J/Ψ and Xc Polarization at Hadron Colliders

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Contents

- Theoretical framework: NRQCD @ NLO
- $J/\psi$ and $Xc$ polarization and CO LDMEs fit
- Conclusion
The short distance parton level cross section is perturbative and process-dependent.

The parton distribution functions and long distance matrix elements $<O_n>$ are non-perturbative but universal.

The long distance matrix elements are matrix elements of four-fermion operators in NRQCD:

$$\langle O_n \rangle = \langle 0^| \chi^\dagger \kappa_n \psi (\sum | H + X ) \langle H + X | \psi^\dagger \kappa_n^\dagger \chi | 0 \rangle$$

The long distance matrix elements are scaled by $\nu$:

$$\nu_c^2 \approx 0.23, \nu_b^2 \approx 0.08$$
To solve the $\Psi'$ surplus puzzle, the color-octet (CO) mechanism was proposed by Braaten and Fleming based on NRQCD factorization.

- The CO yield scales as $p_T^{-4}$ decreases much slower than $p_T^{-8}$ of color-singlet (CS) state.
- A natural explanation of observed $\Psi'$ (and $J/\Psi$ )surplus at the Tevatron.

<table>
<thead>
<tr>
<th>States</th>
<th>$p_T$ behavior at LO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^3S_1$</td>
<td>$p_T^{-8}$</td>
</tr>
<tr>
<td>$^3S_1$</td>
<td>$p_T^{-4}$</td>
</tr>
<tr>
<td>$^1S_0$</td>
<td>$p_T^{-6}$</td>
</tr>
<tr>
<td>$^3P_J$</td>
<td>$p_T^{-6}$</td>
</tr>
</tbody>
</table>

NRQCD @ LO: $J/\psi(\psi(2S))$ polarization puzzle

- Although it seems to successfully explain the cross sections, CO encounters difficulties when the polarization is also taken into consideration.
- Dominated by gluon fragmentation to $^{3}S_{1}$ at large $p_T$, LO NRQCD predicts a sizable transverse polarization, while the measurement gives almost unpolarized.
- In gluon fragmentation, the spin-flip interaction is suppressed (Cho, Wise (1994)).
NRQCD @ NLO

Kinematical enhancements at large $p_T$

<table>
<thead>
<tr>
<th>Fock states</th>
<th>$p_T$ scaling at NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^3S_1^{[1]}$</td>
<td>$p_T^{-6}$</td>
</tr>
<tr>
<td>$^3S_1^{[8]}$</td>
<td>$p_T^{-4}$</td>
</tr>
<tr>
<td>$^1S_0^{[1,8]}$</td>
<td>$p_T^{-4}$</td>
</tr>
<tr>
<td>$^3P_J^{[1,8]}$</td>
<td>$p_T^{-4}$</td>
</tr>
</tbody>
</table>

- The NLO color-singlet differential cross section is enhanced by 2 order relative to LO $^3S_1^{[1]}$ result at large $p_T$.
  \[ J.M.Campbell \text{ et al. (2007)} \]
- Corrections to $^3P_J^{[1,8]}$ should be more important at large $p_T$.
- The soft gluon radiations are canceled in the channel $^1S_0^{[1,8]}$, thus, the corrections to these channels are not very significant.
NRQCD @ NLO: Fock states

- Two groups calculated it independently: Ma, Wang, Chao (2011) and Butensckön, Kniehl (2011).
- The results of the two groups for the short-distance coefficients agree.

**Decomposition of P-wave channel:**
- At NLO and larger $p_T$, roughly $d\sigma[^3S_1^{[8]}] \sim p_T^{-4}$ and $d\sigma[^1S_0^{[8]}] \sim p_T^{-6}$, and we find the following decomposition holds within error of a few percent:

\[
\hat{d}\sigma[^3P_J^{[8]}] = r_0 \, d\sigma[^1S_0^{[8]}] + r_1 \, d\sigma[^3S_1^{[8]}]
\]

