Effect of Z_b states and bottom meson loops in $\Upsilon(3S, 4S)$ dipion transitions

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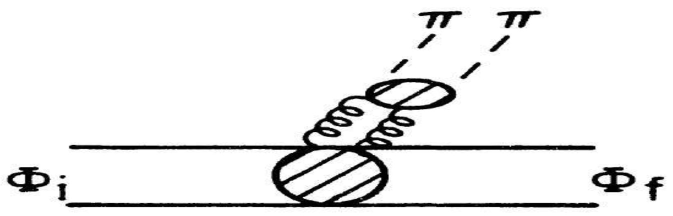
Outline

- $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$
  - Motivation
  - Theoretical framework
    - Heavy meson chiral effective theory
    - Dispersion theory, modified Omn$\acute{e}$s solutions
  - Numerical fits

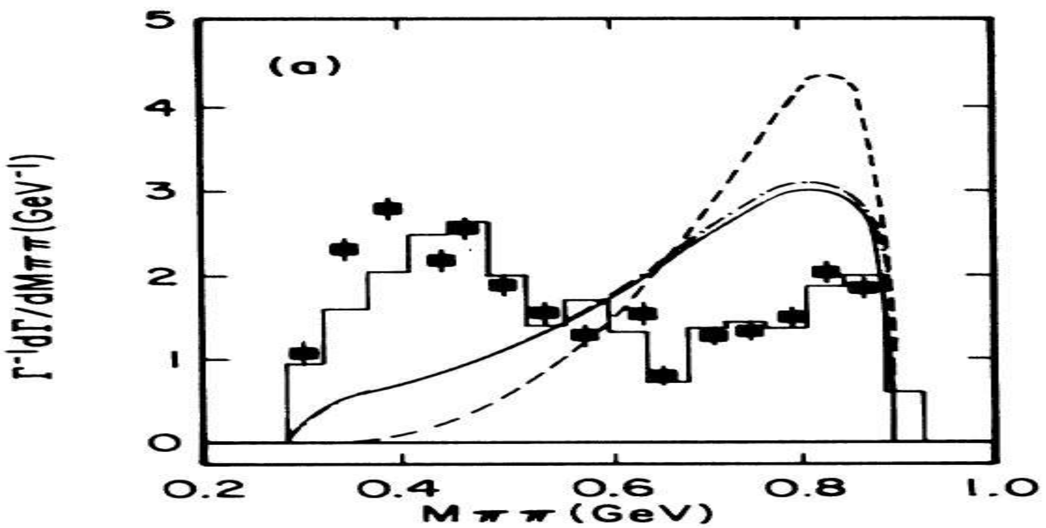
- $\Upsilon(4S) \to \Upsilon(1S, 2S)\pi\pi$
  - Theoretical framework
    - Nonrelativistic effective field theory
    - Two-channel Omn$\acute{e}$s solutions
  - Phenomenology discussion

- Conclusions

- Typical theoretical method to study $\Upsilon(nS) \to \Upsilon(mS)\pi\pi$:
  QCD multipole expansion + soft-pion theorems



- A well-known anomaly for the $\pi\pi$ mass spectra of $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$
  - Experimental data: two-hump behavior
  - Theoretical prediction : a single peak



Various mechanisms to explain this discrepancy

- coupled-channel effects with open-bottom intermediate states
  [H. J. Lipkin and S. F. Tuan, PRL'1988]

- a hypothetical resonance which couples to $\Upsilon\pi$: $b\bar{b}q\bar{q}$, $I^G(J^P) = 1^+(1^+)$
  [V. V. Anisovich, D. V. Bugg, A. V. Sarantsev and B. S. Zou, PRD'1995]

- the $\pi\pi$ resonance [the $f_0(500)$ or σ meson] or strong $\pi\pi$ final-state interaction
  [G. Bélanger, T. A. DeGrand and P. Moxhay, PRD'1989]

