Chiral Lagrangians for mesons with a heavy quark

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Outline

- Background & motivations
- Review of CHPT and heavy-quark meson fields
- Construct the chiral Lagrangians
- Results
- summary

Background & motivation

- HQET + CHPT in low energy

  Heavy quark symmetry + light quark chiral symmetry ⇒ less coupling constants

  The other methods ⇒ calculating coupling constants

- Preparing for the high order calculation

- Studying the new states

  exotic states, hadron spectroscopy

- Extending to double heavy quark states

Building blocks in CHPT

- QCD Lagrangian

$$\mathscr{L} = \mathscr{L}^0_{\rm QCD} + \bar{q}(\not{v} + \not{a}\gamma_5 - s + ip\gamma_5)q$$

$N_f = 2:\ \langle a^\mu \rangle = 0;\quad N_f = 3:\ \langle a^\mu \rangle = \langle v^\mu \rangle = 0$

$\langle \cdots \rangle$ means tracing over the (flavour and/or spinor) space which is not a number

- Building blocks

$$\begin{aligned} u^\mu &= i\{u^\dagger(\partial^\mu - ir^\mu)u - u(\partial^\mu - il^\mu)u^\dagger\}, \\ \chi_\pm &= u^\dagger \chi u^\dagger \pm u\chi^\dagger u, \\ h^{\mu\nu} &= \nabla^\mu u^\nu + \nabla^\nu u^\mu, \\ f_+^{\mu\nu} &= uF_L^{\mu\nu}u^\dagger + u^\dagger F_R^{\mu\nu}u, \\ f_-^{\mu\nu} &= uF_L^{\mu\nu}u^\dagger - u^\dagger F_R^{\mu\nu}u = -\nabla^\mu u^\nu + \nabla^\nu u^\mu, \end{aligned}$$

$r^\mu = v^\mu + a^\mu$, $l^\mu = v^\mu - a^\mu$, $\chi = 2B_0(s + ip)$,

$F_R^{\mu\nu} = \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu]$, $F_L^{\mu\nu} = \partial^\mu l^\nu - \partial^\nu l^\mu - i[l^\mu, l^\nu]$

- Covariant derivative

$$\nabla^\mu X \equiv \partial^\mu X + [\Gamma^\mu, X],\ \Gamma^\mu = \frac{1}{2}\{u^\dagger(\partial^\mu - ir^\mu)u + u(\partial^\mu - il^\mu)u^\dagger\}$$

Heavy-meson fields

- pseudoscalar and vector mesons $(Q\bar{q})$: $a = 1, \cdots, N_f, Q = c$ or b

$$P_a = \begin{cases} (D^0,\ D^+,\ (D_s)) \\ (B^-,\ B^0,\ (B_s)) \end{cases}, \quad P_a^* = \begin{cases} (D^{*0},\ D^{*+},\ (D_s^*)) \\ (B^{*-},\ B^{*0},\ (B_s^*)) \end{cases}$$

- Covariant derivative: $\tilde{P} = P$ or P^*

$$D^\mu \tilde{P}^\dagger = (\partial^\mu + \Gamma^\mu)\tilde{P}^\dagger$$

$$D^\mu \tilde{P} = \tilde{P}(\overleftarrow{\partial}^\mu + \Gamma^{\mu\dagger}) = (\partial^\mu + \Gamma^{\mu *})\tilde{P}$$

$$D^{\mu\nu\cdots\rho} = \frac{1}{n!}(\underbrace{D^\mu D^\nu \cdots D^\rho}_{n} + \text{full permutation of } D\text{'s})$$

- tensor representation

$$H = \frac{1+\not{v}}{2}(P_\mu^* \gamma^\mu + \delta P \gamma_5), \quad \bar{H} = \gamma^0 H^\dagger \gamma^0$$

δ is a relative phase

Purpose & procedure

- Construct the Lagrangians: Relativity/heavy quark symmetry
  - Expand by momentum and/or HQ mass ($M \to \infty$)
  - Requirements: symmetry (Lorentz, P, C, h.c. and/or HQ symmetry)
  - Building blocks properties
  - Linear relations
- Relations between LECs (relativity and heavy quark symmetry)
  - Relations between two kinds of the independent terms
  - Relations or constraints of LECs

Structures of Lagrangians

- Relativity

$$\mathscr{L} = \mathscr{L}_{PP} + \mathscr{L}_{P^*P^*} + \mathscr{L}_{PP^*}$$
$$= \sum_n C_n P \cdots P^\dagger + \sum_n C_n P^* \cdots P^{*\dagger} + \sum_n C_n (\delta P \cdots P^{*\dagger} + \text{h.c.})$$

Ignoring flavour indexes and choosing $\delta = 1$ in the following

- Heavy quark symmetry

$$\mathscr{L} = \sum_n D_n \langle H \cdots \Gamma \bar{H} \rangle$$

$\langle \cdots \rangle$ means tracing over the (flavour and/or spinor) space not a number

Heavy quark symmetry forbids Γ between $\bar{H}$ and H

All the building blocks' properties are in these structures

Properties of the building blocks I

- Chiral rotation (R):

$$u \xrightarrow{R} g_L u h^\dagger = h u g_R^\dagger, \quad u^2 = U,$$

$$X \xrightarrow{R} X' = hXh^\dagger \text{ (} X \text{ are building blocks in CHPT)}$$

$$\tilde{P} \xrightarrow{R} \tilde{P}' = \tilde{P} h^\dagger, \quad \tilde{P}^\dagger \xrightarrow{R} \tilde{P}'^\dagger = h \tilde{P}^\dagger \quad \tilde{P} = P,\ P^* \text{ or } H$$

Properties of the building blocks II

- Chiral dimension (Dim), P, C and h.c. (building blocks)