- As a result, we use two linearly combined LDMEs:

\[
M_{0,r_0}^{J/\psi} = \langle \mathcal{O}^{J/\psi}[^1S_0^{[8]}] \rangle + \frac{r_0}{m_c^2} \langle \mathcal{O}^{J/\psi}[^3P_0^{[8]}] \rangle
\]
\[
M_{1,r_1}^{J/\psi} = \langle \mathcal{O}^{J/\psi}[^3S_1^{[8]}] \rangle + \frac{r_1}{m_c^2} \langle \mathcal{O}^{J/\psi}[^3P_0^{[8]}] \rangle
\]

- For CDF: $r_0 = 3.9$, $r_1 = -0.56$

Ma, Wang, Chao, (2011)
NRQCD @ NLO: Fit LDMEs

Ma, Wang, Chao (2011)

Fit LDMEs using Tevatron Data

<table>
<thead>
<tr>
<th>$p_T^{cut}$ GeV</th>
<th>$H$</th>
<th>$\langle \mathcal{O}^H \rangle$ GeV$^3$</th>
<th>$M_{1,r_1}^H$ 10$^{-2}$ GeV$^3$</th>
<th>$M_{0,r_0}^H$ 10$^{-2}$ GeV$^3$</th>
<th>$\chi^2/d.o.f.$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>$J/\psi$</td>
<td>1.16</td>
<td>0.05 ± 0.02</td>
<td>7.4 ± 1.9</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>$\psi'$</td>
<td>0.76</td>
<td>0.12 ± 0.03</td>
<td>2.0 ± 0.6</td>
<td>0.56</td>
</tr>
<tr>
<td>5</td>
<td>$J/\psi$</td>
<td>1.16</td>
<td>0.16 ± 0.05</td>
<td>5.2 ± 1.3</td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td>$\psi'$</td>
<td>0.76</td>
<td>0.17 ± 0.04</td>
<td>1.1 ± 0.3</td>
<td>2.2</td>
</tr>
</tbody>
</table>

- Agree with data only for $p_T > 7$ GeV, but the agreement is very good
- The smallness of $M_1$ for $J/\psi$ due to the steep experimental curve, which is roughly shows the $p_T^{-6}$-$p_T^{-5}$ behavior in the moderate $p_T$ region.

Could the smallness of $M_1$ for $J/\psi$ account for the nearly non-polarized CDFII data?
J/ψ and Xc Polarization
Polarization observables

1. \( J/\Psi \ (J=1) \rightarrow l^+ l^- \):

\[
N = \frac{3 \frac{d}{dp_T} \cos^2 \theta}{N + \frac{d}{dp_T}},
\]

\[
N = \frac{d}{dp_T} \cos \theta + \frac{d}{dp_T} \sin \theta
\]

2. \( \chi_c \ (J=1) \rightarrow J/\Psi \ \gamma \):

\[
N = \frac{3 \frac{d}{dp_T}}{(1+3d)N + (1+3d) \frac{d}{dp_T}}
\]

\[
N = \frac{d_{11}}{dp_T} + \frac{d_{00}}{dp_T} + \frac{d_{11}}{dp_T}
\]

3. \( \chi_c \ (J=2) \rightarrow J/\Psi \ \gamma \):

\[
N = \frac{3 \frac{d}{dp_T}}{(1+3d)N + (1+3d) \frac{d}{dp_T}}
\]

\[
N = \frac{d_{11}}{dp_T} + \frac{d_{00}}{dp_T} + \frac{d_{11}}{dp_T}
\]

\[
N = \frac{2}{10}
\]

\[
= \frac{1}{10} \left( 2 + 3a_2 J^2 + 6a_3 J^2 - 4a_4 J^2 + 7a_5 J^2 \right)
\]

\[
= \frac{1}{10} \left( 2 + 3a_2 J^2 + 6a_3 J^2 - 4a_4 J^2 + 7a_5 J^2 \right)
\]

\[
= \frac{1}{10} \left( 2 + 3a_2 J^2 + 6a_3 J^2 - 4a_4 J^2 + 7a_5 J^2 \right)
\]
Polarization observables II

\[
\chi_c J \rightarrow J/\psi \gamma \rightarrow l^+ l^- \gamma (Z' = v2):
\]
\[
N + 3 \frac{d}{dp_T} \cos J', \mu 1 + \cos^2 J', \mu 1 + \cos^2 J'.
\]