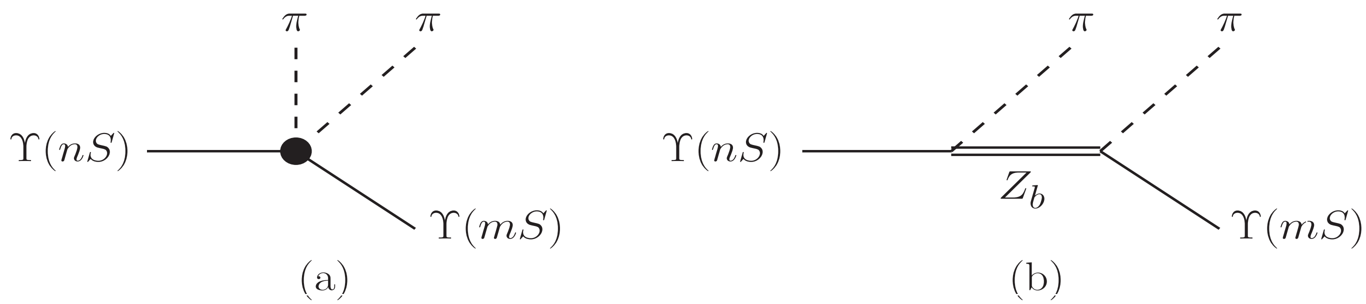
- relativistic corrections
  [M. B. Voloshin, PRD'2006]

In 2011, $Z_b(10610)^\pm$ and $Z_b(10650)^\pm$ were observed in the decay processes $\Upsilon(5S) \to \Upsilon(nS)\pi^+\pi^-$ ($n = 1, 2, 3$) and $\Upsilon(5S) \to h_b(mP)\pi^+\pi^-$ ($m = 1, 2$) by the Belle Collaboration. Their quantum numbers are indeed $I^G(J^P) = 1^+(1^+)$.

Our strategy:

- dispersion theory, based on unitarity, analyticity and crossing symmetry $\Rightarrow$ account for the $\pi\pi$ rescattering in a model-independent way
- consider the effects of the measured two Z_bs, which provide a left-hand-cut contribution
- the subtraction constants are obtained by matching the dispersive amplitudes to heavy meson chiral effective theory

- tree-level calculation



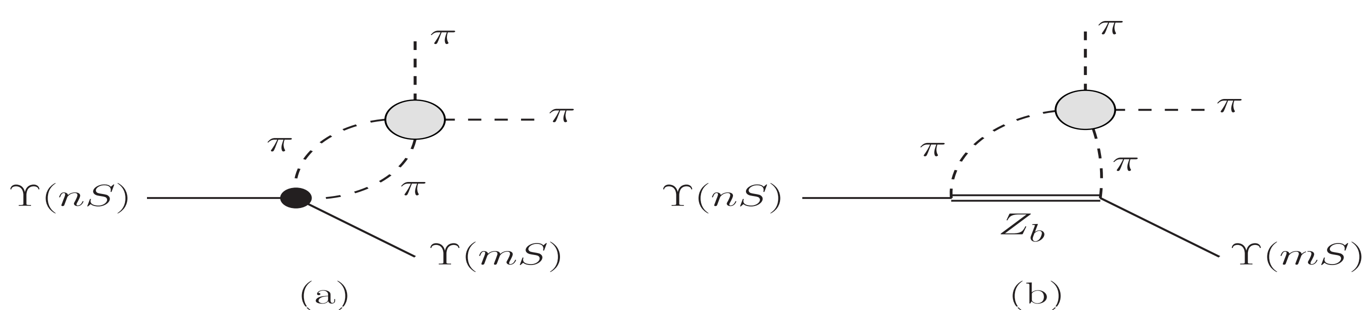
in the heavy-quark limit, the heavy quarks' spin decouples: $J \equiv \Upsilon \cdot \sigma + \eta_b$

$$\mathscr{L}_{\Upsilon\Upsilon'\pi\pi} = \frac{c_1}{2}\langle J^\dagger J'\rangle\langle u_\mu u^\mu\rangle + \frac{c_2}{2}\langle J^\dagger J'\rangle\langle u_\mu u_\nu\rangle v^\mu v^\nu + \text{h.c.} \tag{1}$$

