Generators BCVEGPY and GENXICC for doubly heavy mesons and baryons

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Developed in about ten years

Generators for Bc meson and Xicc baryon events


==== BCVEGPY ↑ ======== GENXICC ↓ ====

Directly related works for hadronic production of Bc meson and baryons

4) "Hadronic Production Of Bc Meson Induced By The Heavy Quarks Inside The Collision Hadrons" >> Phys.Rev.D 72, 114009 (2005)
To agree with the purpose of the present conference, the main purpose of the present talk is to provide a detailed introduction to the Improved Helicity Amplitude Technologies.

To improve the efficiency of the generators.
• Mechanisms for the Bc hadronic production

A) gluon-gluon fusion ------- dominant (our main concern)
   color-singlet: S-wave: Bc (1), Bc*(~2.6); P-wave: Bc*(~ 0.5)
   color-octet: S-wave: Bc+Bc* (~0.1)

B) quark-antiquark annihilation ----- must be light quark
   color-singlet: S-wave: Bc+Bc* (~0.1)

There are also extrinsic and intrinsic heavy quark mechanisms, in the generators, we do not consider them so far, which provide contributions in lower pt regions.
QCD factorization picture

Differential cross-section for subprocess

\[
\frac{1}{2^{11} \times 3} \frac{(2\pi)^4 |M|^2}{4(k_1 \cdot k_2)} \times d\Phi_3(k_1 + k_2; P, q_{b1}, q_{c2}),
\]

Three-body phase-space

\[
d\Phi_3(k_1 + k_2; P, q_{b1}, q_{c2}) = \delta^4(k_1 + k_2 - P - q_{b1} - q_{c2}) \frac{d^3 \vec{P}}{(2\pi)^3 2E_P} \frac{d^3 q_{b1}}{(2\pi)^3 2E_{q_{b1}}} \frac{d^3 q_{c2}}{(2\pi)^3 2E_{q_{c2}}},
\]
Xicc similar to Bc case
Model: Diquark \Rightarrow Baryon

\[ HME_i = \langle q_0 | c_4 + m_c | q_0 \rangle \hat{\Gamma}_i (q_3 - m_c) | q_0 \rangle, \]
\[ HME_i = -\langle q_0 (-\lambda_2) | c_4 + m_c | q_0 (-\lambda_2) \rangle, \]
\[ \langle p \lambda_1 | k_1 \cdots k_n | q_\lambda_2 \rangle = (-1)^{n+1} \langle q_\lambda_2 | k_1 \cdots k_n | p \rangle. \]

Quark lines

Color factor

| TABLE V. The square of the six independent color factors (including the cross terms) for $gg \rightarrow (cc)_3 [1^3 S_0] + c + \bar{c}$, $(C_{nij} \times C_{nij}^*)$ with $m, n = (1, 2, \cdots, 6)$, respectively. |
|---|---|---|---|---|---|---|
| $C_{1ij}$ | $C_{2ij}$ | $C_{3ij}$ | $C_{4ij}$ | $C_{5ij}$ | $C_{6ij}$ |
| $C_{1ij}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ |
| $C_{2ij}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ |
| $C_{3ij}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ |
| $C_{4ij}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ |
| $C_{5ij}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ |
| $C_{6ij}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ |

| TABLE VI. The square of the six independent color factors (including the cross terms) for $gg \rightarrow (cc)_3 [1^3 S_0] + c + \bar{c}$, $(C_{nij} \times C_{nij}^*)$ with $m, n = (1, 2, \cdots, 6)$, respectively. |
|---|---|---|---|---|---|---|
| $C_{1ij}$ | $C_{2ij}$ | $C_{3ij}$ | $C_{4ij}$ | $C_{5ij}$ | $C_{6ij}$ |
| $C_{1ij}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| $C_{2ij}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| $C_{3ij}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| $C_{4ij}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| $C_{5ij}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| $C_{6ij}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
1) Helicity amplitude approach

To get the numerical value at the amplitude level

2) Detailed processes for the approach


The amplitude independent of the reference light-like momentum
Replacing the gluon polarization to its momentum, the amplitude must be zero.
Basic units

inner product

\[ \langle k_1 \cdot k_2 \rangle = \langle k_{1-} | k_{2+} \rangle = \sqrt{k_{1-} k_{2+} e^{i \varphi_1}} - \sqrt{k_{1+} k_{2-} e^{i \varphi_2}} \]
\[ = k_{1\perp} \sqrt{k_{2+}} - k_{2\perp} \sqrt{k_{1+}}, \]

spinor product

\[ \langle k_{1+} | k_3 | k_{2+} \rangle = \langle k_{1+} | k_3 \rangle \langle k_3 | k_{2+} \rangle \]
\[ = \frac{1}{\sqrt{k_{1+} k_{2+}}} (k_{1+} k_{2+} k_{3-} - k_{1+} k_{2\perp} k_{3\perp} - k_{1\perp} k_{2+} k_{3\perp} + k_{1\perp} k_{2\perp} k_{3+}) \]

\[ P' = P - \frac{v^2}{2P_{00}} q_{00} \]
Divide the whole amplitude into several gauge invariant groups, and simplify each group with proper gauges.