1. \( \chi_c 1 \rightarrow J/\psi \gamma \rightarrow l^+ l^- \gamma (Z' = v2) \):

\[
N = \frac{d}{dp_T} \frac{6(a_2^{J=1})^2}{(5 - 6(a_2^{J=1})^2)^2} \frac{d}{dp_T} \frac{d}{dp_T} \cos J', \mu 1 + \cos^2 J', \mu 1 + \cos^2 J'.
\]

2. \( \chi_c 2 \rightarrow J/\psi \gamma \rightarrow l^+ l^- \gamma (Z' = v2) \):

\[
N = \frac{d}{dp_T} \frac{15}{7} \frac{14(a_2^{J=2})^2}{(7 - 14(a_2^{J=2})^2)^2} \frac{d}{dp_T} \frac{5(a_3^{J=2})^2}{(5 - 6(a_3^{J=2})^2)^2} \frac{d}{dp_T} \cos J', \mu 1 + \cos^2 J', \mu 1 + \cos^2 J'.
\]

\[
N = \frac{2}{s} \frac{d}{dp_T} \frac{d}{dp_T} \frac{d}{dp_T} \frac{d}{dp_T} \cos J', \mu 1 + \cos^2 J', \mu 1 + \cos^2 J'.
\]

M2, E3
Complete NLO correction for $\Psi$ – polarization

Chao, Ma, Shao, Wang, Zhang (2012)

- Negative transverse component of $^3P_J^{[8]}$ channel may cancel that of $^3S_1^{[8]}$ channel, leading to mainly unporlarized $J/\Psi$.

- Strong correlation between the cross section and the polarization is observed:
  
  \[ d\sigma_{\text{tot}} \sim M_1/pT^4 + \ldots, \quad \text{with } r1 = -0.56 \]
  
  \[ d\sigma_{11} \sim M_1'/pT^4 + \ldots, \quad \text{with } r1' = -0.52 \]
Fit CO LDMEs: “fit” V.S. “scan+fit”

Hadroproduction unpolarized data can only fit two LDMEs combination unambiguity (pT>7GeV).

\[ M_{0,r_0} = \left\langle \mathcal{O}(S_0^{[8]}) \right\rangle + \frac{r_0}{m_c^2} \left\langle \mathcal{O}(\mathcal{P}_0^{[8]}) \right\rangle \]
\[ M_{1,r_1} = \left\langle \mathcal{O}(S_1^{[8]}) \right\rangle + \frac{r_1}{m_c^2} \left\langle \mathcal{O}(\mathcal{P}_0^{[8]}) \right\rangle \]

[Y.Q. Ma & K. Wang & K. T. Chao (2011)]

<table>
<thead>
<tr>
<th>( \left\langle \mathcal{O}(S_1^{[1]}) \right\rangle )</th>
<th>( M_{0,r_0} )</th>
<th>( M_{1,r_1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>GeV(^3)</td>
<td>( 10^{-2}\text{GeV}^3 )</td>
<td>( 10^{-2}\text{GeV}^3 )</td>
</tr>
<tr>
<td>1.16</td>
<td>7.4 \pm 1.9</td>
<td>0.05 \pm 0.02</td>
</tr>
</tbody>
</table>

\[ \chi^2 \text{ fit to } \frac{d\sigma}{dp_T} \text{ and } \lambda_\phi \]

<table>
<thead>
<tr>
<th>( \left\langle \mathcal{O}(S_1^{[1]}) \right\rangle )</th>
<th>( \left\langle \mathcal{O}(S_0^{[8]}) \right\rangle )</th>
<th>( \left\langle \mathcal{O}(S_1^{[8]}) \right\rangle )</th>
<th>( \frac{\left\langle \mathcal{O}(\mathcal{P}_0^{[8]}) \right\rangle}{m_c^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>GeV(^3)</td>
<td>( 10^{-2}\text{GeV}^3 )</td>
<td>( 10^{-2}\text{GeV}^3 )</td>
<td>( 10^{-2}\text{GeV}^3 )</td>
</tr>
<tr>
<td>1.16</td>
<td>8.9 \pm 0.98</td>
<td>0.30 \pm 0.12</td>
<td>0.56 \pm 0.21</td>
</tr>
<tr>
<td>1.16</td>
<td>11</td>
<td>0</td>
<td>2.4</td>
</tr>
</tbody>
</table>

First line: fit three LDMEs using yield and polarization data; Second and third line: “scan” all possible value of \(<\mathcal{O}(1S_0^{[8]})>\) and then fit the other two LDMEs. Using condition: LDMEs \( >0 \).
J/ψ polarization @ LHC

- Reasonable good for the pT distribution of cross section up to 70 GeV at the LHC.
- To predict unpolarized results at hadron colliders, a linear combination of CO LDMEs similar to $M_1$ (i.e., $M_1'$) should be near zero.
- Only direct J/ψ contributions.

