Effective field theory at lepton colliders

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(Technion)

Hefei, 22 November 2018
FeynRules/MadGraph School on Collider Phenomenology

Effective field theory at lepton colliders

1. Effective field theory
2. Future lepton colliders
3. Global Higgs analysis
4. Global top-quark analysis
   - pair production
   - single FCNC production
5. Global top&Higgs analysis

1. Effective field theory

EFT essence

Like good musicians, good physicists know which scales are relevant for which process.
Burgess, `hep-th/0701053`

Focus on the most relevant physics

- take small parameters to zero/large ones to infinity
- compute corrections systematically in a perturbative way

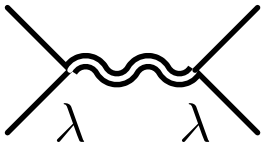
Exploit separation of scales in QFTs

e.g. perform low-energy expansions

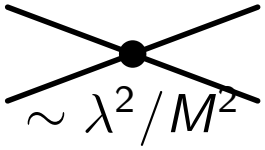
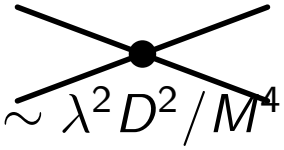
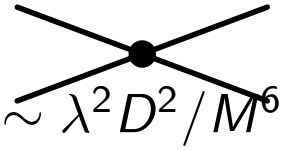
Basic expectation: very heavy physics should have a limited impact on low-energy phenomena

Appelquist-Carrazone: $(E/M)^n$ suppression (in unbroken gauge theories)

Integrating out: introducing a cutoff generates a tower of operators



$$\sim \frac{\lambda^2}{s - M^2} \sim \frac{-\lambda^2}{M^2}\left(1 - \frac{s}{M^2} + \ldots\right) \qquad \text{for } \sqrt{s} \ll M$$

$\sim$  $+$  $+$  $+ \ldots$

$\sim \lambda^2/M^2$ $\quad \sim \lambda^2 D^2/M^4$ $\quad \sim \lambda^2 D^2/M^6$

Top-down or bottom-up

Top-down

The more complete theory is known and calculable.
$\rightarrow$ simplify computations
$\rightarrow$ systematize the computation of corrections
$\rightarrow$ resum logarithms of scale ratios through operators' RG running
e.g. EW Hamiltonian, HQEF, SCET, ...

Bottom-up

The more complete theory is either unknown or not calculable.
e.g. non-perturbative QCD, Fermi theory (back then), BSM, ...

how? 1. construct **all** lowest-dimensional operators with:

- a given field content
- symmetries (e.g. Lorentz + gauge + global)

2. determine C_i/Λ^{d-4} coefficients from measurements

$\rightarrow$ a general and systematic parametrization of the theory space

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> *(...) if one writes down the most general possible Lagrangian, including all terms consistent with assumed symmetry principles, (...) the result will simply be the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetry.*
> [*Phenomenological Lagrangians*, Weinberg '79]

The standard-model effective field theory

Field content: q, u, d, l, e, φ
Symmetries: Lorentz + $SU(3)_C \times SU(2)_L \times U(1)_Y$

The SM Lagrangian contains all "renormalizable" operators ($d \leq 4$)!
(except $G^{\mu\nu}\tilde{G}_{\mu\nu}$ on which stringent constraints exist)

The bottom-up EFT principles seem to apply!
And, a separation of scale seem to exist between SM and BSM.

$\longrightarrow$ Let's proceed with $d > 4$!

"Non-renormalizable" operators

We learn:

- $d < 4$: "super renormalizable"
- $d = 4$: "renormalizable"
- $d > 4$: "non-renormalizable"

"Non-renormalizable" operators inserted in higher-and-higher loop diagrams require more-and-more UV counterterms.

Still, to order $1/\Lambda^n$ in the EFT expansion, only a finite number of counterterms from operators of $d \leq n + 4$ is required to any loop order.

is sufficient for all practical purposes!

$\rightarrow$ all possible operators of $d \leq n + 4$ are in general required

$\rightarrow$ they mix under RG evolution!

$\rightarrow$ a complete (and non-redundant) basis is needed

Operator basis construction

1. Construct all possible operators
2. Remove redundancies
   - Dirac algebra, including Fierz identities

     e.g. $(\bar{\psi}_1\gamma^\mu P_L\psi_2)\,(\bar{\psi}_3\gamma_\mu P_R\psi_4) = -2\,(\bar{\psi}_1 P_L\psi_4)\,(\bar{\psi}_3 P_R\psi_2)$
   - integration by part
   - field redefinitions ("equivalence theorem")

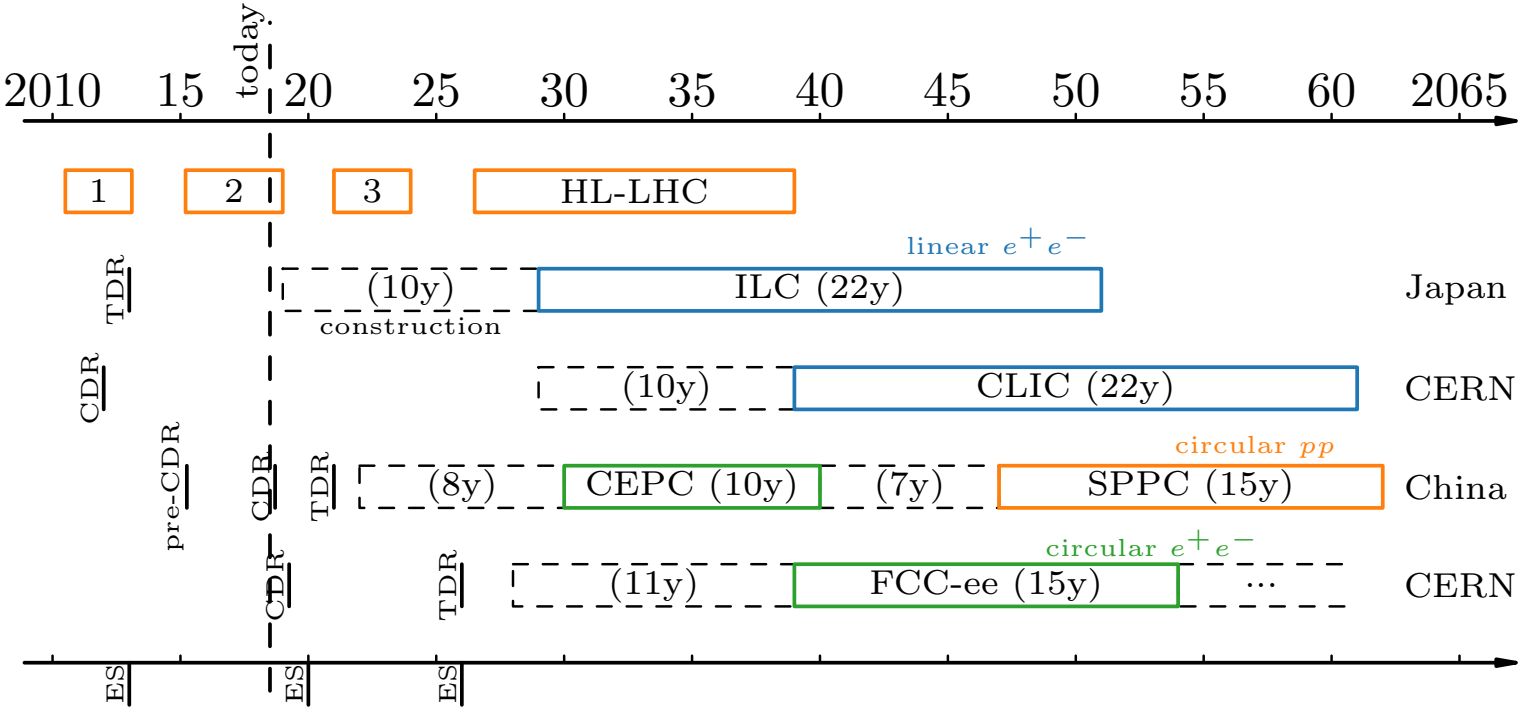
     Take
     $$\mathcal{L}[\phi] = \mathcal{L}^4[\phi] + \ldots + \frac{1}{\Lambda^{d-4}}\mathcal{L}^d[\phi] + \ldots$$
     do a non-linear field redefinition $\phi \to \phi + \frac{f[\phi]}{\Lambda^{d-4}}$:
     $$\mathcal{L}[\phi] \to \mathcal{L}^4[\phi] + \ldots + \frac{1}{\Lambda^{d-4}}\left(f[\phi]\frac{\delta\mathcal{L}^4}{\delta\phi}[\phi] + \mathcal{L}^d[\phi]\right) + \ldots$$
     $\to$ One can effectively remove any combination of operators proportional to the lowest-order classical EOM ($\delta\mathcal{L}^4/\delta\phi$) and induce only higher $1/\Lambda$-order corrections.

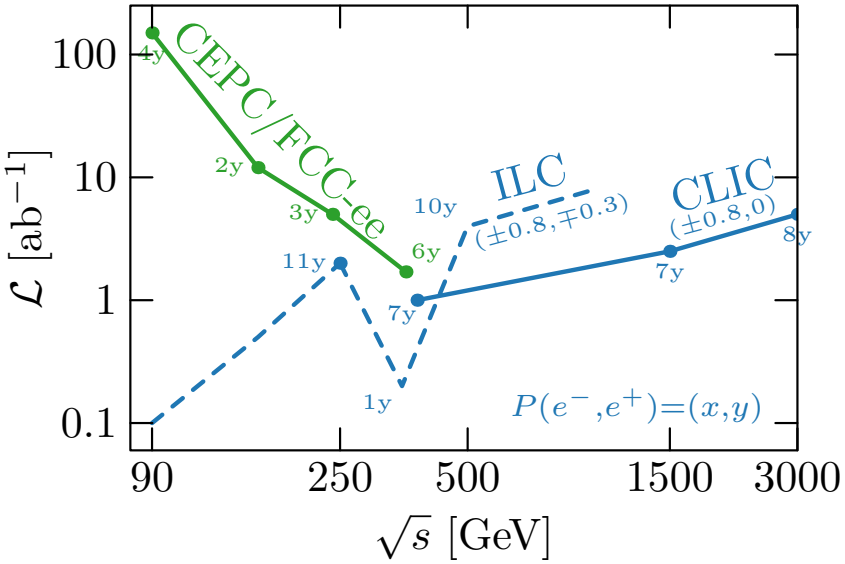
For the SMEFT

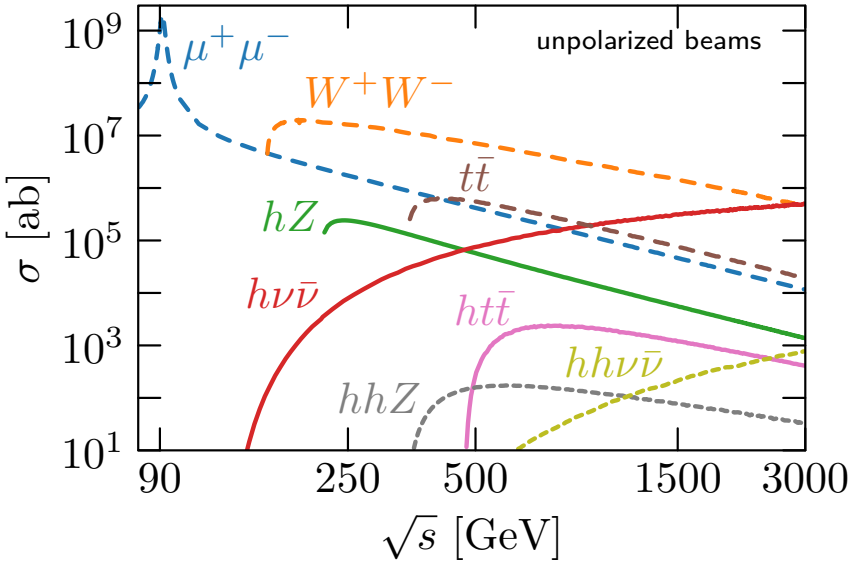
Warsaw, SILH, Higgs, ... bases of dimension-six operators

2. Future lepton colliders

Future lepton colliders



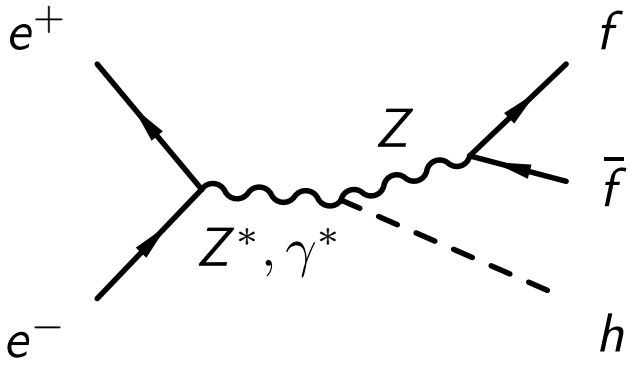




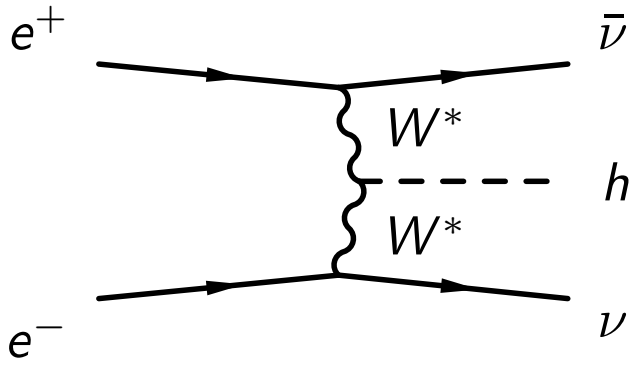
3. Global Higgs analysis

[1704.02333]
[1711.03978]

Introduction

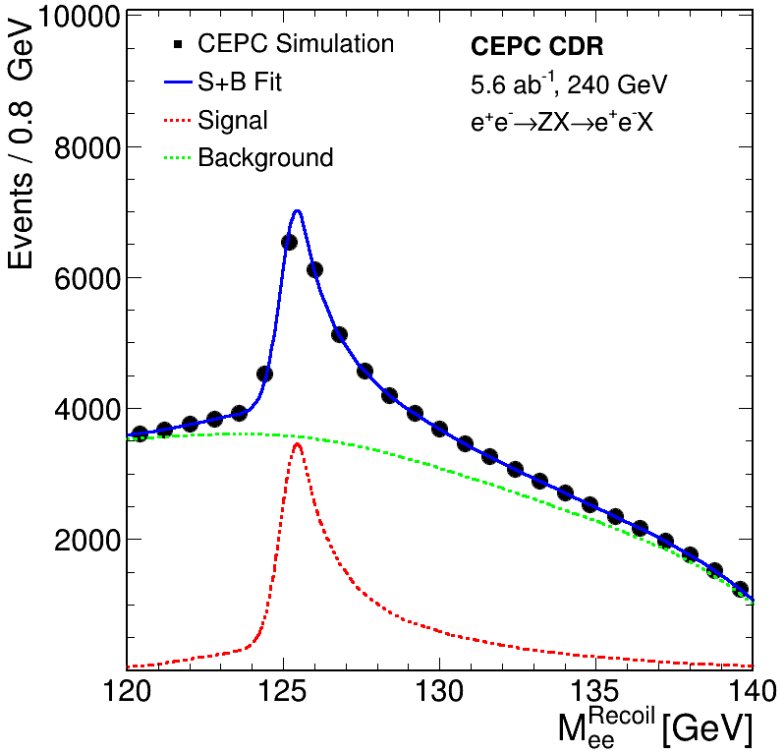


Higgsstrahlung



W-boson fusion

[CEPC CDR, 14 Nov]



recoil mass$^2 = s - 2E_{f\bar{f}}\sqrt{s} + m^2_{f\bar{f}}$

Property	Estimated Precision
m_H	5.9 MeV
Γ_H	3.1%
$\sigma(ZH)$	0.5%
$\sigma(\nu\bar{\nu}H)$	3.2%

Decay mode	$\sigma(ZH) \times$ BR	BR
$H \to b\bar{b}$	0.27%	0.56%
$H \to c\bar{c}$	3.3%	3.3%
$H \to gg$	1.3%	1.4%
$H \to WW^*$	1.0%	1.1%
$H \to ZZ^*$	5.1%	5.1%
$H \to \gamma\gamma$	6.8%	6.9%
$H \to Z\gamma$	15%	15%
$H \to \tau^+\tau^-$	0.8%	1.0%
$H \to \mu^+\mu^-$	17%	17%
$H \to$ inv	–	$< 0.30\%$

up to permil precision!

Global EFT setup

- employ the Higgs basis of dim-6 operators
- focus mostly on Higgs-related processes:

$$e^+e^- \to \; hZ,\; W^+W^- \qquad \text{(incl. angular distributions)}$$
$$h\nu\bar{\nu},\; ht\bar{t},\; hhZ,\; hh\nu\bar{\nu}$$
$$h \to ZZ^*,\; WW^*,\; \gamma\gamma,\; \gamma Z,\; gg,\; b\bar{b},\; c\bar{c},\; \tau^+\tau^-,\; \mu^+\mu^-$$

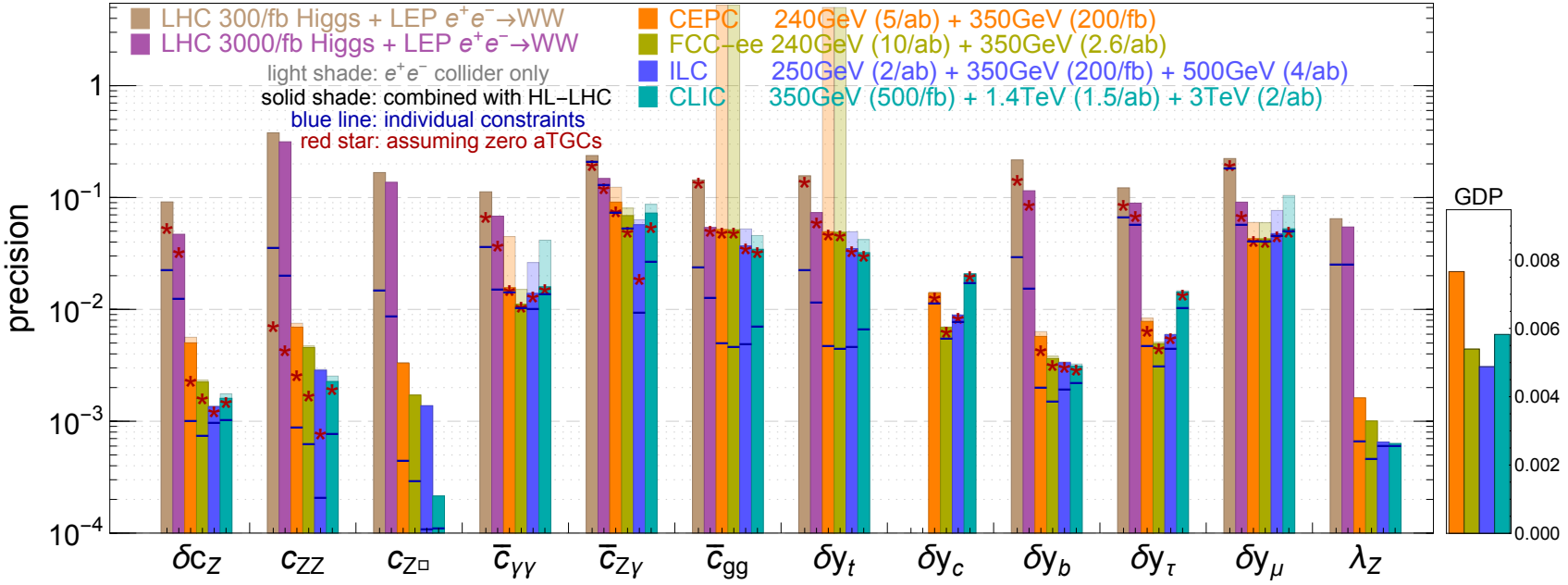
- only relax flavour universality to distinguish Yukawa's
- assume CPV, EW parameters, dipole operators are well constrained

$\longrightarrow$ 13 EFT d.o.f.:

$\Gamma_{xy}/\Gamma_{xy}^{\rm SM} \sim 1 \pm 2\bar{c}_{xy} + \ldots$

$$\delta c_Z\,,\quad c_{ZZ}\,,\quad c_{Z\Box}\,,$$
$$\bar{c}_{\gamma\gamma}\,,\quad \bar{c}_{Z\gamma}\,,\quad \bar{c}_{gg}\,,$$
$$\delta y_t\,,\quad \delta y_c\,,\quad \delta y_b\,,\quad \delta y_\tau\,,\quad \delta y_\mu\,,$$
$$\lambda_Z\,, \delta\kappa_\lambda$$

Global constraints, without Higgs self-coupling



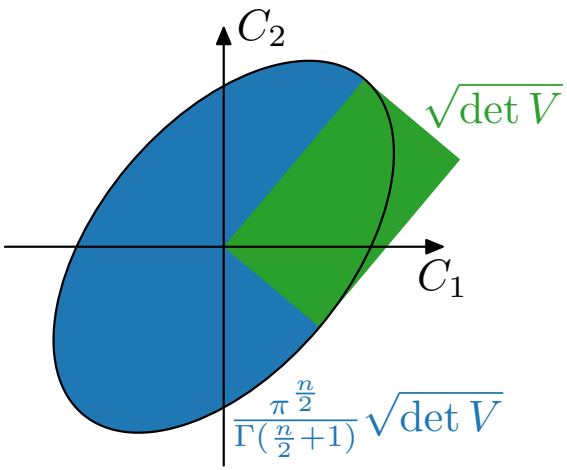
- importance of complementary measurements
  (different c.o.m. energies, polarizations, distributions)
- importance of diboson measurement precision
  (not studied much by exp. collaborations)
- order of magnitude improvement wrt LHC
  (especially on δc_Z, δc_{ZZ}, $\delta c_{Z\Box}$, δy_b, δy_τ, λ_Z)
- LHC helps for $\bar{c}_{\gamma\gamma}$, δy_μ, and δy_t (below 500 GeV!)

