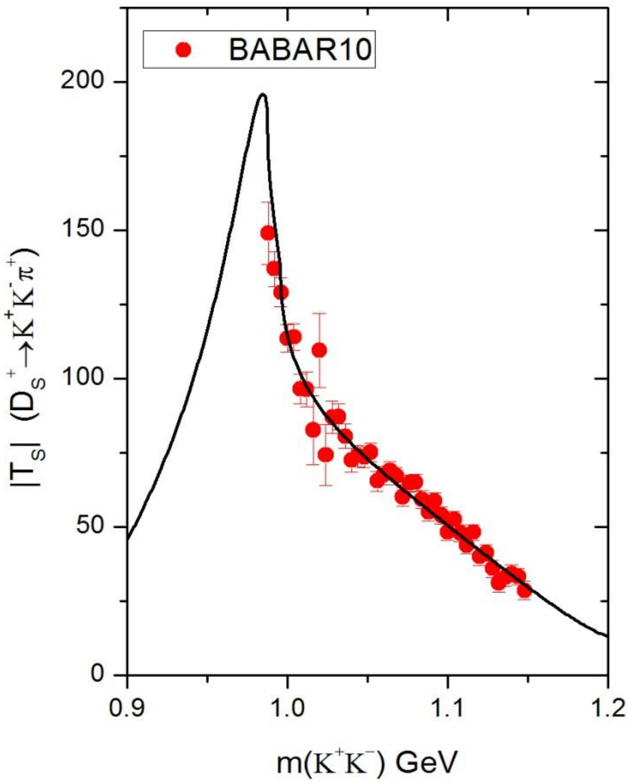
- Notice that the r.h.c is begin from s_R=2GeV^2, due to the unknown phases.

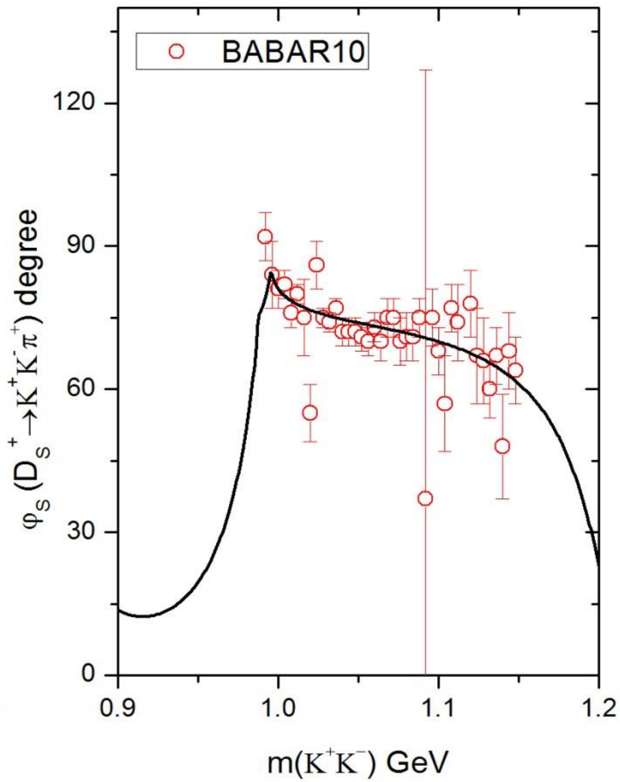
BABAR && BES

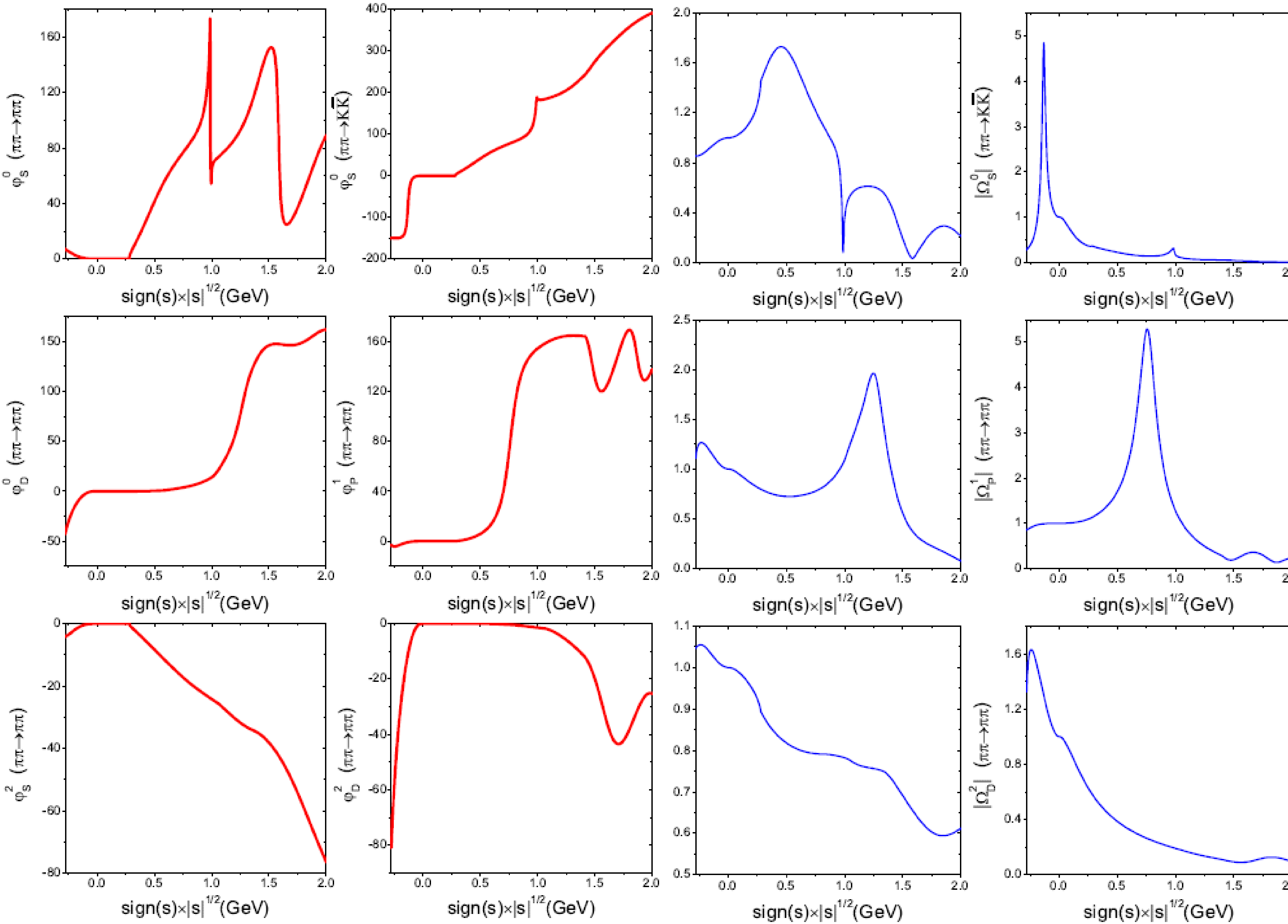
- **$\pi\pi$ - KK scattering inputs**
  - KK threshold region is important as it is around $f_0(980)$.