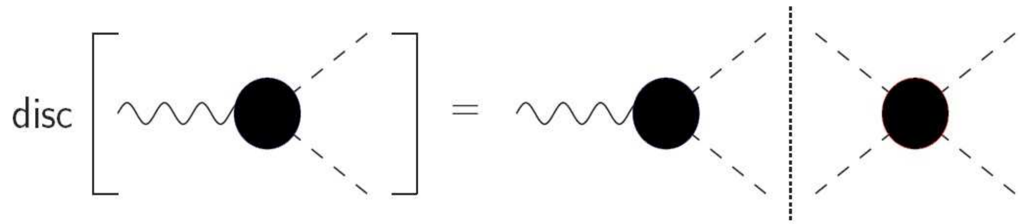
$$\mathscr{L}_{Z_b\Upsilon\pi} = C_Z \Upsilon^i \langle {Z^i}^\dagger u_\mu\rangle v^\mu + \text{h.c.} \tag{2}$$

where $v^\mu = (1, \mathbf{0})$: the velocity of the heavy quark.

- pion-pion final-state interactions



- elastic unitarity



$$\frac{1}{2i}\mathrm{disc}\, F_l(s) = \mathrm{Im}\, F_l(s) = F_l(s) \sin \delta_l^I(s) e^{-i\delta_l^I(s)}\,. \qquad (3)$$

Watson's theorem: phase of $F_l(s)$ is just $\delta_l^I(s)$, the elastic $\pi\pi$ phase shift

- traditional solution to this homogeneous integral equation [Omnès, Nuovo Cim'1958]

$$F_l(s) = P_n(s)\Omega_l^I(s)\,, \qquad \Omega_l^I(s) = \exp\left\{\frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{dx}{x}\frac{\delta_l^I(x)}{x-s}\right\}. \qquad (4)$$

$P_n(s)$: polynomial; $\Omega_l^I(s)$: Omnès function

- inhomogeneous Omnès problem [A. V. Anisovich and H. Leutwyler,, PLB'1996]

$$\operatorname{Im} M_l(s) = \left[M_l(s) + \hat{M}_l(s)\right] \sin\delta_l^I(s) e^{-i\delta_l^I(s)} \,. \tag{5}$$

$$M_l(s) = \Omega_l^I(s)\left\{P_l^{n-1}(s) + \frac{s^n}{\pi}\int_{4m_\pi^2}^{\infty}\frac{dx}{x^n}\frac{\hat{M}_l(x)\sin\delta_l^I(x)}{|\Omega_l^I(x)|(x-s)}\right\}, \tag{6}$$

  - $\hat{M}_l(s)$: left-hand-cut contributions, here approximated by partial-wave projection of Z_b exchange only
  - $P_l^{n-1}(s)$: subtraction polynomial, determined by matching to heavy meson chiral effective theory
  - $\delta_l^I(x)$: $\pi\pi$ phase shift as input, isospin I=0, S-wave and D-wave

- number of subtractions to make the dispersive integrals converge

  - $\Omega_l^I(s) \sim s^{-k}$, if δ_l^I approaches $k\pi$ for large s: $\delta_{0,2}^0(s) \to \pi, \Omega_{0,2}^0(s) \sim 1/s$
  - $\hat{M}_l(s) \sim s$ in the range 1 GeV$^2 \lesssim s \ll m_\Upsilon^2$

  Three subtractions are sufficient.

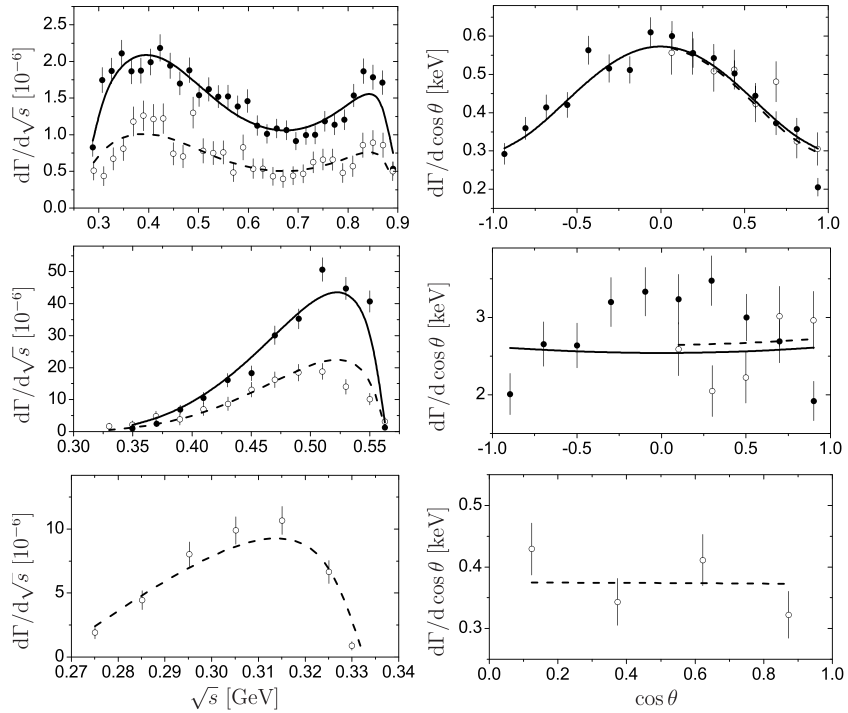
– chiral representations $M_0^\chi(s)$, $M_2^\chi(s) \sim s^2$, can be covered by the degree of the subtractions