Global determinant parameter

In a n-dimensional Gaussian fit, with covariance matrix V,

$$\text{GDP} \equiv \sqrt[2n]{\det V}$$

provides a geometric average of the constraints strength.



Interestingly, GDP ratios are operator-basis independent!

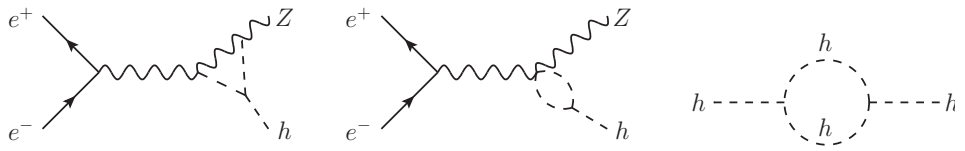
- as the volume scales linearly with coefficient normalization
- as the volume is invariant under rotations

$\Longrightarrow$ convenient to assess constraints strengthening

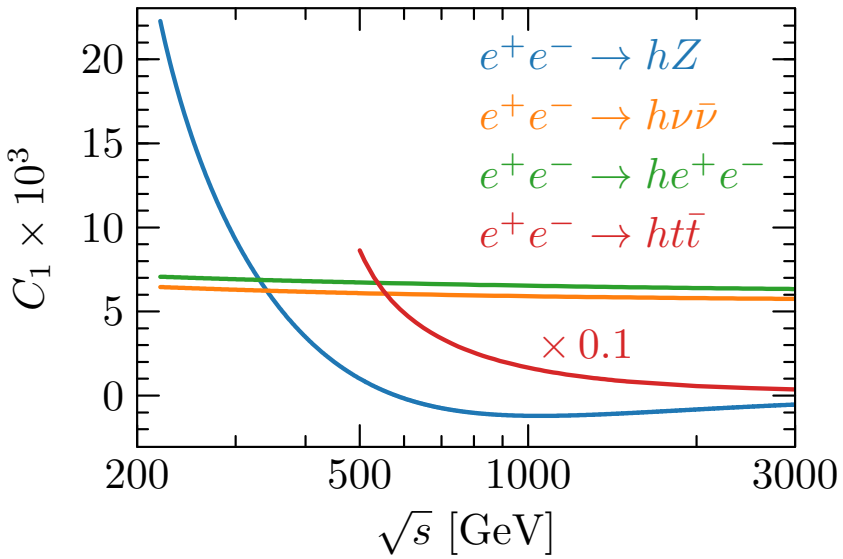
Higgs self-coupling at low energies

- NLO sensitivity (finite and gauge-invariant NLO EW subset)
- dominated by $e^+e^- \to hZ$ at threshold

[McCullough '13]
[Gorbahn, Haisch '16]
[Degrassi et al. '16]
[Bizon et al '16]
[Degrassi et al. '17]
[Kribs et al '17]

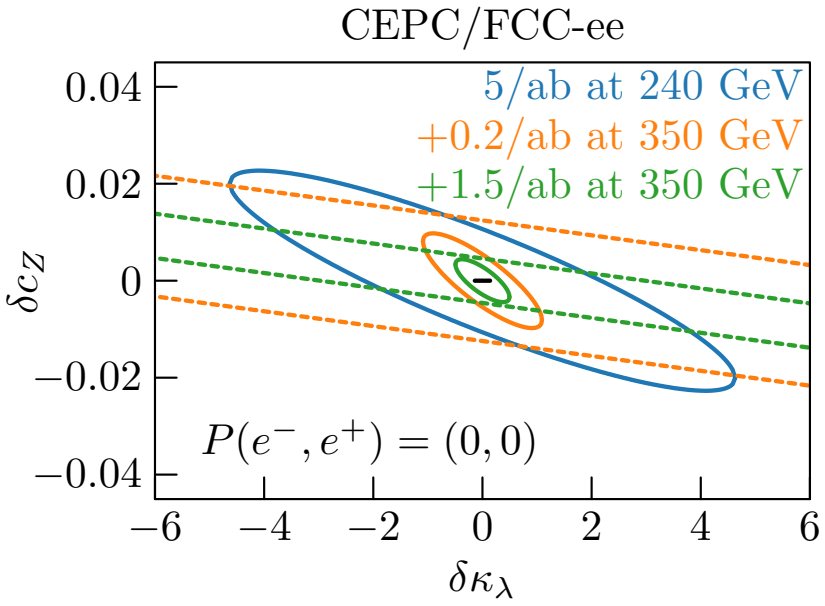


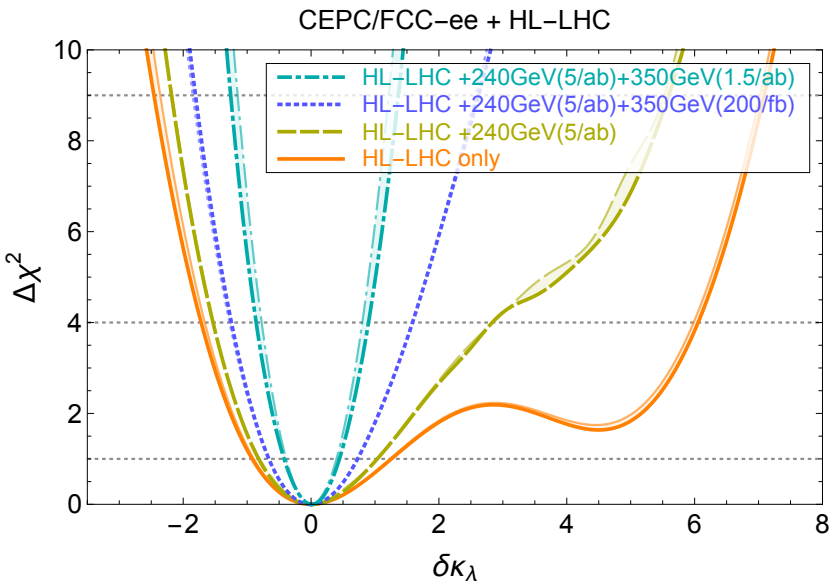
$$\Sigma_{\rm NLO}/\Sigma_{\rm NLO}^{\rm SM} \simeq 1 + (C_1 - 0.0031)\,\delta\kappa_\lambda + \ldots$$



$\to$ few permil hZ measurement naively implies a few 10% constraint

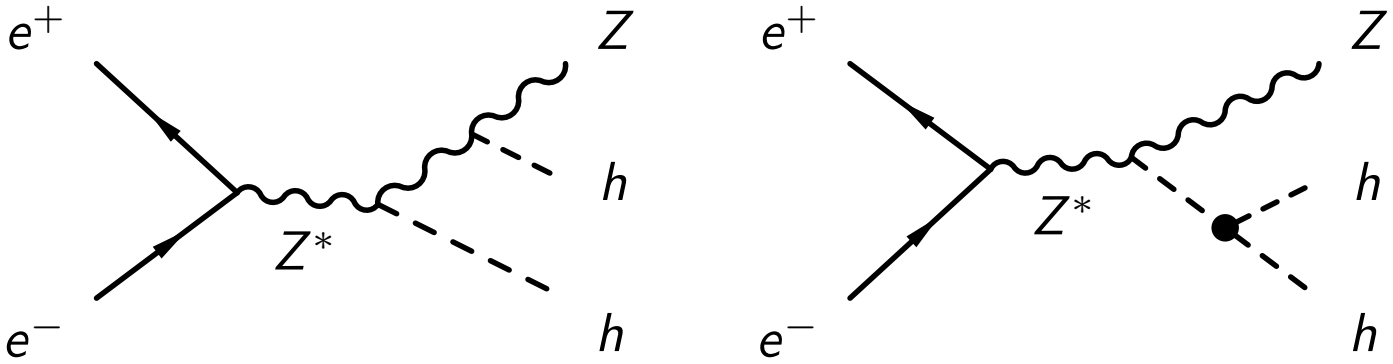
- individual $\Delta\chi^2$=1 limit (14%) much tighter than global ones (460, 110, 50%)
- 350 GeV run necessary to lift approximate degeneracies, without LHC



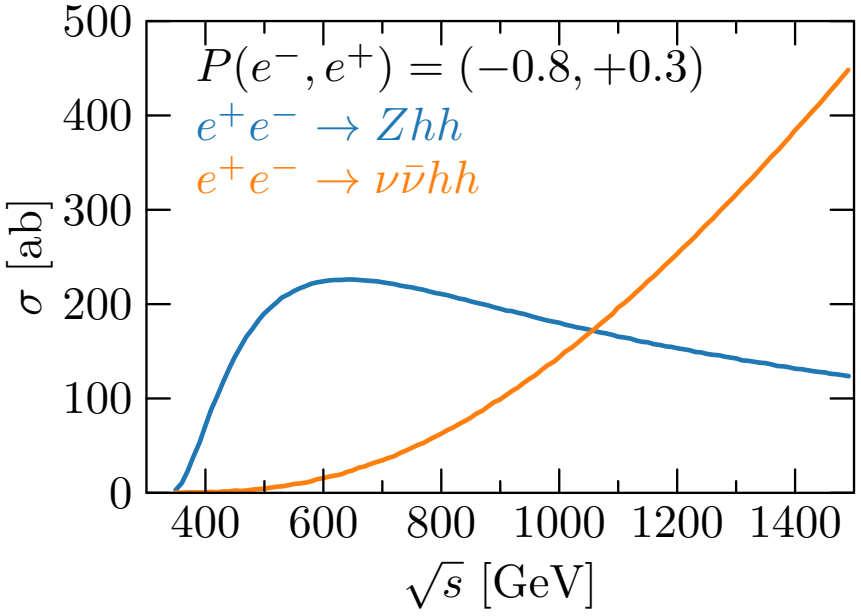


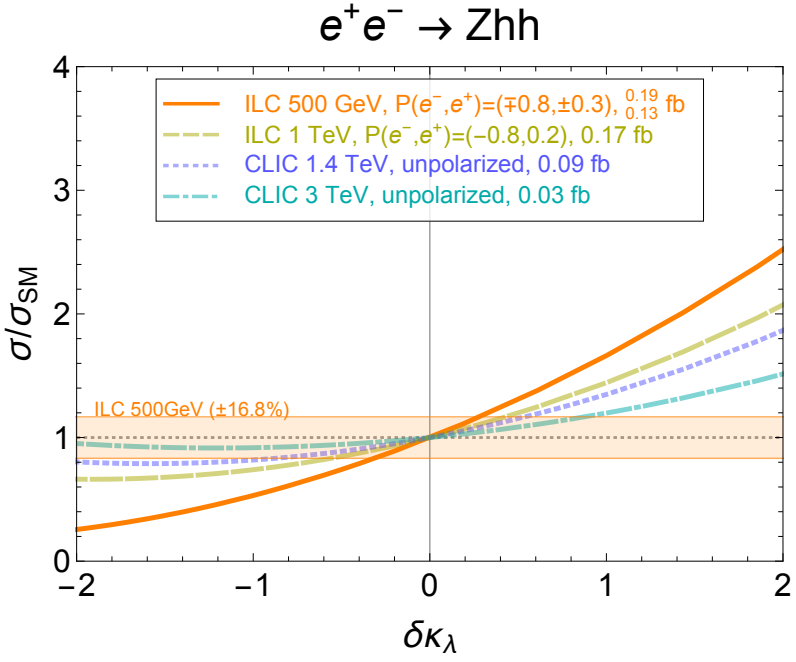
- second LHC minimum already resolved by a 240 GeV run
- constraints dominated by lepton colliders for 1.5 ab^{-1} at 350 GeV ($\sim$ 40%)

Higgs self-coupling at high energies



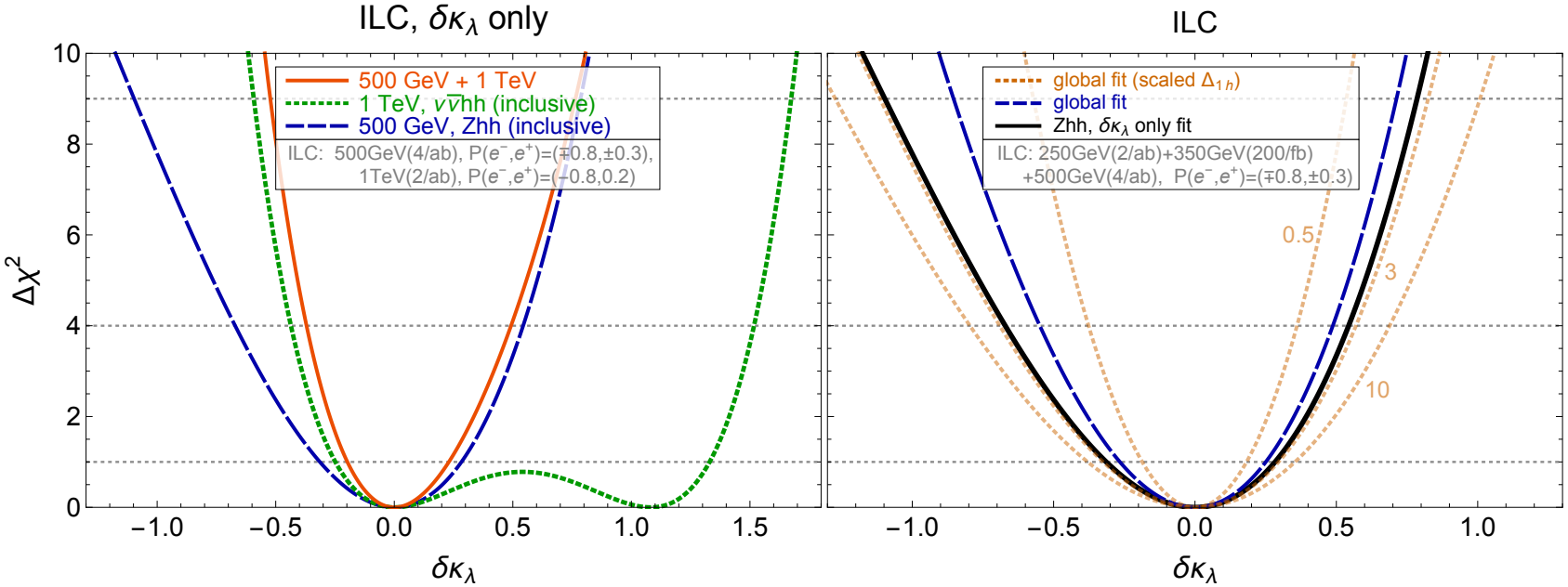
- two hh production modes: double Higgsstrahlung and WW-fusion
- sensitivity to $\delta\kappa_\lambda$ decreases with $\sqrt{s}$





ILC

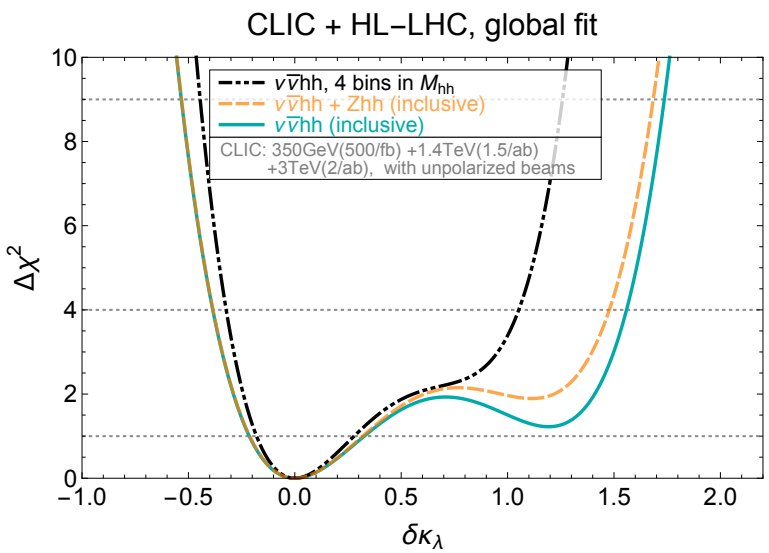
- perfect complementarity between 500 GeV and 1 TeV runs
- both individual and global $\Delta\chi^2$=1 limits $\sim$ 20%
- though, single Higgs measurements could have an impact