	Dim	P	C	h.c.
u^μ	1	$-u_\mu$	$(u^\mu)^T$	u^μ
$h^{\mu\nu}$	2	$-h_{\mu\nu}$	$(h^{\mu\nu})^T$	$h^{\mu\nu}$
$\chi_\pm$	2	$\pm\chi_\pm$	$(\chi_\pm)^T$	$\pm\chi_\pm$
$f_\pm^{\mu\nu}$	2	$\pm f_{\pm\mu\nu}$	$\mp(f_\pm^{\mu\nu})^T$	$f_\pm^{\mu\nu}$
P	0	$-P$	$(P^\dagger)^T$	$P^\dagger$
$P^{*\mu}$	0	P^*_μ	$(P^{*\mu\dagger})^T$	$P^{*\mu\dagger}$
$D^\mu P$	0	$-D_\mu P$	$(D^\mu P^\dagger)^T$	$(D^\mu P)^\dagger$
$D^\mu P^{*\nu}$	0	$D_\mu P^*_\nu$	$(D^\mu P^{*\nu\dagger})^T$	$(D^\mu P^{*\nu})^\dagger$
$\varepsilon^{\mu\nu\lambda\rho}$	0	$-\varepsilon_{\mu\nu\lambda\rho}$	$\varepsilon^{\mu\nu\lambda\rho}$	$\varepsilon^{\mu\nu\lambda\rho}$

Properties of the building blocks III

- Chiral dimension (Dim), P, C and h.c. (γ matrices)

For heavy quark symmetry

	Dim	P	C	h.c.
1	0	$+$	$+$	$+$
γ_5	1	$-$	$+$	$-$
γ^μ	0	$+$	$-$	$+$
$\gamma_5\gamma^\mu$	0	$-$	$+$	$+$
$\sigma^{\mu\nu}$	0	$+$	$-$	$+$

Linear relations I

- Partial integration $\tilde{P} = P$ or P^*

$$0 \doteq \tilde{P}\overleftarrow{D}^{\mu}\mathcal{O}^{\cdots}\tilde{P}^{\dagger} + \tilde{P}\mathcal{O}^{\cdots}D^{\mu}\tilde{P}^{\dagger}$$

"$\doteq$" means that both sides are equal if high order terms are ignored

- Schouten identity

$$\varepsilon^{\mu\nu\lambda\rho}A^{\sigma} - \varepsilon^{\sigma\nu\lambda\rho}A^{\mu} - \varepsilon^{\mu\sigma\lambda\rho}A^{\nu} - \varepsilon^{\mu\nu\sigma\rho}A^{\lambda} - \varepsilon^{\mu\nu\lambda\sigma}A^{\rho} = 0$$

- Equations of motion:

$$\nabla^{\mu}u_{\mu} = \frac{i}{2}\left(\chi_{-} - \frac{1}{N_f}\langle\chi_{-}\rangle\right)$$

$$(D^2 + M_P^2)P^{\dagger} \doteq 0, \quad (D^2 + M_{P^*}^2)P_{\mu}^{*\dagger} \doteq 0$$

$$D^{\mu}P_{\mu}^{*\dagger} \doteq 0$$

$$v^{\mu}P_{\mu}^{*\dagger} = 0 \quad \text{(HQET)}$$

Linear relations II

- All possible structures of γ matrix and ε

For heavy quark symmetry

$$
\begin{aligned}
\mathcal{O}(p^1)\ :& 1, \\
\mathcal{O}(p^2)\ :& D^{\mu}, \gamma_5\gamma^{\mu}, \\
\mathcal{O}(p^3)\ :& 1, D^{\mu\nu}, \gamma_5\gamma^{\mu}D^{\nu}, \sigma^{\mu\nu}, \\
\mathcal{O}(p^4)\ :& 1, D^{\mu\nu}, D^{\mu\nu\lambda\rho}, \gamma_5\gamma^{\mu}D^{\nu}, \gamma_5\gamma^{\mu}D^{\nu\lambda\rho}, \sigma^{\mu\nu}, \sigma^{\mu\nu}D^{\lambda\rho}, \\
& \epsilon^{\mu\nu\lambda\rho}, \epsilon^{\mu\nu\lambda\rho}D_{\rho}{}^{\sigma}.
\end{aligned}
$$

Linear relations III

- Covariant derivatives and Bianchi identity

$$0 = \nabla^\mu \nabla^\nu O - \nabla^\nu \nabla^\mu O - \frac{1}{4} u^\mu u^\nu O + \frac{1}{4} u^\nu u^\mu O + \frac{i}{2} f_+^{\mu\nu} O + \frac{1}{4} O u^\mu u^\nu - \frac{1}{4} O u^\nu u^\mu - \frac{i}{2} O f_+^{\mu\nu}$$

$$0 = \nabla^\mu f_+^{\nu\lambda} + \nabla^\nu f_+^{\lambda\mu} + \nabla^\lambda f_+^{\mu\nu}$$

- Cayley-Hamilton relations:

$$0 = AB + BA - A\langle B\rangle - B\langle A\rangle - \langle AB\rangle + \langle A\rangle\langle B\rangle \quad N_f = 2$$

$$\begin{aligned} 0 = {} & ABC + ACB + BAC + BCA + CAB + CBA - AB\langle C\rangle - AC\langle B\rangle \\ & - BA\langle C\rangle - BC\langle A\rangle - CA\langle B\rangle - CB\langle A\rangle - A\langle BC\rangle - B\langle AC\rangle - C\langle AB\rangle \\ & - \langle ABC\rangle - \langle ACB\rangle + A\langle B\rangle\langle C\rangle + B\langle A\rangle\langle C\rangle + C\langle A\rangle\langle B\rangle \\ & + \langle A\rangle\langle BC\rangle + \langle B\rangle\langle AC\rangle + \langle C\rangle\langle AB\rangle - \langle A\rangle\langle B\rangle\langle C\rangle \quad N_f = 3 \end{aligned}$$