- S- and D-wave amplitudes

$$\begin{aligned} M_0(s) = \quad & \Omega_0^0(s)\Bigg\{ -\frac{2}{F_\pi^2}\Big[c_1\Big(s-2m_\pi^2\Big)+\frac{c_2}{2}\Big(s+\mathbf{q}^2\Big(1-\frac{\sigma_\pi^2}{3}\Big)\Big)\Big] \\ & \quad + \sum_{i=1,2} C_{nm,i}\frac{s^3}{\pi}\int_{4m_\pi^2}^\infty \frac{dx}{x^3}\frac{\bar{M}_{0i}(x)\sin\delta_0^0(x)}{|\Omega_0^0(x)|(x-s)}\Bigg\}, \\ M_2(s) = \quad & \Omega_2^0(s)\Bigg\{\frac{2}{3F_\pi^2}c_2\mathbf{q}^2\sigma_\pi^2 + \sum_{i=1,2} C_{nm,i}\frac{s^3}{\pi}\int_{4m_\pi^2}^\infty \frac{dx}{x^3}\frac{\bar{M}_{2i}(x)\sin\delta_2^0(x)}{|\Omega_2^0(x)|(x-s)}\Bigg\}. \end{aligned} \quad (7)$$

- fit with the $Z_{bi}\Upsilon(nS)\pi$ coupling strengths as free parameters

the Z_{b1} and Z_{b2} are strongly correlated in the fit, therefore, we use only one Z_b state.



CLEO [PRD'2007]
from top to bottom:
$\Upsilon(3S) \to \Upsilon(1S)\pi\pi$,
$\Upsilon(2S) \to \Upsilon(1S)\pi\pi$,
$\Upsilon(3S) \to \Upsilon(2S)\pi\pi$.

	$\Upsilon(3S) \to \Upsilon(1S)\pi\pi$	$\Upsilon(2S) \to \Upsilon(1S)\pi\pi$	$\Upsilon(3S) \to \Upsilon(2S)\pi^0\pi^0$
c_1	-0.025 ± 0.001	0.09 ± 0.05	-0.6 ± 0.1
c_2	0.026 ± 0.001	0.04 ± 0.08	0.2 ± 0.3
$C_{nm,1}$	0.145 ± 0.006	1.3 ± 1.4	3.7 ± 2.6
$\frac{\chi^2}{\text{d.o.f}}$	$\frac{108.18}{87-3} = 1.29$	$\frac{101.68}{40-3} = 2.75$	$\frac{12.18}{11-3} = 1.52$

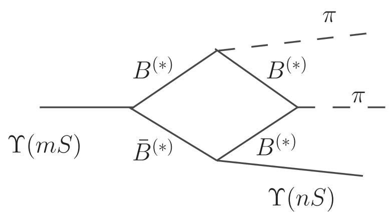
$\Upsilon(4S) \to \Upsilon(1S, 2S)\pi^+\pi^-$

