The Decay Constants $f_{D_s}$ and $f_{D^+}$ from Lattice QCD

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Lattice QCD is an \textit{ab initio} method

\[ S_{\text{Dirac}} = \bar{\psi}(\not{D} + m)\psi \]

Procedure (executive summary):

- Choose the bare coupling constant, \( g_0 \), extract \( \alpha_s \) at short distance e.g. from plaquette.
- Determine lattice spacing, \( a \), from e.g. HQ potential.
- Quark masses \( m_u, m_d, m_s, m_c \) and \( m_b \) determined by reproducing masses of \( \pi, K, \eta_c \) and \( \eta_b \) mesons.
- QCD parameters now completely fixed in other computations.
“Gold-plated” quantities in LQCD

Lattice QCD technology for process is understood, not computationally expensive and statistical signal to noise ratio is good.

- Stable particles not near threshold.
- At most one stable hadron in both initial and final states.
- Low to moderate momentum transfer in process.

Spectroscopy, leptonic decays, semileptonic decays and neutral meson mixing!
Leptonic and semileptonic decays plus mixing...

\[
\begin{pmatrix}
|V_{ud}| & |V_{us}| & |V_{ub}| \\
\pi \to \ell \bar{\nu} & K \to \ell \bar{\nu} & B \to \pi \ell \bar{\nu}
\end{pmatrix}
\begin{pmatrix}
|V_{cd}| & |V_{cs}| & |V_{cb}|

D \to \ell \bar{\nu} & D_s \to \ell \bar{\nu} & B \to D^* \ell \bar{\nu}
\end{pmatrix}
\begin{pmatrix}
|V_{td}| & |V_{ts}| & |V_{tb}|

D \to \pi \ell \bar{\nu} & D \to K \ell \bar{\nu} & B \to D \ell \bar{\nu}
\end{pmatrix}
\begin{pmatrix}
B-\bar{B} \text{ mixing:} \\
\hat{B}_{B_d} \text{ and } f_B \\
B_s-\bar{B}_s \text{ mixing:} \\
\hat{B}_{B_s} \text{ and } f_{B_s}
\end{pmatrix}

K-\bar{K} \text{ mixing: } |\epsilon_K| \sim B_K \eta(1 - \bar{\rho})
Charm systems and lattice QCD

Charmonium and $D$ mesons are ideal systems for testing lattice QCD methods.

- Abundant and relatively precise experimental data.
- Test lattice technologies for both heavy and light quarks.
- Same techniques used for bottom: $m_c \rightarrow m_b$.
- CKM physics and possible new physics signals,
  e.g. nonstandard leptonic $D_s$ decays, Dobrescu and Kronfeld, \texttt{arXiv:0803.0512} – the “$f_{D_s}$ puzzle” circa 2007.

Three talks at CHARM 2010: Charmonium (C. Detar), semileptonic decays (H. Na) and decay constants (this talk).
Experiment determines the product $f_{Dq}|V_{cq}|$,

$$\Gamma(D_q \rightarrow \ell \nu) = \frac{G_f^2}{8\pi} m_\ell^2 \left(1 - \frac{m_\ell^2}{M_{Dq}^2}\right)^2 M_{Dq} f_{Dq}^2 |V_{cq}|^2,$$

while the lattice can compute $f_{Dq}$ from first principles.

A complete lattice calculation of the $f_{Dq}$ must address:

- dynamical (sea) quark effects,
- discretization effects and finding the continuum limit $a \rightarrow 0$,
- chiral extrapolation to the physical light quarks,
- tuning errors in determinations of “$a$” and quark masses.
Lattice studies of $f_{D_s}$ and $f_{D^+}$

Focus on results from three collaborations with features:

<table>
<thead>
<tr>
<th>collab.</th>
<th>name</th>
<th>$n_f$</th>
<th>sea quarks</th>
<th>valence quarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPQCD</td>
<td>MILC</td>
<td>2 + 1</td>
<td>asqtad</td>
<td>HISQ</td>
</tr>
<tr>
<td>FNAL/MILC</td>
<td>MILC</td>
<td>2 + 1</td>
<td>asqtad</td>
<td>asqtad, FNAL clover</td>
</tr>
<tr>
<td>ETMC</td>
<td>ETMC</td>
<td>2</td>
<td>tw-mass</td>
<td>tw-mass, tw-mass</td>
</tr>
</tbody>
</table>


**ETMC:** B. Blossier, *et al.*, arXiv:0904.0954
Neglecting vacuum polarization ($n_f = 0$, quenched QCD) leads to 10-20% uncertainties.

Effects from a quenched strange quark, e.g. in the ETMC $n_f = 2$ study, are difficult to estimate \textit{a priori}.