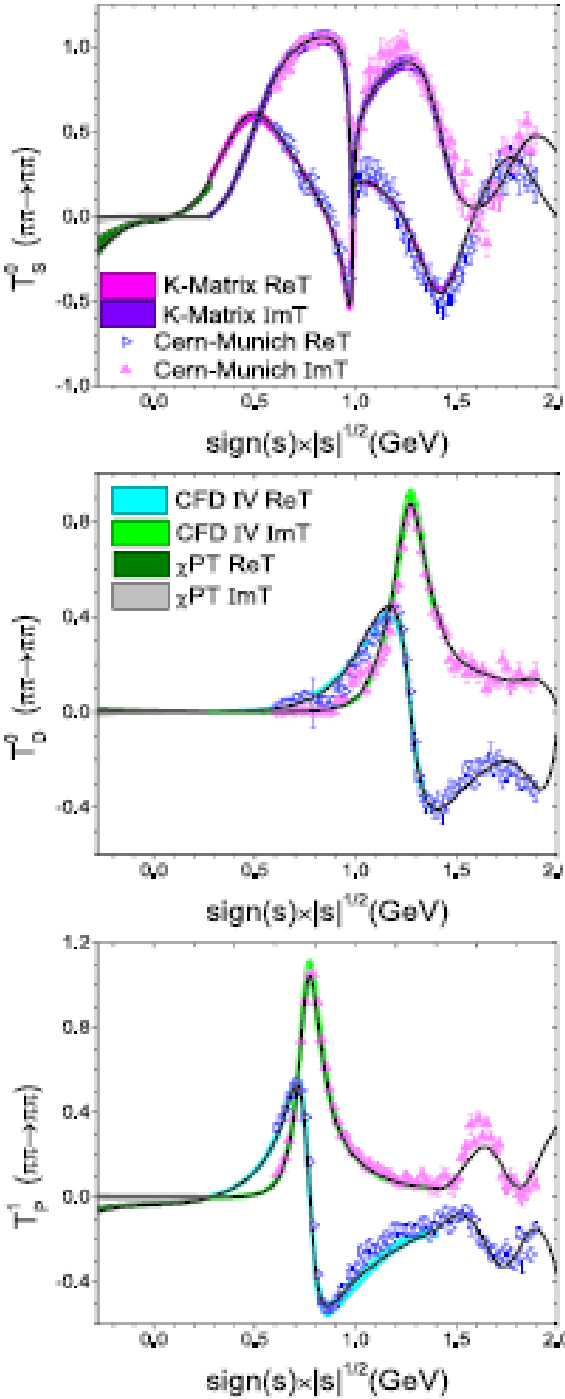


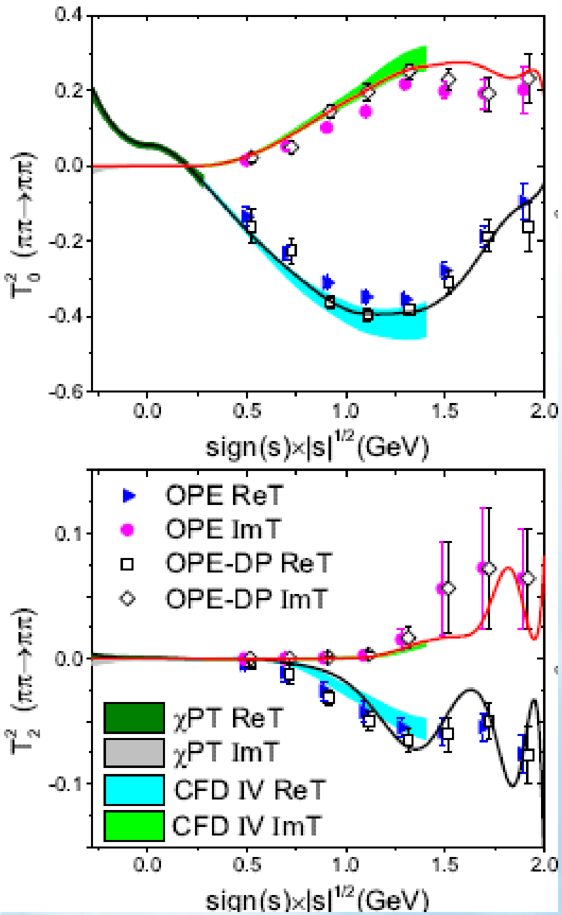




φ_S^0 ($\pi\pi\to\pi\pi$)
φ_S^0 ($\pi\pi\to K\bar{K}$)
$|\Omega_S^0|$ ($\pi\pi\to K\bar{K}$)
φ_D^0 ($\pi\pi\to\pi\pi$)
φ_P^1 ($\pi\pi\to\pi\pi$)
$|\Omega_P^1|$ ($\pi\pi\to\pi\pi$)
φ_S^2 ($\pi\pi\to\pi\pi$)
φ_D^2 ($\pi\pi\to\pi\pi$)
$|\Omega_D^2|$ ($\pi\pi\to\pi\pi$)
sign(s)×|s|^1/2(GeV)