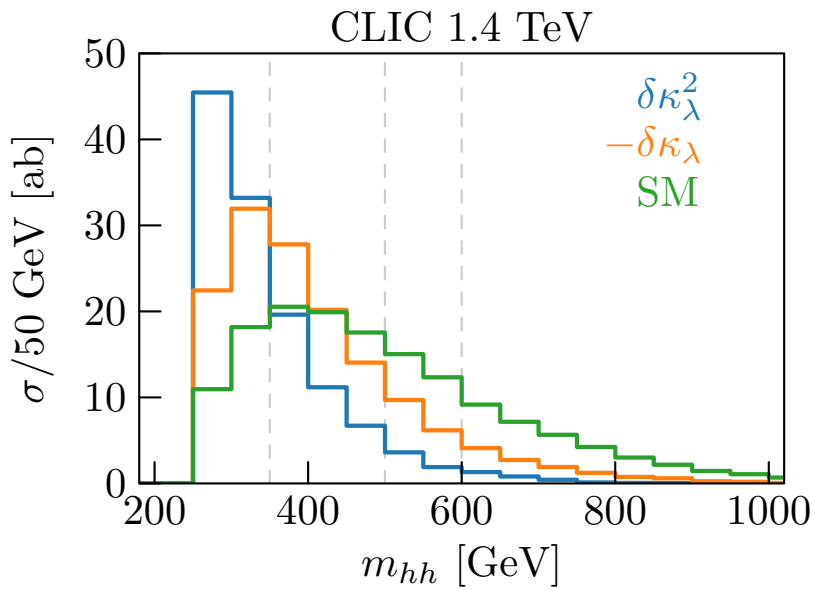


CLIC

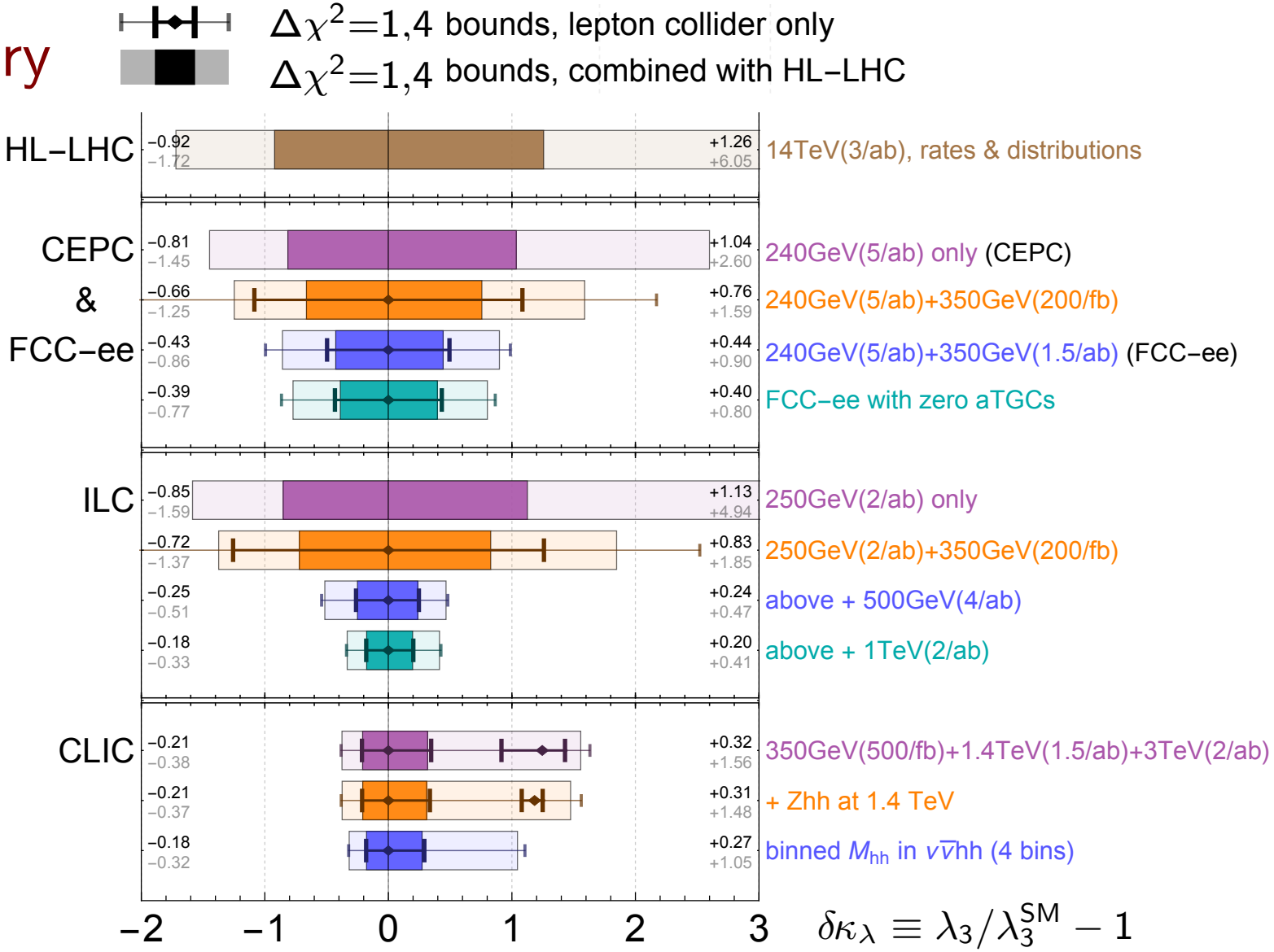
- missing $e^+e^- \to Zhh$ to constrain positive $\delta\kappa_\lambda$
- exploiting m_{hh} invariant mass, instead
- both individual and global $\Delta\chi^2$=1 limits $\sim -20, +30\%$

[Contino et al '13]





Summary



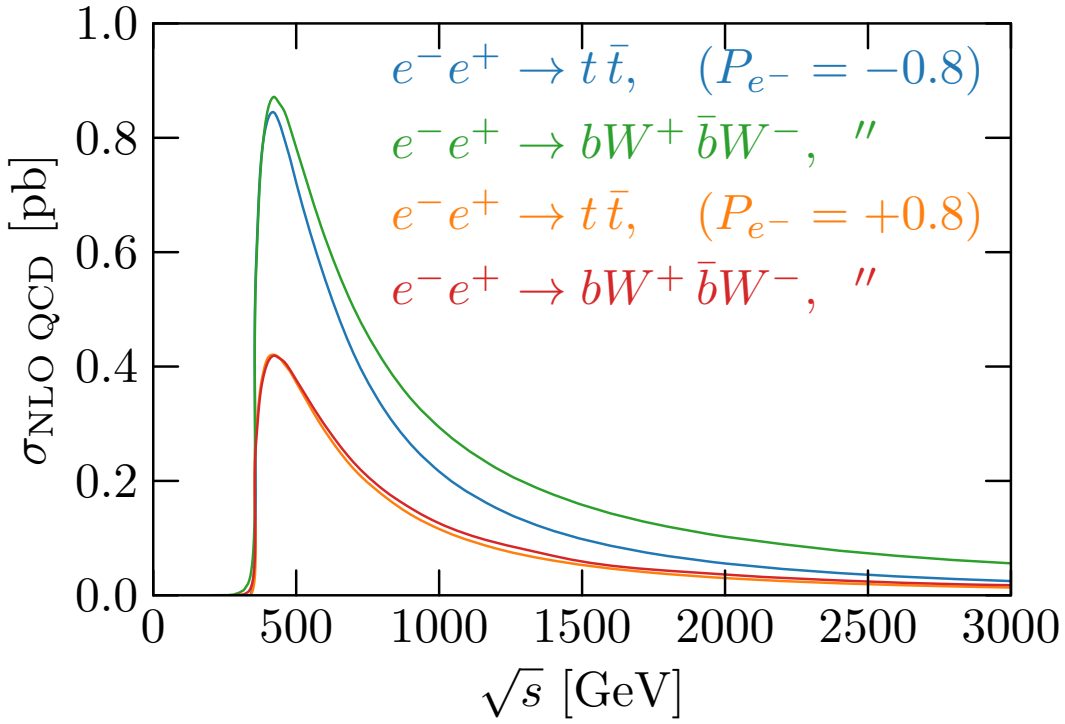
- robust indirect constraints at low energy require a global analysis
  - → $\sim 75\%$ precision with $0.2\,\mathrm{ab}^{-1}$ at $350\,\mathrm{GeV}$, $\sim 40\%$ with $1.5\,\mathrm{ab}^{-1}$
- single-Higgs measurements could affect direct high-energy determinations
  - → $\sim 20\%$ precision with $500\,\mathrm{GeV} + 1\,\mathrm{TeV}$ runs

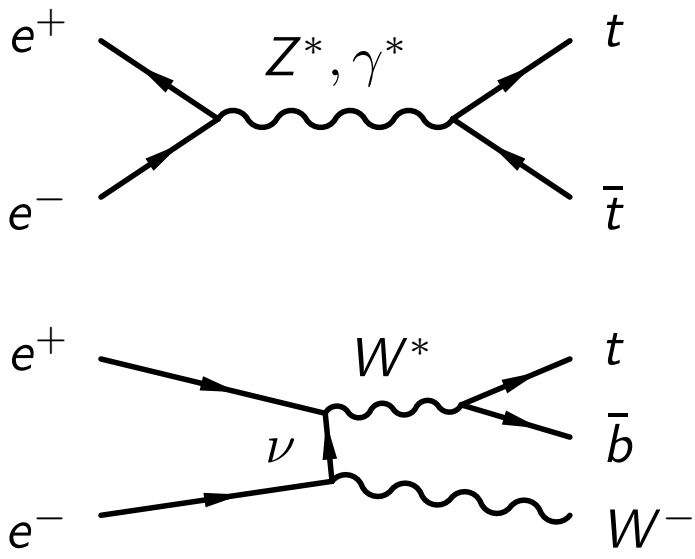
4. Global top-quark analysis

Pair production

[1807.02121]

Top-quark pair production





$+\ W^+W^- \to t\,\bar{t}$
catching up at multi-TeV
w/ unitarity breaking effects
[Grojean, Wulzer, You, Zhang]

- σ peaked at about 380 GeV
- enhanced for a left-handed beam
- fall-off as $1/s$
- single-top contribution increasingly important

LHC TOP WG EFT standards

[1802.07237]

Joint theory effort under the auspices of the LHC TOP WG, with extensive feedback from experimentalists.

Make reasonable assumptions

- focus a priori on processes and operators involving top quarks
- determine which contributions are relevant
- prioritize the study of flavour structures

Fix notation

- define d.o.f. natural for top physics at the LHC
- fix notation, normalization, and indicative allowed ranges
- provide simulation tools as TH/EXP interface [UFO model]

Discuss analysis strategies (one example)

- address the challenges of a global EFT
- highlight useful experimental outputs

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Translate your results in these standards to ease comparisons and combinations!

Discuss analysis strategies (one example)

- address the challenges of a global EFT
- highlight useful experimental outputs

Up-sector SMEFT

[Grzadkowski et al '10]

$$\mathcal{L}_{\text{EFT}} = \sum_i \frac{C_i}{\Lambda^2} O_i$$

Two-quark operators:

Scalar:	$O_{u\varphi} \equiv \bar{q} u \, \tilde{\varphi} \quad \varphi^\dagger \varphi,$		
Vector:	$O^1_{\varphi q} \equiv \bar{q}\gamma^\mu q \quad \varphi^\dagger i\overleftrightarrow{D}_\mu \varphi$	$\equiv O^+_{\varphi q} + O^V_{\varphi q} - O^A_{\varphi q},$	
	$O^3_{\varphi q} \equiv \bar{q}\gamma^\mu \tau^I q \quad \varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi$	$\equiv O^+_{\varphi q} - O^V_{\varphi q} + O^A_{\varphi q}$	(CC also)
	$O_{\varphi u} \equiv \bar{u}\gamma^\mu u \quad \varphi^\dagger i\overleftrightarrow{D}_\mu \varphi$	$\equiv O^V_{\varphi q} + O^A_{\varphi q}$	
	$O_{\varphi ud} \equiv \bar{u}\gamma^\mu d \quad \tilde{\varphi}^\dagger i\overleftrightarrow{D}_\mu \varphi,$		(CC only, m_b int.)
Tensor:	$O_{uB} \equiv \bar{q}\sigma^{\mu\nu} u \, \tilde{\varphi} \, g_Y B_{\mu\nu},$	$\equiv O_{uA} - \tan\theta_W O_{uZ}$	
	$O_{uW} \equiv \bar{q}\sigma^{\mu\nu}\tau^I u \, \tilde{\varphi} \, g_W W^I_{\mu\nu},$	$\equiv O_{uA} + \text{cotan}\,\theta_W O_{uZ}$	(CC also)
	$O_{dW} \equiv \bar{q}\sigma^{\mu\nu}\tau^I d \, \tilde{\varphi} \, g_W W^I_{\mu\nu},$		(CC only, m_b int.)
	$O_{uG} \equiv \bar{q}\sigma^{\mu\nu} T^A u \, \tilde{\varphi} \, g_s G^A_{\mu\nu}.$		(NLO only)

Two-quark–two-lepton operators:

Scalar:	$O^S_{lequ} \equiv \bar{l} e \;\; \varepsilon \;\; \bar{q} u,$		(CC also, m_e int.)
	$O_{ledq} \equiv \bar{l} e \quad \bar{d} q,$		(CC only, m_e int.)
Vector:	$O^1_{lq} \equiv \bar{l}\gamma_\mu l \quad \bar{q}\gamma^\mu q$	$\equiv O^+_{lq} + O^V_{lq} - O^A_{lq},$	
	$O^3_{lq} \equiv \bar{l}\gamma_\mu \tau^I l \quad \bar{q}\gamma^\mu \tau^I q$	$\equiv O^+_{lq} - O^V_{lq} + O^A_{lq},$	(CC also)
	$O_{lu} \equiv \bar{l}\gamma_\mu l \quad \bar{u}\gamma^\mu u$	$\equiv O^V_{lq} + O^A_{lq},$	
	$O_{eq} \equiv \bar{e}\gamma^\mu e \quad \bar{q}\gamma_\mu q$	$\equiv O^V_{eq} - O^A_{eq},$	
	$O_{eu} \equiv \bar{e}\gamma_\mu e \quad \bar{u}\gamma^\mu u$	$\equiv O^V_{eq} + O^A_{eq},$	
Tensor:	$O^T_{lequ} \equiv \bar{l}\sigma_{\mu\nu} e \quad \varepsilon \quad \bar{q}\sigma^{\mu\nu} u.$		(CC also, m_e int.)

Up-sector SMEFT

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Vector:

$$O^1_{\varphi q} \equiv \bar{q}\gamma^\mu q \quad \varphi^\dagger i\overleftrightarrow{D}_\mu \varphi \quad \equiv O^+_{\varphi q} + O^V_{\varphi q} - O^A_{\varphi q},$$

$$O^3_{\varphi q} \equiv \bar{q}\gamma^\mu \tau^I q \quad \varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi \quad \equiv O^+_{\varphi q} - O^V_{\varphi q} + O^A_{\varphi q} \qquad \text{(CC also)}$$

$$O_{\varphi u} \equiv \bar{u}\gamma^\mu u \quad \varphi^\dagger i\overleftrightarrow{D}_\mu \varphi \quad \equiv O^V_{\varphi q} + O^A_{\varphi q}$$

$$O_{\varphi ud} \equiv \bar{u}\gamma^\mu d \quad \tilde{\varphi}^\dagger i\overleftrightarrow{D}_\mu \varphi, \qquad \text{(CC only, } m_b \text{ int.)}$$

Tensor:

$$O_{uB} \equiv \bar{q}\sigma^{\mu\nu} u \, \tilde{\varphi} \, g_Y B_{\mu\nu}, \quad \equiv O_{uA} - \tan\theta_W O_{uZ}$$

$$O_{uW} \equiv \bar{q}\sigma^{\mu\nu} \tau^I u \, \tilde{\varphi} \, g_W W^I_{\mu\nu}, \quad \equiv O_{uA} + \cot\theta_W O_{uZ} \qquad \text{(CC also)}$$

$$O_{dW} \equiv \bar{q}\sigma^{\mu\nu} \tau^I d \, \tilde{\varphi} \, g_W W^I_{\mu\nu}, \qquad \text{(CC only, } m_b \text{ int.)}$$

$$O_{uG} \equiv \bar{q}\sigma^{\mu\nu} T^A u \, \tilde{\varphi} \, g_s G^A_{\mu\nu} \qquad \text{(NLO only)}$$

8+2 CPV dof's probed linearly in $e^+e^- \to t\bar{t} \to bW^+\bar{b}W^-$ at LO and in the $m_b/m_t \to 0$ limit.

Two-quark–two-lepton operators:

Scalar:

$$O^S_{lequ} \equiv \bar{l} e \; \varepsilon \; \bar{q} u, \qquad \text{(CC also, } m_e \text{ int.)}$$

$$O_{ledq} \equiv \bar{l} e \quad \bar{d} q, \qquad \text{(CC only, } m_e \text{ int.)}$$

Vector:

$$O^1_{lq} \equiv \bar{l}\gamma_\mu l \quad \bar{q}\gamma^\mu q \quad \equiv O^+_{lq} + O^V_{lq} - O^A_{lq},$$

$$O^3_{lq} \equiv \bar{l}\gamma_\mu \tau^I l \quad \bar{q}\gamma^\mu \tau^I q \quad \equiv O^+_{lq} - O^V_{lq} + O^A_{lq}, \qquad \text{(CC also)}$$

$$O_{lu} \equiv \bar{l}\gamma_\mu l \quad \bar{u}\gamma^\mu u \quad \equiv O^V_{lq} + O^A_{lq},$$

$$O_{eq} \equiv \bar{e}\gamma^\mu e \quad \bar{q}\gamma_\mu q \quad \equiv O^V_{eq} - O^A_{eq},$$

$$O_{eu} \equiv \bar{e}\gamma_\mu e \quad \bar{u}\gamma^\mu u \quad \equiv O^V_{eq} + O^A_{eq},$$

Tensor:

$$O^T_{lequ} \equiv \bar{l}\sigma_{\mu\nu} e \quad \varepsilon \quad \bar{q}\sigma^{\mu\nu} u. \qquad \text{(CC also, } m_e \text{ int.)}$$

Anomalous vertices

$$t\bar{t}\gamma: \quad \gamma_\mu \overbrace{\left(F_{1V}^{\gamma} + \gamma_5 F_{1A}^{\gamma}\right)}^{\sim\phi} + \frac{\sigma_{\mu\nu} i q^\nu}{2m_t} \overbrace{\left(F_{2V}^{\gamma} + i\gamma_5 F_{2A}^{\gamma}\right)}^{\sim\mathrm{Re,Im}\{C_{uA}\}}$$

$$t\bar{t}Z: \quad \gamma_\mu \overbrace{\left(F_{1V}^{Z} + \gamma_5 F_{1A}^{Z}\right)}^{\sim C_{\varphi q}^{V}, C_{\varphi q}^{A}} + \frac{\sigma_{\mu\nu} i q^\nu}{2m_t} \overbrace{\left(F_{2V}^{Z} + i\gamma_5 F_{2A}^{Z}\right)}^{\sim\mathrm{Re,Im}\{C_{uZ}\}}$$

$$t\bar{b}W: \quad \gamma_\mu \overbrace{\left(F_{1V}^{W} + \gamma_5 F_{1A}^{W}\right)}^{\sim C_{\varphi q}^{+} - \frac{1}{2}(C_{\varphi q}^{V} - C_{\varphi q}^{A}) \pm C_{\varphi ud}} + \frac{\sigma_{\mu\nu} i q^\nu}{2m_t} \overbrace{\left(F_{2V}^{W} + i\gamma_5 F_{2A}^{W}\right)}^{\sim s_W^2 C_{uA} + s_W c_W C_{uZ} \pm C_{dW}^{*}}$$

Insufficiencies:

- miss four-fermion operators,
- conflict with gauge invariance,
  do not allow for radiative corrections to be computed,
- complex couplings where the tree-level EFT prescribes real ones,
- hide correlations induced by gauge invariance,
  preclude the combination of measurements in various sectors

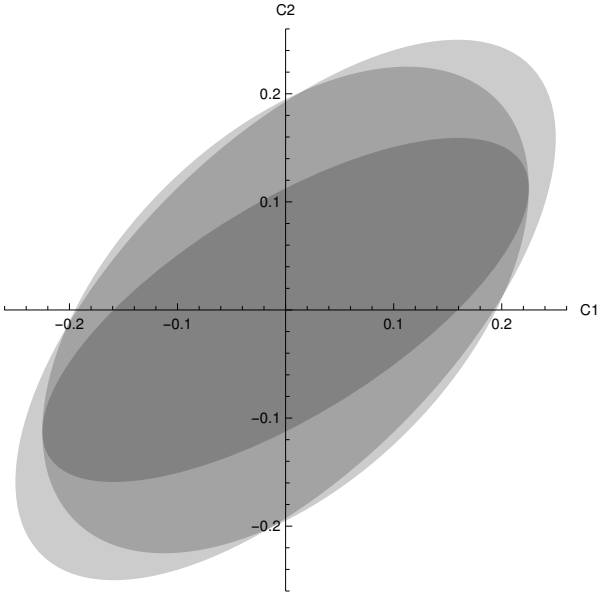
Statistically optimal observables

[Atwood,Soni '92]
[Diehl,Nachtmann '94]

minimize the one-sigma ellipsoid in EFT parameter space

(*joint efficient* set of estimators, saturating the Cramér-Rao bound: $V^{-1} = I$,like MEM)