The heavier charm mass motivates a perturbative bound on effects from quenched charm. HPQCD estimates this error to be $\ll 0.01\%$ for $f_{D_s}$.

Note: MILC/FNAL, HPQCD and ETMC are now generating gauge configurations including dynamical charm ($n_f = 2 + 1 + 1$)
The MILC collaboration has made publicly available sets of gluon configurations having three flavors dynamical quarks [A. Bazavov, et al., arXiv:0903.3598].

- Quenching is no longer dominant systematic.
- One flavor \( m_h \approx m_s \), two flavors \( m_s/10 \leq m_l \leq m_s \).
- Numerically less expensive than other methods.
- Lighter quarks reduce “chiral” extrapolation systematics.
- Leading gluons errors \( \mathcal{O}(\alpha_s^2 a^2) \) and “Asqtad” improved staggered quarks \( \mathcal{O}(\alpha_s a^2) \).
- Lattice spacings of 0.045, 0.06, 0.09, 0.12 and 0.15 fm.
Open science

- Sets (ensembles) of gauge configurations are expensive to generate, requiring large amounts of time on the fastest computers in the world.

- The MILC, ETMC and other sets of configurations are openly shared worldwide via the ILDG, the International Lattice Data Grid.

- A rich set of open source LQCD application codes are available in the MILC and Chroma codes which use the USQCD SciDAC portable LQCD libraries.
Valence quarks are improved

**MILC/FNAL**

- **light**: Asqtad improved stag. leading errors $O(\alpha_s a^2)$
- **charm**: Clover in FNAL interpretation $O(\alpha_s a^2 \Lambda^2, a^4 \Lambda^4)$

**HPQCD**

HISQ (Highly Improved Stag. Quark) for both light and charm.

- **light**: $O(\alpha_s a^2)$, though smaller than asqtad.
- **charm**: leading error $O(\alpha_s a^2 m_c^2)$

**ETMC**

Twisted-mass quarks for both light and charm.

$$L_{doublet} = \bar{\chi} \left( \partial + m_q + i\mu_q \gamma_5 \tau^3 \right) \chi$$

At tuned twist, $O(a^2 \mu_q^2)$ errors for $q = \text{light and charm}$.
Extrapolation in $a^2$ setting quarks to their physical masses

- most precise lattice result
- new result for $f_{D_s}$ only
- $2\sigma$ higher than 2007 result!
- now at five lattice spacings
- better tuning of quark masses and lattice spacing
- full $f_{D^+}$ update to follow?
- significant lattice spacing dependence

<table>
<thead>
<tr>
<th>$a$ [fm]</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>7.8%</td>
</tr>
<tr>
<td>0.12</td>
<td>4.2%</td>
</tr>
<tr>
<td>0.09</td>
<td>1.7%</td>
</tr>
</tbody>
</table>
Lattice spacing recalibration on MILC lattices

Distance $r_1$ defined by the HQ potential

- older $\Upsilon(2S-1S)$ gave a larger $r_1$ (top two values)
- recent MILC and HPQCD give a lower $r_1$ (bottom two values)

- net effect on $f_{D_s}$ is smaller than naive rescaling since quark masses must be retuned
FNAL/MILC $f_{D_s}$ and $f_{D^+}$ extrapolation

Fit at finite $a$ and simulated sea quarks

Extrap. $a \to 0$ and all $m_q \to$ physical

- $a = 0.09, 0.12$ and $0.15$ fm
- eleven sets of gluon configurations
- $\chi$ logs not apparent at finite “a”

PRELIMINARY result
- physical $f_{D_q}$ indicated in red
- “full QCD” subset of data points overlay the extrapolation
## Detailed error budgets

### HPQCD

<table>
<thead>
<tr>
<th>Source</th>
<th>$f_{DS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>statistics /valence tuning</td>
<td>0.57</td>
</tr>
<tr>
<td>$r_1/a$ (lat. spacing)</td>
<td>0.15</td>
</tr>
<tr>
<td>$r_1$</td>
<td>0.57</td>
</tr>
<tr>
<td>$a^2$ extrap.</td>
<td>0.40</td>
</tr>
<tr>
<td>sea-quark extrap.</td>
<td>0.34</td>
</tr>
<tr>
<td>finite vol.</td>
<td>0.10</td>
</tr>
<tr>
<td>$m_{\eta_S}$ ($m_s$ tune)</td>
<td>0.13</td>
</tr>
<tr>
<td>QED in $D_S$</td>
<td>0.10</td>
</tr>
<tr>
<td>QED and annih. $m_{\eta_c}$</td>
<td>0.00</td>
</tr>
<tr>
<td>quenched charm</td>
<td>0.00</td>
</tr>
<tr>
<td>Total</td>
<td>1.0</td>
</tr>
</tbody>
</table>