our amplitudes

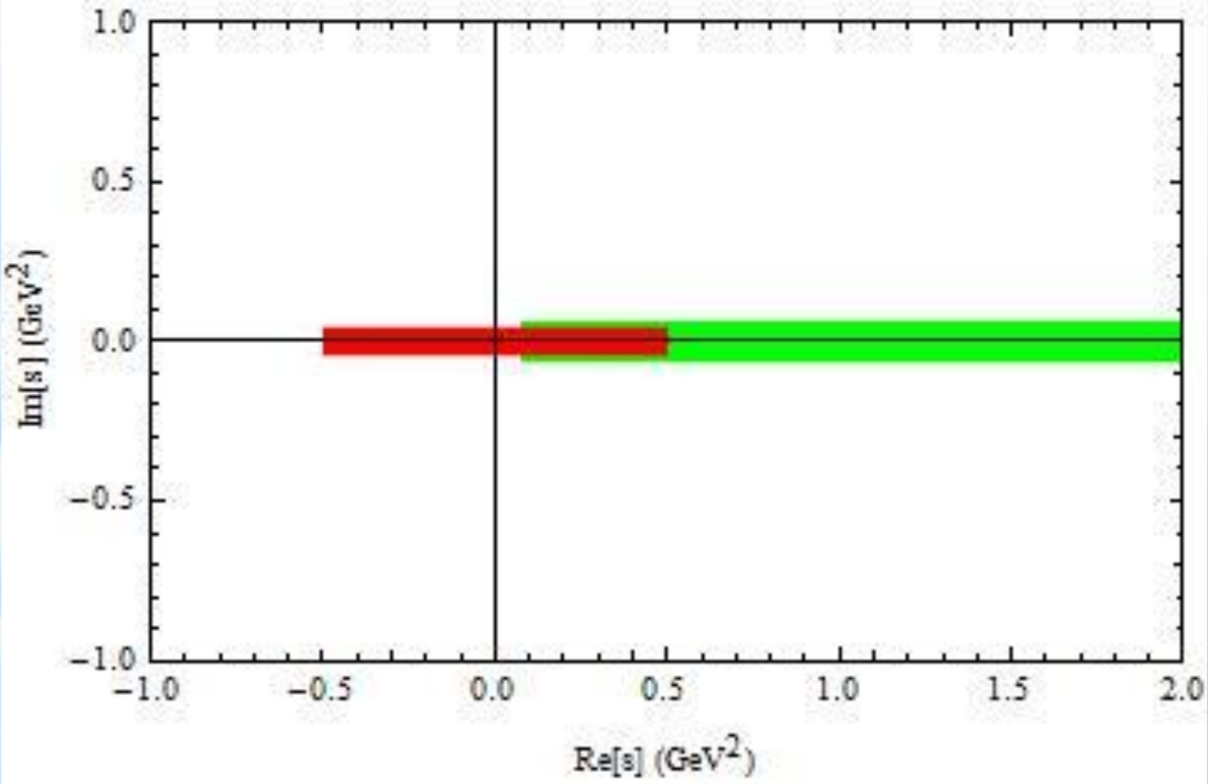




- **ππ** scattering amplitudes on the real axis.
- The fit is of high quality. Even in the prediction region $s \in [-4M_\pi^2, 0]$
- We also fit to the Roy-like equations' on the complex s-plane.

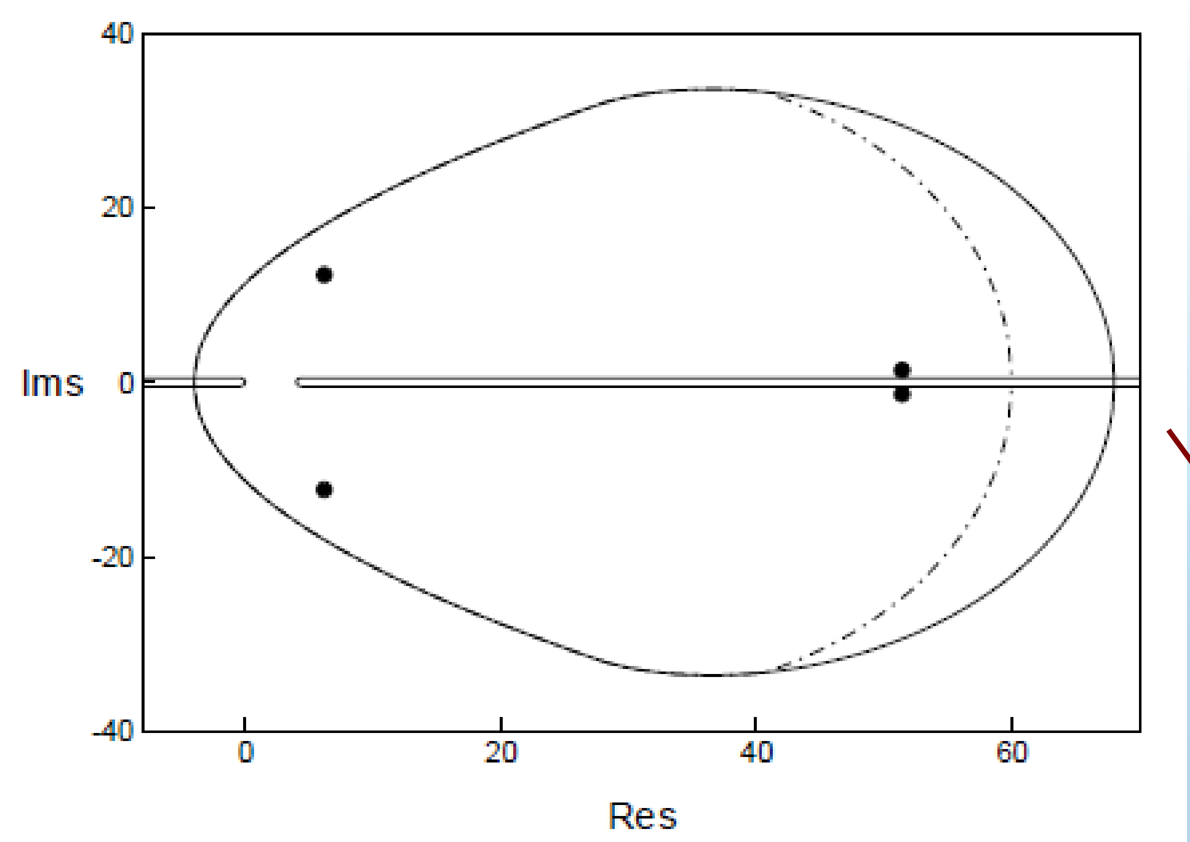
where K-Matrix, ChPT, etc. works

- K-Matrix only works at $[4m_\pi^2, 2]\mathrm{GeV}^2$ and a bit far away from the real axis.
- ChPT will be work in $[-0.5, 0.5]\mathrm{GeV}^2$ and a bit deeper in the complex plane.