For small C_i, with a phase-space distribution $\sigma(\Phi) = \sigma_0(\Phi) + \sum_i C_i\, \sigma_i(\Phi)$, the stat. opt. obs. are the average values of $O_i(\Phi) = \sigma_i(\Phi)/\sigma_0(\Phi)$.



e.g. $\sigma(\phi) = 1 + \cos(\phi) + C_1 \sin(\phi) + C_2 \sin(2\phi)$

1. asymmetries: $O_i \sim \text{sign}\{\sin(i\phi)\}$
2. moments: $O_i \sim \sin(i\phi)$
3. statistically optimal: $O_i \sim \dfrac{\sin(i\phi)}{1 + \cos\phi}$

$\Longrightarrow$ area ratios 1.9 : 1.7 : 1

Previous applications in $e^+e^- \to t\,\bar{t}$, on different distributions:
[Grzadkowski, Hioki '00] [Janot '15] [Khiem et al '15]

Setup details

resonant $e^+e^- \to t\,\bar{t} \to bW^+\,\bar{b}W^-$
$m_b/m_t \to 0$, analytical LO observable def.
effective stat. efficiencies determined with full sim.
in semi-leptonic final state

CLIC-like scenario

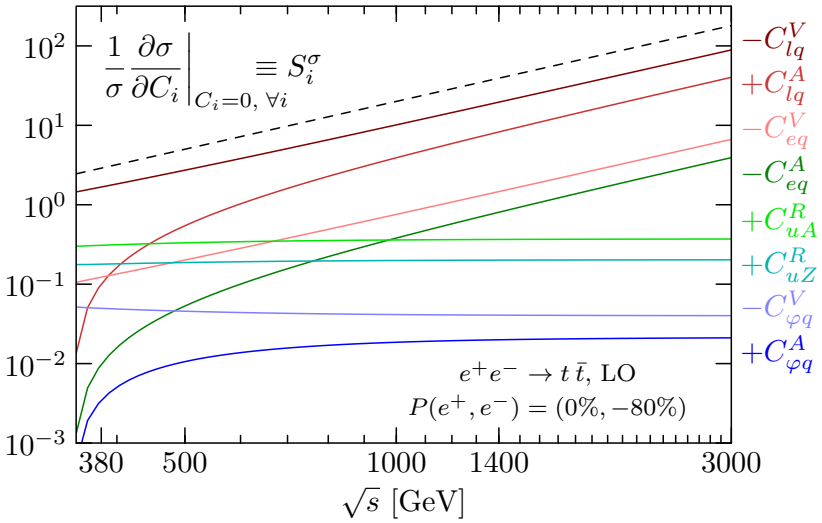
$500\,\mathrm{fb}^{-1}$ at $\sqrt{s} = 380\,\mathrm{GeV}$
$1.5\,\mathrm{ab}^{-1}$ at $\sqrt{s} = 1.4\,\mathrm{TeV}$
$3\,\mathrm{ab}^{-1}$ at $\sqrt{s} = 3\,\mathrm{TeV}$

shared btw $P(e^+, e^-) = (0\%, \mp 80\%)$

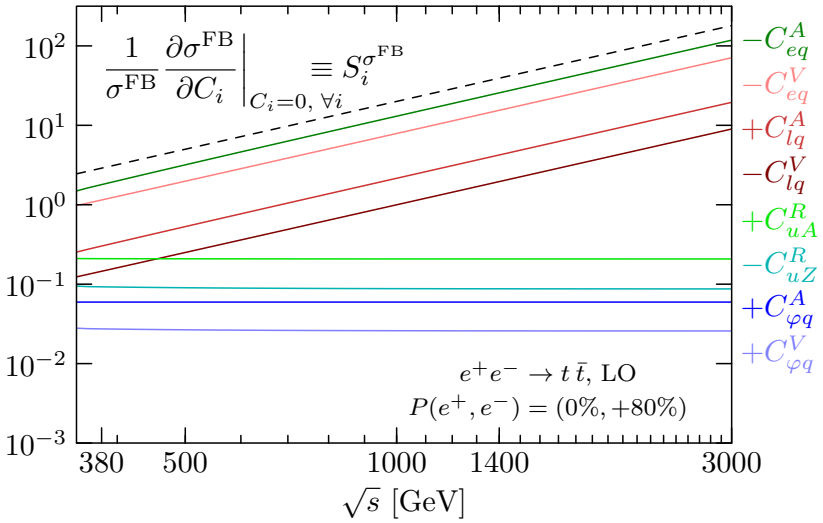
Newest CLIC luminosities: 1, 2.5, $5\,\mathrm{ab}^{-1}$, with a 80/20% share btw polarizations at high energies.
[LCWS, 22 Oct. 2018]

Sensitivity vs. $\sqrt{s}$

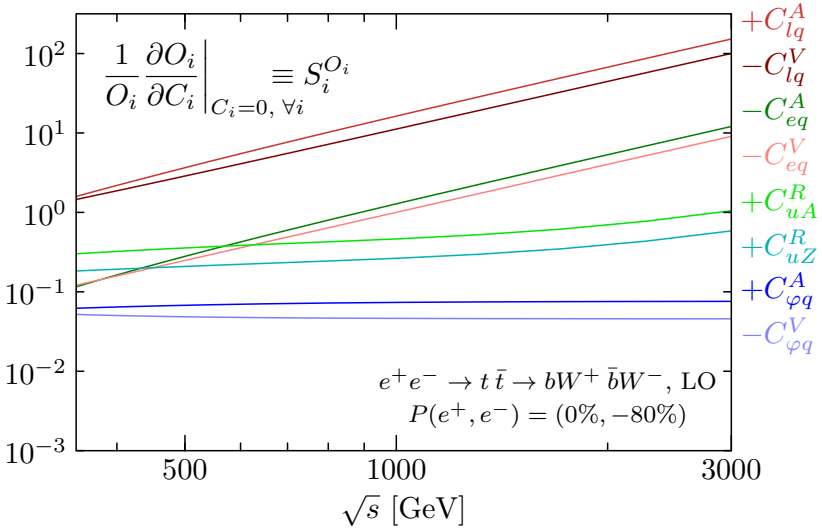
Total cross section (left pol.):

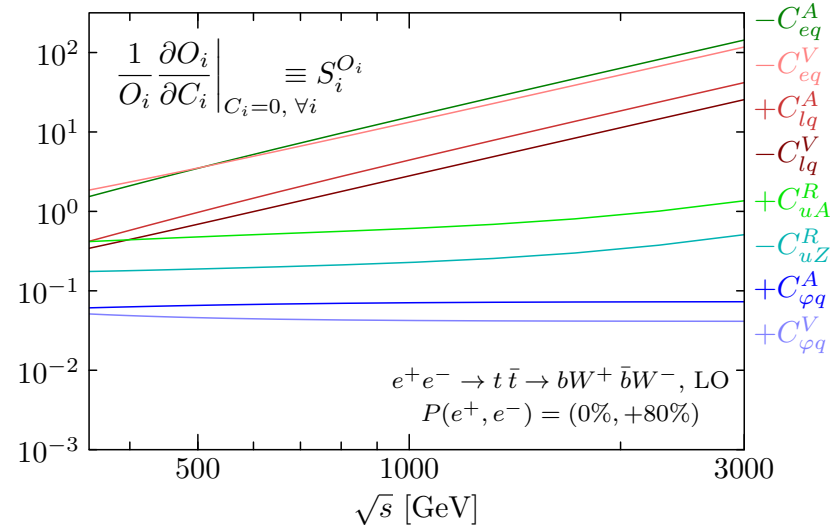


FB-integrated cross section (right pol.):



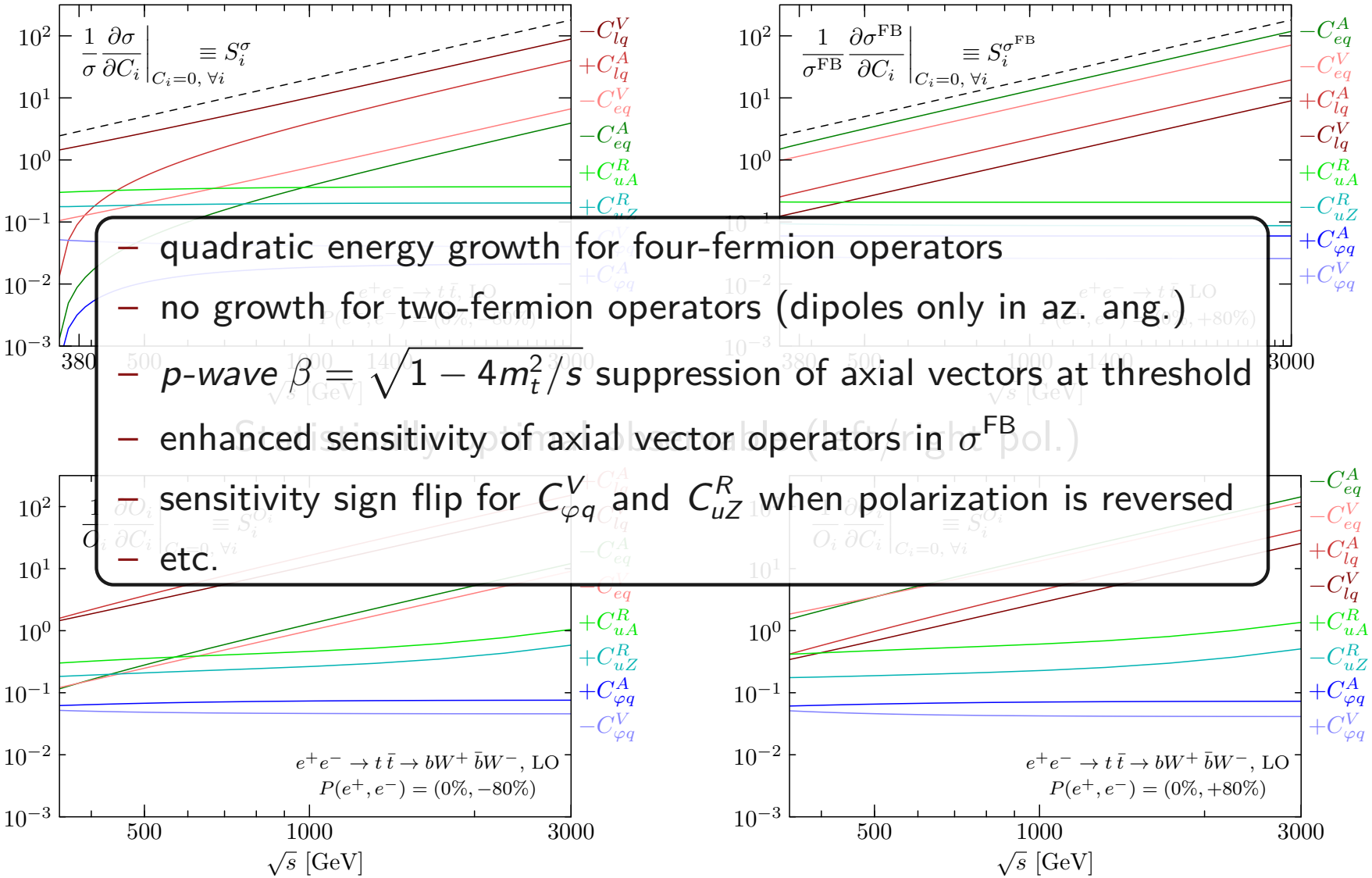
Statistically optimal observable (left/right pol.)





Sensitivity vs. $\sqrt{s}$

Total cross section (left pol.): FB-integrated cross section (right pol.):



- quadratic energy growth for four-fermion operators
- no growth for two-fermion operators (dipoles only in az. ang.)
- *p-wave* $\beta = \sqrt{1 - 4m_t^2/s}$ suppression of axial vectors at threshold
- enhanced sensitivity of axial vector operators in $\sigma^{\rm FB}$
- sensitivity sign flip for $C_{\varphi q}^V$ and C_{uZ}^R when polarization is reversed
- etc.

Helicity amplitude decomposition in $bW^+\bar{b}W^-$

[Jacob,Wick '59]

[Schmidt '95]

Production amplitudes:

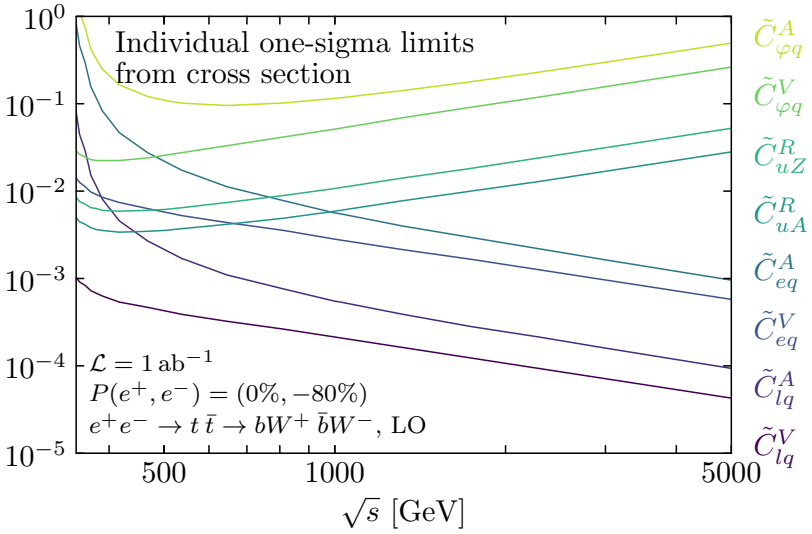
$$
\begin{aligned}
++ &: A_1 \sim \frac{2m_t}{\sqrt{s}} V + \sqrt{s}\,(D - \beta\tilde{D}) \\
-- &: A_2 \sim \frac{2m_t}{\sqrt{s}} V + \sqrt{s}\,(D + \beta\tilde{D}) \\
+- &: A_3 \sim (V + \beta A) + 2m_t D \\
-+ &: A_4 \sim (V - \beta A) + 2m_t D
\end{aligned}
$$

In terms of $\Omega = \{\theta_0, \theta_1, \phi_1, \theta_2, \phi_2\}$ helicity angles:

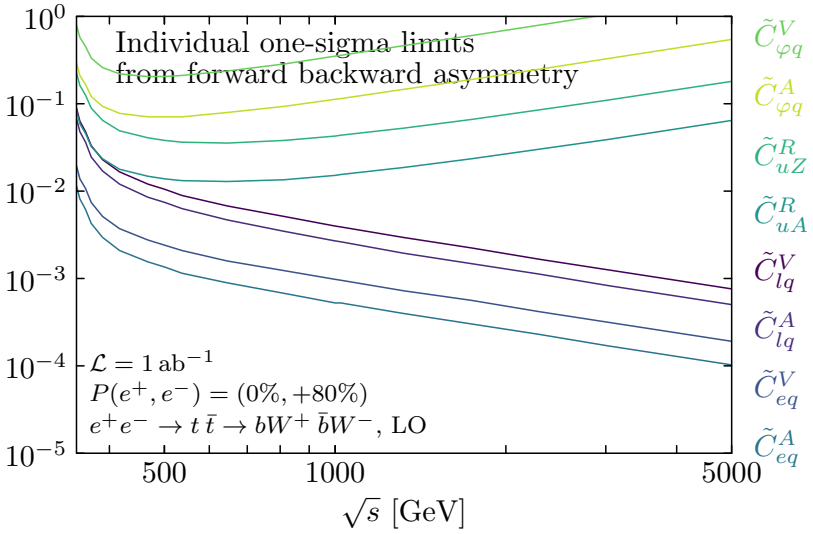
$$
\frac{d\sigma}{d\Omega} \propto
\begin{array}{lllllllll}
+3/4 & & (|A_3|^2 + |A_4|^2) & |a_2|^2 + |a_4|^2 & |b_1|^2 + |b_3|^2 & (1+\cos^2\theta_0) & & & \\
+3/4 & & (|A_3|^2 - |A_4|^2) & |a_2|^2 + |a_4|^2 & |b_1|^2 - |b_3|^2 & (1+\cos^2\theta_0) & & \cos\theta_2 & \\
+3/4 & & (|A_3|^2 - |A_4|^2) & |a_2|^2 - |a_4|^2 & |b_1|^2 + |b_3|^2 & (1+\cos^2\theta_0) & \cos\theta_1 & & \\
+3/4 & & (|A_3|^2 + |A_4|^2) & |a_2|^2 - |a_4|^2 & |b_1|^2 - |b_3|^2 & (1+\cos^2\theta_0) & \cos\theta_1 & \cos\theta_2 & \\
-3/2 & & (|A_3|^2 - |A_4|^2) & |a_2|^2 + |a_4|^2 & |b_1|^2 + |b_3|^2 & \cos\theta_0 & & & \\
-3/2 & & (|A_3|^2 + |A_4|^2) & |a_2|^2 + |a_4|^2 & |b_1|^2 - |b_3|^2 & \cos\theta_0 & & \cos\theta_2 & \\
-3/2 & & (|A_3|^2 + |A_4|^2) & |a_2|^2 - |a_4|^2 & |b_1|^2 + |b_3|^2 & \cos\theta_0 & \cos\theta_1 & & \\
-3/2 & & (|A_3|^2 - |A_4|^2) & |a_2|^2 - |a_4|^2 & |b_1|^2 - |b_3|^2 & \cos\theta_0 & \cos\theta_1 & \cos\theta_2 & \\
+3/2 & & (|A_1|^2 + |A_2|^2) & |a_2|^2 + |a_4|^2 & |b_1|^2 + |b_3|^2 & \sin^2\theta_0 & & & \\
-3/2 & & (|A_1|^2 - |A_2|^2) & |a_2|^2 + |a_4|^2 & |b_1|^2 - |b_3|^2 & \sin^2\theta_0 & & \cos\theta_2 & \\
+3/2 & & (|A_1|^2 - |A_2|^2) & |a_2|^2 - |a_4|^2 & |b_1|^2 + |b_3|^2 & \sin^2\theta_0 & \cos\theta_1 & & \\
-3/2 & & (|A_1|^2 + |A_2|^2) & |a_2|^2 - |a_4|^2 & |b_1|^2 - |b_3|^2 & \sin^2\theta_0 & \cos\theta_1 & \cos\theta_2 & \\
+3/2 & \sqrt{2} & \mathrm{Re}\{A_1^* A_4\} & |a_2|^2 - |a_4|^2 & |b_3|^2 & \sin\theta_0(1+\cos\theta_0) & \sin\theta_1 & (1+\cos\theta_2) & \cos\phi_1 \\
+3/2 & \sqrt{2} & \mathrm{Re}\{A_1^* A_4\} & |a_2|^2 - |a_4|^2 & |b_1|^2 & \sin\theta_0(1+\cos\theta_0) & \sin\theta_1 & (1-\cos\theta_2) & \cos\phi_1 \\
+3/2 & \sqrt{2} & \mathrm{Re}\{A_2^* A_3\} & |a_2|^2 - |a_4|^2 & |b_1|^2 & \sin\theta_0(1-\cos\theta_0) & \sin\theta_1 & (1+\cos\theta_2) & \cos\phi_1 \\
+3/2 & \sqrt{2} & \mathrm{Re}\{A_2^* A_3\} & |a_2|^2 - |a_4|^2 & |b_3|^2 & \sin\theta_0(1-\cos\theta_0) & \sin\theta_1 & (1-\cos\theta_2) & \cos\phi_1 \\
-3/2 & \sqrt{2} & \mathrm{Re}\{A_2^* A_4\} & |a_4|^2 & |b_1|^2 - |b_3|^2 & \sin\theta_0(1+\cos\theta_0) & (1+\cos\theta_1) & \sin\theta_2 & \cos\phi_2 \\
-3/2 & \sqrt{2} & \mathrm{Re}\{A_2^* A_4\} & |a_2|^2 & |b_1|^2 - |b_3|^2 & \sin\theta_0(1+\cos\theta_0) & (1-\cos\theta_1) & \sin\theta_2 & \cos\phi_2 \\
-3/2 & \sqrt{2} & \mathrm{Re}\{A_1^* A_3\} & |a_2|^2 & |b_1|^2 - |b_3|^2 & \sin\theta_0(1-\cos\theta_0) & (1+\cos\theta_1) & \sin\theta_2 & \cos\phi_2 \\
-3/2 & \sqrt{2} & \mathrm{Re}\{A_1^* A_3\} & |a_4|^2 & |b_1|^2 - |b_3|^2 & \sin\theta_0(1-\cos\theta_0) & (1-\cos\theta_1) & \sin\theta_2 & \cos\phi_2 \\
-3 & & \mathrm{Re}\{A_1^* A_2\} & |a_2|^2 - |a_4|^2 & |b_1|^2 - |b_3|^2 & \sin^2\theta_0 & \sin\theta_1 & \sin\theta_2 & \cos(\phi_1+\phi_2) \\
-3/2 & & \mathrm{Re}\{A_3^* A_4\} & |a_2|^2 - |a_4|^2 & |b_1|^2 - |b_3|^2 & \sin^2\theta_0 & \sin\theta_1 & \sin\theta_2 & \cos(\phi_1-\phi_2) \\
 & & & & & & & & \\
+3/2 & \sqrt{2} & \mathrm{Im}\{A_1^* A_4\} & |a_2|^2 - |a_4|^2 & |b_3|^2 & \sin\theta_0(1+\cos\theta_0) & \sin\theta_1 & (1+\cos\theta_2) & \sin\phi_1 \\
+3/2 & \sqrt{2} & \mathrm{Im}\{A_1^* A_4\} & |a_2|^2 - |a_4|^2 & |b_1|^2 & \sin\theta_0(1+\cos\theta_0) & \sin\theta_1 & (1-\cos\theta_2) & \sin\phi_1 \\
-3/2 & \sqrt{2} & \mathrm{Im}\{A_2^* A_3\} & |a_2|^2 - |a_4|^2 & |b_1|^2 & \sin\theta_0(1-\cos\theta_0) & \sin\theta_1 & (1+\cos\theta_2) & \sin\phi_1 \\
-3/2 & \sqrt{2} & \mathrm{Im}\{A_2^* A_3\} & |a_2|^2 - |a_4|^2 & |b_3|^2 & \sin\theta_0(1-\cos\theta_0) & \sin\theta_1 & (1-\cos\theta_2) & \sin\phi_1 \\
+3/2 & \sqrt{2} & \mathrm{Im}\{A_2^* A_4\} & |a_4|^2 & |b_1|^2 - |b_3|^2 & \sin\theta_0(1+\cos\theta_0) & (1+\cos\theta_1) & \sin\theta_2 & \sin\phi_2 \\
+3/2 & \sqrt{2} & \mathrm{Im}\{A_2^* A_4\} & |a_2|^2 & |b_1|^2 - |b_3|^2 & \sin\theta_0(1+\cos\theta_0) & (1-\cos\theta_1) & \sin\theta_2 & \sin\phi_2 \\
-3/2 & \sqrt{2} & \mathrm{Im}\{A_1^* A_3\} & |a_2|^2 & |b_1|^2 - |b_3|^2 & \sin\theta_0(1-\cos\theta_0) & (1+\cos\theta_1) & \sin\theta_2 & \sin\phi_2
\end{array}
$$