Linear relations IV

- Contact terms: Only in the p^4 order

relativity

$$P\langle F_L^{\mu\nu} F_L^{\mu\nu}\rangle P^\dagger + H.c. \qquad P\langle F_L^{\mu\nu} F_L^{\mu\lambda}\rangle DDP^{\dagger\nu\lambda} + H.c.$$

$$P^{*\mu}\langle F_L^{\mu\nu} F_L^{\nu\lambda}\rangle P^{*\dagger\lambda} + H.c. \qquad P^{*\mu}\langle F_L^{\nu\lambda} F_L^{\nu\lambda}\rangle P^{*\dagger\mu} + H.c.$$

$$P^{*\mu}\langle F_L^{\mu\nu} F_L^{\lambda\rho}\rangle DDP^{*\dagger\nu\lambda\rho} + H.c. \qquad P^{*\mu}\langle F_L^{\nu\lambda} F_L^{\nu\rho}\rangle DDP^{*\dagger\lambda\rho\mu} + H.c.$$

$$P\langle \chi\chi^\dagger\rangle P^\dagger \qquad P\,\det\chi P^\dagger + H.c. \quad N_f = 2$$

$$P^{*\mu}\langle \chi\chi^\dagger\rangle P^{*\dagger\mu} \qquad P^{*\mu}\det\chi P^{*\dagger\mu} + H.c. \quad N_f = 2$$

heavy quark symmetry

$$H\langle F_L^{\mu\nu} F_L^{\mu\nu}\rangle \bar{H} + H.c. \qquad H\langle F_L^{\mu\nu} F_L^{\mu\lambda}\rangle DD\bar{H}^{\nu\lambda} + H.c.$$

$$H\langle \chi\chi^\dagger\rangle \bar{H} \qquad H\,\det\chi \bar{H} + H.c. \quad N_f = 2$$

Constructing chiral Lagrangian

- Piling building blocks (all terms)
- C and Hermitian conjugate is invariant
- Picking out contact terms
- Removing linear depend terms
- By computer

Results

$O(p^1)$ order

- Relativity

$$\mathscr{L}^{(1)} = D_\mu P D^\mu P^\dagger - M_P^2 P P^\dagger - \frac{1}{2}(D^\mu P^{*\nu} - D^\nu P^{*\mu})(D_\mu P_\nu^{*\dagger} - D_\nu P_\mu^{*\dagger}) + f_Q(P u^\mu {P^{*\dagger}}_\mu + \text{H.c.}) + \frac{1}{2} g_Q \varepsilon_{\mu\nu\lambda\rho}(P^{*\rho} u^\lambda (D_\mu P_\nu^{*\dagger} - D_\nu P_\mu^{*\dagger}) + \text{H.c.})$$

Tung-Mow Yan, et al., PRD46, 1148(1992)

- Heavy quark symmetry

$$\mathscr{L}_{HQ}^{(1)} = \langle H i v^\mu D_\mu \bar{H} \rangle + g \langle H u_\lambda \bar{H} \gamma_5 \gamma^\lambda \rangle$$

Mark B. Wise, et al., PRD45, R2188 (1992)

$O(p^1)$ order: Relations of LECs I

- Heavy quark masses approach: $M = M_P = M_{P^*}$

- Preprocessing: Unifying dimensions of relativistic LECs (C_n)

$$\tilde{C}_n = C_n M^m, \quad \text{such as} \quad \tilde{g}_Q = g_Q M$$

m is the number of covariant derivatives on heavy meson fields

The dimensions of the relativistic LECs in $O(p^n)$ order are all $2 - n$

- Rescale H: $H \to \sqrt{M} e^{-iMv\cdot x} H$

The dimensions of the heavy quark symmetry LECs in $O(p^n)$ order are all $1 - n$

$O(p^1)$ order: Relations of LECs II

- Two methods to get LECs relations in the heavy quark symmetry:
  Method I: $P \to H$

$$
\begin{aligned}
\mathscr{L}^{(1)} \to & \langle H i v^\mu D_\mu \bar{H} \rangle + (\frac{1}{2M}\tilde{g}_Q - \frac{1}{4M} f_Q) \langle H u_\lambda \bar{H} \gamma_5 \gamma^\lambda \rangle \\
& + (\frac{1}{2M} g_Q + \frac{1}{4M} f_Q) \langle H u_\lambda \gamma_5 \gamma^\lambda \bar{H} \rangle \\
\Rightarrow \quad & 2M g_Q = 2\tilde{g}_Q = f_Q = 2Mg
\end{aligned}
$$

Method II: $H \to P$ We choose this

$$
\mathscr{L}^{(1)}_{HQ} \to \cdots - 2g\varepsilon^{\mu\nu\lambda\rho} \langle {P^*}_\mu u_\lambda D_\rho {P^{*\dagger}}_\nu + H.c. \rangle + 2gM(\langle P u_\mu P^{*\dagger\mu} \rangle + H.c.)
$$

$$
\Rightarrow f_Q = 2M g_Q = 2Mg
$$

Relations of LECs

Generally, at the $O(p^n)$ order

$$\mathscr{L}^{(n)} = \sum_{k=1}^{p} C_k O_k = \sum_{k=1}^{p} \tilde{C}_k \tilde{O}_k, \quad \tilde{C}_k = C_k M^m, \quad \tilde{O}_k = O_k / M^m$$

$$\mathscr{L}_{HQ}^{(n)} = \sum_{l=1}^{q} D_l P_l = M \sum_{l=1}^{q} D_l \sum_{k=1}^{p} A_{lk} \tilde{O}_k$$