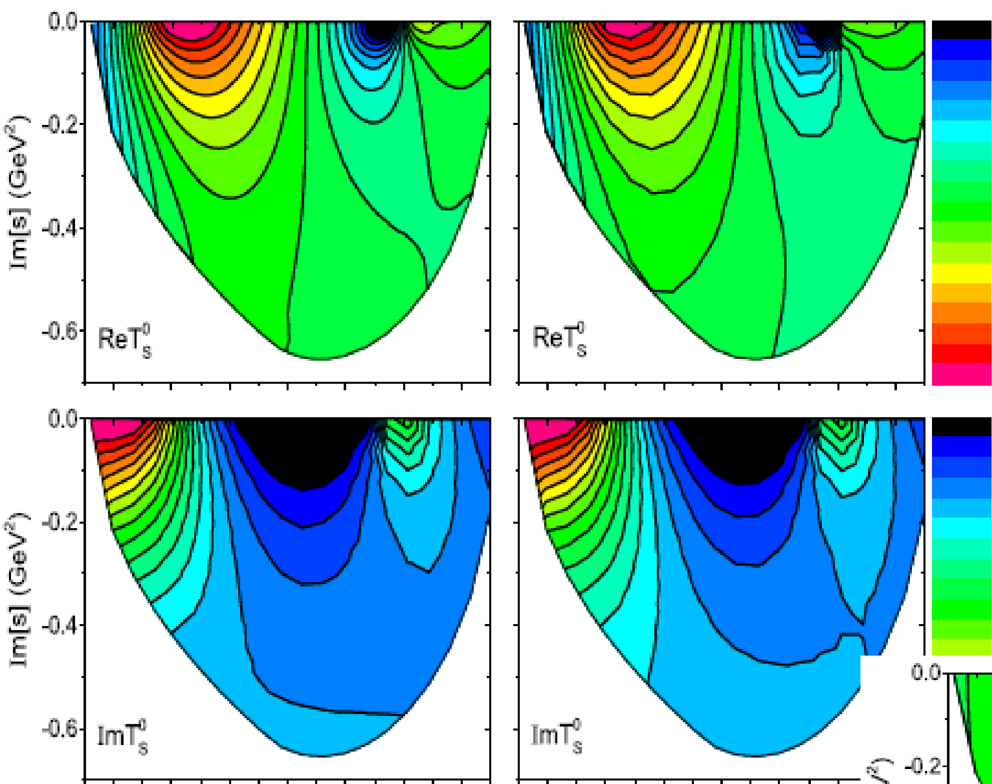


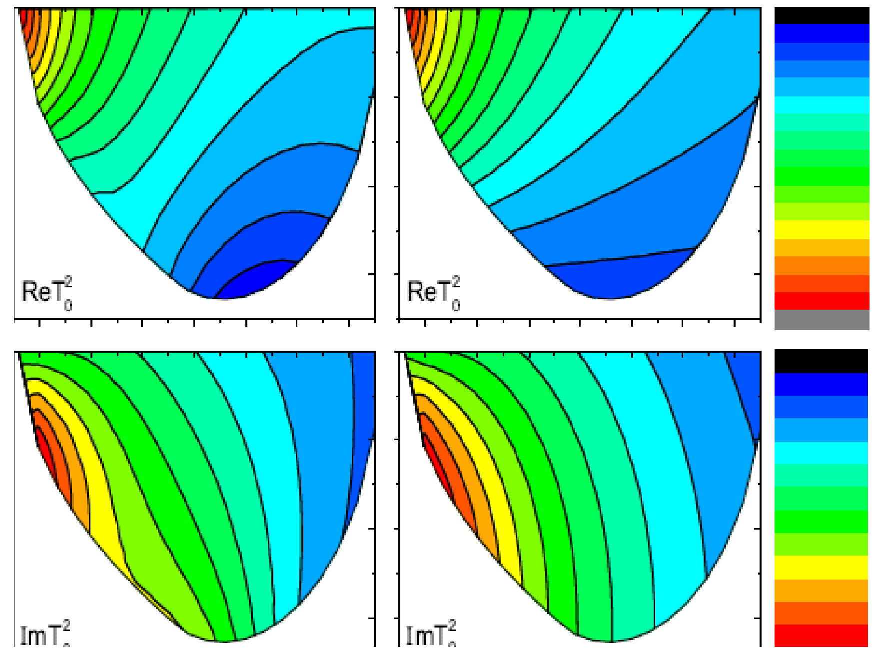
Roy Equation domain

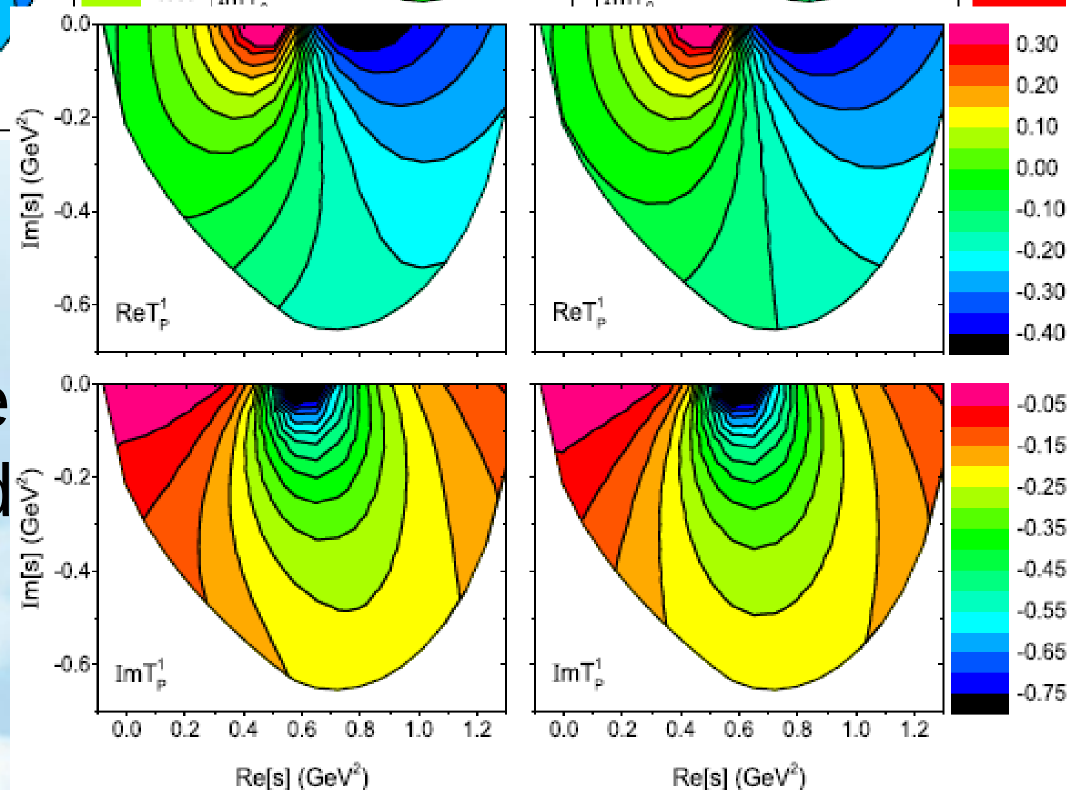
- Given elasticity, they are less conclusive above KKbar threshold. The unit is m_π^2,



I. Caprini, G. Colangelo and H. Leutwyler, PRL96 (2006) 132001







- This makes sure the polynomials absorb the contribution of l.h.c and distant r.h.c properly.

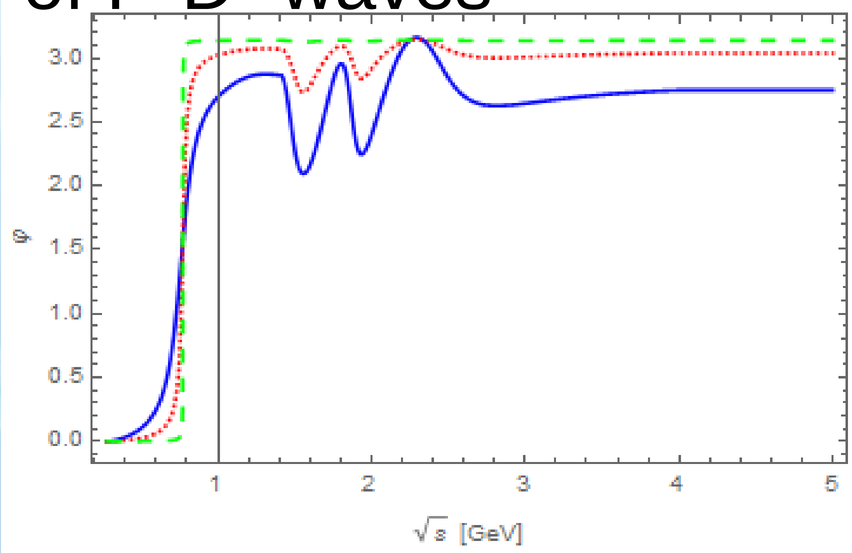
Nc dependence

- The real part of χPT amplitudes is O(Nc^{-1}) and that of imaginary part is O(Nc^{-2}). We thus define