Find out all the independent fermion lines “bases”

Simplify the fermion lines “bases”

Expand all the Feynman amplitude over these bases and find out the corresponding coefficients

Great improvement! for massless lines

Numerical calculation

Problem/complexity: 1: how to choose gauge in each group; 2: stability of numerical calculation; 3: no help for the massive fermion lines.

```
\phi^{\pm}(k, q) = \frac{\sqrt{2}}{\langle q_+ | g_+ \rangle} \left[ |k_+ \rangle \langle g_+ | + |g_+ \rangle \langle k_+ | \right]
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\phi^{\pm}_\mu(k, g) = \frac{\langle k_\pm, g | g_\pm \rangle}{\sqrt{2} \langle g_\pm | k_\pm \rangle}
```
Corrected Feynman rule needed for decomposing:

\[ g_f \, a_{\alpha e} \Gamma_{\mu \nu \delta}^{\mu \nu \delta}(P, S, K) = g_f \, a_{\alpha e} \Gamma_{\mu \nu \delta}^{\mu \nu \delta}(P, S, K) + g_f \, a_{\alpha e} G_{\mu \nu \delta}^{\mu \nu \delta}(P, S, K), \]

\[ -i g V_{\lambda \mu \nu \delta}^{\lambda \mu \nu \delta}(P, Q, K_1, k_2) = -i g V_{\lambda \mu \nu \delta}^{\lambda \mu \nu \delta}(P, Q, K_1, k_2) - i g G_{\lambda \mu \nu \delta}^{\lambda \mu \nu \delta}(P, Q, K_1, k_2), \]

where \( T_{\mu \nu \delta}^{\mu \nu \delta}(P, S, K) \) and \( V_{\lambda \mu \nu \delta}^{\lambda \mu \nu \delta}(P, Q, K_1, k_2) \) are the primary Feynman rules:

\[ T_{\mu \nu \delta}^{\mu \nu \delta}(P, S, K) = (P - S)^{\delta \nu} g^{\mu \nu} + (S - K)^{\mu} g^{\delta \nu} + (K - P)^{\nu} g^{\delta \mu}, \]

\[ V_{\lambda \mu \nu \delta}^{\lambda \mu \nu \delta}(P, Q, K_1, k_2) = f_{\alpha e} f_{\beta e} (g_{\lambda \nu} g_{\mu \delta} - g_{\lambda \delta} g_{\mu \nu}) + f_{\alpha e} f_{\beta e} (g_{\lambda \nu} g_{\mu \delta} - g_{\lambda \delta} g_{\mu \nu}) + f_{\alpha e} f_{\beta e} (g_{\lambda \nu} g_{\mu \delta} - g_{\lambda \delta} g_{\mu \nu}), \]

and \( G_{\mu \nu \delta}^{\mu \nu \delta}(P, S, K) \) and \( G_{\lambda \mu \nu \delta}^{\lambda \mu \nu \delta}(P, Q, K_1, k_2) \) are the modified part:

\[ G_{\mu \nu \delta}^{\mu \nu \delta}(P, S, K) = (\pm) (P^\mu P^\nu P^\delta / (P \cdot K) + S^\mu S^\nu S^\delta / (S \cdot K)), \]

\[ G_{\lambda \mu \nu \delta}^{\lambda \mu \nu \delta}(P, Q, K_1, k_2) = -f_{\alpha e} f_{\beta e} f_{\delta e} (S_1^\lambda S_1^\mu S_1^\nu S_1^\delta / (S_1 \cdot K_1) (S_1 \cdot K_2)) + f_{\alpha e} f_{\beta e} f_{\delta e} (S_2^\lambda S_2^\mu S_2^\nu S_2^\delta / (S_2 \cdot K_1) (S_2 \cdot K_2)). \]
Replacing the polarization vector by the gluon momentum

**APPENDIX A: GAUGE INVARIANCE OF THE \(cb\) SUBSET**

Totally there are four gauge invariant subset \(cc, bb, cb, bc\), we list here the demonstration of the gauge invariance of the \(cb\) and the \(cc\) subset, while the gauge invariance of the other two subsets can be easily demonstrated by the gluon symmetry.

The involved matrix elements are \(M_{cb1}, M_{cb2}, M_{cb3}, M_{cb4}, M_{cc1}, M_{cc2}, M_{cc3}, M_{cc4}, M_{bc1}, M_{bc2}, M_{bc3}, M_{bc4}, M_{oc1}, M_{oc2}, M_{oc3}\). To demonstrate the gauge invariance we set \(\beta_1 = 1\) and \(\beta_2 = \frac{1}{s_2}\), then we obtain for the primary matrix elements:

\[
M_{cb1} = (-C_{3ij})\bar{u}_b \gamma_\alpha \psi P_\gamma \gamma_\alpha v_a \cdot \frac{1}{s_2},
\]

\[
M_{cb2} = (C_{3ij})\bar{u}_b \gamma_\alpha \psi P_\gamma \gamma_\alpha v_a \cdot \frac{1}{s_2},
\]

\[
M_{cb3} = (C_{3ij})\bar{u}_b \gamma_\alpha \psi P_\gamma \gamma_\alpha v_a \cdot \frac{1}{s_2},
\]

\[
M_{cb4} = (C_{3ij} - C_{3ij})\bar{u}_b \gamma_\alpha \psi P_\gamma \gamma_\alpha v_a \cdot \frac{1}{s_2},
\]

\[
(M_{cc1})_{cb} = (C_{3ij} - C_{3ij})\left(-\bar{u}_b \gamma_\alpha \psi P_\gamma \gamma_\alpha v_a \cdot \frac{1}{s_2} + \bar{u}_b \gamma_\alpha \psi P_\gamma \gamma_\alpha v_a \cdot \frac{1}{s_2}\right),
\]

\[
(M_{cc2})_{cb} = -\frac{1}{s_2} + 1 + \frac{1}{s_2} = 0,
\]

\[
(M_{cc3})_{cb} = -\frac{1}{s_2} + 1 + \frac{1}{s_2} = 0,
\]

\[
(M_{cc4})_{cb} = (C_{3ij} - C_{3ij})\left(\bar{u}_b \gamma_\alpha \psi P_\gamma \gamma_\alpha v_a \cdot \frac{1}{s_2} + \bar{u}_b \gamma_\alpha \psi P_\gamma \gamma_\alpha v_a \cdot \frac{1}{s_2}\right),
\]