Individual limits vs. $\sqrt{s}$, for $1\,\mathrm{ab}^{-1}$

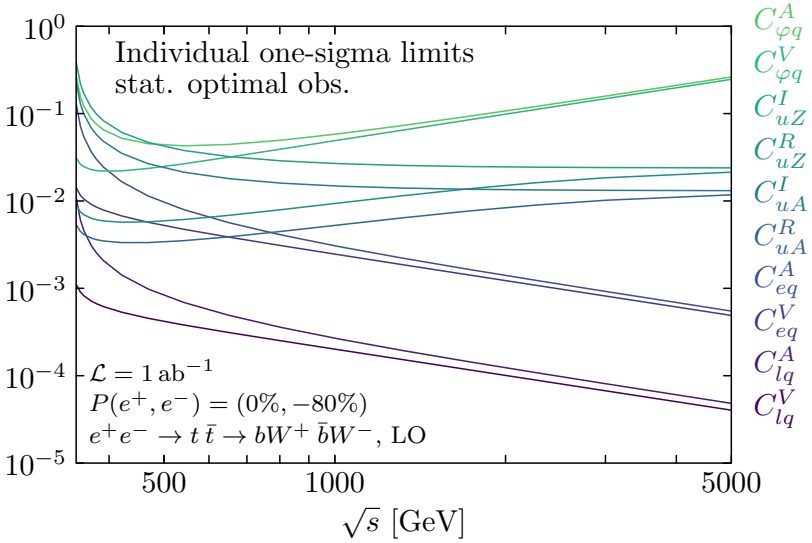
Total cross section (left pol.):

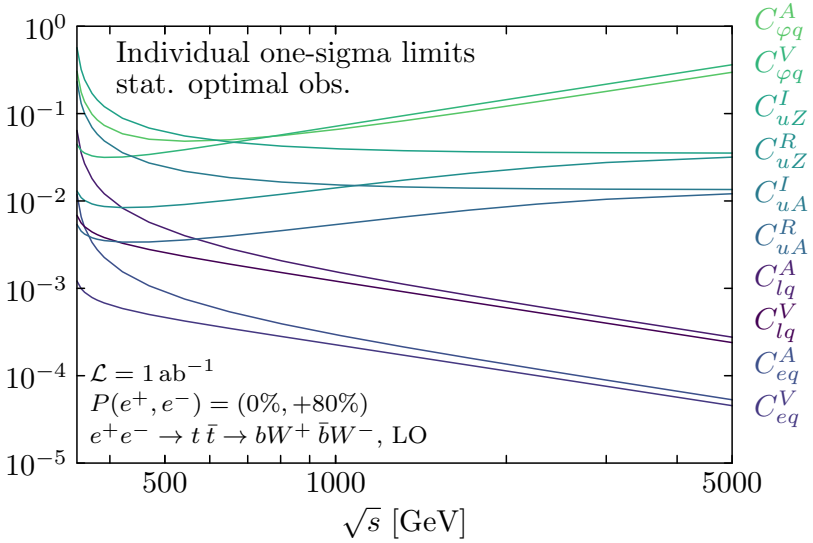


FB-integrated cross section (right pol.):



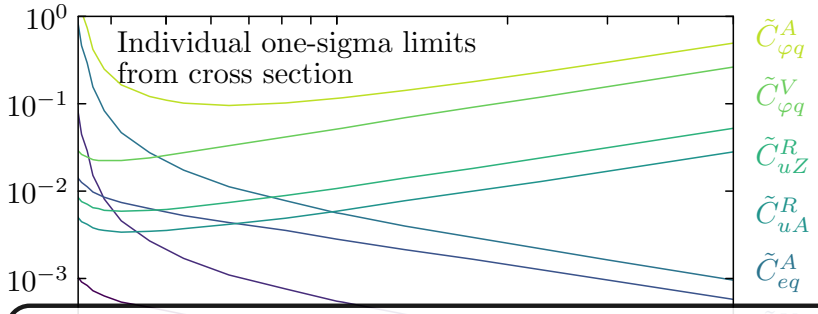
Statistically optimal observable (left/right pol.)



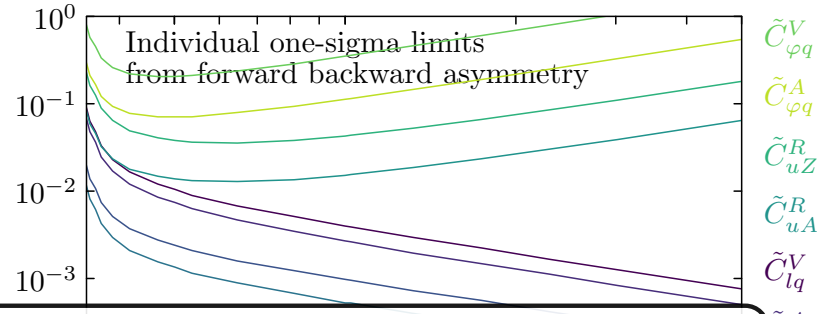


Individual limits vs. $\sqrt{s}$, for $1\,\mathrm{ab}^{-1}$

Total cross section (left pol.):

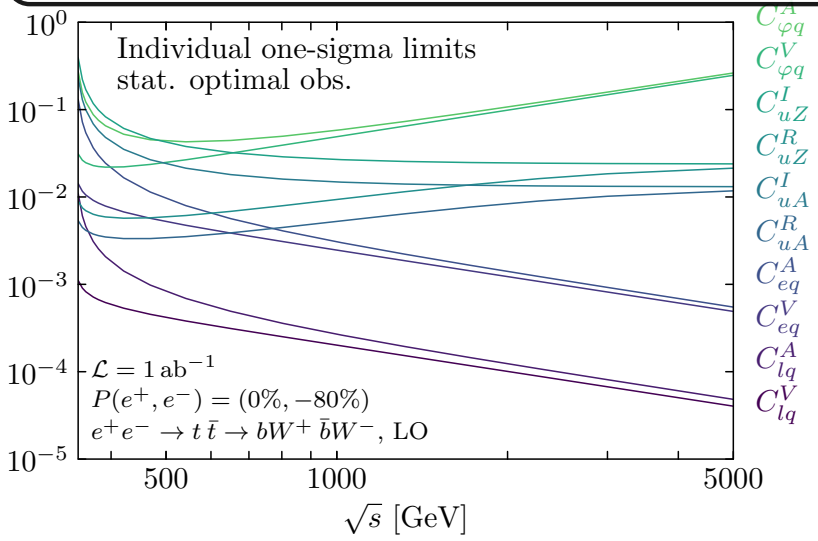


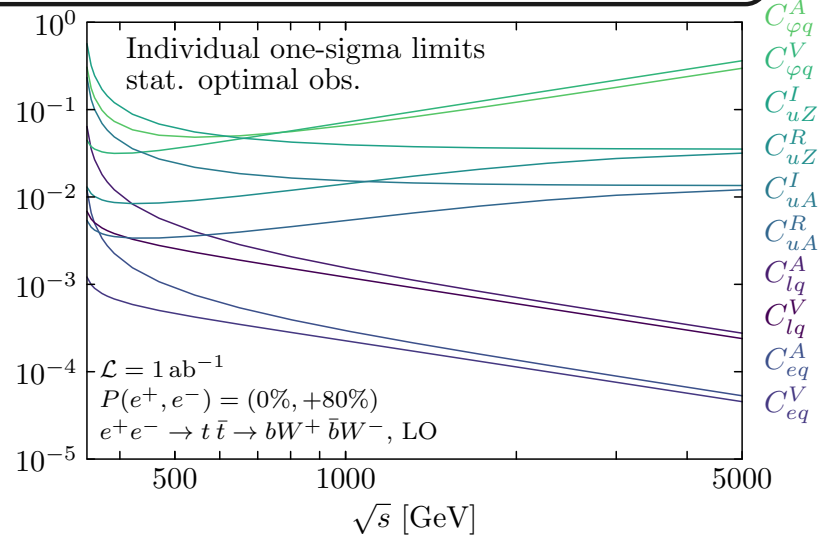
FB-integrated cross section (right pol.):



Energy helps
- a lot for four-fermion operators
- a bit for CPV dipoles
- only up to about 500 GeV for vectors and dipoles

(A luminosity growing linearly with $\sqrt{s}$ only changes this picture qualitatively above 3 TeV.)

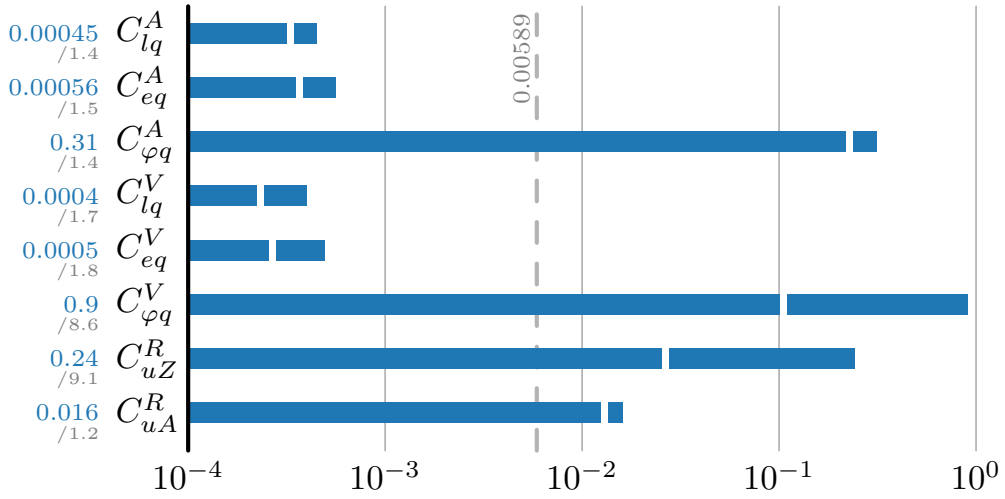


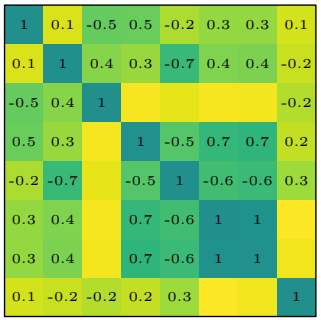


CLIC global reach

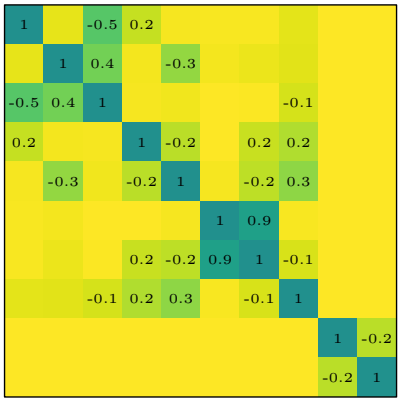
- in TeV^{-2}, $\Delta\chi^2 = 1$
- white marks: individual constraints
- dashed vertical lines: GDPs
- gray numbers: global/individual ratios

$\sigma + A^{\mathrm{FB}}$:

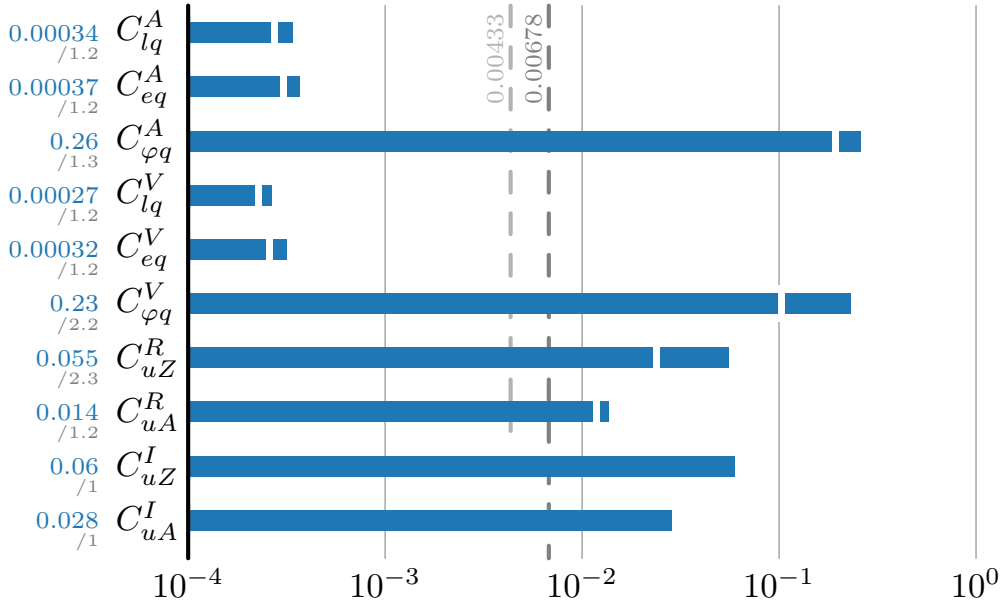




correlation matrices



Statistically optimal observables:



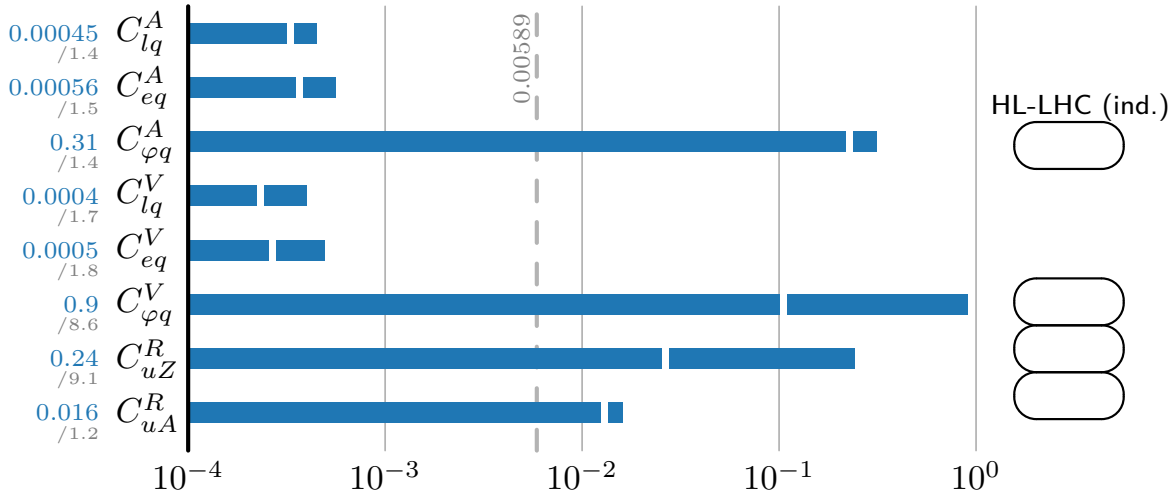
factor of 1.6 improvement of the 8-coefficient GDP

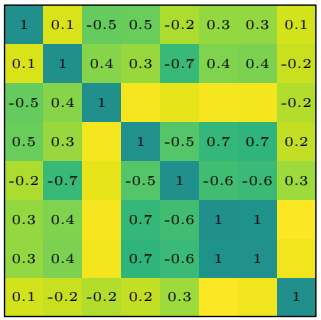
with linear coefficient dependence only

CLIC global reach

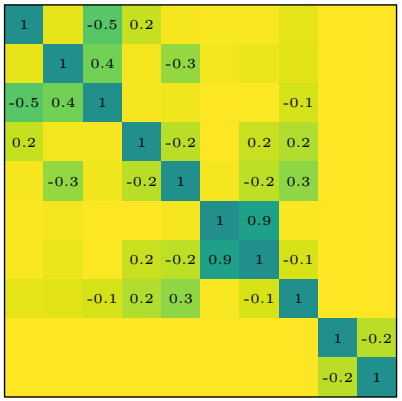
- in TeV^{-2}, $\Delta\chi^2 = 1$
- white marks: individual constraints
- dashed vertical lines: GDPs
- gray numbers: global/individual ratios

$\sigma + A^{\mathrm{FB}}$:

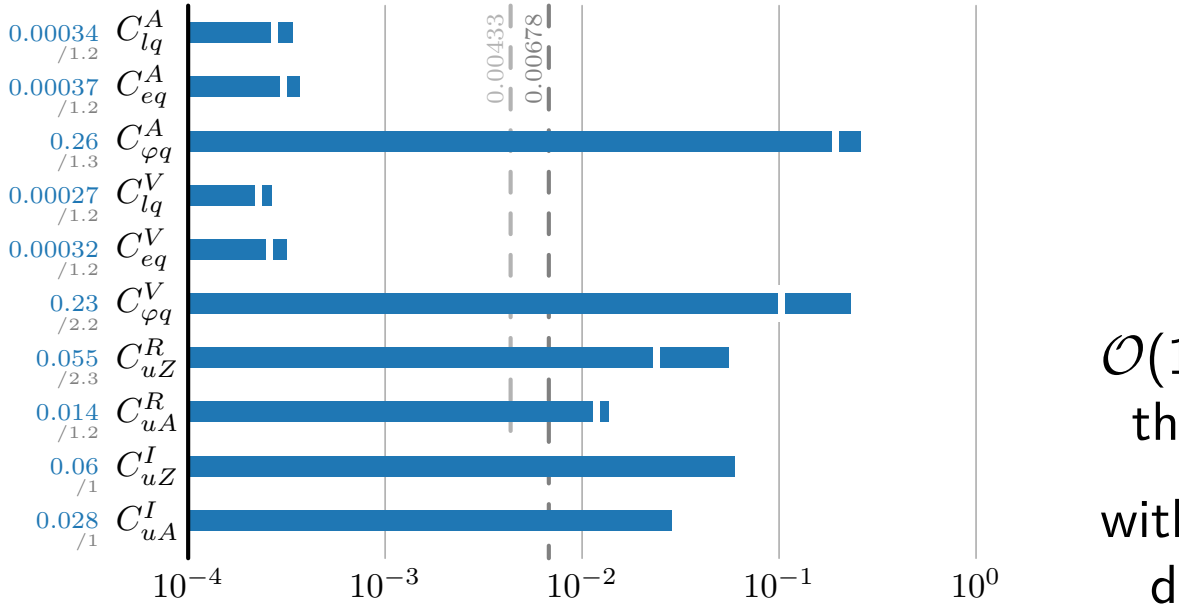




correlation matrices



Statistically optimal observables:



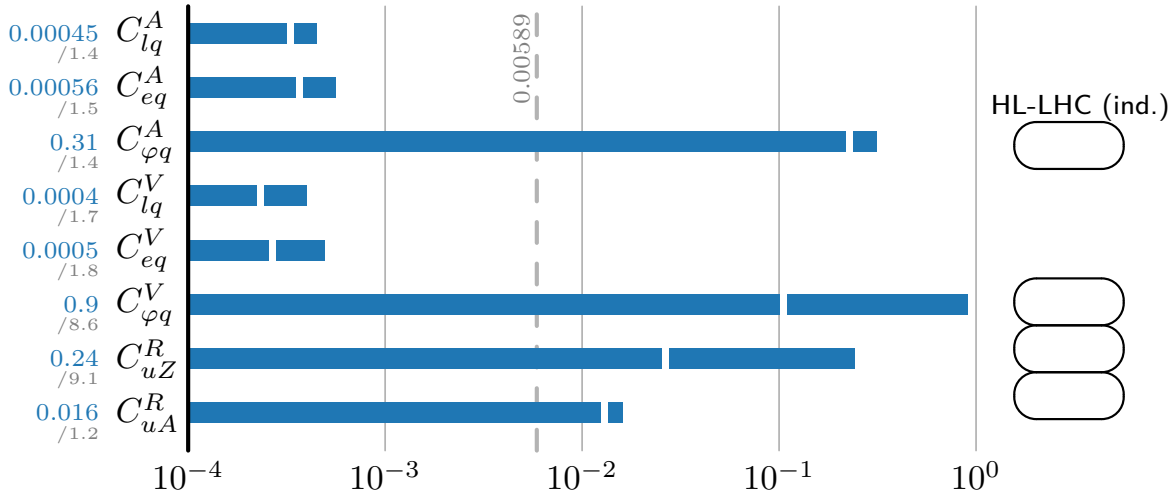
$\mathcal{O}(10^{1-3})$ better than HL projections

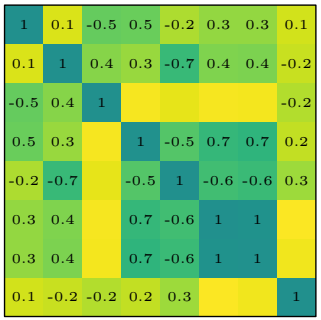
with linear coefficient dependence only

CLIC global reach

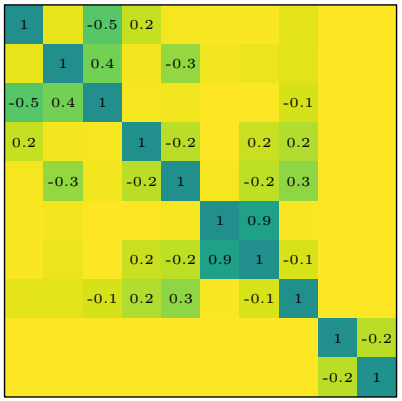
- in TeV^{-2}, $\Delta\chi^2 = 1$
- white marks: individual constraints
- dashed vertical lines: GDPs
- gray numbers: global/individual ratios

$\sigma + A^{\mathrm{FB}}$:





correlation matrices



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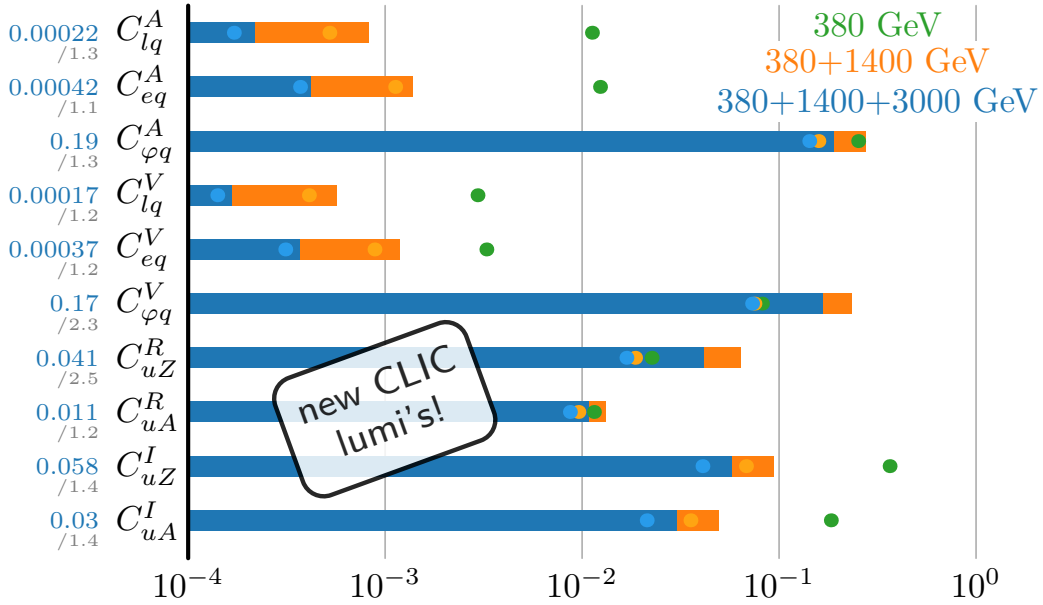

runs at two $\sqrt{s}$ are required

with linear coefficient dependence only

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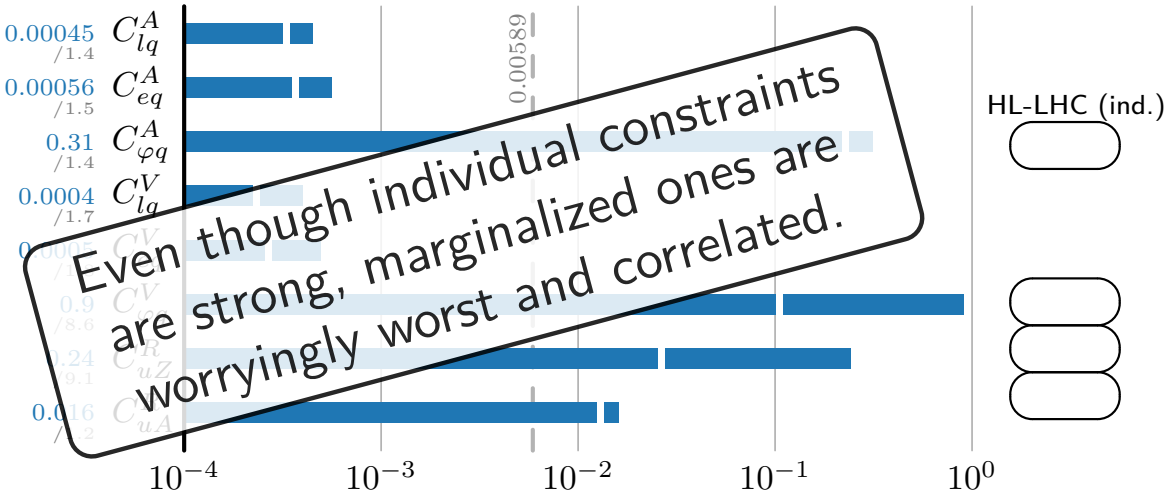
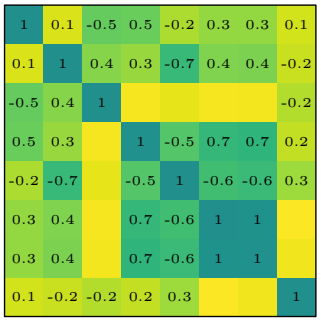
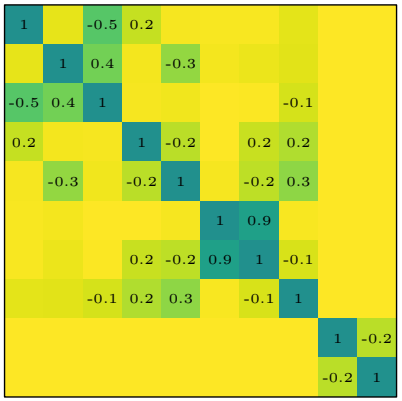


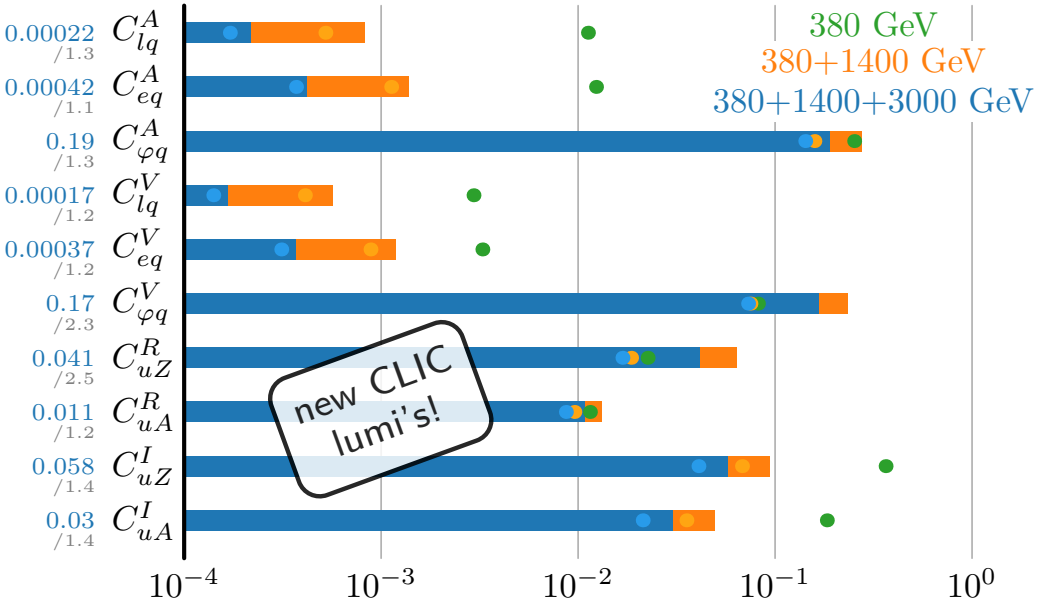
Even though individual constraints are strong, marginalized ones are worryingly worst and correlated.



correlation matrices



Statistically optimal observables:



new CLIC lumi's!

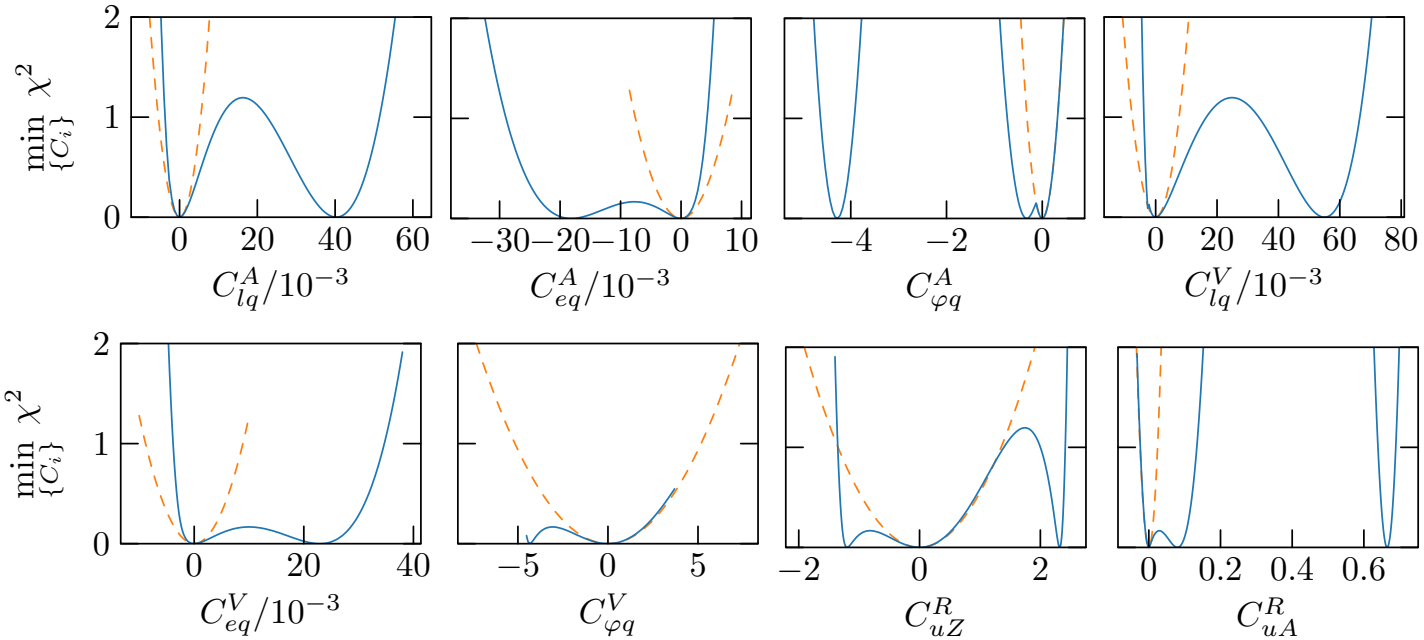
runs at two $\sqrt{s}$ are required

with linear coefficient dependence only

Robustness of the linear EFT truncation

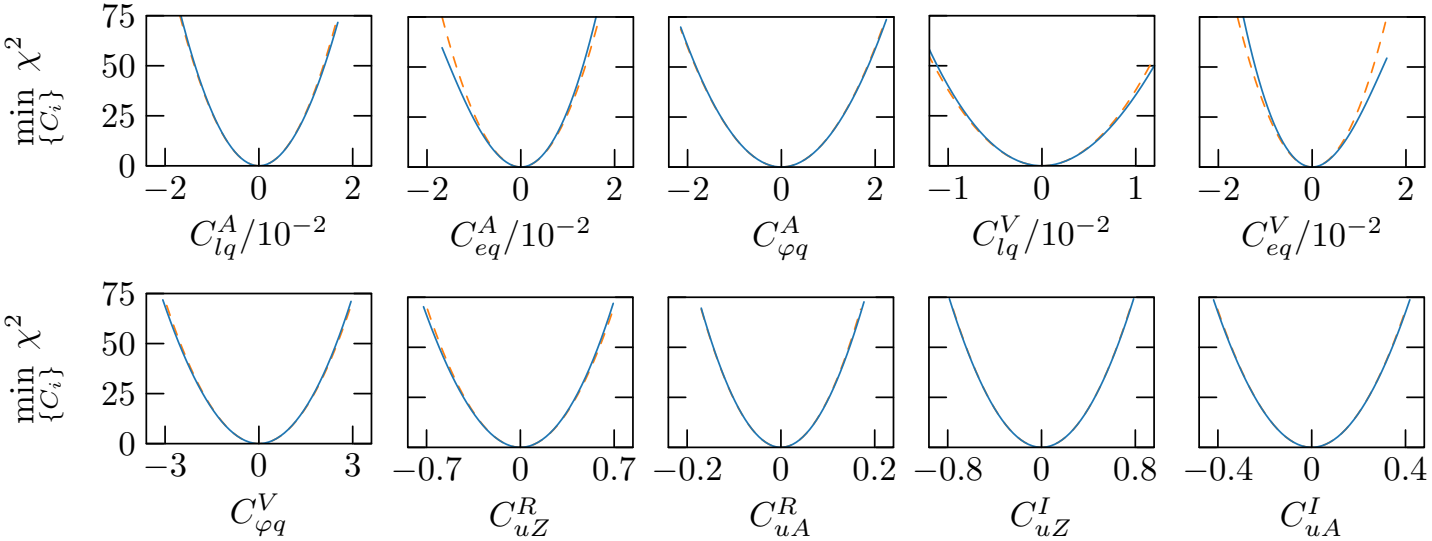
$\sigma + A^{\mathrm{FB}}$:

dashed: linear dependence only
plain: linear+quadratic dep.



Statistically optimal observables:

Note the vertical scale!