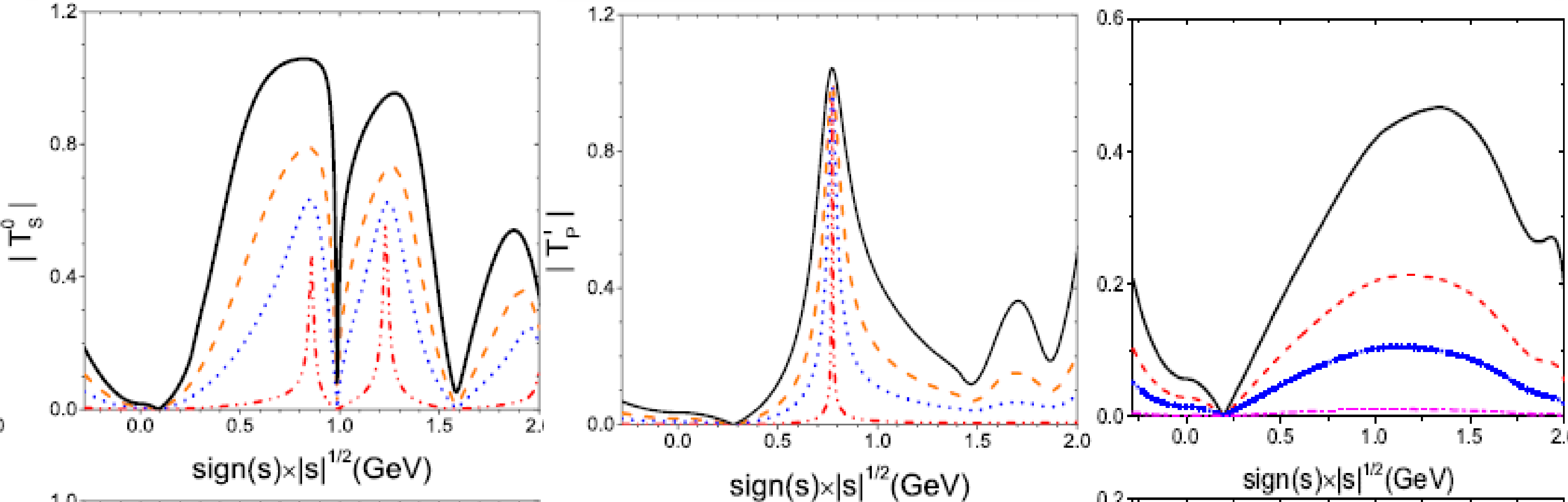
$$\varphi(s, N_C) = \arctan\left[\frac{3}{N_C}\tan\varphi(s)\right]$$

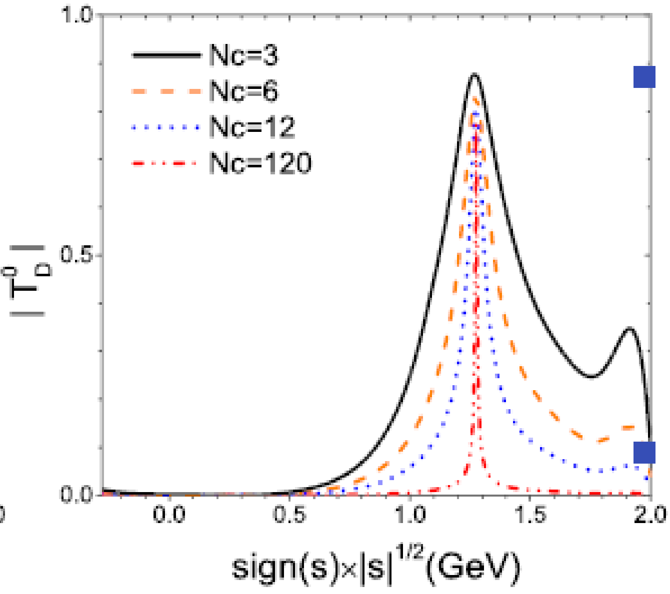
$$P_J^I(s, N_C) = \frac{3}{N_C}P_J^I(s)$$

- This also ensures the phase of P-D- waves behaves as that of BW formalism in the large Nc limit.