\[
(M_{bc1})_{cb} = (C_{3ij} - C_{3ij})\left(-\bar{u}_b \gamma_\alpha \psi P_\gamma \gamma_\alpha v_a \cdot \frac{1}{s_2} + \bar{u}_b \gamma_\alpha \psi P_\gamma \gamma_\alpha v_a \cdot \frac{1}{s_2}\right),
\]

\[
(M_{bc2})_{cb} = -\frac{1}{s_2} + 1 + \frac{1}{s_2} = 0,
\]

\[
(M_{bc3})_{cb} = (C_{3ij} - C_{3ij})\left(\bar{u}_b \gamma_\alpha \psi P_\gamma \gamma_\alpha v_a \cdot \frac{1}{s_2} + \bar{u}_b \gamma_\alpha \psi P_\gamma \gamma_\alpha v_a \cdot \frac{1}{s_2}\right),
\]

where \(i, j\) are the quark's color indexes and \((M_{oc1})_{cb}\) is the part of \(M_{cc1}\) that attributes to the \(cc\) subset, and so on.

While for the matrix elements containing the modified part, by carefully fixed the sign of the modified part of 3-gluon vertex, we obtain

\[
(M_{cc1})_{cb} + (M_{cc2})_{cb} = (C_{3ij} - C_{3ij})\bar{u}_b \gamma_\alpha \psi P_\gamma \gamma_\alpha v_a \cdot \frac{1}{s_2},
\]

\[
(M_{cc1})_{cb} + (M_{cc2})_{cb} = (C_{3ij} - C_{3ij})\bar{u}_b \gamma_\alpha \psi P_\gamma \gamma_\alpha v_a \cdot \frac{1}{s_2},
\]

\[
(M_{cc1})_{cb} + (M_{cc2})_{cb} = (C_{3ij} - C_{3ij})\bar{u}_b \gamma_\alpha \psi P_\gamma \gamma_\alpha v_a \cdot \frac{1}{s_2},
\]

\[
(M_{cc1})_{cb} + (M_{cc2})_{cb} = (C_{3ij} - C_{3ij})\bar{u}_b \gamma_\alpha \psi P_\gamma \gamma_\alpha v_a \cdot \frac{1}{s_2},
\]

When add all these terms together, we get the desired result that all of them are cancelled out exactly.
Replacing the polarization vector by the gluon momentum

**APPENDIX B: GAUGE INVARIANCE OF THE \(cc\) SUBSET**

For the \(cc\) subset, the involved matrix elements are \(M_{cc1}, M_{cc2}, \ldots, M_{cc8}, M_{cc1}, M_{cc2}, M_{cc1}, M_{cc2}, M_{cc3}, M_{cc4}\). To demonstrate the gauge invariance we also set \(f_1 = \bar{f}_1\) and \(f_2 = \bar{f}_2\), then we obtain for the primary matrix elements:

\[
M_{cc1} + M_{cc2} + M_{cc7} = (C_{13j} + C_{23j})\bar{u}_b\gamma_5\gamma_\mu p_\mu v_a \frac{1}{2s_b}, \tag{B1}
\]

\[
M_{cc3} = (-C_{33j})\bar{u}_b\gamma_5\gamma_\mu p_\mu v_a \frac{1}{s_b}, \tag{B2}
\]

\[
M_{cc4} = (-C_{33j})\bar{u}_b\gamma_5\gamma_\mu p_\mu v_a \frac{1}{s_b}, \tag{B3}
\]

\[
M_{cc6} + M_{cc8} + M_{cc6} = (C_{33j} + C_{33j} - 2C_{33j})\bar{u}_b\gamma_5\gamma_\mu p_\mu v_a \frac{1}{2s_b}, \tag{B4}
\]

\[
\langle M_{cc1} \rangle_{cc} = (C_{33j} - C_{33j}) \frac{1}{s_b(s_a - s_b)}(\bar{u}_b\gamma_5 p_\mu k_\mu v_a + (s_1 - s_b)\bar{u}_b\gamma_5\gamma_\mu p_\mu v_a), \tag{B5}
\]

\[
\langle M_{cc2} \rangle_{cc} = (C_{33j} - C_{33j}) \frac{1}{s_b(s_a - s_b)}(\bar{u}_b\gamma_5 p_\mu k_\mu v_a + (s_2 - s_b)\bar{u}_b\gamma_5\gamma_\mu p_\mu v_a), \tag{B6}
\]

\[
\langle M_{cc3} \rangle_{cc} = (C_{33j}) \frac{1}{s_b(s_a - s_b)}(\bar{u}_b\gamma_5 p_\mu k_\mu v_a + (s_1 - s_b)\bar{u}_b\gamma_5\gamma_\mu p_\mu v_a), \tag{B7}
\]

\[
\langle M_{cc4} \rangle_{cc} = (C_{33j}) \frac{1}{s_b(s_a - s_b)}(\bar{u}_b\gamma_5 p_\mu k_\mu v_a + (s_2 - s_b)\bar{u}_b\gamma_5\gamma_\mu p_\mu v_a), \tag{B8}
\]

\[
\langle M_{cc6} \rangle_{cc} = (C_{33j} - C_{33j} - C_{33j}) \left( \frac{s_1 + s_2}{s_b(s_a - s_b)(s_a - s_b)}(\bar{u}_b\gamma_5 p_\mu k_\mu v_a + \frac{s_a - s_2}{s_b(s_a - s_b)}\bar{u}_b\gamma_5\gamma_\mu p_\mu v_a) \right), \tag{B9}
\]

\[
\langle M_{cc8} \rangle_{cc} = (C_{33j} - C_{33j} - C_{33j}) \left( \frac{s_1 + s_2}{s_b(s_a - s_b)(s_a - s_b)}(\bar{u}_b\gamma_5 p_\mu k_\mu v_a + \frac{s_a - s_2}{s_b(s_a - s_b)}\bar{u}_b\gamma_5\gamma_\mu p_\mu v_a) \right), \tag{B10}
\]