We can obtain

$$\tilde{C}_k = M \sum_{l} A_{kl} D_l, \quad A_{kl} \text{ are dimensionless}$$

and $p - q$ constraint conditions of $\tilde{C}_k$

$O(p^2)$ order: Relativity I

O_n	$SU(2)$	$SU(3)$	O_n	$SU(2)$	$SU(3)$
$Pu^\mu u_\mu P^\dagger$	1	1	$P^{*\mu}\langle u^\nu u_\nu\rangle {P^{*\dagger}}_\mu$		12
$Pu^\mu u^\nu D_{\mu\nu}P^\dagger$	2	2	$P^{*\mu}\langle u^\nu u^\lambda\rangle D_{\nu\lambda}{P^{*\dagger}}_\mu$		13
$P\langle u^\mu u_\mu\rangle P^\dagger$		3	$iP^{*\mu}{f_{+\mu}}^\nu {P^{*\dagger}}_\nu$	9	14
$P\langle u^\mu u^\nu\rangle D_{\mu\nu}P^\dagger$		4	$iP^{*\mu}\langle {f_{+\mu}}^\nu\rangle {P^{*\dagger}}_\nu$	10	
$P\chi_+P^\dagger$	3	5	$P^{*\mu}\chi_+{P^{*\dagger}}_\mu$	11	15
$P\langle\chi_+\rangle P^\dagger$	4	6	$P^{*\mu}\langle\chi_+\rangle {P^{*\dagger}}_\mu$	12	16
$P^{*\mu}u_\mu u^\nu {P^{*\dagger}}_\nu$	5	7	$\varepsilon^{\mu\nu\lambda\rho}Pu_\mu u_\nu D_\lambda {P^{*\dagger}}_\rho$	13	17
$P^{*\mu}u^\nu u_\mu {P^{*\dagger}}_\nu$	6	8	$P{f_-}^{\mu\nu}D_\mu {P^{*\dagger}}_\nu$	14	18
$P^{*\mu}u^\nu u_\nu {P^{*\dagger}}_\mu$	7	9	$Ph^{\mu\nu}D_\mu {P^{*\dagger}}_\nu$	15	19
$P^{*\mu}u^\nu u^\lambda D_{\nu\lambda}{P^{*\dagger}}_\mu$	8	10	$i\varepsilon^{\mu\nu\lambda\rho}Pf_{+\mu\nu}D_\lambda {P^{*\dagger}}_\rho$	16	20
$P^{*\mu}\langle u_\mu u^\nu\rangle {P^{*\dagger}}_\nu$		11	$i\varepsilon^{\mu\nu\lambda\rho}P\langle f_{+\mu\nu}\rangle D_\lambda {P^{*\dagger}}_\rho$	17	

PP^* terms ignore the Hermitian parts.

$O(p^2)$ order: Relativity II

$$\begin{aligned}\mathcal{L}^{(2)} =\; & -2[c_0\langle PP^\dagger\rangle\langle\chi_+\rangle - c_1\langle P\chi_+P^\dagger\rangle - c_2\langle PP^\dagger\rangle\langle u^\mu u_\mu\rangle - c_3\langle Pu^\mu u_\mu P^\dagger\rangle \\ & + \frac{c_4}{m_P^2}\langle\mathcal{D}_\mu P\mathcal{D}_\nu P^\dagger\rangle\langle\{u^\mu,u^\nu\}\rangle + \frac{c_5}{m_P^2}\langle\mathcal{D}_\mu P\{u^\mu,u^\nu\}\mathcal{D}_\nu P^\dagger\rangle \\ & + \frac{c_6}{m_P^2}\langle\mathcal{D}_\mu P[u^\mu,u^\nu]\mathcal{D}_\nu P^\dagger\rangle] + 2[\tilde{c}_0\langle P^*_\mu P^{*\mu\dagger}\rangle\langle\chi_+\rangle - \tilde{c}_1\langle P^*_\mu\chi_+P^{*\mu\dagger}\rangle \\ & - \tilde{c}_2\langle P^*_\mu P^{*\mu\dagger}\rangle\langle u^\mu u_\mu\rangle - \tilde{c}_3\langle P^*_\nu u^\mu u_\mu P^{*\nu\dagger}\rangle \\ & + \frac{\tilde{c}_4}{m_{P^*}^2}\langle\mathcal{D}_\mu P^*_\alpha\mathcal{D}_\nu P^{*\alpha\dagger}\rangle\langle\{u^\mu,u^\nu\}\rangle + \frac{\tilde{c}_5}{m_{P^*}^2}\langle\mathcal{D}_\mu P^*_\alpha\{u^\mu,u^\nu\}\mathcal{D}_\nu P^{*\alpha\dagger}\rangle \\ & + \frac{\tilde{c}_6}{m_{P^*}^2}\langle\mathcal{D}_\mu P^*_\alpha[u^\mu,u^\nu]\mathcal{D}_\nu P^{*\alpha\dagger}\rangle],\end{aligned}$$

M. Altenbuchinger, L. S. Geng and W. Weise, PRD89, 014026(2014)

$O(p^2)$ order: Heavy quark symmetry

P_n	$SU(2)$	$SU(3)$
$\langle H u^\mu u_\mu \bar{H}\rangle$	1	1
$\langle H u^\mu u^\nu v_\mu v_\nu \bar{H}\rangle$	2	2
$i\langle H u^\mu u^\nu \sigma_{\mu\nu} \bar{H}\rangle + \text{H.c.}$	3	3
$\langle H \langle u^\mu u_\mu\rangle \bar{H}\rangle$		4
$\langle H \langle u^\mu u^\nu\rangle v_\mu v_\nu \bar{H}\rangle$		5
$\langle H {f_+}^{\mu\nu} \sigma_{\mu\nu} \bar{H}\rangle$	4	6
$\langle H \langle {f_+}^{\mu\nu}\rangle \sigma_{\mu\nu} \bar{H}\rangle$	5	
$\langle H \chi_+ \bar{H}\rangle$	6	7
$\langle H \langle\chi_+\rangle \bar{H}\rangle$	7	8

$O(p^2)$ order: Relations of LECs ($N_f = 3$)

LECs in heavy quark symmetry

$$\tilde{C}_1^{(2)} = -2D_1^{(2)},\ \tilde{C}_2^{(2)} = 2D_2^{(2)},\ \tilde{C}_3^{(2)} = -2D_4^{(2)},\ \tilde{C}_4^{(2)} = 2D_5^{(2)},$$

$$\tilde{C}_5^{(2)} = -2D_7^{(2)},\ \tilde{C}_6^{(2)} = -2D_8^{(2)},\ \tilde{C}_7^{(2)} = -4D_3^{(2)},\ \tilde{C}_8^{(2)} = 4D_3^{(2)}$$

$$\tilde{C}_9^{(2)} = 2D_1^{(2)},\ \tilde{C}_{10}^{(2)} = -2D_2^{(2)},\ \tilde{C}_{11}^{(2)} = 0,\ \tilde{C}_{12}^{(2)} = 2D_4^{(2)}$$

$$\tilde{C}_{13}^{(2)} = -2D_5^{(2)},\ \tilde{C}_{14}^{(2)} = 4D_6^{(2)},\ \tilde{C}_{15}^{(2)} = 2D_7^{(2)},\ \tilde{C}_{16}^{(2)} = 2D_8^{(2)}$$

$$\tilde{C}_{17}^{(2)} = -4D_3^{(2)},\ \tilde{C}_{18}^{(2)} = 0,\ \tilde{C}_{19}^{(2)} = 0,\ \tilde{C}_{20}^{(2)} = 2D_6^{(2)}$$

