Effects of Higgs in Electroweak Chiral Lagrangian

Qing Wang

Tsinghua University, Beijing

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Effects of Higgs in Electroweak Chiral Lagrangian

Model Independent Description of New Physics

Equivalence between Linear and Nonlinear Realizations of EWCL with Higgs

Extended EWCL and Integrating out Higgs

Effects of Higgs in EWCL
Unknown **Scalar field**: linear representation of $SU(2)_L \otimes U(1)_Y = 3$ longitudinal $W^\pm, Z + 1$ Higgs
Fig. 1: The flowering of the Higgs physics that is expected to bloom at the TeV scale.
Model Independent Description of New Physics

Effective Electroweak Chiral Lagrangian – EWCL

- It respect present status of SM:
  Higgs not found! Its self coupling and Yukawa interactions are not tested!

- It describe present status of experiment:
  there are three goldstone bosons and they couple with fermions!

- It is a kind of Higgsless Standard Model:
  Without higgs, EW symmetry and its breaking must be realized nonlinearly!

- Without Higgs makes theory model independent:
  Underlying models are parameterized by coefficients of EWCL.
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EW Chiral Lagrangian: boson part

\[ L_{\text{Scalar}} \rightarrow L_{\text{EWCL}}^{\text{boson}} \]

\[ T = U^3 U^\dagger \quad V_\mu = (D_\mu U)U^\dagger \quad D_\mu U = \partial_\mu U + ig_2 W_\mu U - ig_1 B_\mu \tau^3/2 \]

\[ L_{\text{EWCL}}^{\text{boson}} = -(f^2/4)\text{tr}(V_\mu V^\mu) + (f^2/4)\beta_1\text{tr}(TV_\mu)^2 + (1/2)\alpha_1 g_2 g_1 B_\mu \nu \text{tr}(TW^{\mu\nu}) \]

\[ + (i/2)\alpha_2 g_1 B_\mu \nu \text{tr}(T[V_\mu, V_\nu]) + i\alpha_3 g_2 \text{tr}(W_\mu \nu [V_\mu, V_\nu]) + \alpha_4 \text{tr}(V_\mu V_\nu)^2 + \alpha_5 \text{tr}(V_\mu V^\mu)^2 \]

\[ + \alpha_6 \text{tr}(V_\mu V_\nu)\text{tr}(TV^\mu)\text{tr}(TV_\nu) + \alpha_7 \text{tr}(V_\mu V^\mu)\text{tr}(TV_\nu)\text{tr}(TV^\nu) + (1/4)\alpha_8 g_2^2 \text{tr}(TW_{\mu\nu})^2 \]

\[ + (i/2)\alpha_9 g_2 \text{tr}(TW_{\mu\nu})\text{tr}(T[V_\mu, V_\nu]) + (1/2)\alpha_10 \text{tr}(TV_\mu)\text{tr}(TV_\nu)^2 \]

\[ + \alpha_{11} g_2 \epsilon^{\mu\nu\rho\lambda} \text{tr}(TV_\mu)\text{tr}(V_\nu W_\rho W_\lambda) + \alpha_{12} \text{tr}(TV_\mu)\text{tr}(V_\nu W_{\mu\nu}) + \alpha_{13} g_2 g_1 \epsilon^{\mu\nu\rho\sigma} B_\mu \nu \text{tr}(TW_{\rho\sigma}) \]

\[ + \alpha_{14} g_2^2 \epsilon^{\mu\nu\rho\sigma} \text{tr}(TW_{\mu\nu})\text{tr}(TW_{\rho\sigma}) + 2i\alpha_{15} g_2 \epsilon^{\mu\nu\rho\sigma} \text{tr}(W_{\mu\nu} V_\rho V_\sigma) \]

\[ + \alpha_{16} g_2^2 \epsilon^{\mu\nu\rho\sigma} \text{tr}(TW_{\mu\nu})\text{tr}(TW_{\rho\sigma}) + 2i\alpha_{17} g_2 g_1^2 \epsilon^{\mu\nu\rho\sigma} \text{tr}(TW_{\mu\nu}\text{tr}(TV_\rho V_\sigma) \]

\[ + \alpha_{18} g_1 \epsilon^{\mu\nu\rho\sigma} B_\mu \nu B_{\rho\sigma} + \alpha_{19} g_2^2 \epsilon^{\mu\nu\rho\sigma} \text{tr}(W_{\mu\nu} W_{\rho\sigma}) + (g_1^2/4) Z_1 B_\mu \nu B^{\mu\nu} + (g_2^2/2) Z_2 \text{tr}(W_{\mu\nu} W^{\mu\nu}) + O(p^6) \]
**Model Independent Description of New Physics**