\[
\langle M_{cc10} \rangle_{cc} = (C_{33j} + C_{33j} - C_{33j}) \left( \frac{1}{s_b(s_a - s_b)}(2\bar{u}_b\gamma_5 p_\mu k_\mu v_a - 2\bar{u}_b\gamma_5\gamma_\mu p_\mu v_a) + \frac{s_2 - s_1}{2s_b(s_a - s_b)}\bar{u}_b\gamma_5\gamma_\mu p_\mu v_a \right), \tag{B11}
\]

\[
\frac{1}{(s_a - s_b)(s_1 - s_b)} + \frac{1}{(s_b - s_1)(s_a - s_1)} + \frac{1}{(s_b - s_a)(s_1 - s_a)} = 0
\]
Replacing the polarization vector by the gluon momentum

\[
(M_{\alpha\beta})_{\alpha\alpha} = \frac{1}{s_b(s_a - s_b)} \left( (2C_{2ij} - 2C_{3ij} + C_{3ij} - C_{2ij} - C_{3ij} + C_{2ij}) \bar{u}_b \gamma_5 \gamma_P \gamma_5 \gamma_P \gamma_v a + (2C_{2ij} - 2C_{3ij} + C_{3ij} - C_{2ij} + C_{3ij} - C_{2ij} - C_{3ij} + C_{2ij}) \frac{s_b}{2} \bar{u}_b \gamma_5 \gamma_P \gamma_v a \right). \quad (B12)
\]

Adding all these terms together and by using the relation

\[ s_b + s_a - s_1 - s_2 - s_b = 0, \]

we obtain

\[
M_{\alpha \alpha 1} + \ldots + M_{\alpha \alpha 6} + (M_{\alpha \alpha 1})_{\alpha \alpha} + \ldots + (M_{\alpha \alpha 2})_{\alpha \alpha} + (M_{\alpha \alpha 3})_{\alpha \alpha} + \ldots + (M_{\alpha \alpha 4})_{\alpha \alpha} =
\]

\[
(c_{3ij} - C_{2ij} + C_{5ij}) \bar{u}_b \gamma_5 \gamma_P \gamma_5 \gamma_P \gamma_v a \frac{s_b - 2s_a}{s_b(s_1 - s_b)} + (c_{3ij} - C_{2ij} + C_{5ij}) \bar{u}_b \gamma_5 \gamma_P \gamma_v a \frac{s_b - 2s_a}{s_b(s_2 - s_b)(s_a - s_b)}. \quad (B13)
\]

For the matrix elements involving the modified part, we have

\[
M_{\alpha \alpha} + M_{\alpha \alpha} = C_{\alpha \alpha 1} \frac{1}{2s_b} \bar{u}_b \gamma_5 \gamma_P \gamma_5 \gamma_P \gamma_v a, \quad (B14)
\]

\[
(M_{\alpha \alpha 3})_{\alpha \alpha} = -C_{\alpha \alpha 2} \frac{1}{2s_b} \bar{u}_b \gamma_5 \gamma_P \gamma_5 \gamma_P \gamma_v a, \quad (B15)
\]

\[
(M_{\alpha \alpha 1} + M_{\alpha \alpha 2})_{\alpha \alpha} = C_{\alpha \alpha 1} \frac{1}{s_b(s_b - s_a)} \bar{u}_b \gamma_5 \gamma_P \gamma_5 \gamma_P \gamma_v a, \quad (B16)
\]

\[
(M_{\alpha \alpha 1} + M_{\alpha \alpha 2})_{\alpha \alpha} = C_{\alpha \alpha 2} \frac{1}{s_b(s_b - s_a)} \bar{u}_b \gamma_5 \gamma_P \gamma_5 \gamma_P \gamma_v a, \quad (B17)
\]

\[
(M_{\alpha \alpha 3})_{\alpha \alpha} = C_{\alpha \alpha 1} \frac{s_1 - s_b - s_a}{s_b(s_a - s_b)(s_1 - s_b)} \bar{u}_b \gamma_5 \gamma_P \gamma_5 \gamma_P \gamma_v a, \quad (B18)
\]

\[
(M_{\alpha \alpha 3})_{\alpha \alpha} = C_{\alpha \alpha 2} \frac{s_2 - s_b - s_a}{s_b(s_a - s_b)(s_a - s_b)} \bar{u}_b \gamma_5 \gamma_P \gamma_5 \gamma_P \gamma_v a, \quad (B19)
\]

\[
(M_{\alpha \alpha 4})_{\alpha \alpha} = -C_{\alpha \alpha 2} \frac{1}{s_b(s_a - s_b)} \bar{u}_b \gamma_5 \gamma_P \gamma_5 \gamma_P \gamma_v a - C_{\alpha \alpha 1} \frac{1}{s_b(s_a - s_b)} \bar{u}_b \gamma_5 \gamma_P \gamma_5 \gamma_P \gamma_v a. \quad (B20)
\]

where the color factors

\[
C_{\alpha \alpha 1} = C_{3ij} - C_{3ij} - C_{3ij},
C_{\alpha \alpha 2} = C_{1ij} - C_{3ij} - C_{3ij},
C_{\alpha \alpha 3} = C_{1ij} + C_{3ij} - C_{2ij} - C_{3ij}. \quad (B21)
\]

When adding all these modified terms together we obtain

\[
M_{\alpha \alpha} + M_{\alpha \alpha} + (M_{\alpha \alpha 1})_{\alpha \alpha} + \ldots + (M_{\alpha \alpha 2})_{\alpha \alpha} + (M_{\alpha \alpha 3})_{\alpha \alpha} + \ldots + (M_{\alpha \alpha 4})_{\alpha \alpha} =
\]

\[
(c_{3ij} - C_{2ij} + C_{5ij}) \bar{u}_b \gamma_5 \gamma_P \gamma_5 \gamma_P \gamma_v a \frac{2s_a - s_b}{s_b(s_1 - s_b)(s_a - s_b)} + (c_{3ij} - C_{1ij} + C_{5ij}) \bar{u}_b \gamma_5 \gamma_P \gamma_5 \gamma_P \gamma_v a \frac{2s_a - s_b}{s_b(s_2 - s_b)(s_a - s_b)}. \quad (B22)
\]

Adding Eq. (B13) and Eq. (B22) together, we get the desired result.
Contributions from the extra terms can not equal to zero for the massive case!