Constraint conditions

$$0 = \tilde{C}_1^{(2)} + \tilde{C}_9^{(2)},\ 0 = \tilde{C}_2^{(2)} + \tilde{C}_{10}^{(2)},\ 0 = \tilde{C}_3^{(2)} + \tilde{C}_{12}^{(2)},$$

$$0 = \tilde{C}_4^{(2)} + \tilde{C}_{13}^{(2)},\ 0 = \tilde{C}_5^{(2)} + \tilde{C}_{15}^{(2)},\ 0 = \tilde{C}_6^{(2)} + \tilde{C}_{16}^{(2)},$$

$$0 = \tilde{C}_7^{(2)},\ 0 = \tilde{C}_8^{(2)} + \tilde{C}_{17}^{(2)},\ 0 = \tilde{C}_{11}^{(2)},\ 0 = \tilde{C}_{14}^{(2)},$$

$$0 = \tilde{C}_{18}^{(2)},\ 0 = \tilde{C}_{19}^{(2)}$$

$O(p^2)$ order: Relations of LECs $(N_f = 2)$

LECs in heavy quark symmetry

$$\tilde{c}_1^{(2)} = -2d_1^{(2)},\ \tilde{c}_2^{(2)} = 2d_2^{(2)},\ \tilde{c}_3^{(2)} = -2d_6^{(2)},\ \tilde{c}_4^{(2)} = -2d_7^{(2)},$$
$$\tilde{c}_5^{(2)} = -4d_3^{(2)},\ \tilde{c}_6^{(2)} = 4d_3^{(2)},\ \tilde{c}_7^{(2)} = 2d_1^{(2)},\ \tilde{c}_8^{(2)} = -2d_2^{(2)},$$
$$\tilde{c}_9^{(2)} = 4d_4^{(2)},\ \tilde{c}_{10}^{(2)} = 4d_5^{(2)},\ \tilde{c}_{11}^{(2)} = 2d_6^{(2)},\ \tilde{c}_{12}^{(2)} = 2d_7^{(2)},$$
$$\tilde{c}_{13}^{(2)} = -4d_3^{(2)},\ \tilde{c}_{14}^{(2)} = 0,\ \tilde{c}_{15}^{(2)} = 0,\ \tilde{c}_{16}^{(2)} = 2d_4^{(2)},\ \tilde{c}_{17}^{(2)} = 2d_5^{(2)},$$

Constraint conditions

$$0 = \tilde{c}_1^{(2)} + \tilde{c}_7^{(2)},\ 0 = \tilde{c}_2^{(2)} + \tilde{c}_8^{(2)},\ 0 = \tilde{c}_3^{(2)} + \tilde{c}_{11}^{(2)},$$
$$0 = \tilde{c}_4^{(2)} + \tilde{c}_{12}^{(2)},\ 0 = \tilde{c}_5^{(2)},\ 0 = \tilde{c}_6^{(2)} + \tilde{c}_{13}^{(2)},$$
$$0 = \tilde{c}_9^{(2)},\ 0 = \tilde{c}_{10}^{(2)},\ 0 = \tilde{c}_{14}^{(2)},\ 0 = \tilde{c}_{15}^{(2)}$$