**EW Chiral Lagrangian: fermion part**

\[
L_i = \begin{pmatrix} \nu_i \\ E_i \end{pmatrix}, \quad Q_i = \begin{pmatrix} U_i \\ D_i \end{pmatrix}, \quad \Phi = U \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}
\]

\[
\mathcal{L}_{\text{Yukawa}} \rightarrow \mathcal{L}_{\text{EWCL}}^{\text{fermion}} = \overline{L}_i \begin{pmatrix} 0 & 0 \\ 0 & f_e \end{pmatrix} U_P R L_i + \overline{Q}_i \begin{pmatrix} f_u^i & 0 \\ 0 & f_d^i \end{pmatrix} U_P R Q_i + \text{h.c.}
\]

\[
+ f_{ij}^\nu L^\alpha T C L_j^\beta \Phi^\alpha \Phi'^\beta \epsilon^{\alpha\alpha'} \epsilon^{\beta\beta'} + \text{h.c.} \quad \text{Neutrino Majorana mass}
\]

\[
+ \text{high dimension terms}
\]

**high dimension terms:** self interactions; interactions with gauge fields!
Model Independent Description of New Physics

Experimental Tests

\[ S = -16\pi\Pi'_{3B}(0) = -16\pi\alpha_1 \]
\[ \alpha T = \frac{e^2[\Pi_{11}(0) - \Pi_{33}(0)]}{c^2 s^2 m_Z^2} = 2\beta_1 \]
\[ U = 16\pi[\Pi'_{11}(0) - \Pi'_{33}(0)] = -16\pi\alpha_8 \]

\[
\frac{\mathcal{L}_{WWV}}{g_{WWV}} = i g_V^V (W_{\mu\nu} V_{-\mu} V_{\nu} - W_{\mu\nu} W_{+\mu} V_{\nu}) + i \kappa_V W_{\mu}^+ W_{\nu}^- V_{\mu\nu} - g_4^V W_{\mu}^+ W_{\nu}^- (\partial_{\mu} V_{\nu} + \partial_{\nu} V_{\mu})
\]
\[
+ g_5^V \epsilon_{\mu\nu\rho\sigma} [W_{\mu}^+ (\partial_{\rho} W_{\nu}^-) - (\partial_{\rho} W_{\mu}^+) W_{\nu}^-] V_{\lambda} + i \kappa_V W_{\mu}^+ W_{\nu}^- \tilde{V}_{\mu\nu}
\]

\[ g_1^Z - 1 = \frac{\beta_1}{c^2 - s^2} + \frac{e^2\alpha_1}{c^2(c^2 - s^2)} + \frac{e^2\alpha_3}{s^2 c^2} \quad g_1^\gamma - 1 = 0 \quad g_5^Z = \frac{e^2\alpha_{11}}{s^2 c^2} \quad g_5^\gamma = 0 \]

\[ \kappa_{\gamma} - 1 = \frac{e^2(-\alpha_1 + \alpha_2 + \alpha_3 - \alpha_8 + \alpha_9)}{s^2} \quad \kappa_{Z} - 1 = \frac{\beta_1}{c^2 - s^2} + \frac{e^2\alpha_1}{c^2(c^2 - s^2)} + \frac{e^2(\alpha_1 - \alpha_2)}{c^2} + \frac{e^2(\alpha_3 - \alpha_8 + \alpha_9)}{s^2} \]
Model Independent Description of New Physics

Two Future Possibilities

In Future, next generation colliders will all work at TeV energy region

- **Possibility One:** No new particle is found in TeV energy region worst case
  violate unitarity
  **EWCL works !**

- **Possibility Two:** New particle is found in TeV energy region
  $Z'$, SUSY partners, Higgs, ... people will be excited and busy
Model Independent Description of New Physics

- Discovery of new particle needs time: at least 3 years
- Before discovery of new particle: **EWCL works**!
- Once new particle is found: need go beyond **EWCL** not urgent now