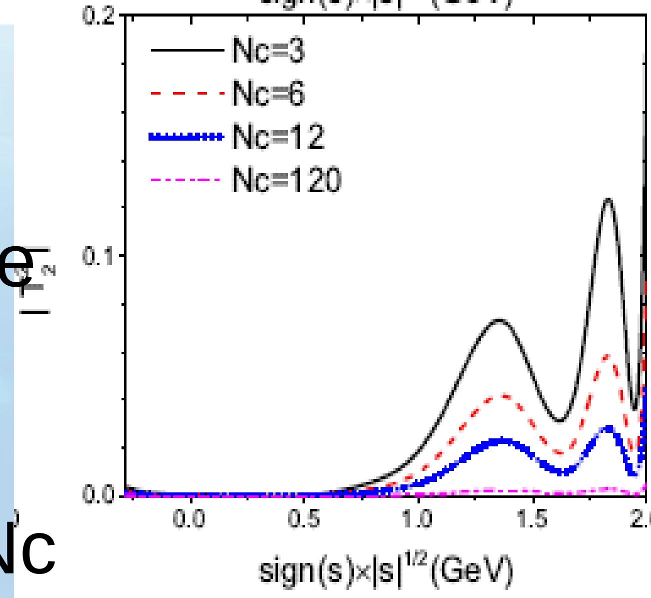


T amplitudes by varying Nc



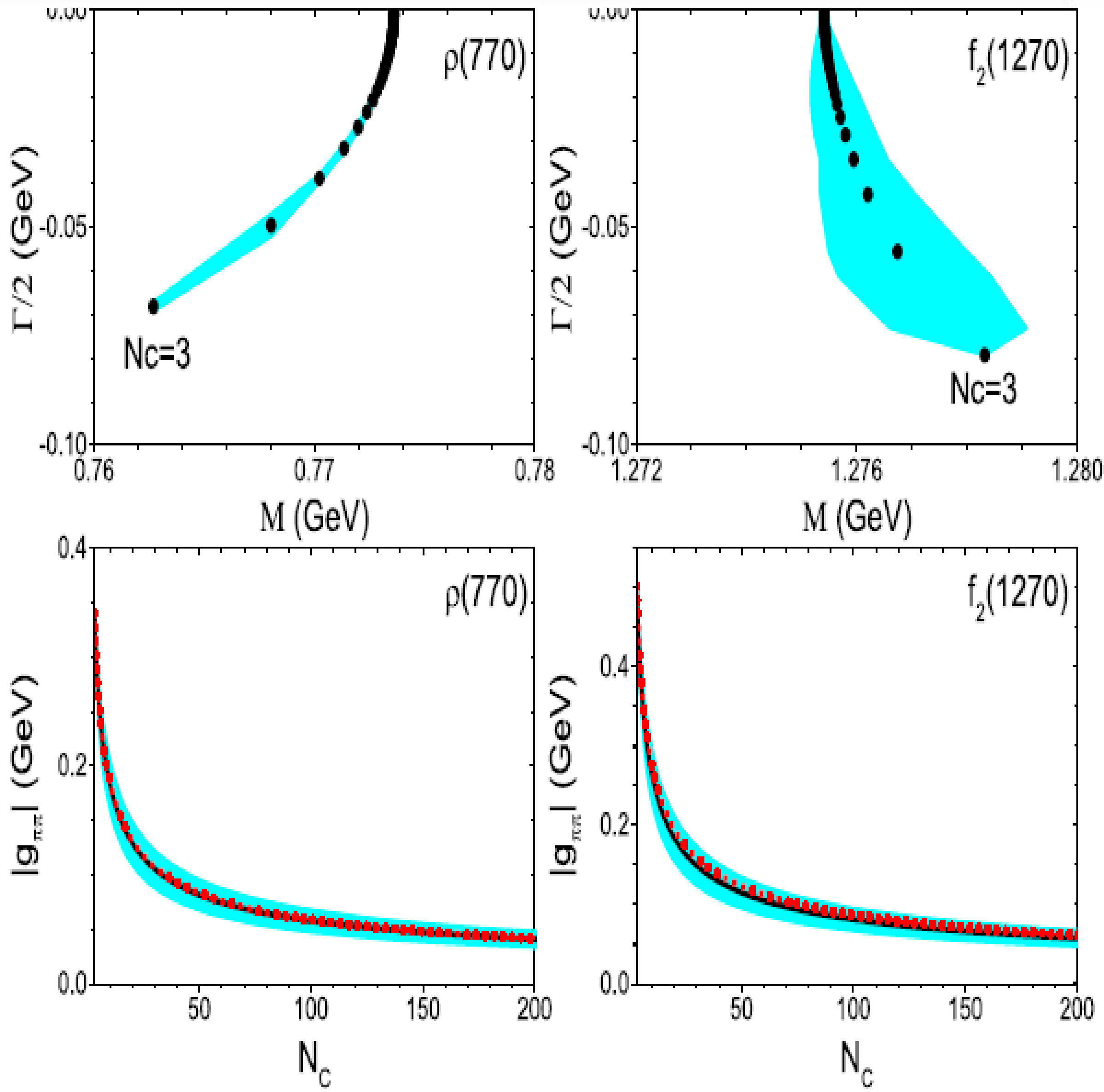


- Peaks of P-, D-waves are narrower, behave as BW type.
- I=2 amplitudes vanish at large Nc



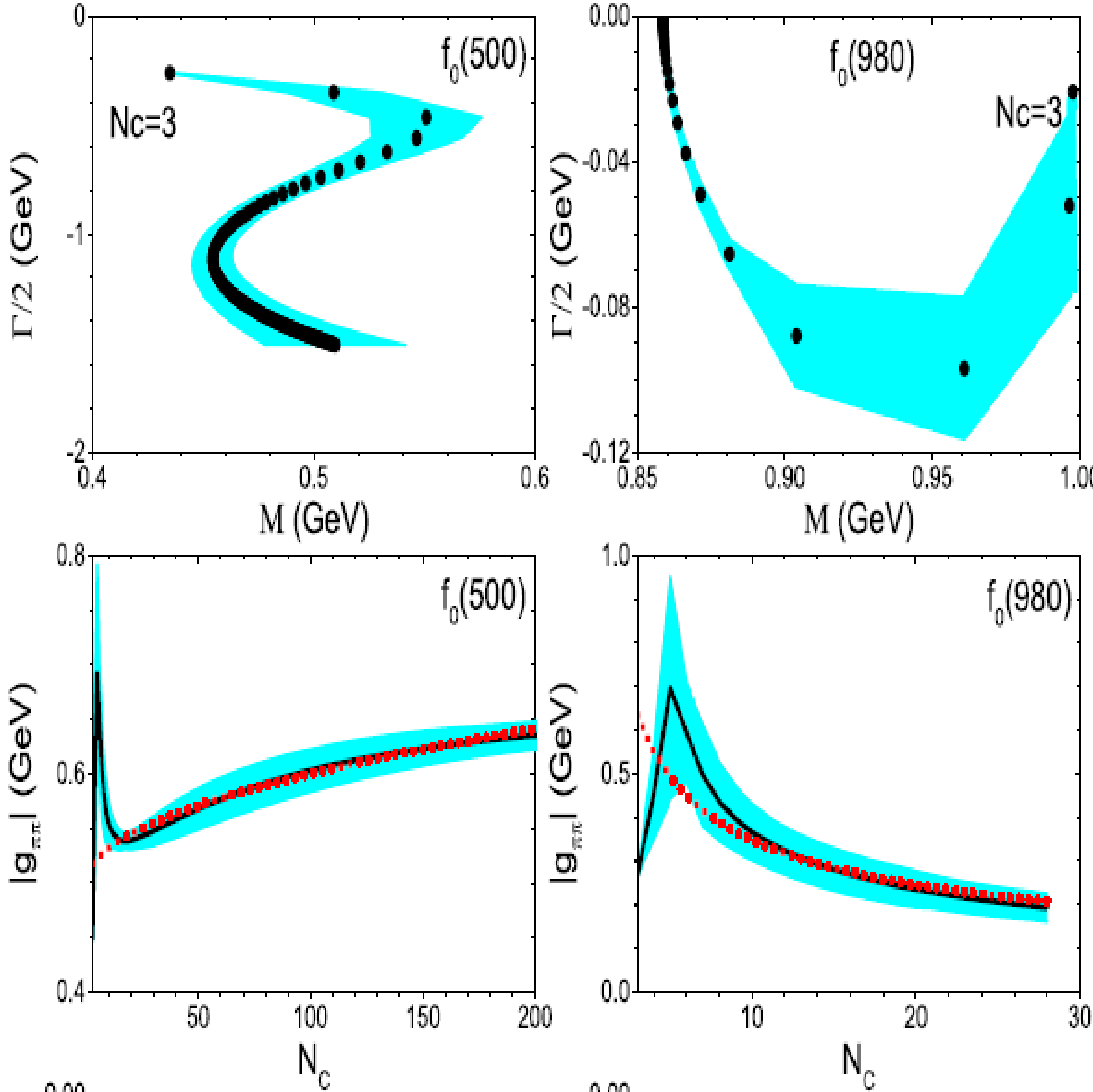
Trajectories of poles and couplings

- ρ, f_2 behave as BW resonances. The masses are of O(1) and widths of O(Nc^{-1}). The couplings to ππ is O($Nc^{-1/2}$).
- This confirms the introduce of Nc dependence is properly.



Light scalars

- The mass of б is of O(1) and the coupling to ππ has $O(Nc^{1/2})$.
- The coupling of $f_0(980)$ contains $O(Nc^{-1/2})$.
- They all have a peak around Nc=5. Implying mixing structure.