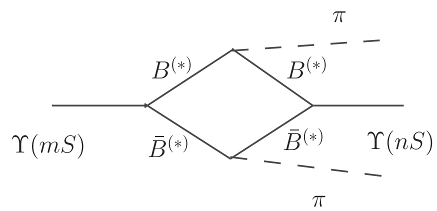
- bottom meson loops may be important

$$m_{\Upsilon(4S)} - 2m_B = 0.021 \text{ GeV}, \qquad \text{branching ratio: } \Upsilon(4S) \to B\bar{B} > 96\%$$

- nonrelativistic effective field theory (NREFT)
  - the velocity of the intermediate bottom meson:
    $$\nu = \sqrt{|m_{\Upsilon(lS)} - m_{B^{(*)}} - m_{B^{(*)}}|/m_{B^{(*)}}}$$
  - the propagator: $[l^2 - m^2_{B^{(*)}}]^{-1} \to [l^0 - \frac{\mathbf{l}^2}{2m_{B^{(*)}}} - m_{B^{(*)}}]^{-1} \sim \nu^{-2}$
  - integral measure: $\int d^4l \sim \nu^5$

- power counting of the loops



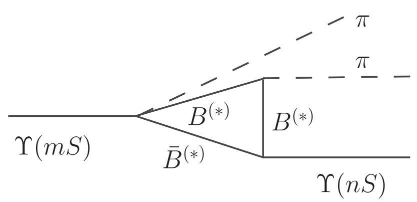


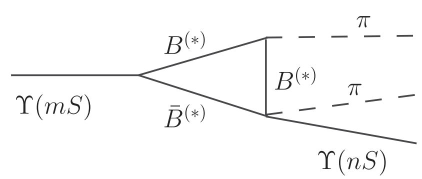
$\Upsilon B^{(*)}\bar{B}^{(*)}$: P wave

$\Upsilon B^{(*)}\bar{B}^{(*)} \times \Upsilon' B^{(*)}\bar{B}^{(*)}$: $\mathcal{O}(\nu^2)$

$B^{(*)}B^{(*)}\pi$: $\langle \bar{H}_a^\dagger \sigma_i \partial^i \Phi_{ab} \bar{H}_b \rangle \sim q_\pi$

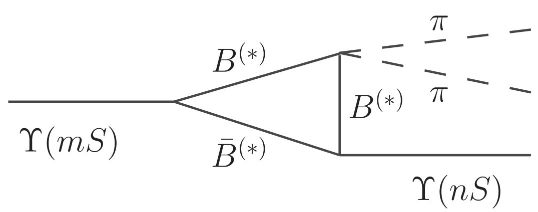
Box: $\nu^5\nu^2 q_\pi^2/\nu^8 = q_\pi^2/\nu$.





$\Upsilon B^{(*)}\bar{B}^{(*)}\pi$: $\langle J \bar{H}_a^\dagger H_b^\dagger \rangle u_{ab}^0 \sim E_\pi \sim q_\pi$