During the time before discovery of new particle: urgent

- Can we estimate effects of new particle below its threshold?
- New particles as virtual particle contribute to physics! \(\Rightarrow\) coefficients of **EWCL**
- **EWCL** is most economical and effective theory to investigate new physics!
Model Independent Description of New Physics

Investigating New Physics in terms of EWCL

- **Experimentally**: test and fix coefficients of the EWCL
  
  need to analyze and choose the best process He,Kuang,Yuan,hep-ph/9704276

- **Theoretically**: calculate coefficients of the EWCL
  
  need to perform computation: integrate out new particles QCD experience
  
  PRD61,54011(00); PRD66,14019(02); PLB532,240(02); PLB560,188(03)

Each underlying model is corresponding to a group of coefficients!
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<th>Operators</th>
<th>$\mathcal{L}_{1,13}$</th>
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Model Independent Description of New Physics

Calculate Coefficients of EWCL from Different Models

- SM: need to integrate out Higgs
- One doublet technicolor model: reproduce scale up result
- One family technicolor model: find difference with scale up result
- Top color assisted technicolor model
- Little higgs and higgsless models . . . . . .

Hard and tedious work !

Model dependent research !

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Model Independent Description of New Physics

Investigating New Physics beyond EWCL

- Model dependent research:  
- Model independent research: The first discovered particles may be \( Z', h, \rho \)
  
  - Adding in EWCL a \( Z' \)
  
  - Adding in EWCL a Higgs: either linearly or nonlinearly
  
  - Adding in EWCL a vector boson: Like QCD CL with \( \rho \)
Equivalence between Linear and Nonlinear Realizations of EWCL with Higgs

Adding in Higgs to EWCL: Two Types of Effective Theory

- **Linear Realization:** \( \text{SM} + \text{high dimension operators}! \)
  \[
  \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_n \frac{f_n}{\Lambda^2_H} \mathcal{O}_n
  \]

- **Nonlinear Realization:** \( \text{electroweak chiral Lagrangian} + \text{higgs} \)
  \[
  \Phi^c \equiv i\tau^2 \Phi^* \quad \Sigma \equiv (\Phi^c, \Phi) \equiv \frac{h + v}{\sqrt{2}} U \quad U = e^{i\pi i \tau^i}, \; i = 1, 2, 3
  \]

They are equivalent or not?

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Equivalence between Linear and Nonlinear Realizations of EWCL with Higgs

**Dimension Six Operators in Linear Realization**

\[
\mathcal{O}_{DW} = Tr([D_\mu, \hat{W}_\nu][D^\mu, \hat{W}^{\nu\rho}])
\]

\[
\mathcal{O}_{BW} = \Phi^+ \hat{B}_{\mu\nu} \hat{W}^{\mu\nu} \Phi
\]

\[
\mathcal{O}_{WW} = Tr(\hat{W}_{\mu\nu} \hat{W}^{\nu\rho} \hat{W}^\mu)
\]

\[
\mathcal{O}_{BB} = \Phi^+ \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \Phi
\]

\[
\mathcal{O}_B = (D_\mu \Phi)^+ \hat{B}^{\mu\nu} (D_\nu \Phi)
\]

\[
\mathcal{O}_{\Phi,1} = [(D_\mu \Phi)^+ \Phi] [\Phi^+ D^\mu \Phi]
\]

\[
\mathcal{O}_{WW} = \Phi^+ \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \Phi
\]

\[
\mathcal{O}_W = (D_\mu \Phi)^+ \hat{W}^{\mu\nu} (D_\nu \Phi)
\]

\[
\mathcal{O}_{\Phi,2} = \frac{1}{2} \partial_\mu (\Phi^+ \Phi) \partial^\mu (\Phi^+ \Phi)
\]

\[
\mathcal{O}_{\Phi,3} = \frac{1}{3} (\Phi^+ \Phi)^3
\]

\[
\mathcal{O}_{\Phi,4} = (\Phi^+ \Phi) [(D_\mu \Phi)^+ (D^\mu \Phi)]
\]

\[
D_\mu = \partial_\mu + ig T^a W^a_\mu + ig' Y B_\mu
\]

\[
\hat{W}_{\mu\nu} = ig T^a W^a_{\mu\nu} \quad \hat{B}_{\mu\nu} = ig' B_{\mu\nu}
\]
Equivalence between Linear and Nonlinear Realizations of EWCL with Higgs