$O(p^3)$ order: Relativity I

O_n	$SU(2)$	$SU(3)$	O_n	$SU(2)$	$SU(3)$
$\varepsilon^{\mu\nu\lambda\rho} P u_\mu u_\nu u_\lambda D_\rho P^\dagger$	1	1	$iP^{*\mu} \langle \nabla_\mu f_+{}^{\nu\lambda} \rangle D_\nu P^{*\dagger}{}_\lambda$ + H.c.	35	
$\varepsilon^{\mu\nu\lambda\rho} P \langle u_\mu u_\nu u_\lambda \rangle D_\rho P^\dagger$		2	$iP^{*\mu} \langle \nabla^\nu f_{+\nu}{}^{\lambda} \rangle D_\lambda P^{*\dagger}{}_\mu$	36	
$P u^\mu f_{-\mu}{}^\nu D_\nu P^\dagger$ + H.c.	2	3	$\varepsilon^{\mu\nu\lambda\rho} P^*{}_\mu u_\nu \chi_+ D_\lambda P^{*\dagger}{}_\rho$ + H.c.	37	44
$P u^\mu h_\mu{}^\nu D_\nu P^\dagger$ + H.c.	3	4	$\varepsilon^{\mu\nu\lambda\rho} P^*{}_\mu \langle u_\nu \chi_+ \rangle D_\lambda P^{*\dagger}{}_\rho$	38	45
$P u^\mu h^{\nu\lambda} D_{\mu\nu\lambda} P^\dagger$ + H.c.	4	5	$\varepsilon^{\mu\nu\lambda\rho} P^*{}_\mu \langle \chi_+ \rangle u_\nu D_\lambda P^{*\dagger}{}_\rho$		46
$i\varepsilon^{\mu\nu\lambda\rho} P f_{+\mu\nu} u_\lambda D_\rho P^\dagger$ + H.c.	5	6	$iP^{*\mu} u^\nu \chi_- D_\nu P^{*\dagger}{}_\mu$ + H.c.	39	47
$i\varepsilon^{\mu\nu\lambda\rho} P \langle f_{+\mu\nu} \rangle u_\lambda D_\rho P^\dagger$	6		$i\varepsilon^{\mu\nu\lambda\rho} P^*{}_\mu \chi_{-\nu} D_\lambda P^{*\dagger}{}_\rho$	40	48
$i\varepsilon^{\mu\nu\lambda\rho} P \langle f_{+\mu\nu} u_\lambda \rangle D_\rho P^\dagger$		7	$i\varepsilon^{\mu\nu\lambda\rho} P^*{}_\mu \langle \chi_{-\nu} \rangle D_\lambda P^{*\dagger}{}_\rho$	41	49
$iP \nabla^\mu f_{+\mu}{}^\nu D_\nu P^\dagger$	7	8	$P u^\mu u_\mu u^\nu P^{*\dagger}{}_\nu$	42	50
$iP \langle \nabla^\mu f_{+\mu}{}^\nu \rangle D_\nu P^\dagger$	8		$P u^\mu u^\nu u_\mu P^{*\dagger}{}_\nu$	43	51
$iP u^\mu \chi_- D_\mu P^\dagger$ + H.c.	9	9	$P u^\mu u^\nu u_\nu P^{*\dagger}{}_\mu$		52
$\varepsilon^{\mu\nu\lambda\rho} P^*{}_\mu u_\nu u_\lambda u^\sigma D_\rho P^{*\dagger}{}_\sigma$ + H.c.	10	10	$P u^\mu u^\nu u^\lambda D_{\mu\nu} P^{*\dagger}{}_\lambda$	44	53
$\varepsilon^{\mu\nu\lambda\rho} P^*{}_\mu u_\nu u_\lambda u^\sigma D_\sigma P^{*\dagger}{}_\rho$ + H.c.	11	11	$P u^\mu u^\nu u^\lambda D_{\mu\lambda} P^{*\dagger}{}_\nu$	45	54
$\varepsilon^{\mu\nu\lambda\rho} P^*{}_\mu u_\nu u^\sigma u_\lambda D_\rho P^{*\dagger}{}_\sigma$ + H.c.	12	12	$P u^\mu u^\nu u^\lambda D_{\nu\lambda} P^{*\dagger}{}_\mu$		55
$\varepsilon^{\mu\nu\lambda\rho} P^*{}_\mu u_\nu u^\sigma u_\sigma D_\lambda P^{*\dagger}{}_\rho$ + H.c.	13	13	$P \langle u^\mu u_\mu \rangle u^\nu P^{*\dagger}{}_\nu$		56
$\varepsilon^{\mu\nu\lambda\rho} P^*{}_\mu u^\sigma u_\nu u_\lambda D_\rho P^{*\dagger}{}_\sigma$ + H.c.		14	$P \langle u^\mu u_\mu u^\nu \rangle P^{*\dagger}{}_\nu$		57
$\varepsilon^{\mu\nu\lambda\rho} P^*{}_\mu u_\nu u^\sigma u^\delta D_{\lambda\sigma\delta} P^{*\dagger}{}_\rho$ + H.c.	14	15	$P \langle u^\mu u^\nu \rangle u^\lambda D_{\mu\nu} P^{*\dagger}{}_\lambda$		58
$\varepsilon^{\mu\nu\lambda\rho} P^*{}_\mu \langle u_\nu u_\lambda u^\sigma \rangle D_\rho P^{*\dagger}{}_\sigma$ + H.c.		16	$P \langle u^\mu u^\nu u^\lambda \rangle D_{\mu\nu} P^{*\dagger}{}_\lambda$		59
$\varepsilon^{\mu\nu\lambda\rho} P^*{}_\mu \langle u_\nu u^\sigma \rangle u_\lambda D_\rho P^{*\dagger}{}_\sigma$ + H.c.		17	$\varepsilon^{\mu\nu\lambda\rho} P u_\mu f_{-\nu\lambda} P^{*\dagger}{}_\rho$	46	60
$\varepsilon^{\mu\nu\lambda\rho} P^*{}_\mu \langle u_\nu u^\sigma \rangle u_\lambda D_\sigma P^{*\dagger}{}_\rho$		18	$\varepsilon^{\mu\nu\lambda\rho} P f_{-\mu\nu} u_\lambda P^{*\dagger}{}_\rho$	47	61
$\varepsilon^{\mu\nu\lambda\rho} P^*{}_\mu \langle u_\nu u^\sigma \rangle u_\sigma D_\lambda P^{*\dagger}{}_\rho$		19	$\varepsilon^{\mu\nu\lambda\rho} P u_\mu f_{-\nu}{}^\sigma D_{\lambda\sigma} P^{*\dagger}{}_\rho$	48	62
$\varepsilon^{\mu\nu\lambda\rho} P^*{}_\mu \langle u_\nu u^\sigma \rangle u^\delta D_{\lambda\sigma\delta} P^{*\dagger}{}_\rho$		20	$\varepsilon^{\mu\nu\lambda\rho} P f_{-\mu\nu} u^\sigma D_{\lambda\sigma} P^{*\dagger}{}_\rho$	49	63
$P^{*\mu} u_\mu f_-{}^{\nu\lambda} D_\nu P^{*\dagger}{}_\lambda$ + H.c.	15	21	$\varepsilon^{\mu\nu\lambda\rho} P u_\mu h_\nu{}^\sigma D_{\lambda\sigma} P^{*\dagger}{}_\rho$	50	64
$P^{*\mu} u^\nu f_{-\mu}{}^\lambda D_\nu P^{*\dagger}{}_\lambda$ + H.c.	16	22	$\varepsilon^{\mu\nu\lambda\rho} P h_\mu{}^\sigma u_\nu D_{\lambda\sigma} P^{*\dagger}{}_\rho$	51	65
$P^{*\mu} u^\nu f_{-\mu}{}^\lambda D_\lambda P^{*\dagger}{}_\nu$ + H.c.	17	23	$\varepsilon^{\mu\nu\lambda\rho} P \langle u_\mu f_{-\nu\lambda} \rangle P^{*\dagger}{}_\rho$		66
$P^{*\mu} u^\nu f_{-\nu}{}^\lambda D_\lambda P^{*\dagger}{}_\mu$ + H.c.	18	24	$\varepsilon^{\mu\nu\lambda\rho} P \langle u_\mu f_{-\nu}{}^\sigma \rangle D_{\lambda\sigma} P^{*\dagger}{}_\rho$		67