Semi-local duality

- local duality: the Regge exchange in the crossed channel is dual to the contribution of direct channel resonances.
- In the real world only semi-local duality is satisfied: It is hold on average through FESR.

$$\int_{\nu_1}^{\nu_2} d\nu F_n^{I_t}(\nu,t)_{\rm resonances} \simeq \int_{\nu_1}^{\nu_2} d\nu F_n^{I_t}(\nu,t)_{\rm Regge}$$

- In UχPT it is tested at large Nc. The semi-local duality is violated at Nc=15-30.

Pelaez *et al*. PRD84 (2011) 096006, б has qq component

Guo *et al*. PRD86 (2012) 054006, $f_0(980)$, $f_2(1270)$, ρ(1450), …

- Then is semi-local duality satisfied in our method?

Nc=3

- With the ratio

$$R_n^I(t) = \frac{\int_{\nu_1}^{\nu_2} d\nu F_n^{I_t}(\nu, t)}{\int_{\nu_1}^{\nu_3} d\nu F_n^{I_t}(\nu, t)}.$$

- One can check the semi-local duality at Nc=3.
- With uncertainty they are compatible with each other.

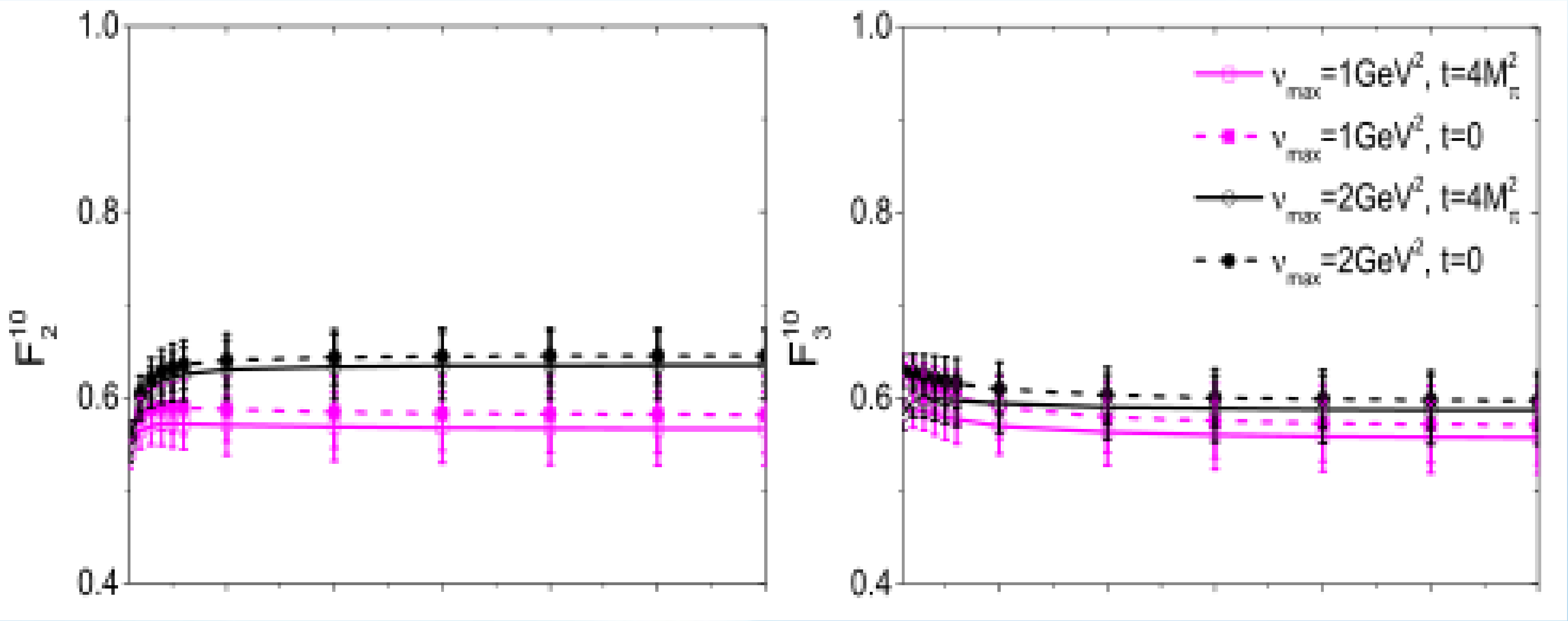
	n	$I_t = 0$		$I_t = 1$	
		$t = 4M_\pi^2$	$t = 0$	$t = 4M_\pi^2$	$t = 0$
S,P,D	0	0.431(116)	0.430(122)	0.381(162)	0.396(183)
	1	0.656(85)	0.668(85)	0.619(131)	0.649(139)
	2	0.842(40)	0.865(34)	0.829(73)	0.866(69)
	3	0.948(12)	0.968(8)	0.948(32)	0.973(26)
Regge	0	0.225	0.233	0.325	0.353
	1	0.425	0.445	0.578	0.642
	2	0.705	0.765	0.839	0.908
	3	0.916	0.958	0.966	0.990

- When n is small, higher partial waves can not be ignored.
- When n is large, the contribution of resonances are suppressed.
- We focus on n=1-3

Nc up to 180

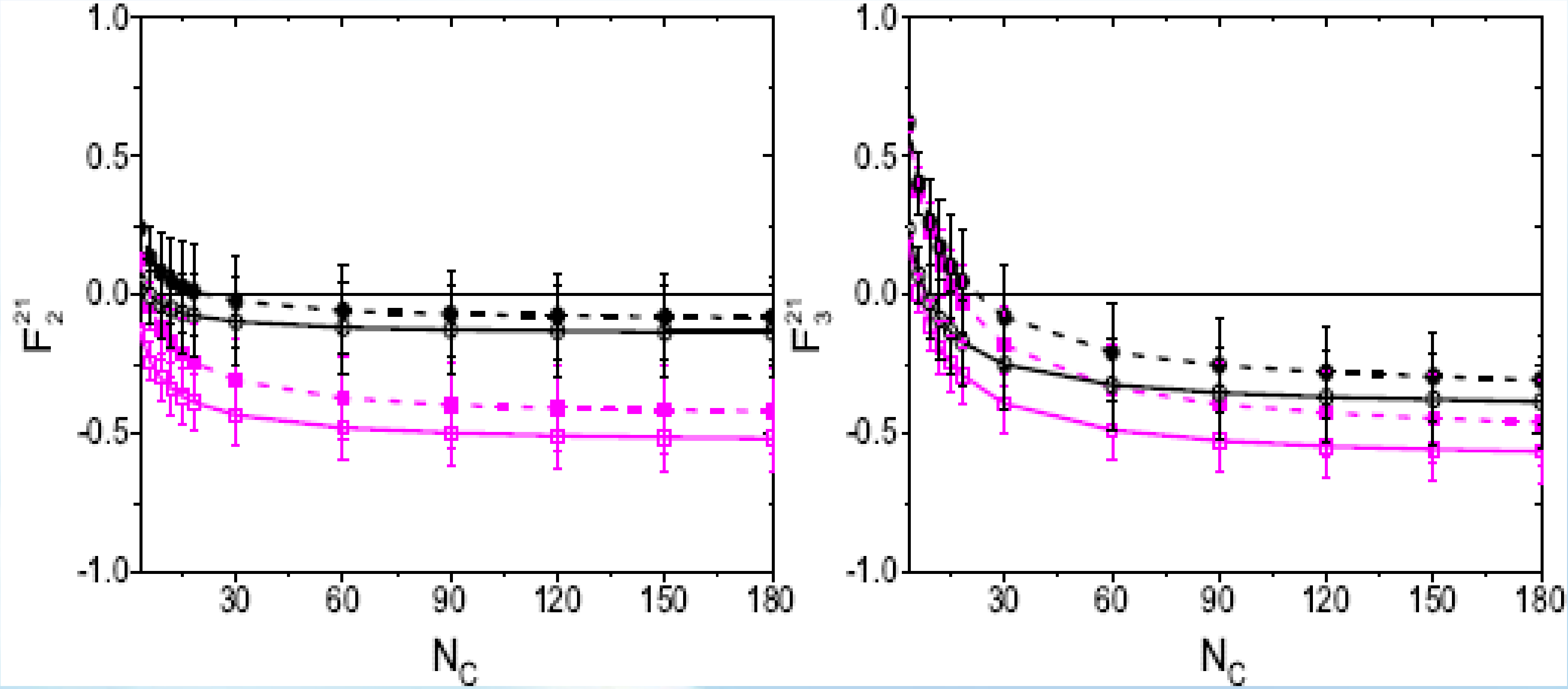
- Semi-local duality implies that F^{10}_n=2/3 and F^{21}_n=0, where