**O(p^4) Operators in Nonlinear Realization**

\[
\begin{align*}
l_4^1 & \equiv B_{\mu\nu} \text{Tr}(TW^{\mu\nu}) \\
l_4^3 & \equiv \text{Tr}(W_{\mu\nu}[V^\mu, V^\nu]) \\
l_4^5 & \equiv [\text{Tr}(V_\mu V^\mu)]^2 \\
l_4^7 & \equiv \text{Tr}(V_\mu V^\mu) \text{Tr}(TV_\nu) \text{Tr}(TV^\nu) \\
l_4^9 & \equiv \text{Tr}(TW_{\mu\nu}) \text{Tr}(T[V^\mu, V^\nu]) \\
l_4^{11} & \equiv \epsilon_{\mu\nu\rho\lambda} \text{Tr}(TV^\mu) \text{Tr}(V^\nu W_{\rho\lambda}) \\
l_4^{13} & \equiv \epsilon_{\mu\nu\rho\lambda} B^{\mu\nu} \text{Tr}(TW^{\rho\lambda}) \\
l_4^2 & \equiv B_{\mu\nu} \text{Tr}(T[V^\mu, V^\nu]) \\
l_4^4 & \equiv [\text{Tr}(V_\mu V_\nu)]^2 \\
l_4^6 & \equiv \text{Tr}(V_\mu V_\nu) \text{Tr}(TV^\mu) \text{Tr}(TV^\nu) \\
l_4^8 & \equiv [\text{Tr}(TW_{\mu\nu})]^2 \\
l_4^{10} & \equiv [\text{Tr}(TV_\mu) \text{Tr}(TV_\nu)]^2 \\
l_4^{12} & \equiv \text{Tr}(TV^\mu) \text{Tr}(V_\nu W^{\mu\nu}) \\
l_4^{14} & \equiv \epsilon_{\mu\nu\rho\lambda} \text{Tr}(TW^{\mu\nu}) \text{Tr}(TW^{\rho\lambda})
\end{align*}
\]
Equivalence between Linear and Nonlinear Realizations of EWCL with Higgs

**Linear Realization** $\Rightarrow$ **Nonlinear Realization**

\[
2(D_\mu \Phi)^+(D_\nu \Phi) = \partial_\mu h^2 + h^2 Tr(TV_\mu) \\
2\Phi^+ W_{\mu\nu} \Phi = h^2 Tr(TW_{\mu\nu}) \\
2(D_\mu \Phi)^+(D_\nu \Phi) = h^2 [Tr(TV_\mu V_\nu) - Tr(V_\mu V_\nu)] + 2(\partial_\mu h)(\partial_\nu h) \\
2(D_\mu \Phi)^+ W^{\mu\nu} (D_\nu \Phi) = h^2 Tr(W^{\mu\nu} V_\mu V_\nu) - (\partial_\mu h^2) Tr(W^{\mu\nu} V_\nu) \\
2\Phi^+ W^{\nu\rho} (D_\mu \Phi) = h^2 [Tr(TV^{\mu} W^{\nu\rho}) + Tr(V^{\mu} W^{\nu\rho})] \\
2(D_\mu \Phi)^+ W^{\nu\rho} \Phi = h^2 [Tr(TV^{\mu} W^{\nu\rho}) - Tr(V^{\mu} W^{\nu\rho})]
\]
Equivalence between Linear and Nonlinear Realizations of EWCL with Higgs

Nonlinear Realization $\Rightarrow$ Linear Realization

\[
\text{Tr}(TV_\mu) = (\Phi^+ \Phi)^{-1} \left[ 2(D_\mu \Phi)^+ \Phi - \partial_\mu (\Phi^+ \Phi) \right]
\]

\[
\text{Tr}(TW_{\mu\nu}) = 2(\Phi^+ \Phi)^{-1} [\Phi^+ W_{\mu\nu} \Phi]
\]

\[
\text{Tr}(V_\mu V_\nu) = \frac{1}{2} (\Phi^+ \Phi)^{-2} \partial_\mu (\Phi^+ \Phi) \partial_\nu (\Phi^+ \Phi) - (\Phi^+ \Phi)^{-1} [(D_\mu \Phi)^+ (D_\nu \Phi) + h.c.]
\]