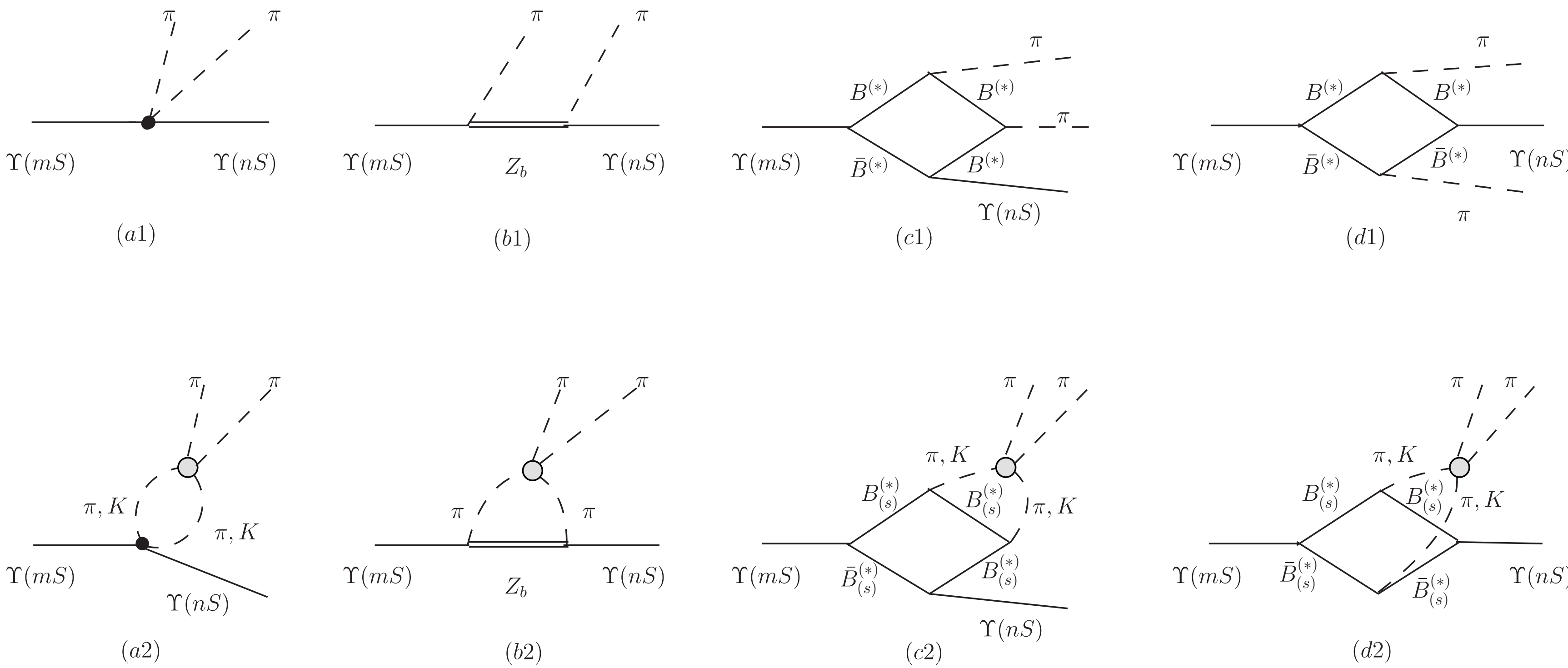
Triangle, I: $\nu^5 q_\pi^3/(\nu^6) = q_\pi^3/\nu$



$B^{(*)}B^{(*)}\pi\pi$: $PC = ++$ pion pairs $\mathcal{O}(q_\pi^2)$

Triangle, II: $\nu^5\nu^2 q_\pi^2/\nu^6 = \nu q_\pi^2$

Box diagrams are dominant among the loops.



- two-channel unitarity conditions

$$\mathrm{Im}\,\mathbf{M}_0(s) = 2iT_0^{0*}(s)\Sigma(s)\left[\mathbf{M}_0(s) + \hat{\mathbf{M}}_0(s)\right], \quad (8)$$

$$\mathbf{M}_0(s) = \begin{pmatrix} M_0^{\pi}(s) \\ \frac{2}{\sqrt{3}}M_0^{K}(s) \end{pmatrix}, \quad \hat{\mathbf{M}}_0(s) = \begin{pmatrix} \hat{M}_0^{\pi}(s) \\ \frac{2}{\sqrt{3}}\hat{M}_0^{K}(s) \end{pmatrix}. \quad (9)$$