$O(p^3)$ order: Relativity II

$P^{*\mu}u_\mu h^{\nu\lambda}D_\nu P^{*\dagger}{}_\lambda$ + H.c.	19	25	$\varepsilon^{\mu\nu\lambda\rho}P\langle u_\mu h_\nu{}^\sigma\rangle D_{\lambda\sigma}P^{*\dagger}{}_\rho$		68
$P^{*\mu}u^\nu h_\mu{}^\lambda D_\nu P^{*\dagger}{}_\lambda$ + H.c.	20	26	$P\nabla^\mu f_{-\mu}{}^\nu P^{*\dagger}{}_\nu$	52	69
$P^{*\mu}u^\nu h_\mu{}^\lambda D_\lambda P^{*\dagger}{}_\nu$ + H.c.	21	27	$P\nabla^\mu f_-{}^{\nu\lambda}D_{\mu\nu}P^{*\dagger}{}_\lambda$	53	70
$P^{*\mu}u^\nu h_\nu{}^\lambda D_\lambda P^{*\dagger}{}_\mu$ + H.c.	22	28	$P\nabla^\mu h^{\nu\lambda}D_{\mu\nu}P^{*\dagger}{}_\lambda$	54	71
$P^{*\mu}u^\nu h^{\lambda\rho}D_{\nu\lambda\rho}P^{*\dagger}{}_\mu$ + H.c.	23	29	$iPf_+{}^{\mu\nu}u_\mu P^{*\dagger}{}_\nu$	55	72
$P^{*\mu}\langle u_\mu f_-{}^{\nu\lambda}\rangle D_\nu P^{*\dagger}{}_\lambda$ + H.c.		30	$iPu^\mu f_{+\mu}{}^\nu P^{*\dagger}{}_\nu$	56	73
$P^{*\mu}\langle u^\nu f_{-\mu}{}^\lambda\rangle D_\nu P^{*\dagger}{}_\lambda$		31	$iPf_+{}^{\mu\nu}u^\lambda D_{\mu\lambda}P^{*\dagger}{}_\nu$	57	74
$P^{*\mu}\langle u_\mu h^{\nu\lambda}\rangle D_\nu P^{*\dagger}{}_\lambda$ + H.c.		32	$iPu^\mu f_+{}^{\nu\lambda}D_{\mu\nu}P^{*\dagger}{}_\lambda$	58	75
$\varepsilon^{\mu\nu\lambda\rho}P^*{}_\mu\nabla_\nu f_{-\lambda}{}^\sigma D_\rho P^{*\dagger}{}_\sigma$ + H.c.	24	33	$iP\langle f_+{}^{\mu\nu}\rangle u_\mu P^{*\dagger}{}_\nu$	59	
$\varepsilon^{\mu\nu\lambda\rho}P^*{}_\mu\nabla_\nu f_{-\lambda}{}^\sigma D_\sigma P^{*\dagger}{}_\rho$	25	34	$iP\langle f_+{}^{\mu\nu}\rangle u^\lambda D_{\mu\lambda}P^{*\dagger}{}_\nu$	60	
$\varepsilon^{\mu\nu\lambda\rho}P^*{}_\mu\nabla_\nu h^{\sigma\delta}D_{\lambda\sigma\delta}P^{*\dagger}{}_\rho$	26	35	$iP\langle f_+{}^{\mu\nu}u_\mu\rangle P^{*\dagger}{}_\nu$		76
$i\varepsilon^{\mu\nu\lambda\rho}P^*{}_\mu f_{+\nu\lambda}u^\sigma D_\rho P^{*\dagger}{}_\sigma$ + H.c.	27	36	$iP\langle f_+{}^{\mu\nu}u^\lambda\rangle D_{\mu\lambda}P^{*\dagger}{}_\nu$		77
$i\varepsilon^{\mu\nu\lambda\rho}P^*{}_\mu f_{+\nu\lambda}u^\sigma D_\sigma P^{*\dagger}{}_\rho$ + H.c.	28	37	$i\varepsilon^{\mu\nu\lambda\rho}P\nabla_\mu f_{+\nu}{}^\sigma D_{\lambda\sigma}P^{*\dagger}{}_\rho$	61	78
$i\varepsilon^{\mu\nu\lambda\rho}P^*{}_\mu f_{+\nu}{}^\sigma u_\lambda D_\rho P^{*\dagger}{}_\sigma$ + H.c.	29	38	$i\varepsilon^{\mu\nu\lambda\rho}P\langle\nabla_\mu f_{+\nu}{}^\sigma\rangle D_{\lambda\sigma}P^{*\dagger}{}_\rho$	62	
$i\varepsilon^{\mu\nu\lambda\rho}P^*{}_\mu f_{+\nu}{}^\sigma u_\sigma D_\lambda P^{*\dagger}{}_\rho$ + H.c.	30	39	$Pu^\mu\chi_+P^{*\dagger}{}_\mu$	63	79
$i\varepsilon^{\mu\nu\lambda\rho}P^*{}_\mu\langle f_{+\nu\lambda}\rangle u^\sigma D_\rho P^{*\dagger}{}_\sigma$ + H.c.	31		$P\chi_+u^\mu P^{*\dagger}{}_\mu$	64	80
$i\varepsilon^{\mu\nu\lambda\rho}P^*{}_\mu\langle f_{+\nu}{}^\sigma\rangle u_\lambda D_\rho P^{*\dagger}{}_\sigma$ + H.c.	32		$P\langle u^\mu\chi_+\rangle P^{*\dagger}{}_\mu$	65	81
$i\varepsilon^{\mu\nu\lambda\rho}P^*{}_\mu\langle f_{+\nu\lambda}u^\sigma\rangle D_\rho P^{*\dagger}{}_\sigma$ + H.c.		40	$P\langle\chi_+\rangle u^\mu P^{*\dagger}{}_\mu$		82
$i\varepsilon^{\mu\nu\lambda\rho}P^*{}_\mu\langle f_{+\nu}{}^\sigma u_\lambda\rangle D_\rho P^{*\dagger}{}_\sigma$ + H.c.		41	$iP\chi_-{}^\mu P^{*\dagger}{}_\mu$	66	83
$iP^{*\mu}\nabla_\mu f_+{}^{\nu\lambda}D_\nu P^{*\dagger}{}_\lambda$ + H.c.	33	42	$iP\langle\chi_-{}^\mu\rangle P^{*\dagger}{}_\mu$	67	84
$iP^{*\mu}\nabla^\nu f_{+\nu}{}^\lambda D_\lambda P^{*\dagger}{}_\mu$	34	43			

PP^* terms ignore the Hermitian parts.