\[
\text{Tr}(TV_\mu V_\nu) = (\Phi^+ \Phi)^{-1} [(D_\mu \Phi)^+ (D_\nu \Phi) - h.c.]
\]

\[
\text{Tr}(V^\mu W^{\nu\rho}) = (\Phi^+ \Phi)^{-1} [-(D^\mu \Phi)^+ W^{\nu\rho} \Phi + h.c.]
\]

\[
\text{Tr}(TV^\mu W^{\nu\rho}) = (\Phi^+ \Phi)^{-1} [(D^\mu \Phi)^+ W^{\nu\rho} \Phi + h.c.]
\]

\[
\text{Tr}(W^{\mu\nu} V_\mu V_\nu) = 2(\Phi^+ \Phi)^{-1} [(D_\mu \Phi)^+ W^{\mu\nu} (D_\nu \Phi)]
\]

\[
+ (\Phi^+ \Phi)^{-2} \partial_\mu (\Phi^+ \Phi) [-(D^\mu \Phi)^+ W^{\nu\rho} \Phi + h.c.]
\]
Equivalence between Linear and Nonlinear Realizations of EWCL with Higgs

- Mathematically linear and nonlinear realizations are equivalent

- \( m_h \propto \text{higgs self coupling} \) in linear realization

- Perturbation expansion converge \( \Rightarrow \) light higgs

- Discussion of light higgs favors linear realization

- Heavy higgs can only be discussed in nonlinear realization

- Nonlinear realization can discuss light higgs in principle
Extended EWCL and Integrating out Higgs

**EEWCL:** EWCL include in Higgs field

- Writing done most general EEWCL previous works only focus on special terms
- Integrating out Higgs field
- Investigate its effects on EWCL coefficients

**Problems**

- Higgs dependence in EEWCL is rather arbitrary
- Higgs field cannot be exactly integrated out!
- Can we make estimations on Higgs Effects?
### Li’S FROM RESONANCE EXCHANGE

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<th>$L_i^r(M_\rho)$</th>
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$^a$) Input  
$^b$) Short-Distance Constraints
Extended EWCL and Integrating out Higgs

- Take low energy expansion
- VEV part of Higgs field is order of $p^0$
- Quantum fluctuation part of Higgs field $h$ is at least order of $p^2$
- Only accurate to 1-loop precision
- Use dimensional regularization
- Apply equation of motion
Extended EWCL and Integrating out Higgs

All terms contribute to $p^4$ EWCL at 1-loop

\[
\mathcal{L}^{(2)} = m^2[(f_1 - f_3 \delta c) A^2_\mu + (f_2 - f_4 \delta c) Tr(V^2_\mu)]
\]

\[
\delta c = \frac{a}{32\pi^2} \left( \frac{1}{2-D} - \gamma + 1 + \ln \frac{4\pi \mu^2}{m^2} \right)
\]

\[
\mathcal{L}^{(4)} = -\frac{1}{2}m^2(1-a\delta c)h^2 + mh[(f_3-f_5 \delta c) A^2_\mu + (f_4-f_6 \delta c) Tr(V^2_\mu)] + [g^i_0-(g^i_0)' \delta c] l^i_4
\]

\[
\mathcal{L}^{(6)} = \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{6}amh^3 + \frac{1}{2}f_5 h^2 A^2_\mu + \frac{1}{2}f_6 h^2 Tr(V^2_\mu)
\]

\[
\mathcal{L}^{(8)} = -\frac{1}{12}bh^4 + \frac{1}{6m}h^3[f_7 A^2_\mu + f_8 Tr(V^2_\mu)] + \frac{1}{2}(g^i_0)'' h^2 l^i_4 + \frac{1}{2m^2}g^k_2 (\partial_\mu h)(\partial_\nu h) l^{\mu\nu}_2
\]

\[
\mathcal{L}^{(10)} = \frac{1}{2}(g^k_2)' m^{-3}(\partial_\mu h)(\partial_\nu h) h l^{k\mu\nu}_2
\]
Extended EWCL and Integrating out Higgs

\[ A_\mu = tr(TV_\mu) \]

\[ l_{2 \mu \nu}^1 = Tr(TV^\mu)Tr(TV^\nu) \]  \[ l_{2 \mu \nu}^2 = Tr(V^\mu V^\nu) \]

\[ l_{3 \mu}^1 = Tr(TV^\mu)Tr(V^\nu V_\nu) \]  \[ l_{3 \mu}^2 = Tr(TV^\nu)Tr(V^\mu V_\nu) \]