### FNAL/MILC

<table>
<thead>
<tr>
<th>Source</th>
<th>$f_{DS}$</th>
<th>$f_{D^+}$</th>
<th>$f_{DS}/f_{D^+}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>stat. + disc. effects</td>
<td>2.9</td>
<td>3.6</td>
<td>1.1</td>
</tr>
<tr>
<td>chiral extrapolation</td>
<td>0.8</td>
<td>1.4</td>
<td>1.2</td>
</tr>
<tr>
<td>inputs $r_1$, $m_s$, $m_d$ and $m_u$</td>
<td>0.7</td>
<td>0.8</td>
<td>0.1</td>
</tr>
<tr>
<td>input $m_c$ or $m_b$</td>
<td>1.2</td>
<td>1.0</td>
<td>0.2</td>
</tr>
<tr>
<td>$Z_{V}^{hh}$ and $Z_{V}^{qq}$</td>
<td>1.0</td>
<td>1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>higher-order $\rho_{A_4}$</td>
<td>0.3</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>finite volume</td>
<td>0.2</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Total</td>
<td>3.5</td>
<td>4.2</td>
<td>1.7</td>
</tr>
</tbody>
</table>

### Predicted improvements

The Decay Constants $f_{DS}$ and $f_{D^+}$ from Lattice QCD
ETMC $f_{D_S}$ extrapolation ($n_f = 2$)

- **top:** extrapolation in both $m_i^{\text{sea}}$ and $a$ for $\phi_s = f_{D_S} \sqrt{m_{D_S}}$
- **bottom:** extrapolation of ratio $\phi_s / \phi_d$
- bulk of many syst. errors cancel in ratio
- both $SU(2)$ and $SU(3)$ chiral P.Th. fits shown
- lattice spacings $a = 0.065, 0.085$ and $0.10$ fm
Three flavor $f_{D_s}$ results differ at the $1.4\sigma$ level

### Results in 2 + 1 flavor QCD

<table>
<thead>
<tr>
<th>collaboration</th>
<th>$f_{D_s}$ [MeV]</th>
<th>$f_{D^+}$ [MeV]</th>
<th>$f_{D_s}/f_{D^+}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPQCD</td>
<td>248.0 ± 2.5</td>
<td>213 ± 4</td>
<td>1.164 ± 0.018</td>
</tr>
<tr>
<td>FNAL/MILC</td>
<td>261.4 ± 9.2</td>
<td>220.3 ± 9.3</td>
<td>1.19 ± 0.02</td>
</tr>
</tbody>
</table>

### Results in 2 flavor QCD

<table>
<thead>
<tr>
<th>collaboration</th>
<th>$f_{D_s}$ [MeV]</th>
<th>$f_{D^+}$ [MeV]</th>
<th>$f_{D_s}/f_{D^+}$</th>
</tr>
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<tbody>
<tr>
<td>ETMC</td>
<td>244 ± 8</td>
<td>197 ± 9</td>
<td>1.24 ± 0.03</td>
</tr>
</tbody>
</table>

HPQCD: $f_{D^+}$ based on older ratio and updated $f_{D_s}$

FNAL/MILC: PRELIMINARY
Brief history of $f_{D_s}$

- Gray bands lattice three-flavor avg.
- Yellow bands expt. avg.
- Leftmost ($t = 0$) result accompanied by successful prediction of $f_{D^+}$ by FNAL/MILC.
- HPQCD 2007 ($t \approx 2$) result provoked the “$f_{D_s}$ puzzle” (3.8$\sigma$ discrepancy).
- Lattice avg. has come up.
- Expt. has come down.

Kronfeld, arXiv:0912.0543 + updates
Comparisons of lattice to recent experiment

- Includes recent $f_{D_s}$ update from BaBar.
- My unofficial expt. average pending HFAG $f_{D_s}$ update.
- HPQCD and expt. $f_{D_s}$ differ at about the 1.5\,$\sigma$ level.

**BaBar:** P. del Amo Sanches, *et al.*, arXiv:1008.4080

**Belle:** K. Abe, *et al.*, arXiv:0709.1340

**CLEO:** D. Cassel, ICHEP 2010, Paris
BES-III and future lattice

Bounds on 2-Higgs doublet (type-II)

- 1-2% decay constant measurements by BES-III a welcome challenge for lattice!
- HPQCD: update to $f_{D^+}$?
- FNAL/MILC: extend asqtad to finer lattices and higher statistics.
- FNAL/MILC/HPQCD: HISQ valence+sea quarks with $n_f = 2 + 1 + 1$.
- ETMC: A four dynamical flavor prelim. $f_{D_s}$ shown at LATTICE2010.


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