All the quark mass tends to zero

\[ \bar{u}_b f_{1e}^{\lambda_4} v_b = \bar{u}_c \tilde{f}_c^{\lambda_4} v_c = 0, \quad M_{total}^c = 0. \]
Step by step, change all the space-like momenta into light-like.

The massive fermions have time-like momenta $q_i$ ($i = 1, 2$) and $\tilde{q}_i$ are directly connected to $|q_{0\lambda_i}\rangle$ or $\langle q_{0\lambda_i}|$ as in Eq.(28), we may introduce the light-like momenta by defining

$$q'_i = q_i - \frac{q_i^2}{2q_i \cdot q_0}. \quad \tilde{q}'_i = |q'_i\rangle \langle q'_i| = |q_i^+\rangle |q_i^-\rangle + |q_i^-\rangle |q_i^+\rangle.$$

Then $\tilde{q}_i$ for massive fermions can be replaced by the massless ones, $\tilde{q}'_i$, without any consequences. This is due to the relations $\langle q_0|q_{0\lambda_i}\rangle = 0$ or $\langle q_{0\lambda_i}|q_0 = 0$.

Unified gauge: The arbitrary reference light-like momentum in the massive spinor, the polarization vector and in all the intermediate space-like momentum transformation can be taken to be the same.

In this way the amplitude can be fully simplified.

Condensed results that are expressed by the spinor inner product and spinor products.
Construct all the Feynman with all the independent QED-like Feynman diagrams, with the help of the quark-antiquark and gluon-gluon symmetries.

Complete the program based on the Feynman diagrams.

Most Effective Generator!
3) More details of our approach

Helicity Amplitude Approach

A、basic idea, decomposition of the Feynman diagrams

How to Decompose 36 Feynmans?
Skeleton QED-like

B、five groups of diagrams, according to its topologies

Find nine basic Feynman diagrams
Find eight quark lines
Nine basic diagrams

Gluon–gluon and quark–quark exchange

Feynman diagrams that can be directly grouped into the $cc$ subset. Here $i$ and $j$

Feynman diagrams that can be directly grouped into the $bb$ subset. Here $i$ and $j$
Feynman diagrams that can be directly grouped into the \( cb \) or \( bc \) subsets, where.
C, decompose the three gluon vertex

FIG. 6: The three-gluon coupling vertex is decomposed as in Eq.(16): the first two terms are the ‘basic QED-like’ terms and the ‘remaining’ terms are expressed by several extra basic functions.
Cut off the color factor and the scalar part of the propagator

\[ M_{\delta \mu}^{\text{QCD}} \simeq \gamma_\delta (k_1 - Q' + M) \gamma_\mu - \gamma_\mu (k_2 - Q + M) \gamma_\delta + (Q' - M)(\gamma_\delta \gamma_\mu - g_{\mu \delta}) \]

\[ + (\gamma_\mu \gamma_\delta - g_{\mu \delta})(Q + M) + k_{2 \delta} \gamma_\mu - k_{1 \mu} \gamma_\delta \]

\[ M^{\text{QCD}}_{\mu \delta a}(k_1, k_2, Q, Q') \simeq \bar{u}(Q') \gamma_\alpha (k_1 + k_2 - Q + M) (\gamma_\delta (k_1 - Q + M) \gamma_\mu - \gamma_\mu (k_2 - Q + M) \gamma_\delta) u(Q) + (C_1 \cdot X) + \ldots \]

where

\[ C_1 = m_1^2 + m_2^2 + 2k_1 \cdot k_2 - 2Q \cdot k_1 - 2Q \cdot k_2 \]

\[ X = \bar{u}(Q') \gamma_\alpha (\gamma_\delta \gamma_\mu - g_{\mu \delta}) u(Q) \]
All the diagrams can be expressed by its inner structures without introduce extra ones.

Note: Extra functions X and Y are from four-gluon-vertex decompasation

\[
M_{\mu\alpha}^{abcd}(k_1, k_2, Q, Q') = \bar{u}(Q') \left( \gamma_5(Q' - k_2 + M)\gamma_\mu - \gamma_\mu(Q' - k_1 + M)\gamma_5 \right) \\
\times \left( (Q' - k_1 - k_2 + M)\gamma_\alpha \bar{u}(Q) + \langle c2 : Y \rangle + \ldots \right)
\]
Where \((c_i, d_i, e_i, f_i) = \pm 1\); \(X_i\) can be expressed by the defined basic function.
D. Amplitude simplification

(A). general form for the helicity amplitude

\[ M^{(\lambda_1, \lambda_4, \lambda_5, \lambda_6)}_{\ell_1, \ell_4, \ell_5, \ell_6}(q_{b1}, q_{b2}, q_{c1}, q_{c2}, k_1, k_2) = \sum_{\lambda_{2,3}} C_{\ell_1} X_{\ell_4} D_1 B^{(\lambda_1, \lambda_2, \lambda_4, \lambda_5, \lambda_6)}_{\ell_1}(q_{b1}, q_{b2}, q_{c1}, q_{c2}, k_1, k_2) \cdot \]

\[ D_2 B^{(\lambda_2, \lambda_3)}_{\ell_2}(q_{b2}, q_{c1}), \]  

\[ (22) \]

Factorization

\[ gg \rightarrow b + \bar{b} + c + \bar{c} \]

\[ c + b \rightarrow B_c \]