\[ l_{3 \mu}^3 = Tr(TV^\nu)Tr(TV^\mu V_\nu) \]  \[ l_{3 \mu}^4 = Tr(TV_\nu)Tr(TW^{\mu \nu}) \]

\[ l_{3 \mu}^5 = B^{\mu \nu}Tr(TV_\nu) \]  \[ l_{3 \mu}^6 = Tr(TW^{\mu \nu}V_\nu) \]

\[ l_{3 \mu}^7 = Tr(W^{\mu \nu}V_\nu) \]
Extended EWCL and Integrating out Higgs

Integrating out Higgs: loop expansion

\[ \Gamma^{1\text{loop}} = \int d^4x \mathcal{L}_{EEWCL} + \frac{i}{2} \ln \text{Det} \hat{\mathcal{D}} \]

\[ \hat{\mathcal{D}}(x, y) \equiv \frac{\delta^2 S}{\delta h(x) \delta h(y)} = -[\partial^2_x + m^2 - A(x) + C_{\mu\nu}(x) \partial^\mu_x \partial^\nu_x] \delta(x - y) \]

\[ A(x) = -am h(x) + f_5 A_\mu^2(x) + f_6 Tr[V_\mu^2(x)] - bh^2(x) + f_7 m^{-1} h(x) A_\mu^2(x) + f_8 m^{-1} h(x) Tr[V_\mu^2(x)] + m^{-2}(g_0^i)'' l_4^i(x) \]

\[ C^{\mu\nu}(x) = g_2^k m^{-2} l_2^k_{\mu\nu}(x) + (g_2^k)' m^{-3} h l_2^k_{\mu\nu}(x) \]
Extended EWCL and Integrating out Higgs

Integrating out Higgs: loop expansion

\[ \Gamma^{1\text{loop}} = \int d^4 x \mathcal{L}_{\text{EEWCL}} + \frac{i}{2} \ln \text{Det} \hat{D} = \int d^4 x [\mathcal{L}^{(2)} + \mathcal{L}^{(2)} + \cdots] \]

\[ \mathcal{L}^{(2)} = m^2 [\bar{f}_1 A^2_\mu + \bar{f}_2 \text{Tr}(V^2_\mu)] \]

\[ \mathcal{L}^{(4)} = -\frac{1}{2} m^2 h^2 + m [\bar{f}_3 h A^2_\mu + \bar{f}_4 h \text{Tr}(V^2_\mu)] + g_0^i l^i_4 \]
Extended EWCL and Integrating out Higgs

\[ L \equiv \frac{1}{2} - \frac{D}{2} - \gamma + \ln \frac{4\pi \mu^2}{m^2} \]

\[ \bar{f}_1 = f_1 + \frac{1}{32\pi^2} \left[ -\frac{(L + 3/2)}{4} g_2^1 - (L + 1)(f_5 + af_3) \right] \]

\[ \bar{f}_2 = f_2 + \frac{1}{32\pi^2} \left[ -\frac{(L + 3/2)}{4} g_2^2 - (L + 1)(f_6 + af_4) \right] \]

\[ m_h^2 = m^2 \left[ 1 - \frac{1}{16\pi^2}(L + 1)(a^2 + b) - \frac{a^2}{32\pi^2} \right] \]

\[ \bar{f}_3 = f_3 + \frac{1}{32\pi^2} \left[ -(L + 1)f_7 - \frac{L + 3/2}{4} (g_1^1)' - \frac{L + 1}{2} ag_1^1 - (2L + 1)af_5 \right] \]

\[ \bar{f}_4 = f_4 + \frac{1}{32\pi^2} \left[ -(L + 1)f_8 - \frac{L + 3/2}{4} (g_2^2)' - \frac{L + 1}{2} ag_2^2 - (2L + 1)af_6 \right] \]
Extended EWCL and Integrating out Higgs

\[ \bar{g}_4^0 = g_0^4 + \frac{1}{32\pi^2} \left[ - (L + 1) \left[ (g_0^4)'' + a(g_0^4)' \right] + \frac{L + 3/2}{8} (g_2^2)^2 \right] \]

\[ \bar{g}_6^0 = g_0^6 + \frac{1}{32\pi^2} \left[ - (L + 1) \left[ (g_0^6)'' + a(g_0^6)' \right] + \frac{L + 3/2}{4} g_2^1 g_2^2 \right] \]