Chao, Ma, Shao, Wang, Zhang (2012)

All three sets of LDMEs can give good predictions for yield!
Comments on our “scan + fit” method

A necessary condition to guarantee the cross section be positive at high $p_T$ is $M_1>0$, comparing with:

<table>
<thead>
<tr>
<th>$\langle O(1S_0^{[8]}) \rangle$</th>
<th>$\langle O(3S_1^{[8]}) \rangle$</th>
<th>$\langle O(3P_0^{[8]}) \rangle$</th>
<th>$M_{0,3.9}$</th>
<th>$M_{1,-0.56}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.9 ± 0.98</td>
<td>0.3 ± 0.12</td>
<td>0.56 ± 0.21</td>
<td>11.1 ± 1.3</td>
<td>-0.016 ± 0.168</td>
</tr>
<tr>
<td>0</td>
<td>1.4</td>
<td>2.4</td>
<td>9.36</td>
<td>0.056</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>0</td>
<td>11</td>
<td>0</td>
</tr>
</tbody>
</table>

**Note:** First line: fit three LDMEs using yield and polarization data; Second (third) line: choose a minimum (maximum) value of $\langle O(1S_0^{[8]}) \rangle$ and then fit the other two LDMEs.

**Observation:**

1. The central value of the first line gives negative cross section ($p_T>300$ GeV for LHC and $p_T>100$ GeV for Tevatron), but positive cross section can be obtained in all $p_T$ region within the error bar.
2. Although $\langle O(1S_0^{[8]}) \rangle$, $\langle O(3S_1^{[8]}) \rangle$ and $\langle O(3P_0^{[8]}) \rangle$ in the second and third lines are not within the error bar of the first line, the corresponding $M_{0,3.9}$ and $M_{1,-0.56}$ are within the error bar.
3. $\langle O(1S_0^{[8]}) \rangle$, $\langle O(3S_1^{[8]}) \rangle$ and $\langle O(3P_0^{[8]}) \rangle$ in the first line may be unphysical!
Comments on our “scan + fit” method

**Conclusion:**

1. Because only $M_{0,r0}$ and $M_{1,r1}$ can be well constrained by the data, using the fit three parameters method alone cannot gives all allowable parameter space for $<O(1S_0^{[8]})>$, $<O(3S_1^{[8]})>$ and $<O(3P_0^{[8]})>$, although it can determine the allowable parameter space for $M_{0,r0}$ and $M_{1,r1}$.

2. A scan + fit method is more suitable to find out all allowable parameter space.

To avoid the fit be trapped in unphysical parameter space, using “scan+fit” method instead of “fit” method!

**Comments:**

- In our “scan+fit” method, we introduce the condition that all LDMEs >0, which we cannot prove.
- However, at least our result means that even under this constraint, all data of $J/\psi$ hadroproduction at large $p_T$ can be described by NLO NRQCD.
Feeddown contributions

- Feeddown contribution mainly from $\Psi(2S)$ and $\chi_\text{cJ}$, all of which are calculated to NLO and their CO LDMEs are determined by fit Tevatron data.
- The transverse momentum difference is considered and approximated as:
  \[ p_T^{J/\psi} \approx p_T^H \times \frac{m_{J/\psi}}{m_H} \]
  with an very small error $O\left(\frac{(m_{J/\psi}-m_H)^2}{m_H^2}\right)$. (We thank J.P. Lansberg and P. Faccioli for helping to test this approximation by simulation.)

At large $p_T$:
- $J/\psi \sim 0.6$
- $\chi_\text{cJ} \sim 0.3$
- $\Psi(2S) \sim 0.1$
$\chi cJ \rightarrow J/\psi \gamma \rightarrow l^+ l^- \gamma$

Shao & Ma & Han & Chao, in progress

![Graphs](image.png)
Summing the feeddown contributions and the direct ones.

