QCD Correction to $J/\psi$ Production at Different Energy Scales

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1 Introduction

2 J/psi production at the B factories
   - double charmonium production
   - Inclusive $J/\psi$ production

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   - $J/\psi$ production in $Z$ decay
   - $J/\psi$ production from $\Upsilon$ Decay

4 $J/\psi$ production at the Tevatron and LHC
   - QCD Correction to color-singlet $J/\psi$ production
   - QCD Correction to color-octet $J/\psi$ production

5 $J/\psi$ production at the HERA

6 Other New Progress

7 Summary
Introduction

- Perturbative and non-perturbative QCD, hadronization, factorization
- Color-singlet and Color-octet mechanism was proposed based on NRQCD since c-quark is heavy.
- Clear signal to detect $J/\psi$.
- Heavy quarkonium production is a good place to testify these theoretical framework.
- But there are still many difficulties.
  - $J/\psi$ photoproduction at HERA
  - $J/\psi$ production at the B factories
  - $J/\psi$ polarization at the Tevatron
- NLO corrections are important.
  - Data on inelastic $J/\psi$ photoproduction are adequately described by the color singlet channel alone at NLO
  - Double charmonium production at the B factories
QCD Correction to $J/\psi$ Production at Different Energy Scales

- $J/\psi$ production at the B factories
- double charmonium production

$$e^+ e^- \rightarrow J/\psi + \eta_c$$

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<td>$K \equiv \sigma^{NLO}/\sigma^{LO} \sim 2$</td>
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<td>Confirmed by the analytic result in PRD77, (2008), B. Gong and J. X. Wang</td>
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<th>Relativistic corrections</th>
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<td>$K \sim 2$</td>
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<td>PRD67, (2007) E. Braaten and J. Lee</td>
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Problem

LO NRQCD prediction indicates that the cross section of this process is large than that of $J/\psi + \eta_c$ production by a factor of 1.8, but no evidence for this process was found at the B factories.

PRL90, (2003) G. T. Bodwin, E. Braaten and J. Lee
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NLO QCD corrections

- Greatly decreased, with a $K$ factor ranging from $-0.31 \sim 0.25$ depending on the renormalization scale.
- Might explain the situation.

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Inclusive $J/\psi$ production

**LO NRQCD Predictions:**

\[
e^+ e^- \to J/\psi + c\bar{c} \quad 0.07 \sim 0.20 \text{pb} \\
e^+ e^- \to J/\psi + gg \quad 0.15 \sim 0.3 \text{pb} \\
e^+ e^- \to J/\psi^{(8)}(3\mathcal{P}_J, 1\mathcal{S}_0) + g \quad 0.3 \sim 0.8 \text{pb}
\]


**Experimental Data:**

- **BARAR**
  \[\sigma[e^+ e^- \to J/\psi + X] = (2.54 \pm 0.21 \pm 0.21) \text{ pb}\]

- **CLEO**
  \[\sigma[e^+ e^- \to J/\psi + X] = (1.9 \pm 0.20) \text{ pb}\]

- **BELLE**
  \[\sigma[e^+ e^- \to J/\psi + X] = (1.45 \pm 0.10 \pm 0.13) \text{ pb}\]
  \[\sigma[e^+ e^- \to J/\psi + c\bar{c} + X] = (0.87^{+0.21}_{-0.19} \pm 0.17) \text{ pb}\]


**New BELLE Data**

\[\sigma[e^+ e^- \to J/\psi + X] = (1.17 \pm 0.02 \pm 0.07) \text{ pb}\]
\[\sigma[e^+ e^- \to J/\psi + c\bar{c}] = (0.74 \pm 0.08^{+0.09}_{-0.08}) \text{ pb}\]
\[\sigma[e^+ e^- \to J/\psi + X_{\text{non}-c\bar{c}}] = (0.43 \pm 0.09 \pm 0.09) \text{ pb}\]

[Pakhlov et al.(2009)]
Cross section at NLO for $e^+e^- \rightarrow J/\psi + gg$

$$\sigma^{(1)} = \sigma^{(0)} \left\{ 1 + \frac{\alpha_s(\mu)}{\pi} \left[ a(\hat{s}) + \beta_0 \ln \left( \frac{\mu}{2m_c} \right) \right] \right\}$$