\[ \bar{g}_5^0 = g_0^5 + \frac{1}{32\pi^2} \left[ - (L + 1) \left[ (g_0^5)'' + a(g_0^5)' \right] + \frac{L}{2} (f_6)^2 + \frac{L + 1}{2} f_6 g_2^2 + \frac{L + 3/2}{16} (g_2^2)^2 \right] \]

\[ \bar{g}_7^0 = g_0^7 + \frac{1}{32\pi^2} \left[ - (L + 1) \left[ (g_0^7)'' + a(g_0^7)' \right] + L f_5 f_6 + \frac{L + 1}{2} (f_5 g_2^2 + f_6 g_2^1) + \frac{L + 3/2}{8} g_2^1 g_2^2 \right] \]

\[ \bar{g}_{10}^0 = g_0^{10} + \frac{1}{32\pi^2} \left[ - (L + 1) \left[ (g_0^{10})'' + a(g_0^{10})' \right] + \frac{L}{2} (f_5)^2 + \frac{L + 1}{2} f_5 g_2^1 + \frac{3(L + 3/2)}{16} (g_2^1)^2 \right] \]

\[ \bar{g}_i^i = g_i^0 + \frac{1}{32\pi^2} \left[ - (L + 1) \left[ (g_i^0)'' + a(g_i^0)' \right] \right] \quad i = 1, 2, 3, 8, 9, 11, 12, 13, 14 \]
Extended EWCL and Integrating out Higgs

\[ h_c = \frac{m}{m_h^2} \left[ \bar{f}_3 \text{Tr}(TV_\mu)\text{Tr}(TV^\mu) + \bar{f}_4 \text{Tr}(V_\mu V^\mu) \right] \]

\[ \mathcal{L}_{EWCL} = m^2 \left[ \bar{f}_1 \text{Tr}(TV_\mu)\text{Tr}(TV^\mu) + \bar{f}_2 \text{Tr}(V_\mu^2) \right] + (g_0^i + \Delta g_0^i) l_4^i \]

\[ \Delta g_0^5 = \frac{m^2}{2m_h^2} (\bar{f}_4)^2 + \delta g_0^5 \]
\[ \Delta g_0^7 = \frac{m^2}{m_h^2} \bar{f}_3 \bar{f}_4 + \delta g_0^7 \]
\[ \Delta g_0^{10} = \frac{m^2}{2m_h^2} (\bar{f}_3)^2 + \delta g_0^{10} \]

\[ \Delta g_0^j = \delta g_0^j \quad j \neq 5, 7, 10 \]

\[ \delta f_i = \bar{f}_i - f_i \]
\[ \delta g_0^i = \bar{g}_0^i - g_0^i \]
\[ \delta m^2 = m_h^2 - m^2 \]
**Effects of Higgs in EWCL**

\[ \mathcal{L}_{EWCL} = m^2 \left[ \bar{f}_1 Tr(TV_\mu)Tr(TV^\mu) + \bar{f}_2 Tr(V^2_\mu) \right] + (g_i^0 + \Delta g^i_0) l_i^4 \]

\[ \Delta g^5_0 = \frac{m^2}{2m_h^2} (\bar{f}^4) + \delta g^5_0 \quad \Delta g^7_0 = \frac{m^2}{m_h^2} \bar{f}_3 \bar{f}_4 + \delta g^7_0 \quad \Delta g^{10}_0 = \frac{m^2}{2m_h^2} (\bar{f}_3)^2 + \delta g^{10}_0 \]

**Effects of Higgs**

♣ From loop   ♠ From equation of motion for Higgs

**Assumption**

higgs will be the next new particle we find in future experiment!