Only slight change of polarization by the feeddown contributions (at large $p_T$: $-0.05 \rightarrow 0.1$).
Prompt $J/\psi$ polarization at LHC:
preliminary estimations

Shao & Ma & Han & Chao, in progress

Similar to the results of direct contributions only
Comparing with other groups

Butenschon and Kniehl, PRL 108 107002 (2012):
• Using global fit LDMEs, which gives negative $\langle O( ^3P_0^{[8]} ) \rangle$.
• Transverse polarizations do not cancel between $^3S_1^{[8]}$ and $^3P_0^{[8]}$ channels, giving transverse polarization.

\[
\langle O^{J^P} \ ( ^1S_0^{[8]} ) \rangle = (4.97 \pm 0.44) \times 10^{-2} \text{GeV}^3, \quad \langle O^{J^P} \ ( ^3S_1^{[8]} ) \rangle = (2.24 \pm 0.59) \times 10^{-3} \text{GeV}^3,
\]
\[
\langle O^{J^P} \ ( ^3P_0^{[8]} ) \rangle = (-1.61 \pm 0.20) \times 10^{-2} \text{GeV}^5.
\]
Comparing with other groups


• Similar to our central value fit, but feeddown contributions are subtracted.

• Feeddown contributions cannot change the polarization too much, as discussed.

• This “fit method” may not be suitable for finding out all possible parameter space.

\[
\left( \langle \mathcal{O}(^1S_0^{[8]}) \rangle, \langle \mathcal{O}(^3S_1^{[8]}) \rangle, \frac{\langle \mathcal{O}(^3P_0^{[8]}) \rangle}{m_c^2} \right) \equiv \frac{\mathcal{O}}{100} \text{ GeV}^3
\]

\[\mathcal{O} = (9.7 \pm 0.9, -0.46 \pm 0.13, -0.95 \pm 0.25),\]

\[\Lambda = (-9.6 \pm 1.0, 1.7 \pm 0.1, -0.37 \pm 0.01),\]
Summary

• Kinematic enhancement of NRQCD amplitudes in hadroproduction $p_T$ spectrum motivated us to do complete NLO level analysis.

• It seems that all hadroproduction data of $J/\Psi$, including both polarization and yield, can be consistent described by NRQCD at NLO level.

• But S-wave and P-wave cancellation is needed (fine tuning?).

• There is still no consistent solution between NRQCD factorization, hadroproduction (polarization and yield) data, photoproduction data, two-photo production data, BELLE $e^+e^-$ data at NLO level.

• $\chi cJ$ polarization at NLO level is also presented.
Back up slides
Comments on the resummations of $\alpha_s \log P_T^2/m_c^2$

Could the resummation of $\alpha_s \log P_T^2/m_c^2$ change the fit of $M_1$, and thus change our explanation of the yield and polarization of $J/\psi$ production?

Quantitatively, Yes!
Qualitatively, No!

$P_T \sim 300$GeV, $\log P_T^2/m_c^2 \sim 10$

- At NLO and larger $p_T$, the short distance coefficient of $M_1$ is dominated by one gluon fragmentation and gives $d\sigma[^3S_1][8] \sim p_T^{-4}$
- The k-factor of $[^3S_1][8]$ channel is small (0.98-1.3) for $5\text{GeV}<p_T<300\text{GeV}$, which means the coefficients of $\alpha_s \log P_T^2/m_c^2$(~1) is not important. Thus, there should not be significant effects of large $p_T$ resummation for this channel.
- The $[^3P_J][8]$ channel at large $p_T$ is also dominated by one gluon fragmentation, so its behavior may be similar to $[^3S_1][8]$: resummation is not very important.
M. Butenschon and B. Kniehl’s fit

M. Butenschon, B. Kniehl, (2011)

1. Use unpolarized data in pp (not include ATLAS and CMS large pT data), γp, γγ and e+e- collisions.

2. pT>1 GeV for γp, γγ data and pT>3 GeV for pp. There is only one data for e+e-.