Bound state

\[ D_2 B^{(\lambda_2, \lambda_3)}_{\ell_2}(q_{b2}, q_{c1}) = \frac{\psi(0) \sqrt{M}}{2\sqrt{m_b m_c}} \delta_{\lambda_2, \lambda_4} (\delta_{\lambda_2^-, \lambda_3} + \delta_{\lambda_2^+, \lambda_3^-}) \left( \frac{M \mp (s_\ell)}{P \cdot q_0} \right) \]

\[ + \frac{\psi(0) \sqrt{M}}{2\sqrt{m_b m_c}} \left( \frac{1}{2P \cdot q_0} \right) (q_0 \lambda_2 \, \xi(s_\ell) \mathcal{F} q_0 \lambda_3) \]  

\[ \text{vector} \]

\[ u_+^s(q) = \frac{1}{\sqrt{2r \cdot q}} (\gamma^+ \cdot m) |q_h \rangle \]

\[ f_{+}^s(k, q) = \frac{\sqrt{2}}{\langle q_{+}| k_{+} \rangle} |k_{+} \rangle \langle q_{+}| + |q_{+} \rangle \langle k_{+} | \]

\[ \bar{u}_-^s(q) = \frac{1}{\sqrt{2r \cdot q}} (\gamma^- \cdot m) |q_- \rangle \]

\[ e_{-}^s(k, q) = \frac{\langle k_{-} | \gamma_{-} | q_{-} \rangle}{\sqrt{2} \langle q_{+} | k_{+} \rangle} \]
(B)、helicity amplitude for the hard scattering process

\[ E_{m,l,k} \quad (m=1,9) \]

\[ E_{m,j,k} \quad (j=1,2,3,4) \]

\[ B_{F_l}^{(k)} (q_1, q_2, q_3, q_4, k_1, k_2) = \sum_{m=1}^{g} \sum_{j=1}^{4} f_{i,m,j} E_{m,j,k} \]

\[ E_{m,1,5} \equiv +E_{m,3,1} \quad (k = (1, \cdots, 4), (9, \cdots, 1)_{2}, (37, \cdots, 40), (45, \cdots, 48)) \]

\[ E_{m,2,5} \equiv +E_{m,4,2} \quad (k = (5, \cdots, 8), (13, \cdots, 16), (33, \cdots, 36), (41, \cdots, 44)) \]

**TABLE I.** The correspondence between \( k = 1, \cdots, 64 \) and \( \lambda_1 = \pm, \lambda_2 = \pm, \lambda_3 = \pm, \lambda_4 = \pm, \lambda_5 = \pm, \lambda_6 = \pm \), which stand for the helicities of the particles in the process.

<table>
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<tr>
<th>( k )</th>
<th>( \lambda_1 )</th>
<th>( \lambda_2 )</th>
<th>( \lambda_3 )</th>
<th>( \lambda_4 )</th>
<th>( \lambda_5 )</th>
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(C) expansion coefficients

**TABLE II:** The expansion coefficients $f_{i,m,j}$ for the functions $B^{(k)}_{E_{i}}(q_{b1}, q_{b2}, q_{c1}, q_{c2}, k_{1}, k_{2})$ which are grouped into the cb subset directly or indirectly through a proper decomposition (the coefficients $f_{i,m,j}$ are not listed here if they are equal to zero in a whole row).

<table>
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<tr>
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<td>0</td>
</tr>
<tr>
<td>$f_{3e,m,j}$</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>1</td>
<td>0</td>
<td>-2($q_{c2} \cdot k_{1}$)</td>
</tr>
<tr>
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<td>0</td>
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<td>2($q_{c1} \cdot k_{1}$)</td>
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<td>0</td>
<td>2($q_{b1} \cdot k_{2}$)</td>
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<td>-2($q_{b2} \cdot k_{2}$)</td>
</tr>
<tr>
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<td>$s_{b} + s_{c} - s_{g}$</td>
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</table>

QED-like

One three-gluon vertex

Two three-gluon vertex

cb subset

Gluon exchange

bc subset
TABLE III: The expansion coefficients $f_{i,m,j}$ for the functions $B^{(k)}_{i}(q_{01},q_{02},q_{e1},q_{e2},k_{1},k_{2})$ which are grouped into the cc subset directly or indirectly through a proper decomposition (the coefficients $f_{i,m,j}$, which are equal to zero in a whole row, are not listed here).

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<td>-2$f$</td>
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<tr>
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<td>0</td>
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<td>2$q_{e1} \cdot k_{1}$</td>
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QED-like

one three-gluon vertex

two three-gluon vertex

cc subset

quark exchange

bb subset

four-gluon vertex
(D)、basic functions

\[ E_{m,j,k}(q_{b1}, q_{b2}, q_{c1}, q_{c2}, k_1, k_2) \]

Basic diagram

a) eight fermion lines \((f_i)\). \((q_0\,—\,\text{light-like reference momentum})\)

\[
f_0(q_1, q_2, \lambda_1, \lambda_2) = \langle q_0 \lambda_1 | (\slashed{q}_1 + m) \gamma_8 (\slashed{q}_2 - m) | q_0 \lambda_2 \rangle
\]

\[
f_1(q_1, q_2, k, \lambda_1, \lambda_2, \lambda_3) = \langle q_0 \lambda_1 | (\slashed{q}_1 + m) \gamma_8 (k - \slashed{q}_2 + m) \gamma_8^\dagger (k, q_0) (\slashed{q}_2 - m) | q_0 \lambda_2 \rangle
\]

\[
f_2(q_1, q_2, k, \lambda_1, \lambda_2, \lambda_3) = \langle q_0 \lambda_1 | (\slashed{q}_1 + m) \gamma_8 (k, q_0) (\slashed{q}_1 - k + m) \gamma_8 (\slashed{q}_2 - m) | q_0 \lambda_2 \rangle
\]