⇒ Some of \( f_i \) and \( g^i_0 \) may be small
Effects of Higgs in EWCL

Higgs decay and four-gauge-boson coupling

\[ \mathcal{L}_{EWCL} \bigg|_{5,7,10} = \frac{\Gamma_{Z^0Z^0}}{2A} Tr(V_\mu V^\mu) \bigg|_5 + \left( \frac{\Gamma_{Z^0Z^0}}{2A} - \frac{\Gamma_{W^+W^-}}{2B} \right) Tr(V_\mu^2) [Tr(TV_\mu^2)]^2 \bigg|_7 \\
+ \left( \frac{\Gamma_{Z^0Z^0}}{8A} + \frac{A \Gamma_{W^+W^-}^2}{8B^2 \Gamma_{Z^0Z^0}} - \frac{\Gamma_{W^+W^-}}{4B} \right) [Tr(TV_\mu)Tr(TV_\nu)]^2 \bigg|_{10} \]

Dominate if we ignore \( g_0^i \) and \( \delta g_0^i \)

\[
A = \frac{e^4 m_h}{32 \pi s^4 c^4} \frac{1 - 4x + 12x^2}{x^2} (1 - x^2)^{\frac{1}{2}} \quad B = \frac{e^4 m_h}{32 \pi s^4 c^4} \frac{1 - 4y + 12y^2}{y^2} (1 - y^2)^{\frac{1}{2}} \\
\Gamma_{Z^0Z^0} = A(f_3 + f_4)^2, \quad \Gamma_{W^+W^-} = B f_4^2 \quad x = \frac{4m_Z^2}{m_h^2}, \quad y = \frac{4m_W^2}{m_h^2} \]
Effects of Higgs in EWCL

higgs mass dependence

\[
\begin{align*}
\frac{d\bar{f}_1}{dm^2} &= \frac{1}{32\pi^2} \frac{1}{m^2} \left[ \frac{1}{4}g_2^1 + f_5 + af_3 \right] \\
\frac{d\bar{f}_2}{dm^2} &= \frac{1}{32\pi^2} \frac{1}{m^2} \left[ \frac{1}{4}g_2^2 + f_6 + af_4 \right] \\
-\beta_1 &= \bar{f}_1 / \bar{f}_2 \quad \Rightarrow \quad m^2 \frac{d\alpha T}{dm^2} = 2 \frac{d\beta_1}{dm^2} \approx \frac{1}{16\pi^2} \frac{1}{f_2} [f_5 + \beta_1 f_6] \\
m^2 \frac{dg_0^1}{dm^2} &= \frac{1}{32\pi^2} [(g_0^1)'' + a(g_0^1)'] \\
\frac{1}{2}gg'\alpha_1 &= \bar{g}_0 \quad \Rightarrow \quad m^2 \frac{d(gg'S)}{dm^2} = -16\pi \frac{dgg'\alpha_1}{dm^2} = -\frac{1}{\pi} [(g_0^1)'' + a(g_0^1)']
\end{align*}
\]
## Effects of Higgs in EWCL

**higgs mass dependence**

| C   | $16\pi^2 \frac{dC}{d\ln m}|_{p^6}$ | $16\pi^2 \frac{dC}{d\ln m}|_{p^8}$ | $16\pi^2 \frac{dC}{d\ln m}|_{p^{10}}$ | $16\pi^2 \frac{dC}{d\ln m}|_{p^{12}}$ |
|-----|------------------------------------|------------------------------------|------------------------------------|------------------------------------|
| $\tilde{f}_1$ | $f_5$ | $\frac{g_2^1}{4}$ | $a f_3$ | $2a f_5$ |
| $\tilde{f}_2$ | $f_6$ | $\frac{g_2^2}{4}$ | $a f_4$ | $2a f_6$ |
| $\tilde{f}_3$ | $f_7$ | | $(g_2^4)'$ | $a(g_0^4)'$ |
| $\tilde{f}_4$ | $f_8$ | | $(g_2^6)'$ | $a(g_0^4)'$ |
| $\tilde{g}_0^4$ | | $(g_0^4)''$ | | $f_4 f_8 + a(g_0^5)' - \frac{(f_6)^2}{2}$ |
| $\tilde{g}_0^6$ | | $(g_0^6)''$ | | $f_3 f_7 + f_4 f_8 + a(g_0^7)' + f_5 f_6$ |
| $\tilde{g}_0^5$ | | | | $f_3 f_7 + a(g_0^{10})' - \frac{(f_5)^2}{2}$ |
| $\tilde{g}_0^7$ | | | | $a(g_0^i)'$ |
Summary

- R&D of Theory and exp all asked for M Ind invest for new physics
- Before higgs is discovered, EWCL is a good tool to do research
- We have calculated effects from comprehensive higgs up to 1-loop
- Wait for exp data and more detail phenomenology analysis
- Going to estimate effects of other new physics particles
Thanks!