$O(p^3)$ order: Heavy quark symmetry

O_n	$SU(2)$	$SU(3)$	O_n	$SU(2)$	$SU(3)$
$\langle H u^\mu u_\mu u^\nu \gamma_5 \gamma_\nu \bar{H}\rangle$ + H.c.	1	1	$\langle H \langle u^\mu f_-{}^{\nu\lambda}\rangle \sigma_{\nu\lambda} v_\mu \bar{H}\rangle$		19
$\langle H u^\mu u^\nu u_\mu \gamma_5 \gamma_\nu \bar{H}\rangle$	2	2	$\langle H \langle u^\mu h^{\nu\lambda}\rangle \sigma_{\mu\nu} v_\lambda \bar{H}\rangle$		20
$\langle H u^\mu u^\nu u^\lambda \gamma_5 \gamma_\mu v_\nu v_\lambda \bar{H}\rangle$ + H.c.	3	3	$\langle H \nabla^\mu f_{-\mu}{}^{\nu} \gamma_5 \gamma_\nu \bar{H}\rangle$	13	21
$\langle H u^\mu u^\nu u^\lambda \gamma_5 \gamma_\nu v_\mu v_\lambda \bar{H}\rangle$	4	4	$\langle H \nabla^\mu f_-{}^{\nu\lambda} \gamma_5 \gamma_\nu v_\mu v_\lambda \bar{H}\rangle$	14	22
$\langle H \langle u^\mu u_\mu\rangle u^\nu \gamma_5 \gamma_\nu \bar{H}\rangle$		5	$\langle H \nabla^\mu h_\mu{}^{\nu} \gamma_5 \gamma_\nu \bar{H}\rangle$	15	23
$\langle H \langle u^\mu u_\mu u^\nu\rangle \gamma_5 \gamma_\nu \bar{H}\rangle$		6	$\langle H \nabla^\mu h^{\nu\lambda} \gamma_5 \gamma_\mu v_\nu v_\lambda \bar{H}\rangle$	16	24
$\langle H \langle u^\mu u^\nu\rangle u^\lambda \gamma_5 \gamma_\mu v_\nu v_\lambda \bar{H}\rangle$		7	$i\langle H f_+{}^{\mu\nu} u_\mu \gamma_5 \gamma_\nu \bar{H}\rangle$ + H.c.	17	25
$\langle H \langle u^\mu u^\nu u^\lambda\rangle \gamma_5 \gamma_\mu v_\nu v_\lambda \bar{H}\rangle$		8	$i\langle H f_+{}^{\mu\nu} u^\lambda \gamma_5 \gamma_\mu v_\nu v_\lambda \bar{H}\rangle$ + H.c.	18	26
$i\varepsilon^{\mu\nu\lambda\rho}\langle H u_\mu u_\nu u_\lambda v_\rho \bar{H}\rangle$	5	9	$\varepsilon^{\mu\nu\lambda\rho}\langle H f_{+\mu\nu} u_\lambda v_\rho \bar{H}\rangle$ + H.c.	19	27
$i\varepsilon^{\mu\nu\lambda\rho}\langle H \langle u_\mu u_\nu u_\lambda\rangle v_\rho \bar{H}\rangle$		10	$\varepsilon^{\mu\nu\lambda\rho}\langle H \langle f_{+\mu\nu}\rangle u_\lambda v_\rho \bar{H}\rangle$	20	
$i\langle H u^\mu f_{-\mu}{}^{\nu} v_\nu \bar{H}\rangle$ + H.c.	6	11	$\varepsilon^{\mu\nu\lambda\rho}\langle H \langle f_{+\mu\nu} u_\lambda\rangle v_\rho \bar{H}\rangle$		28
$\langle H u^\mu f_-{}^{\nu\lambda} \sigma_{\mu\nu} v_\lambda \bar{H}\rangle$ + H.c.	7	12	$\langle H \nabla^\mu f_{+\mu}{}^{\nu} v_\nu \bar{H}\rangle$	21	29
$\langle H u^\mu f_-{}^{\nu\lambda} \sigma_{\nu\lambda} v_\mu \bar{H}\rangle$ + H.c.	8	13	$\langle H \langle \nabla^\mu f_{+\mu}{}^{\nu}\rangle v_\nu \bar{H}\rangle$	22	
$i\langle H u^\mu h_\mu{}^{\nu} v_\nu \bar{H}\rangle$ + H.c.	9	14	$\langle H u^\mu \chi_+ \gamma_5 \gamma_\mu \bar{H}\rangle$ + H.c.	23	30
$i\langle H u^\mu h^\nu{}_\nu v_\mu \bar{H}\rangle$ + H.c.	10	15	$\langle H \langle u^\mu \chi_+\rangle \gamma_5 \gamma_\mu \bar{H}\rangle$	24	31
$i\langle H u^\mu h^{\nu\lambda} v_\mu v_\nu v_\lambda \bar{H}\rangle$ + H.c.	11	16	$\langle H \langle \chi_+\rangle u^\mu \gamma_5 \gamma_\mu \bar{H}\rangle$		32
$\langle H u^\mu h^{\nu\lambda} \sigma_{\mu\nu} v_\lambda \bar{H}\rangle$ + H.c.	12	17	$i\langle H \chi_-{}^{\mu} \gamma_5 \gamma_\mu \bar{H}\rangle$	25	33
$\langle H \langle u^\mu f_-{}^{\nu\lambda}\rangle \sigma_{\mu\nu} v_\lambda \bar{H}\rangle$		18	$i\langle H \langle \chi_-{}^{\mu}\rangle \gamma_5 \gamma_\mu \bar{H}\rangle$	26	34

Relations of $O(p^3)$ order LECs are in the attach file (click here)

$O(p^4)$ order

Relativity

$$\mathscr{L}^{(4)} = \sum_{m=1}^{652} C_m^{(4)} O_m^{(4)}, \quad N_f = 3$$

$$\mathscr{L}^{(4)} = \sum_{m=1}^{401} c_m^{(4)} o_m^{(4)}, \quad N_f = 2$$

Heavy-quark symmetry

$$\mathscr{L}^{(4)} = \sum_{m=1}^{219} D_m^{(4)} P_m^{(4)}, \quad N_f = 3$$

$$\mathscr{L}^{(4)} = \sum_{m=1}^{144} d_m^{(4)} p_m^{(4)}, \quad N_f = 2$$

$O_m^{(4)}$, $o_m^{(4)}$, $P_m^{(4)}$ and $p_m^{(4)}$ are in the attach file (click here)

Summary

- Giving the chiral Lagrangian for mesons with a heavy quark to $O(p^4)$ order
- Giving the relations between the relativistic and the heavy quark symmetry LECs

Thank you!