<table>
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<tr>
<th>$m_c$ (GeV)</th>
<th>$\alpha_s(\mu)$</th>
<th>$\sigma^{(0)}$ (pb)</th>
<th>$a(\hat{s})$</th>
<th>$\sigma^{(1)}$ (pb)</th>
<th>$\sigma^{(1)}/\sigma^{(0)}$</th>
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<tr>
<td>1.4</td>
<td>0.267</td>
<td>0.341</td>
<td>2.35</td>
<td>0.409</td>
<td>1.20</td>
</tr>
<tr>
<td>1.5</td>
<td>0.259</td>
<td>0.308</td>
<td>2.57</td>
<td>0.373</td>
<td>1.21</td>
</tr>
<tr>
<td>1.6</td>
<td>0.252</td>
<td>0.279</td>
<td>2.89</td>
<td>0.344</td>
<td>1.23</td>
</tr>
</tbody>
</table>

Consistent results from two group:
PRL102, (2009) Y. Q. Ma, Y. J. Zhang and K. T. Chao

Relativistic Correction enhance results about a factor 1.3 from two group:
PRD82, (2010). Y. Jia
QCD Correction to $J/\psi$ Production at Different Energy Scales

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- Inclusive $J/\psi$ production

\[ e^+ e^- \rightarrow J/\psi + c\bar{c} \]

\[ \sigma^{(1)} = \sigma^{(0)} \left\{ 1 + \frac{\alpha_s(\mu)}{\pi} \left[ a(\hat{s}) + \beta_0 \ln \left( \frac{\mu}{2m_c} \right) \right] \right\} \]

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<tr>
<td>1.4</td>
<td>0.267</td>
<td>0.224</td>
<td>8.19</td>
<td>0.380</td>
<td>1.70</td>
</tr>
<tr>
<td>1.5</td>
<td>0.259</td>
<td>0.171</td>
<td>8.94</td>
<td>0.298</td>
<td>1.74</td>
</tr>
<tr>
<td>1.6</td>
<td>0.252</td>
<td>0.129</td>
<td>9.74</td>
<td>0.230</td>
<td>1.78</td>
</tr>
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</table>

Cross sections with different charm quark mass $m_c$ with the renormalization scale $\mu = 2m_c$ and $\sqrt{s} = 10.6$ GeV.

More about the scale and comparison with data

Use Brodsky, Lepage and Mackenzie (BLM) scale setting [Brodsky et al. (1983)]

\[
\sigma^{(1)} = \sigma^{(0)}(\mu^*)[1 + \frac{\alpha_s(\mu^*)}{\pi} b(\hat{s})].
\]

<table>
<thead>
<tr>
<th>(m_c) (GeV)</th>
<th>(\alpha_s(\mu^*))</th>
<th>(\sigma^{(0)}) (pb)</th>
<th>(b(\hat{s}))</th>
<th>(\sigma^{(1)}) (pb)</th>
<th>(\sigma^{(1)}/\sigma^{(0)})</th>
<th>(\mu^*) (GeV)</th>
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<tr>
<td>1.4</td>
<td>0.348</td>
<td>0.381</td>
<td>3.77</td>
<td>0.540</td>
<td>1.42</td>
<td>1.65</td>
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<tr>
<td>1.5</td>
<td>0.339</td>
<td>0.293</td>
<td>4.31</td>
<td>0.429</td>
<td>1.47</td>
<td>1.72</td>
</tr>
<tr>
<td>1.6</td>
<td>0.332</td>
<td>0.222</td>
<td>4.90</td>
<td>0.337</td>
<td>1.52</td>
<td>1.79</td>
</tr>
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Cross sections with different charm quark mass \(m_c\). The renormalization scale \(\mu = \mu^* \sim m_c\).
Momentum distribution of inclusive $J/\psi$ production with $\mu = \mu^*$ and $m_c = 1.4$ GeV is taken for the $J/\psi cc$ channel. The contribution from the feed-down of $\psi'$ has been added to all curves by multiplying a factor of 1.29.
QCD Correction to $J/\psi$ Production at Different Energy Scales

$J/\psi$ production at the B factories

Inclusive $J/\psi$ production

Momentum and angular distributions of inclusive $J/\psi$ production.