\[
\langle O^{J/\Psi} (1^1 S_0^{[8]}[8]) \rangle = (4.97 \pm 0.44) \times 10^{-2} \text{GeV}^3, \\
\langle O^{J/\Psi} (3^3 S_1^{[8]}[8]) \rangle = (2.24 \pm 0.59) \times 10^{-3} \text{GeV}^3, \\
\langle O^{J/\Psi} (3^3 P_0^{[8]}[8]) \rangle = (1.61 \pm 0.20) \times 10^{-2} \text{GeV}^5
\]

3. After feeddown was included (pp:36%, γp:15%, γγ:9%, e+e-:26%),

\[
\langle O^{J/\Psi} (1^1 S_0^{[8]}[8]) \rangle = (3.04 \pm 0.35) \times 10^{-2} \text{GeV}^3, \\
\langle O^{J/\Psi} (3^3 S_1^{[8]}[8]) \rangle = (1.68 \pm 0.46) \times 10^{-3} \text{GeV}^3, \\
\langle O^{J/\Psi} (3^3 P_0^{[8]}[8]) \rangle = (-9.08 \pm 1.61) \times 10^{-3} \text{GeV}^5
\]
fit by GWWZ group


• 1. Use unpolarized data and fit central region data by CDF Run II and forward region data by LHCb simultaneously (r0 and r1 in M0 and M1 are slightly different in these two regions!!!).

• 2. pT > 7 GeV.

• 3. Include feeddown from χc and Ψ(2S).

\[
\langle O^{J/} (^1S_0^{[8]}) \rangle = 0.097 \pm 0.009 \text{GeV}^3, \langle O^{J/} (^3S_1^{[8]}) \rangle = (0.46 \pm 0.13) \times 10^{-2} \text{GeV}^3, \\
\langle O^{J/} (^3P_0^{[8]}) \rangle = (0.22 \pm 0.012) \times 10^{-2} \text{GeV}^3, \\
\langle O^{J/} (^3P_1^{[8]}) \rangle = (0.34 \pm 0.12) \times 10^{-2} \text{GeV}^3, \\
\langle O^{J/} (^3P_2^{[8]}) \rangle = (0.945 \pm 0.54) \times 10^{-2} \text{GeV}^5, \\
\langle O^{J/} (^3P_3^{[8]}) \rangle = (0.34 \pm 0.12) \times 10^{-2} \text{GeV}^3
\]

Cancellation is not sufficient to give an unpolarized prediction.
Decomposition of P-wave channel

Because of the large $K$ factor of P-wave channel, $^3S_1^{[8]}$ channel is no longer the unique source at high $p_T$. We find the following decomposition holds within error of a few percent:

$$d\hat{\sigma}[^3P_f^{[8]}] = r_0 \ d\hat{\sigma}[^1S_0^{[8]}] + r_1 \ d\hat{\sigma}[^3S_1^{[8]}]$$

As a result, we will use two linear combined LDMEs:

$$M_{0,r_0}^{J/\psi} = \langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle + \frac{r_0}{m_c^2} \langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle$$

$$M_{1,r_1}^{J/\psi} = \langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle + \frac{r_1}{m_c^2} \langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle$$

<table>
<thead>
<tr>
<th>$\sqrt{s}$ (TeV)</th>
<th>region of $y$</th>
<th>$r_0$</th>
<th>$r_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.96</td>
<td>(0.0, 0.6)</td>
<td>3.9</td>
<td>-0.56</td>
</tr>
<tr>
<td>7</td>
<td>(0.75, 1.5)</td>
<td>3.9</td>
<td>-0.56</td>
</tr>
<tr>
<td>7</td>
<td>(1.50, 2.25)</td>
<td>3.9</td>
<td>-0.59</td>
</tr>
<tr>
<td>7</td>
<td>(0.0, 0.24)</td>
<td>4.1</td>
<td>-0.56</td>
</tr>
<tr>
<td>7</td>
<td>(0.0, 0.12)</td>
<td>4.1</td>
<td>-0.55</td>
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<tr>
<td>7</td>
<td>(1.2, 1.6)</td>
<td>3.9</td>
<td>-0.57</td>
</tr>
<tr>
<td>7</td>
<td>(1.6, 2.4)</td>
<td>3.9</td>
<td>-0.59</td>
</tr>
<tr>
<td>7</td>
<td>(2.5, 4)</td>
<td>3.9</td>
<td>-0.66</td>
</tr>
<tr>
<td>7</td>
<td>(2.2, 5)</td>
<td>4.0</td>
<td>-0.61</td>
</tr>
<tr>
<td>7</td>
<td>(2.5, 3)</td>
<td>4.0</td>
<td>-0.65</td>
</tr>
<tr>
<td>7</td>
<td>(3.5, 5)</td>
<td>4.0</td>
<td>-0.68</td>
</tr>
<tr>
<td>7</td>
<td>(4.4, 5)</td>
<td>4.2</td>
<td>-0.74</td>
</tr>
<tr>
<td>14</td>
<td>(0.0, 3)</td>
<td>3.9</td>
<td>-0.57</td>
</tr>
<tr>
<td>0.2</td>
<td>(0.0, 0.35)</td>
<td>3.8</td>
<td>-0.60</td>
</tr>
<tr>
<td>0.2</td>
<td>(1.2, 2.4)</td>
<td>4.0</td>
<td>-0.66</td>
</tr>
</tbody>
</table>
Uncertainty: decomposition (1)