$$F_n^{II'}(t) = \frac{\int_{\nu_{min}}^{\nu_{max}} d\nu F_n^{I_t}(\nu,t)}{\int_{\nu_{min}}^{\nu_{max}} d\nu F_n^{I'_t}(\nu,t)}$$



Large Nc

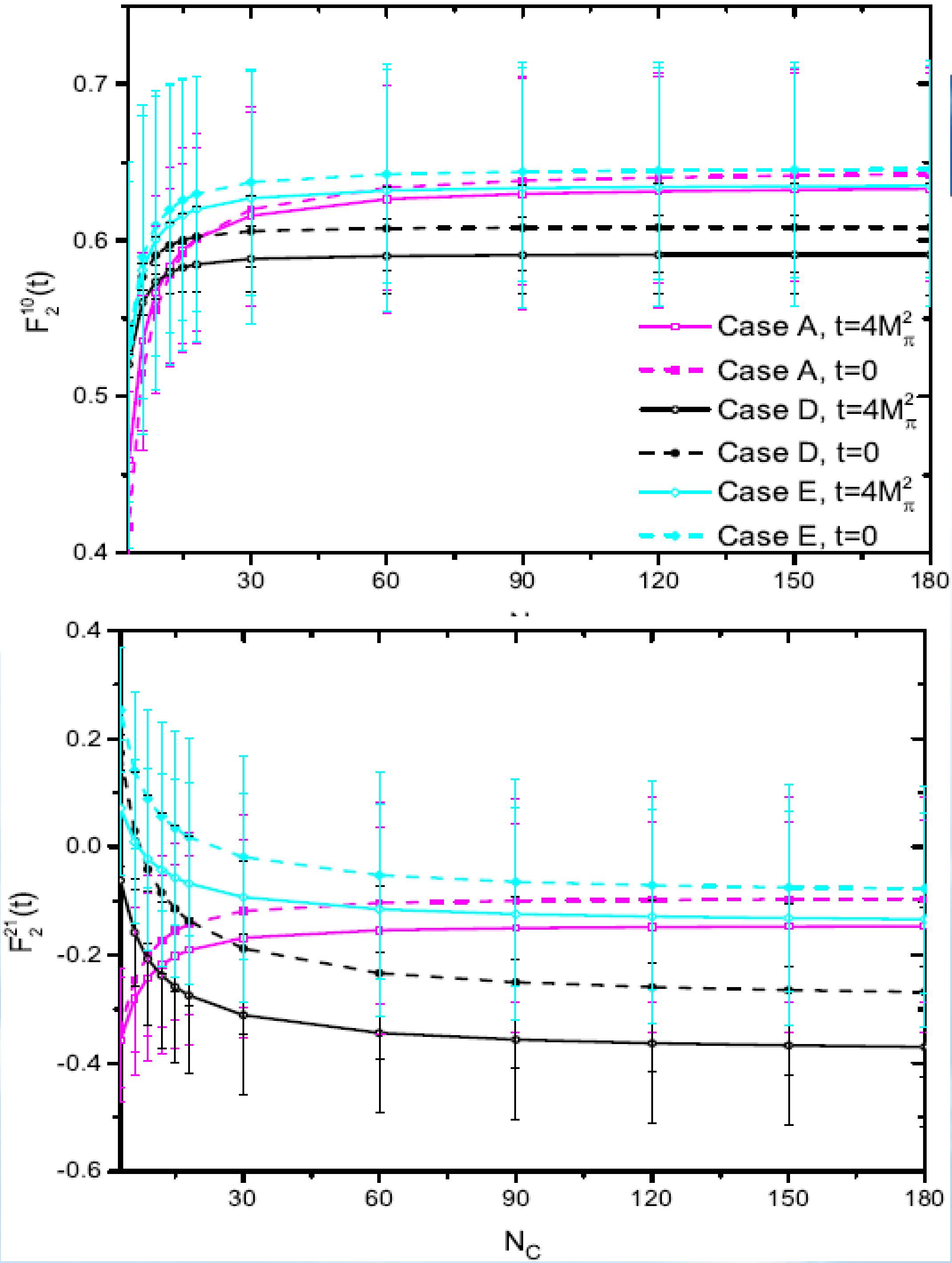
- When Nc is large, it is rather flat and one would expect it lasts until infinity.



Different cases

Case	$F_2^{10}(t)$		$F_2^{21}(t)$	
	$t = 4M_\pi^2$	$t = 0$	$t = 4M_\pi^2$	$t = 0$
O	0.64(5)	0.65(4)	−0.14(16)	−0.06(15)
A	0.63(7)	0.64(7)	−0.15(20)	−0.10(19)
B	0.59(8)	0.59(7)	−0.36(20)	−0.34(19)
C	0.62(8)	0.63(7)	−0.23(20)	−0.19(19)
D	0.59(3)	0.61(3)	−0.37(15)	−0.27(16)
E	0.63(8)	0.65(7)	−0.13(20)	−0.08(19)

- O: original one
- A: no б
- B: no $f_0(980)$
- C: no $f_0(1370)$
- D: no $f_2(1270)$
- E: include [2,4GeV^2]



- б contributes a lot at Nc=3 but not at large Nc.
- Heavier resonances such as ρ(1450), ρ(1700) have small contribution.

6. Summary

duality

Semi-local duality is fulfilled at large Nc. $f_0(980)$, $f_0(1370)$, $f_2(1270)$ have important contribution.

scalars

б contains molecular or tetraquark, $f_0(980)$ should have sizable ss component, they are mixing states.

Nc in FSI

The Nc dependence proposed is a reliable way.

prospects

This method could be helpful for future study include FSI of hadrons.

Thank You For your patience !