The contribution from the feed-down of $\psi'$ has been added to all curves by multiplying a factor of 1.29.
QCD Correction to $J/\psi$ Production at Different Energy Scales

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Polarization parameter $\alpha$ and angular distribution parameter $A$ of $J/\psi$ as functions of $p$. 
Constraint for color-octect matrix element of $c\bar{c}(^{1}S_{0}^{8}, 3P_{j}^{8})$

$$\sigma[e^+ e^- \rightarrow J/\psi + X_{\text{non}-c\bar{c}}] = (0.43 \pm 0.09 \pm 0.09) \text{ pb}$$

$$\sigma[e^+ e^- \rightarrow J/\psi + X_{\text{non}-c\bar{c}}]^{\text{color-onlyTh}} > (0.43) \text{ pb}$$

$$\sigma[e^+ e^- \rightarrow J/\psi + X_{\text{non}-c\bar{c}}]^{\text{color-octetTh}} > (0.6) \text{ pb}$$

From the contribution of $e^+ e^- \rightarrow J/\psi(^{1}S_{0}^{8}, 3P_{j}^{8}) + g$ at NLO

QCD Correction to $J/\psi$ Production at Different Energy Scales

Experimental and Leading-order Theoretical Results. [Acciarri:1998]

$$Br(Z \to J/\psi_{\text{prompt}} + X) = (2.1^{+1.4}_{-1.2}) \times 10^{-4}$$

Dominant process: $Z \to J/\psi + c\bar{c} + X$, and the total decay width is presented as

$$\Gamma^{NLO}(\mu) = \Gamma^{LO}(\mu)[1 + \frac{\alpha_s(\mu)}{\pi}(A + \beta_0 \ln \frac{\mu}{2m_Q} + Bn_f)].$$

(1)

$$Br^{\text{total}} = (7.3 \sim 10) \times 10^{-5}$$

The situation for $J/\psi$ production in $\Upsilon$ decay

**LO NRQCD Predictions:**

\[ Br(\Upsilon \to J/\psi(3S_1^8) + gg) = 6.2 \times 10^{-4} \] , M. Napsuciale, Phys. Rev. D 57, 5711 (1998)

\[ Br(\Upsilon \to J/\psi + c\bar{c}g) = 5.9 \times 10^{-4} \] , S. Y. Li, Q. B. Xie and Q. Wang, Phys. Lett. B 482, 65 (2000)

\[ Br(\Upsilon \to J/\psi + gg) = \text{order at} \times 10^{-4} \] , ????

**Experimental Data for $Br(\Upsilon \to J/\psi + X)$:**

CLEO $(11 \pm 4 \pm 2) \times 10^{-4}$ Phys. Lett. B 224, 445


CLEO $(6.4 \pm 0.4 \pm 0.6) \times 10^{-4}$ Phys. Rev. D70, 072001(2004)

The situation is quite strange ????

The correct leading order prediction is

\[ B_{Direct}(\Upsilon \to J/\psi + c\bar{c}g) = 3.9 \times 10^{-5}. \]  
\[ Z. \ G. \ He \ and \ J. \ X. \ Wang, \ Phys.Rev.D81:054030,2010. \]

Part of NLO prediction from $\Upsilon \to J/\psi + gg$ is

\[ B_{Direct}(\Upsilon \to J/\psi + gg) = 3.1 \times 10^{-5}. \]  
\[ Z. \ G. \ He \ and \ J. \ X. \ Wang, \ arXiv:1009.1563[hep-ph]]. \]

The full QCD correction for the inclusive $J/\psi$ production in $\Upsilon$ decay would be a very interesting and challenge work for explaining the experimental data.
QCD Correction to color-singlet $J/\psi$ production

$P_t$ distribution of $J/\psi$ production at QCD NLO was calculated in PRL98,252002 (2007), J. Campbell, F. Maltoni F. Tramontano

Some technique problems must be solved to calculate $J/\psi$ polarization

$P_t$ distribution of $J/\psi$ polarization at QCD NLO was calculated in PRL100,232001 (2008), B. Gong and J. X. Wang
QCD Correction to color-singlet $\Upsilon$ production

$\Upsilon$ polarization drastically changes from transverse polarization dominant at LO into longitudinal polarization dominant at NLO

$P_t$ distribution of $\Upsilon$ polarization at QCD NLO was calculated with detail in PRD78 074011 (2008), B. Gong and J. X. Wang

Partly NNLO calculation for $\Upsilon$ production calculated by PRL101, 152001(2008), P. Artoisenet, John M. Campbell, J.P. Lansberg, F. Maltoni, F. Tramontano
NLO QCD corrections to $J/\psi$ production via S-wave color octet states