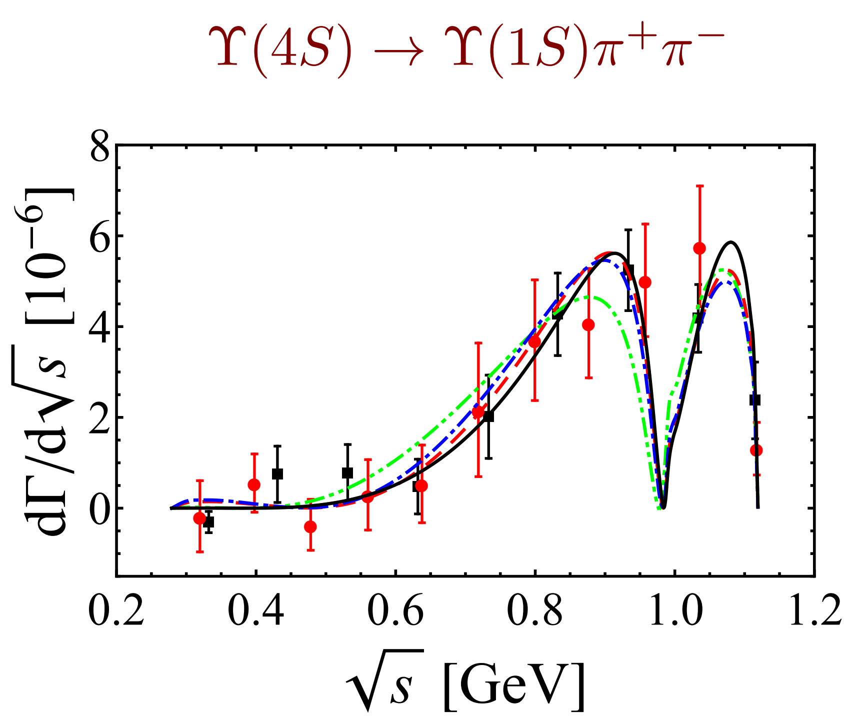
$$T_0^0(s) = \begin{pmatrix} \frac{\eta_0^0(s)e^{2i\delta_0^0(s)}-1}{2i\sigma_\pi(s)} & |g_0^0(s)|e^{i\psi_0^0(s)} \\ |g_0^0(s)|e^{i\psi_0^0(s)} & \frac{\eta_0^0(s)e^{2i\left(\psi_0^0(s)-\delta_0^0(s)\right)}-1}{2i\sigma_K(s)} \end{pmatrix} \quad (10)$$

$$\Sigma(s) = diag\big(\sigma_\pi(s)\theta(s-4m_\pi^2), \sigma_K(s)\theta(s-4m_K^2)\big)\,. \quad (11)$$

- $\delta_0^0(s)$: $\pi\pi$ S-wave isoscalar phase shift
- $|g_0^0(s)|, \psi_0^0(s)$: modulus and phase of $\pi\pi \to K\bar{K}$ S-wave amplitude
- $\eta_0^0(s)$: inelasticity, $= \sqrt{1 - 4\sigma_\pi(s)\sigma_K(s)|g_0^0(s)|^2\theta(s-4m_K^2)}$

$$\mathbf{M}_0(s) = \Omega(s)\left\{\mathbf{M}_0^\chi(s) + \frac{s^3}{\pi}\int_{4m_\pi^2}^\infty \frac{dx}{x^3}\frac{\Omega^{-1}(x)T(x)\Sigma(x)\hat{\mathbf{M}}_0(x)}{x-s}\right\}, \quad (12)$$

$$\mathbf{M}_0^\chi(s) = \left(M_0^{\chi,\pi}(s), 2/\sqrt{3}\, M_0^{\chi,K}(s)\right)^T. \quad (13)$$



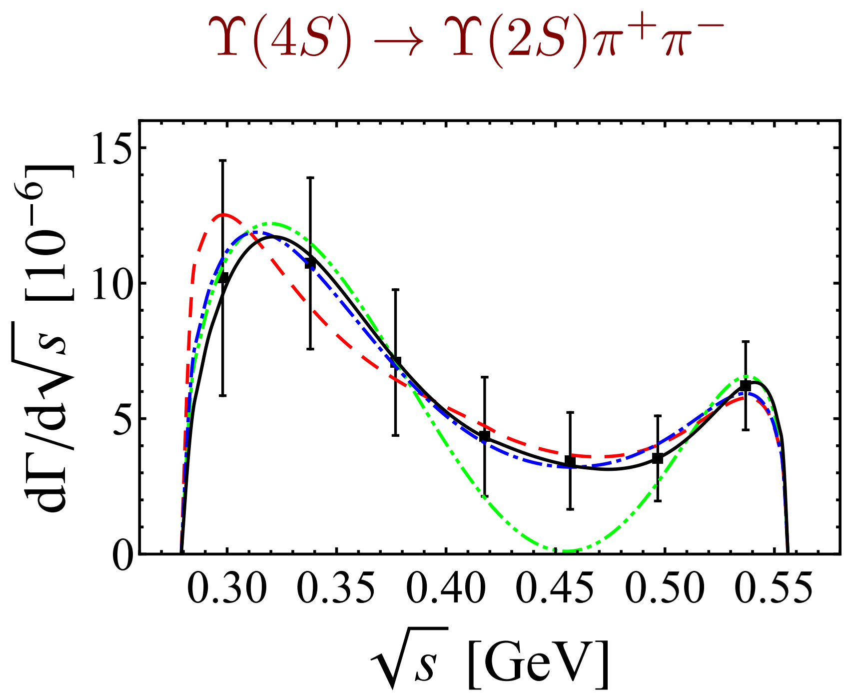
Fit I (green dash-dot-dotted): only contact terms