\[
f_3(q_1, q_2, k, \lambda_1, \lambda_2, \lambda_3) = \langle q_0 \lambda_1 | (\slashed{q}_1 + m) \gamma_8 (k, q_0) (\slashed{q}_2 - m) | q_0 \lambda_2 \rangle
\]

\[
f_4(q_1, q_2, k_1, k_2, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = \langle q_0 \lambda_1 | (\slashed{q}_1 + m) \gamma_8 (k_1 + k_2 - \slashed{q}_2 + m) \gamma_8 (k_1, q_0) \gamma_8 (k_2 - \slashed{q}_2 + m) \gamma_8^\dagger (k_2, q_0) \cdot (\slashed{q}_2 - m) | q_0 \lambda_2 \rangle
\]

\[
f_5(q_1, q_2, k_1, k_2, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = \langle q_0 \lambda_1 | (\slashed{q}_1 + m) \gamma_8 (k_1, q_0) (\slashed{q}_1 - k_1 + m) \gamma_8 (k_2 - \slashed{q}_2 + m) \gamma_8 (k_2, q_0) \cdot (\slashed{q}_2 - m) | q_0 \lambda_2 \rangle
\]

\[
f_6(q_1, q_2, k_1, k_2, \lambda_1, \lambda_3, \lambda_3, \lambda_4) = \langle q_0 \lambda_1 | (\slashed{q}_1 + m) \gamma_8 (k_1, q_0) (\slashed{q}_1 - k_1 + m) \gamma_8 (k_2, q_0) \cdot (\slashed{q}_2 - m) | q_0 \lambda_2 \rangle
\]

\[
f_7(q_1, q_2, k_1, k_2, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = \langle q_0 \lambda_1 | (\slashed{q}_1 + m) \gamma_8 (k_1, q_0) \gamma_8 (k_2, q_0) (\slashed{q}_2 - m) | q_0 \lambda_2 \rangle
\]
b) definition of the nine basic functions

\[
\begin{align*}
E_{1,1,k} &= f_1(q_{c1}, q_{c2}, k_1, \lambda_3, \lambda_4, \lambda_5) \cdot f_2(q_{b1}, q_{b2}, k_2, \lambda_1, \lambda_2, \lambda_6), \\
E_{2,1,k} &= f_2(q_{c1}, q_{c2}, k_1, \lambda_3, \lambda_4, \lambda_5) \cdot f_2(q_{b1}, q_{b2}, k_2, \lambda_1, \lambda_2, \lambda_6), \\
E_{3,1,k} &= f_1(q_{c1}, q_{c2}, k_1, \lambda_3, \lambda_4, \lambda_5) \cdot f_1(q_{b1}, q_{b2}, k_2, \lambda_1, \lambda_2, \lambda_6), \\
E_{4,1,k} &= f_3(q_{c1}, q_{c2}, k_1, \lambda_3, \lambda_4, \lambda_5) \cdot f_1(q_{b1}, q_{b2}, k_2, \lambda_1, \lambda_2, \lambda_6), \\
E_{5,1,k} &= f_3(q_{b1}, q_{b2}, k_2, \lambda_1, \lambda_2, \lambda_5) \cdot f_3(q_{c1}, q_{c2}, k_1, \lambda_3, \lambda_4, \lambda_6), \\
E_{6,1,k} &= f_0(q_{b1}, q_{b2}, \lambda_1, \lambda_2) \cdot f_4(q_{c1}, q_{c2}, k_1, k_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6), \\
E_{7,1,k} &= f_0(q_{b1}, q_{b2}, \lambda_1, \lambda_2) \cdot f_5(q_{c1}, q_{c2}, k_1, k_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6), \\
E_{8,1,k} &= f_0(q_{b1}, q_{b2}, \lambda_1, \lambda_2) \cdot f_6(q_{c1}, q_{c2}, k_1, k_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6), \\
E_{9,1,k} &= f_0(q_{b1}, q_{b2}, \lambda_1, \lambda_2) \cdot f_7(q_{c1}, q_{c2}, k_1, k_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6).
\end{align*}
\]

c) relations between these basic functions

\[
\begin{align*}
E_{1,3,k} &= E_{4,2,k}, & E_{2,3,k} &= E_{2,2,k}, & E_{3,3,k} &= E_{3,2,k}, \\
E_{4,3,k} &= E_{1,2,k}, & E_{5,3,k} &= E_{5,2,k}, & E_{1,4,k} &= E_{4,1,k}, \\
E_{2,4,k} &= E_{2,1,k}, & E_{3,4,k} &= E_{3,1,k}, & E_{4,4,k} &= E_{1,1,k}, \\
E_{5,4,k} &= E_{5,1,k}, & E_{9,4,k} &= E_{9,1,k} + E_{9,2,k} - E_{9,3,k}.
\end{align*}
\]
Two independent functions

\[
\begin{align*}
E_{6,1,k} &= E_{7,1,k} + 2q_{c2} \cdot k_2 E_{9,1,k} - E_{3,2,k} + E_{1,2,k}, \\
E_{6,2,k} &= E_{7,2,k} + 2q_{c2} \cdot k_1 E_{9,2,k} - E_{3,1,k} + E_{1,1,k}, \\
E_{6,3,k} &= E_{7,3,k} + 2q_{b2} \cdot k_2 E_{9,3,k} - E_{3,4,k} + E_{1,4,k}, \\
E_{6,4,k} &= E_{7,4,k} + 2q_{c2} \cdot k_1 E_{9,4,k} - E_{3,3,k} + E_{1,3,k}. \\
\end{align*}
\]