3 tree processes at LO

\begin{align*}
g(p_1) + g(p_2) & \rightarrow J/\psi\left[1S_0^{(8)}, 3S_1^{(8)}\right](p_3) + g(p_4), \\
g(p_1) + q(p_2) & \rightarrow J/\psi\left[1S_0^{(8)}, 3S_1^{(8)}\right](p_3) + q(p_4), \\
q(p_1) + \overline{q}(p_2) & \rightarrow J/\psi\left[1S_0^{(8)}, 3S_1^{(8)}\right](p_3) + g(p_4).
\end{align*}

At NLO

\begin{align*}
gg & \rightarrow J/\psi\left[1S_0^{(8)}, 3S_1^{(8)}\right]gg, \\
gg & \rightarrow J/\psi\left[1S_0^{(8)}, 3S_1^{(8)}\right]q\overline{q}, \\
gq & \rightarrow J/\psi\left[1S_0^{(8)}, 3S_1^{(8)}\right]gq, \\
q\overline{q} & \rightarrow J/\psi\left[1S_0^{(8)}, 3S_1^{(8)}\right]gg, \\
q\overline{q} & \rightarrow J/\psi\left[1S_0^{(8)}, 3S_1^{(8)}\right]q'q', \\
qq & \rightarrow J/\psi\left[1S_0^{(8)}, 3S_1^{(8)}\right]qq, \\
qq' & \rightarrow J/\psi\left[1S_0^{(8)}, 3S_1^{(8)}\right]qq'.
\end{align*}
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- $J/\psi$ production at the Tevatron and LHC
- QCD Correction to color-octet $J/\psi$ production

QCD Correction to color-octet $J/\psi(1S_0, 3S_1)$ production

To fit the Tevatron $p_t$ distribution give more $\langle \mathcal{O}_8^{\psi}(1S_0) \rangle = 0.075$ GeV$^3$ and less $\langle \mathcal{O}_8^{\psi}(3S_1) \rangle = 0.0021$ GeV$^3$ than they are at LO fitting. The experimental data with $p_t < 6$ GeV have to abandon.


Correction to color-octet $J/\psi(1S_0, 3S_1, 3P_1^f)$ production was done recently and gave almost the same prediction for $p_t$ distribution as before without calculation of polarization, by

arXiv:1009.3655, Yan-Qing Ma, Kai Wang, Kuang-Ta Chao
arXiv:1009.5662, Mathias Butenschoen, Bernd A. Kniehl
QCD Correction to $J/\psi$ Production at Different Energy Scales

$J/\psi$ production at the Tevatron and LHC

QCD Correction to color-octet $J/\psi$ production

QCD Correction to color-octet $\Upsilon(1S^8_0, 3S^8_1)$ production

QCD Correction to $J/\psi$ production at HERA.

$P_t$ distribution of production and different scheme of polarization for $J/\psi$ (color-singlet) at QCD NLO was calculated in
PRL102, 142001 (2009), P. Artoisenet, John M. Campbell, F. Maltoni, F. Tramontano,

$P_t$ distribution of production $J/\psi$ (color-octet) at QCD NLO was calculated in

It include p-wave state and some progress in technique must be archived.
Other New Progress

$\chi_{cJ}$ production at hadron colliders with QCD radiative corrections
It include p-wave state and some progress in technique must be archived.

A new factorization scheme for $J/\psi$ hadron production proposed by
J. W. Qiu, et al, Qiu’s talk

Fragmentation function of $c \rightarrow J/\psi$ at QCD NLO was calculated by
B. Gong and J. X. Wang, in prepare
Summary

- For B-factories: NRQCD at NLO of $\alpha_s$ and $\nu$ can well describe $J/\psi$ production data. Strong constraint to the values of color-octect matrix element of $c\bar{c}(^1S^0_0,^3P^8_J)$ to almost zero. The dominant part $c\bar{c}(^3S^8_1)$ for hadron production is still there.

- For $J/\psi$ production in $\Upsilon$ decay, the LO prediction is one order in magnitude smaller than experimental measurement.

- The NLO results for $J/\psi$ production in $z^0$ decay is just half of experimental measurement.

- $c \to J/\psi$ fragmentation function is obtained at NLO level for the first time.

- The polarization problem for $J/\psi$ hadroproduction is still there even at QCD NLO.

- New Progress, ....
Thank you!

P. Pakhlov (Belle) (2004), hep-ex/0412041.


Y.-Q. Ma, Y.-J. Zhang, and K.-T. Chao, Phys. Rev. Lett. 102, 162002 (2009), 0812.5106.