- Two errors induced by decomposing the P-wave channel:
  1) The decomposition has an error of a few percent;
  2) The resulted $r_0$ and $r_1$ vary for different experimental condition.

- The above uncertainties can be determined by using three unconstrained LDMEs to fit data (*we thank G. Bodwin for pointing out this*). Choosing $p_T^{cut} = 7$GeV and by minimizing $\chi^2$ we get:

  \[ O_1 \equiv \langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle = 15.7 \times 10^{-2} \text{ GeV}^3 (\pm 129\%) \]

  \[ O_2 \equiv \langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle = -1.18 \times 10^{-2} \text{ GeV}^3 (\pm 249\%) \]

  \[ O_3 \equiv \frac{\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle}{m_c^2} = -2.28 \times 10^{-2} \text{ GeV}^3 (\pm 239\%) \]

- These LDMEs are obviously unphysically determined. Physical variables are eigenvectors of correlation matrix, which corresponds to linear combinations of these LDMEs:

\[
\begin{pmatrix}
\Lambda_1 \\ \Lambda_2 \\ \Lambda_3
\end{pmatrix} = \begin{pmatrix}
0.96 & -0.14 & -0.26 \\
0.29 & 0.31 & 0.91 \\
0.047 & 0.94 & -0.33
\end{pmatrix} \begin{pmatrix}
O_1 \\ O_2 \\ O_3
\end{pmatrix} = \begin{pmatrix}
15.8 \pm 134\% \\ 2.11 \pm 5.13\% \\ 0.39 \pm 2.45\%
\end{pmatrix} \times 10^{-2} \text{ GeV}^3
\]

Comparison:
\[
\vec{v}_{M_0} = (0.25 \quad 0 \quad 0.97)
\]
\[
\vec{v}_{M_1} = (0 \quad 0.87 \quad -0.48)
\]

$M_{0,r_0}$ and $M_{1,r_1}$ are approximately equivalent to the two well constrained LDMEs:

\[
\begin{align*}
\Lambda_2 & \leftrightarrow M_0 \\
\Lambda_3 & \leftrightarrow M_1
\end{align*}
\]
Predictions between using two LDMEs method and three LDMEs implies:
1) In central region, two methods give almost the same error bar;
2) In forward region, three LDMEs method may underestimates its theoretical uncertainty. (It is interesting to note that the data still seems to locate within the error bar of two LDMEs method predictions.)

Reason: $r_i$ have small differences between CMS and CDF, but larger difference between LHCb and CDF.

It is possible to determine all three LDMEs when data in forward region are sufficient enough!
Uncertainty: $p_T^{cut}$

In the fit procedure, we abandon data with $p_T<7\text{GeV}$, which is essential in our work. There are various reasons for this $p_T$ cut:

- Theoretically:
  1. Small $p_T$ region is dominated by non-perturbative effect because of initial state gluon showers (*Berger, Qiu, Wang, PRD 2005*).
  2. NRQCD factorization is still not proven. But for large $p_T$ region, it was found the factorization holds up to $O(m^4/p_T^4)$ correction (*Kang, Qiu, Sterman, PRL 2011*). So data in large $p_T$ are more confident to describe.