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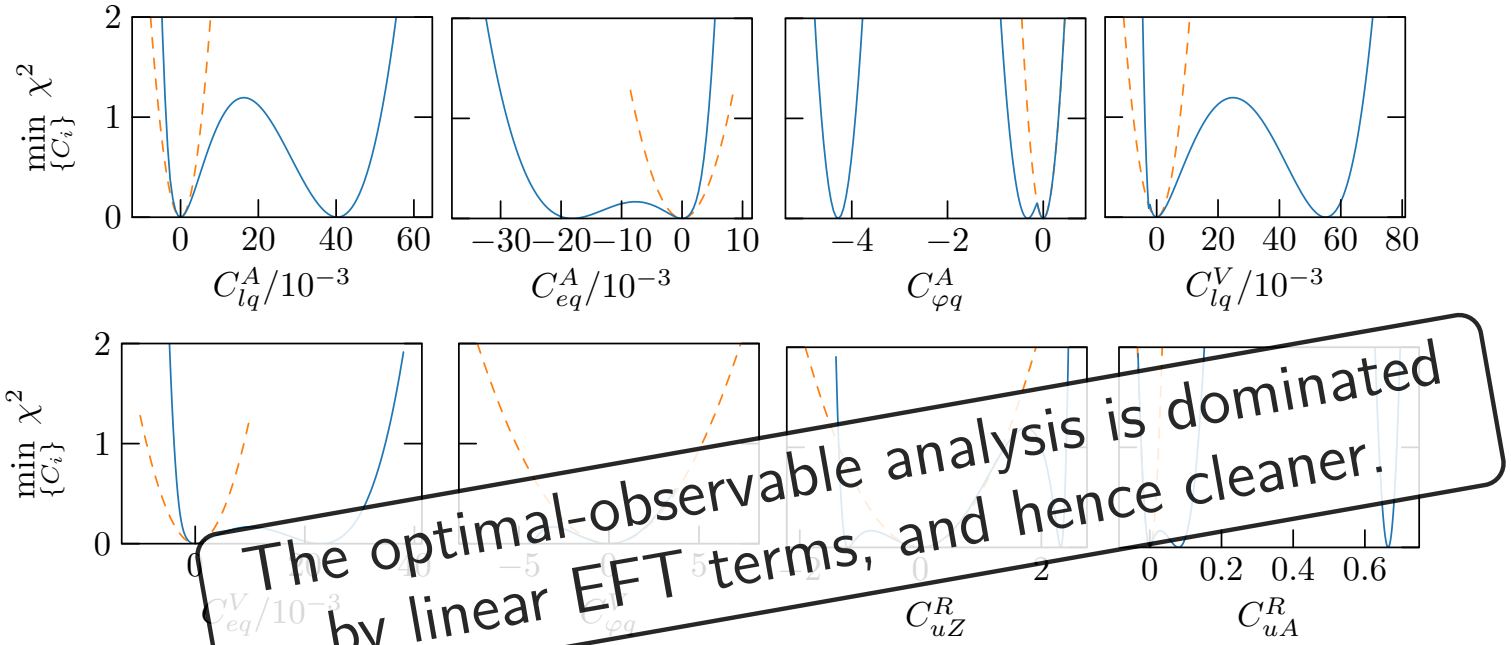
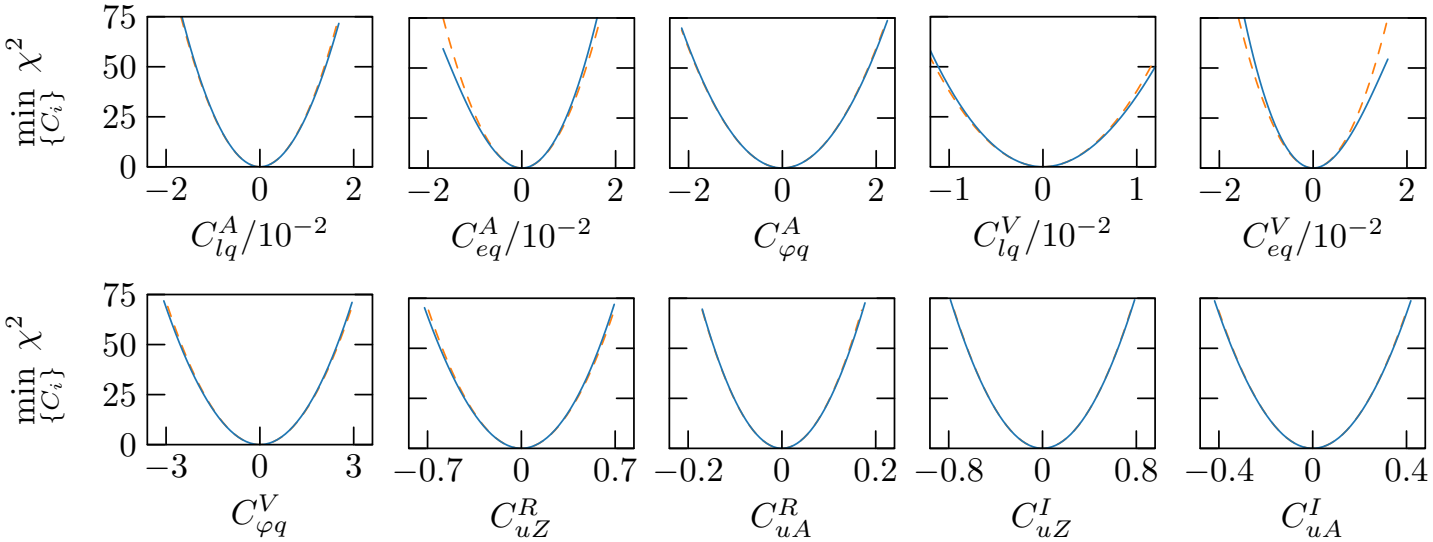


The optimal-observable analysis is dominated by linear EFT terms, and hence cleaner.

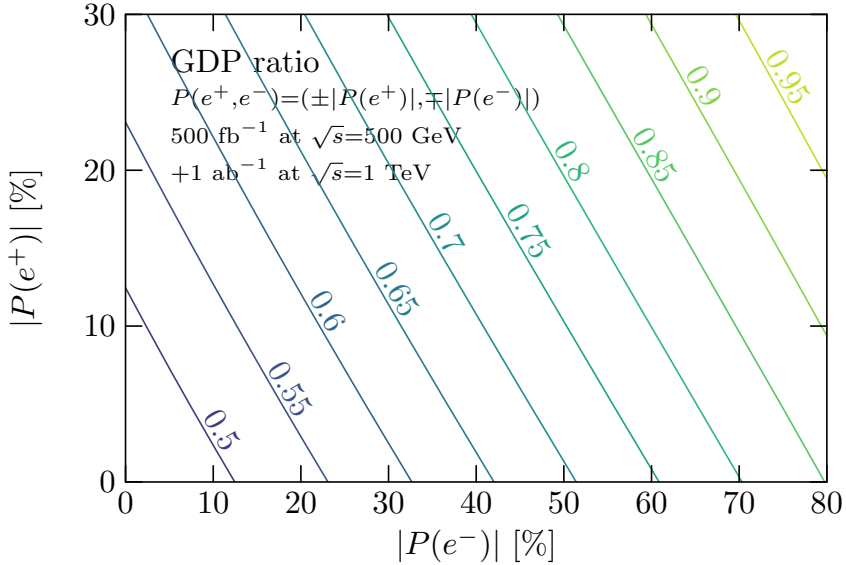
Statistically optimal observables:

Note the vertical scale!



Degrading polarization or energy lever arm

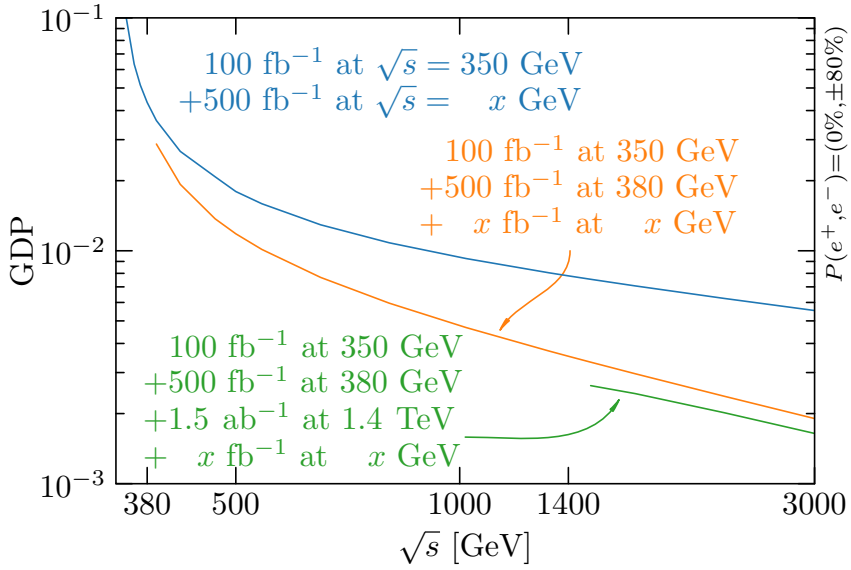
500 GeV + 1 TeV



10% polarization costs $\sim$ 5% of GDP

w.r.t. $P(e^+, e^-) = (\pm 30\%, \mp 80\%)$:

- $P(e^+)$ compensated by 140% lumi
- $P(e^+, e^-)$ 〃 by 460% lumi



$\sqrt{s}$ lever arm is necessary

Interpretation
in composite Higgs models

[1807.10273]

Framework

- The Higgs is composite, pNGB of a new strong sector
- Typical strong sector coupling and mass: $g_\star$, $m_\star$
- Linear mixings between SM states and composite ones: ϵ_u, ϵ_q

$\rightarrow$ dimensional analysis for strong-sector operator coefficients

preserving $\hbar$ and mass dimensions

$$\frac{m_\star^4}{g_\star^2}\, O^{\text{dim-6}}\left(\epsilon_\psi \frac{g_\star}{m_\star^{3/2}}\psi, \frac{g}{m_\star^2}F^{\mu\nu}, \frac{g_\star}{m_\star}\phi\right)$$

up to order-one factors
or justified suppressions

in particular $y_t \simeq \epsilon_u \epsilon_q g_\star$

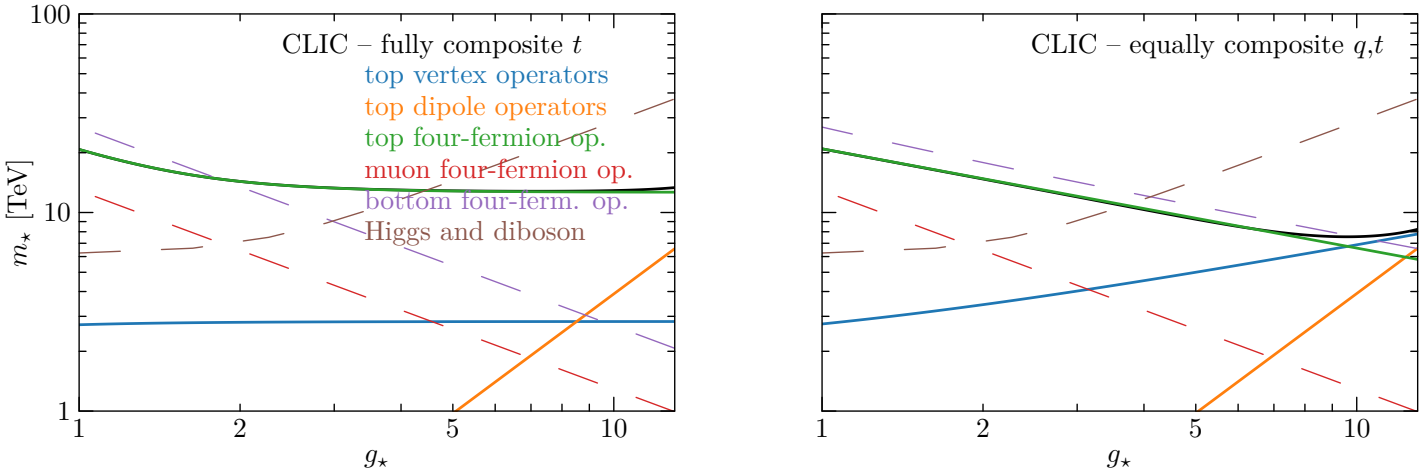
fix either $\epsilon_u = \epsilon_q \simeq \sqrt{\frac{y_t}{g_\star}}$: equally composite top left and right

$\epsilon_u = 1$, $\epsilon_q \simeq \frac{y_t}{g_\star}$: fully composite top right

CLIC prospects

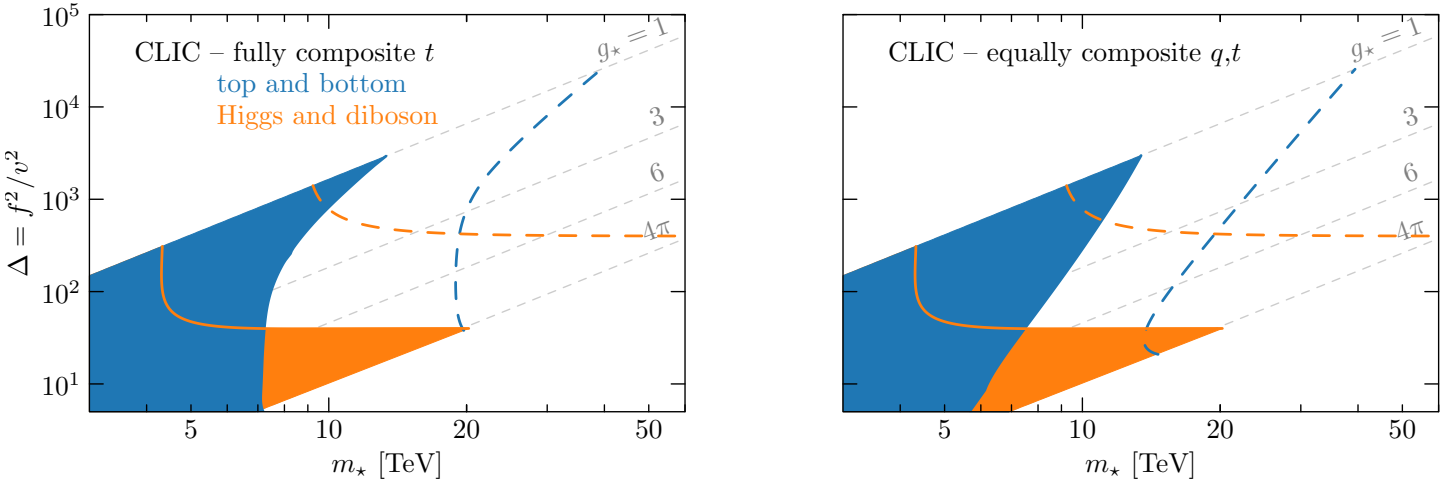
[Higgs fit from 1704.02333]

Different five-sigma sensitivities in (coupling,mass) plane



power counting exactly satisfied

Final five-sigma discovery reach in (mass,tuning) plane



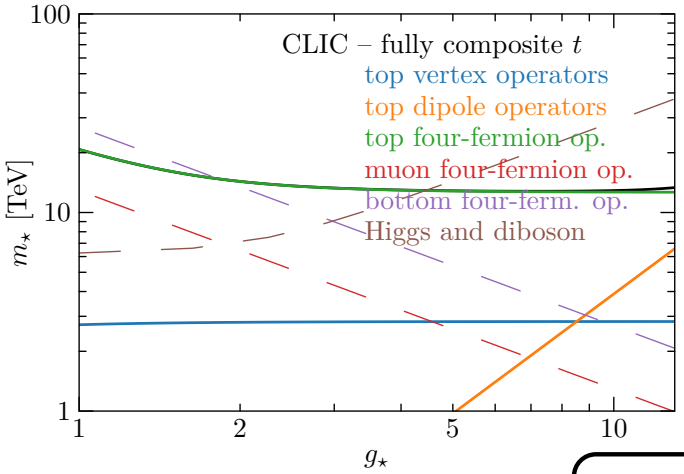
power counting satisfied up to $\pm[1/2, 2]$ $\rightarrow$ filled: pessimistic

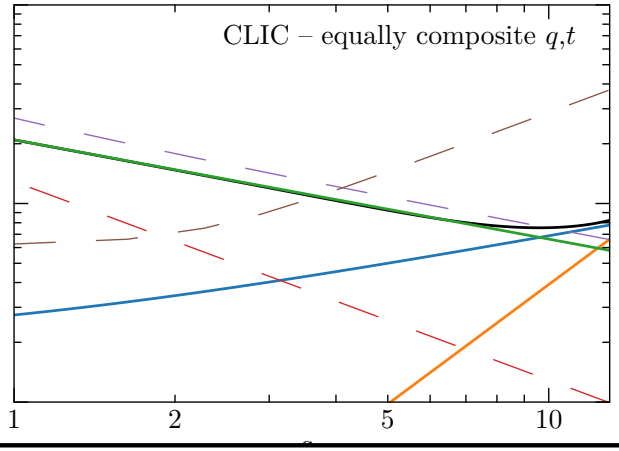
$\rightarrow$ dashed: optimistic

CLIC prospects

[Higgs fit from 1704.02333]

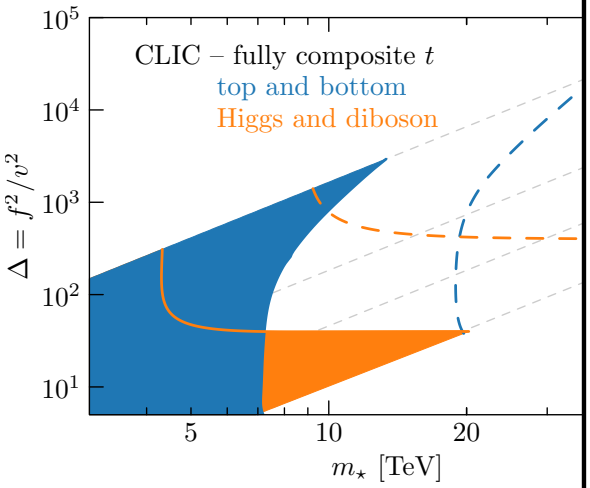
Different five-sigma sensitivities in (coupling,mass) plane





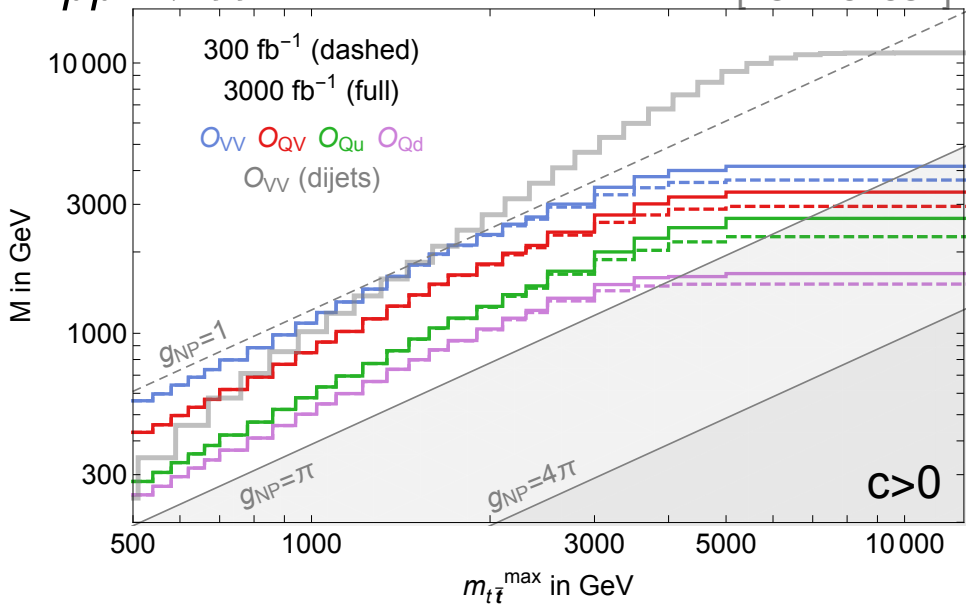
power counting exac

Final five-sigma discovery



power counting satis

$pp \to t\bar{t}$ [1811.04084]



$\to m_\star \lesssim 2\,\text{TeV}\ (t\bar{t})$ and $8\,\text{TeV}/g_\star\ (jj)$