Fit II (red dashed): contact terms + Z_b-exchange

Fit III (blue dot-dashed): contact terms + box diagra

Fit IV (black solid): contact terms + Z_b-exchange + box diagrams

- $f_0(980)$ causes a dip around 1 GeV [D. Y. Chen, X. Liu and X. Q. Li, EPJC'2011]
- Z_b-exchange and box terms can hardly be distinguished



Fit I (green dash-dot-dotted): only contact terms

Fit II (red dashed): contact terms + Z_b-exchange

Fit III (blue dot-dashed): contact terms + box diagra

Fit IV (black solid): contact terms + Z_b-exchange + box diagrams

- including only contact terms roughly reproduces a two-hump structure, while it produces a 0 inside the physical region

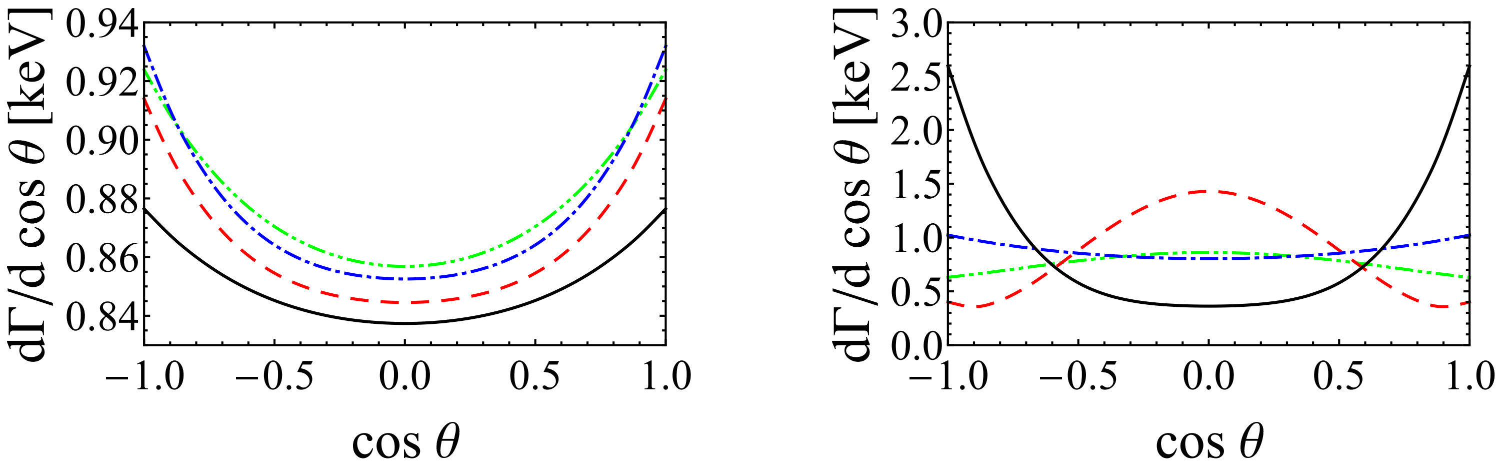


Figure 1: Theoretical predictions of the helicity angular distributions for the decays $\Upsilon(4S) \to \Upsilon(1S)\pi^+\pi^-$ (left) and $\Upsilon(4S) \to \Upsilon(2S)\pi^+\pi^-$ (right).

Angular distribution distinguishes different mechanisms in $\Upsilon(4S) \to \Upsilon(2S)\pi\pi$.

Conclusions

- Dispersion theory can consider strong pion-pion final-state interaction in a model-independent way

- Z_b states can naturally explain the double-peak $\pi\pi$ mass spectrum in $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$ decay

- A dip around 1 GeV caused by the opening of the $K\bar{K}$ in the $\pi\pi$ mass spectrum of $\Upsilon(4S) \to \Upsilon(1S)\pi\pi$ decay

Thanks for your patience