- Experimentally: In the plot, the curvature of data curve is positive at large $p_T$ but negative at small $p_T$, with a turning point at $p_T \approx 6 \text{GeV}$. Thus data below 7GeV cannot be described by perturbative factorization theory.

**Fit:** We perform a $\chi^2$ analysis for $J/\Psi$, and find $\chi^2$/d.o.f. decreases rapidly as the cut increases from 3GeV to 7GeV, and then it almost unchanges when the cut becomes larger:

<table>
<thead>
<tr>
<th>lower $p_T$ cut (GeV)</th>
<th>$\chi^2$/dof</th>
<th>$&lt;\mathcal{O}^3 s_1^{[8]}&gt;_{J/\Psi}$</th>
<th>$&lt;\mathcal{O}^1 s_0^{[8]}&gt;_{J/\Psi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>236.269/16=14.766</td>
<td>0.360089</td>
<td>1.78736</td>
</tr>
<tr>
<td>4</td>
<td>92.9272/12=7.74393</td>
<td>0.250964</td>
<td>3.49161</td>
</tr>
<tr>
<td>5</td>
<td>27.8681/8=3.48351</td>
<td>0.157748</td>
<td>5.1679</td>
</tr>
<tr>
<td>6</td>
<td>9.07871/6=1.51312</td>
<td>0.101501</td>
<td>6.28956</td>
</tr>
<tr>
<td>7</td>
<td>1.31256/4=0.328141</td>
<td>0.049296</td>
<td>7.43362</td>
</tr>
<tr>
<td>8</td>
<td>0.817308/3=0.272436</td>
<td>0.037283</td>
<td>7.71245</td>
</tr>
<tr>
<td>9</td>
<td>0.434183/2=0.217091</td>
<td>0.0226552</td>
<td>8.07939</td>
</tr>
<tr>
<td>10</td>
<td>0.424269/1=0.424269</td>
<td>0.0192824</td>
<td>8.17001</td>
</tr>
</tbody>
</table>
Comparing with BK’s work

- Butenschön and Kniehl (BK) do a similar work for J/ψ production differences from ours include:
  1. BK use both Tevatron data ($p_T^{\text{cut}}=3\text{GeV}$) and HERA data ($p_T^{\text{cut}}=1\text{GeV}$);
  2. BK neglect the feeddown contribution;
  3. BK determine all three CO LDMEs.

- To take advantage of their results, we also neglect feeddown contribution and using three LDMEs to fit (because these choices do not change final results qualitatively, as discussed above), but we still use only Tevatron data with $p_T^{\text{cut}}=7\text{GeV}$.

- $M_{0,r_0}^{J/\psi}$ is well constrained in both two groups, but two results are different significantly. Only difference: using different data!!

☆ Observation: perturbative NRQCD factorization cannot give a consistent description of both Tevatron data ($7\text{GeV} > p_T > 3\text{GeV}$) + HERA data ($p_T > 1\text{GeV}$) and Tevatron data ($p_T > 7\text{GeV}$).
χc Polarization
normalized multipole amplitudes

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$a_2^{J=1} (10^{-2})$</th>
<th>$a_3^{J=2} (10^{-2})$</th>
</tr>
</thead>
</table>
| CLEO[26]            | $-6.26 \pm 0.63 \pm 0.24$ | |}
| Crystal Ball [27]   | $-0.2^{+0.8}_{-2.0}$ | |}
| E835 [29]           | $0.2 \pm 3.2 \pm 0.4$ | |}

- Single quark radiation
- Our choice!
Ratio and LDMEs

\[ \langle O^{cJ} (3 S_1^{[8]}) \rangle = (2J + 1) \times (2.2^{+0.48}_{-0.32}) \times 10^{-3} \text{GeV}^3 \]

\[ \langle O^{cJ} (3 P_J^{[1]}) \rangle = (2J + 1) \frac{3 N_c |R'(0)|^2}{2}, |R'(0)|^2 = 0.075 \text{GeV}^5 \]
$\chi_c J > J/\psi \gamma$
$\chi_c J \to \psi \gamma \to l^+ l^- \gamma$
K factor of short distance coefficient

Ma, Wang, Chao, (2011)