Single top FCNC production

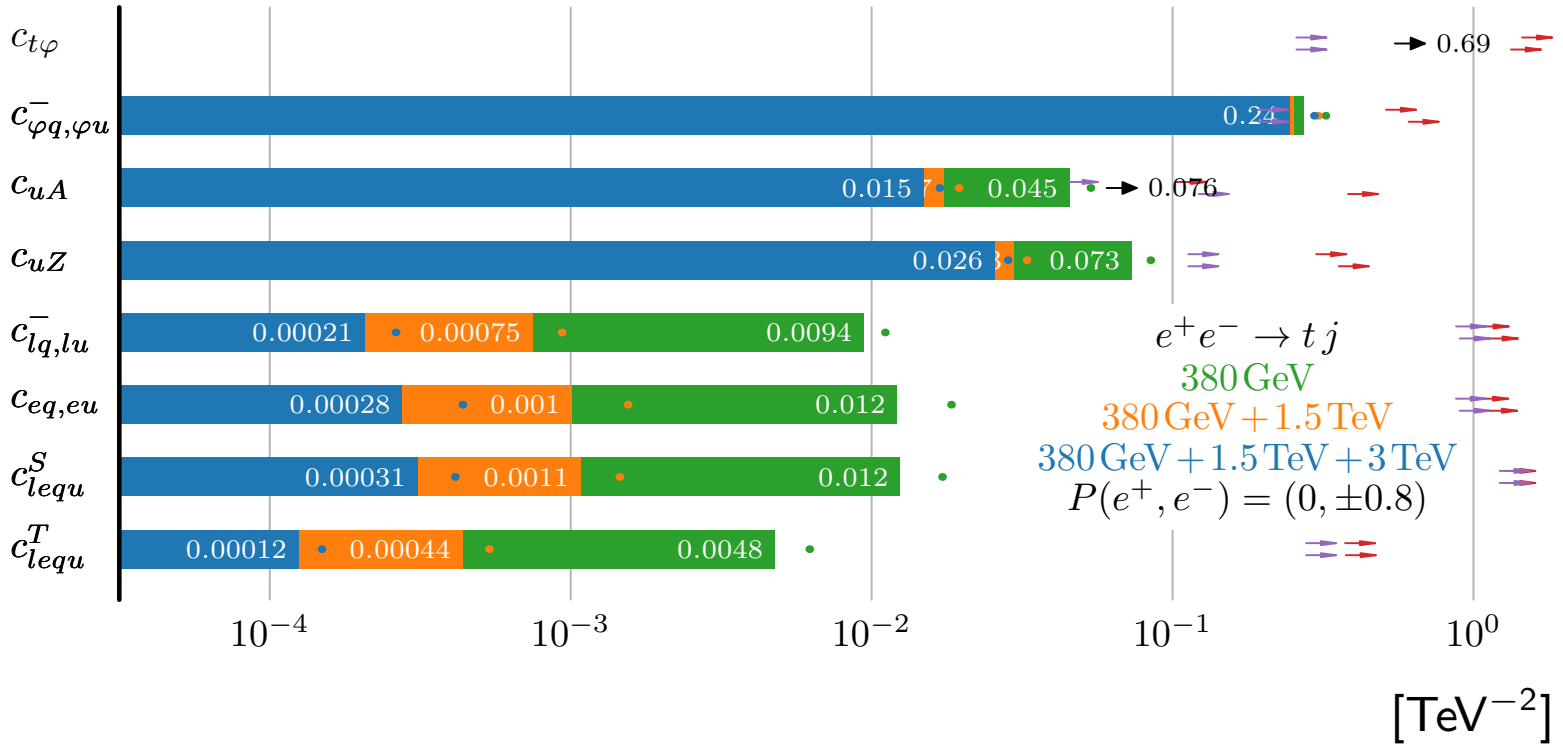
[CLIC BSM YR]

$e^+e^- \to tj$ prospects at CLIC

Marginalized 95% CL constraints

extrapolated from TESLA & FCC studies

[hep-ph/0102197]
[1408.2090]



- compared to decay: black arrows
- compared to current limits up charm
- compared to HL-LHC estimates up charm
- without beam polarization: blobs

$e^+e^- \to tj$ prospects at CLIC

Marginalized 95% CL constraints

extrapolated from TESLA & FCC studies

[hep-ph/0102197]
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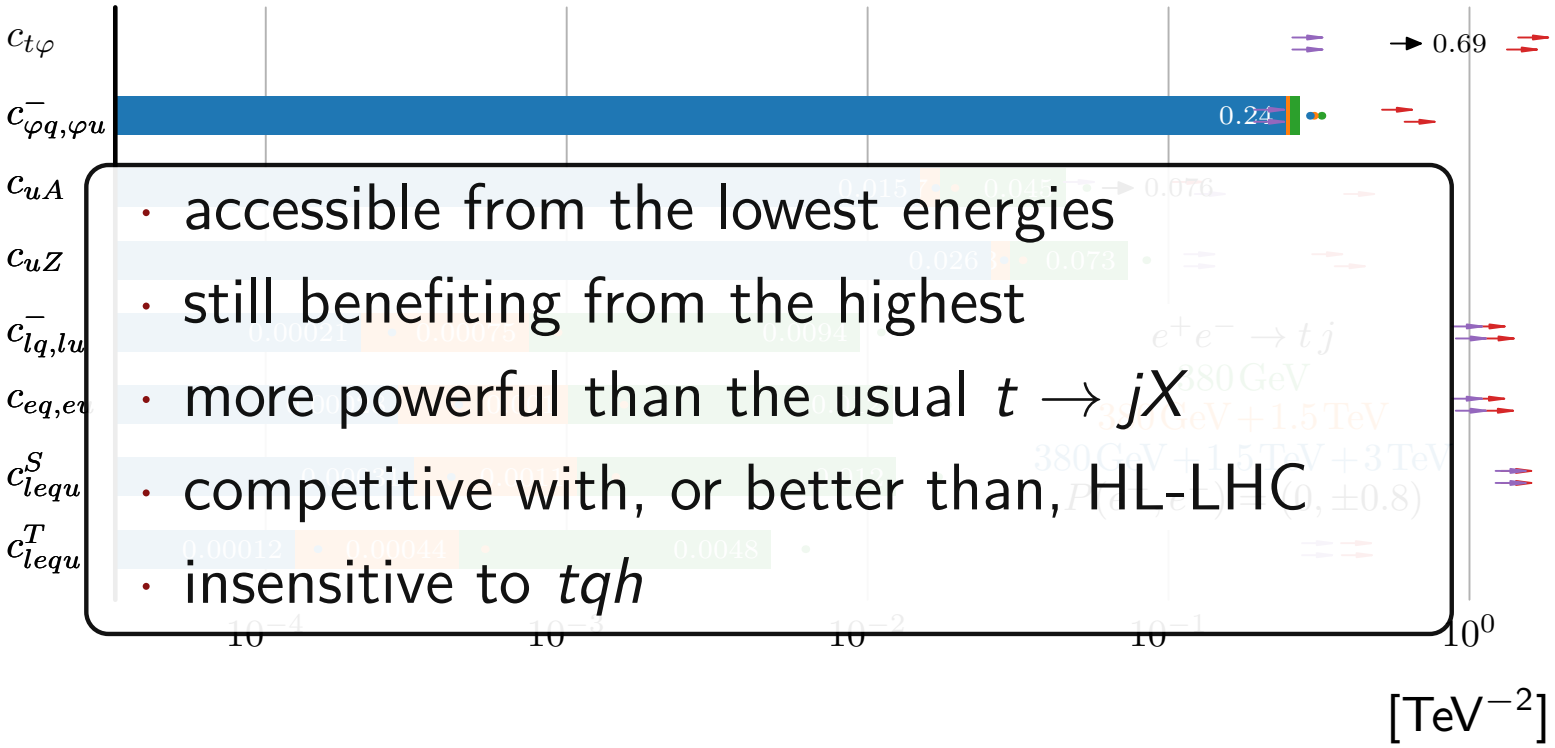


- accessible from the lowest energies
- still benefiting from the highest
- more powerful than the usual $t \to jX$
- competitive with, or better than, HL-LHC
- insensitive to tqh

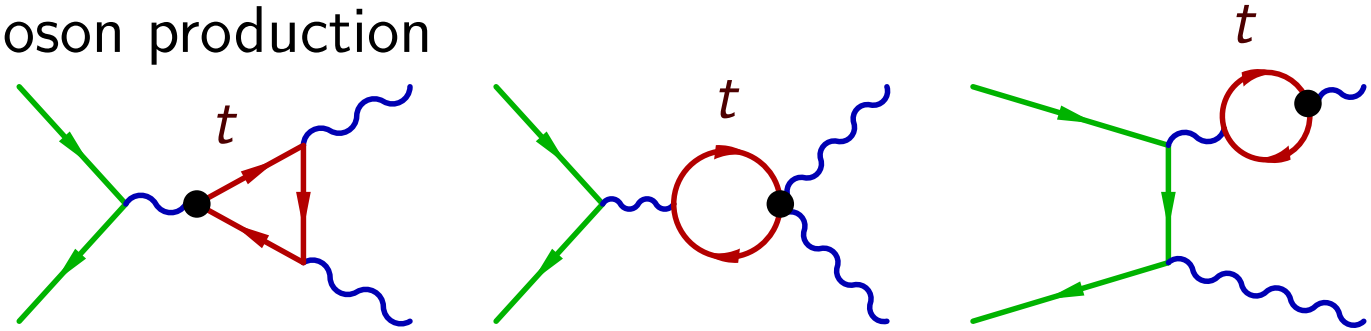
- compared to decay: black arrows
- compared to current limits up charm
- compared to HL-LHC estimates up charm
- without beam polarization: blobs

5. Top&Higgs global analysis

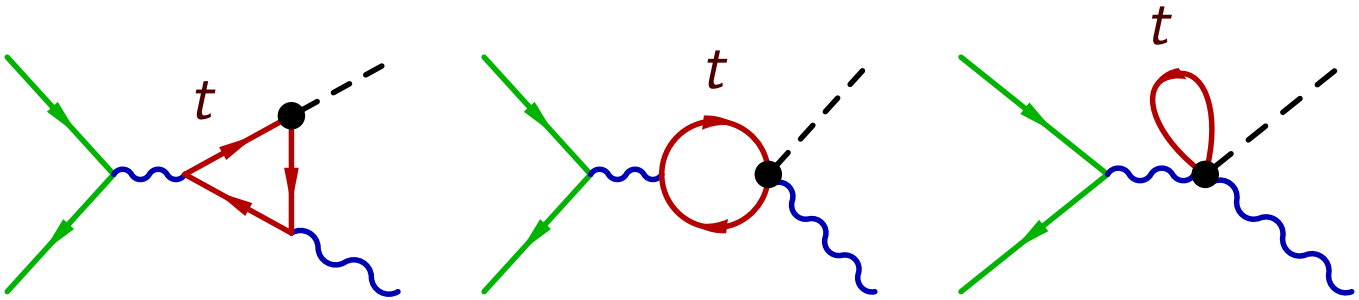
[1809.03520]

Top loops

- At the Z pole [1201.6670]
- In diboson production [1809.03520]



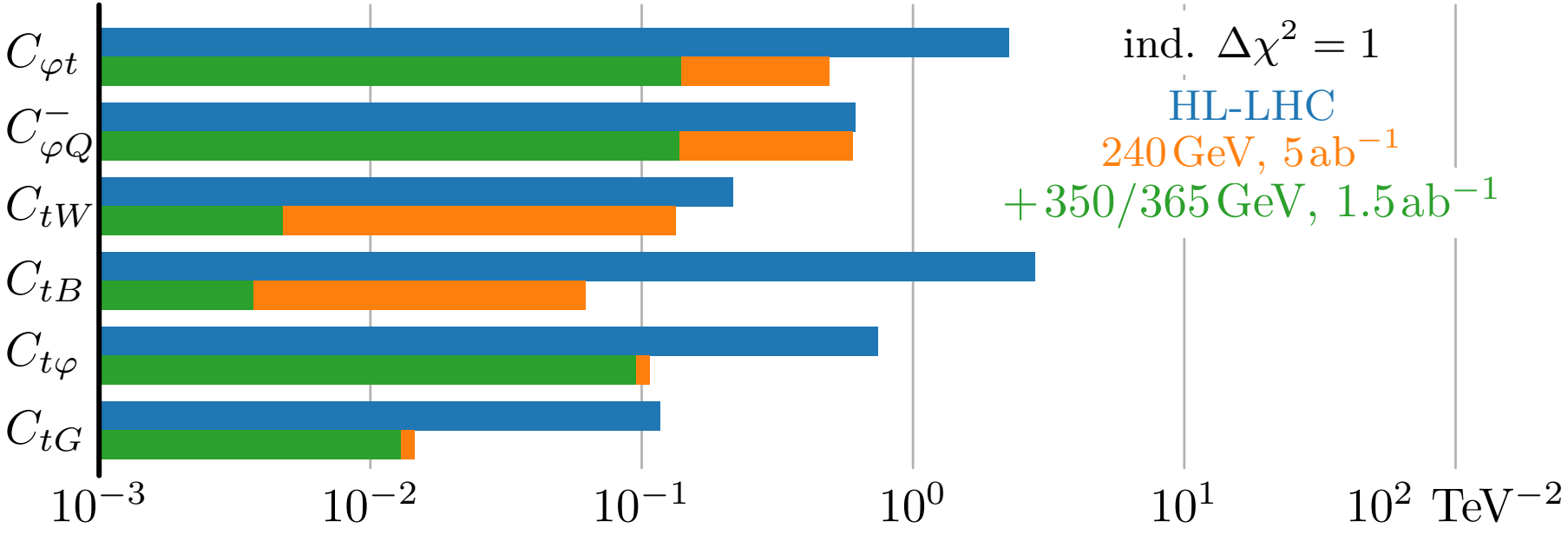
- In Higgs [1804.09766]



Higgsstrahlung, W-fusion and decays

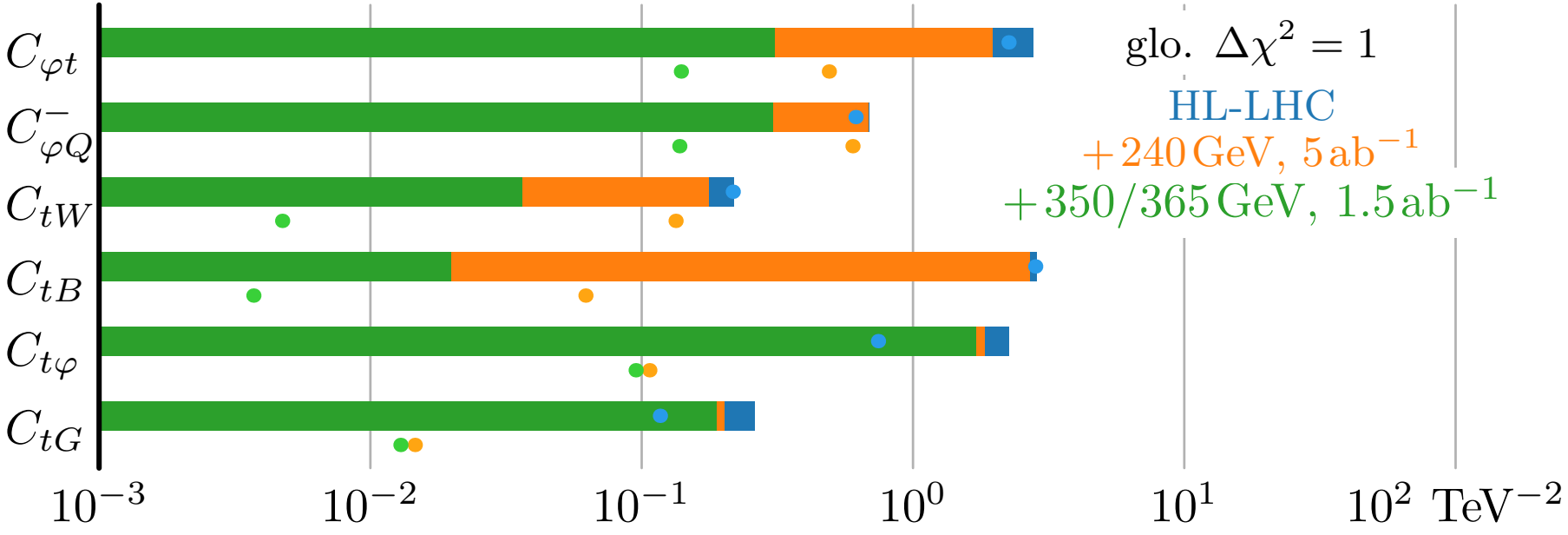
(excluding four-fermion operators, no top loop included in $e^+e^- \to t\bar{t}$)

Individual sensitivities



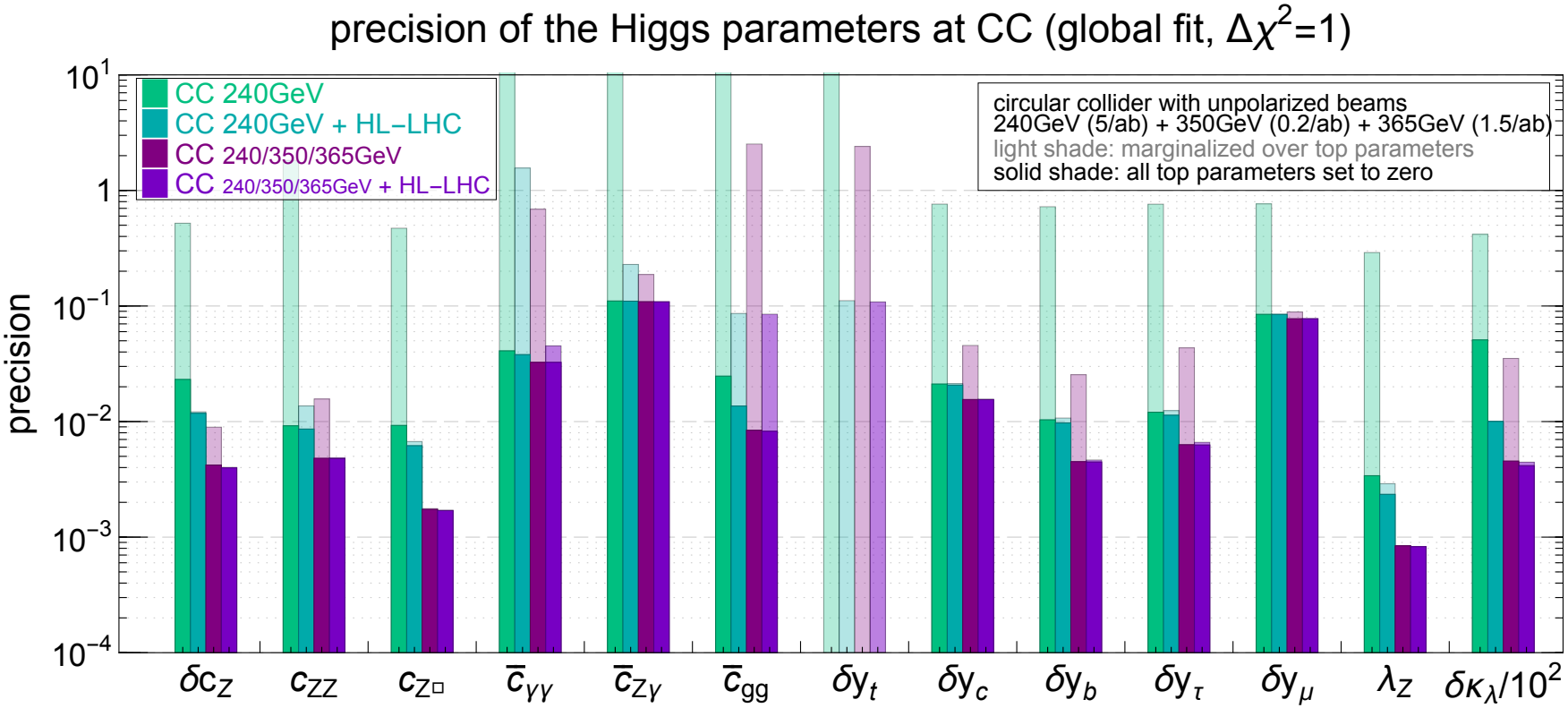
- competitive with the HL-LHC (e.g. on the top Yukawa)
- dominated by Higgs measurements (diboson improves with energy)
- loops in $e^+e^- \to t\bar{t}$ would improve its impact on $C_{t\varphi}$ and C_{tG}
- four-fermion operators to be added...

Global combined reach



- large flat directions with 240 GeV run alone
- more differential distributions would help
- still improves the HL-LHC combination

Contamination in Higgs measurements



- uncertainties on the top have a big effect on the Higgs

6. Conclusions

EFT at future lepton colliders

The SMEFT is an amazing tool that parametrize systematically the theory space beyond the SM.

The top and Higgs escaped the previous generation of lepton colliders and remain to be characterized to the level of EW gauge bosons.

Future machines could deliver clean and precise global EFT analyses which cover a wide variety of BSM scenarios.

Precision buys you:
– large indirect reach to new physics (even beyond LHC one)
– sensitivity to loop corrections (opportunity and worry)