\[
\begin{align*}
\dot{E}_{7,1,k} &= -\dot{E}_{4,1,k} + E_{2,1,k} + E_{8,1,k} - 2q_{c1} \cdot k_2 (2E_{5,1,k} - 2E_{5,2,k} + E_{9,1,k}), \\
\dot{E}_{7,2,k} &= -\dot{E}_{4,2,k} + E_{2,2,k} + E_{8,2,k} - 2q_{c1} \cdot k_2 (2E_{5,2,k} - 2E_{5,1,k} + E_{9,2,k}), \\
\dot{E}_{7,3,k} &= -\dot{E}_{4,3,k} + E_{2,3,k} + E_{8,3,k} - 2q_{b1} \cdot k_1 (2E_{5,3,k} - 2E_{5,4,k} + E_{9,3,k}), \\
\dot{E}_{7,4,k} &= -\dot{E}_{4,4,k} + E_{2,4,k} + E_{8,4,k} - 2q_{b1} \cdot k_2 (2E_{5,4,k} - 2E_{5,3,k} + E_{9,4,k}). \\
\end{align*}
\]
(E), Color rearrangement

\[ M^{(\lambda_1, \lambda_4, \lambda_5, \lambda_6)}(q_{b1}, q_{b2}; q_{c1}, q_{c2}, k_1, k_2) = \sum_{i=1}^{36} M^{(\lambda_1, \lambda_4, \lambda_5, \lambda_6)}(q_{b1}, q_{b2}; q_{c1}, q_{c2}, k_1, k_2) \]

\[ = \sum_{m=1}^{5} C_{mij} M_{m}^{(\lambda_1, \lambda_4, \lambda_5, \lambda_6)}(q_{b1}, q_{b2}; q_{c1}, q_{c2}, k_1, k_2) \]

where

\[ M_{m}' = \sum_{\lambda_2, \lambda_3} M_{F_m}'^{(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6)}(q_{b1}, q_{b2}, q_{c1}, q_{c2}, k_1, k_2) D_2 B_B^{(\lambda_2, \lambda_3)}(q_{b2}, q_{c1}) \]

\( M_{F_m}' \) can be obtained by adding the scalar part of the propagator to the above obtained one.

\[ M_{F_1}' = \frac{D_1}{2}(2(X_{3a} + X_{4c} + X_{5b})E_{1,1,k} - 2X_{4c}E_{2,2,k} - 2X_{4c}E_{3,1,k} + 2X_{4c}E_{4,4,k} - X_{5a}(2E_{5,1,k} - 4E_{5,2,k} + E_{9,1,k} + E_{9,2,k}) + 2(-X_{1g} + X_{5c})E_{6,1,k} \]

\[ + (X_{1b} + X_{1g} + X_{5c})E_{6,2,k} + X_{5c}(E_{8,1,k} - E_{8,2,k}) - X_{2h}E_{8,3,k} + (X_{2f} + X_{2h})E_{8,4,k} + 2X_{4c}E_{5,4,k}q_{b1} \cdot k_2 + 4X_{4c}E_{5,1,k}q_{c2} \cdot k_1 - (X_{1g} + X_{5c})(E_{9,1,k} - E_{9,2,k})f_3 + X_{5c}(4E_{5,1,k} - 4E_{5,2,k} + E_{9,1,k} - E_{9,2,k})f_4 + X_{2h}(-4E_{5,3,k} + 4E_{5,4,k} - E_{9,3,k} + E_{9,4,k})f_5 - E_{9,2,k})f_6 - 2X_{2b}\]

\[ \left\{ E_{2,1,k} + E_{3,1,k} - E_{4,1,k} - E_{5,1,k}(s_c - s_1 + s_b) \right\} \]
\[ M'_{F2} = \frac{D_1}{2} \left( 2(X_{3e} + X_{4a} + X_{5a})E_{1,2,k} - 2X_{4g}E_{2,3,k} - 2X_{4a}E_{3,2,k} + \
2((X_{1a} + X_{1g} + X_{5c})E_{6,1,k} - (X_{1g} + X_{5c})E_{6,2,k} + X_{5c}(E_{8,2,k} - E_{8,1,k}) + (X_{2e} + X_{2h})E_{8,3,k} - X_{2h}E_{8,4,k}) - X_{5d}(2E_{5,2,k} + E_{9,1,k} + E_{9,2,k}) + 2X_{4g}(E_{4,3,k} + 2E_{5,3,k}q_{b1} \cdot k_1) + \
4X_{4a}E_{5,2,k}q_{c2} \cdot k_2 + (X_{1g} + X_{5c})(E_{9,1,k} - E_{9,2,k})f_3 + \
X_{5c}(4E_{5,2,k} - E_{9,1,k} + E_{9,2,k})f_4 + 4E_{5,1,k} \cdot (X_{5d} - X_{5c}f_4) + \
X_{2h}(4E_{5,3,k} - 4E_{5,4,k} + E_{9,3,k} - E_{9,4,k})f_6 - 2X_{5a}(E_{2,2,k} + E_{3,2,k} - E_{4,2,k} - E_{5,2,k}(-s_2 + s_b + s_c)) \right), \]

\[ M'_{F3} = \frac{D_1}{2} \left( -2(X_{4c} + X_{5b})E_{1,1,k} + 2X_{4e}E_{2,4,k} + 2(X_{3c} + X_{4c})E_{3,1,k} + \
2X_{3d}E_{4,1,k} + 2(X_{5c}E_{6,1,k} - X_{5c}E_{6,2,k} + X_{2a}E_{6,4,k} + \
X_{2g}(-E_{6,3,k} + E_{6,4,k}) + X_{1d}E_{7,2,k} + X_{2d}E_{7,4,k} - \
(X_{1h} + X_{5c})E_{8,1,k} + (X_{1f} + X_{1h} + X_{5c})E_{8,2,k}) + X_{5d}(2E_{5,1,k} - 4E_{5,2,k} + E_{9,1,k} + E_{9,2,k}) - 2X_{4e}(E_{4,4,k} + 2E_{5,4,k}q_{b1} \cdot k_2) \
-4X_{4c}E_{5,1,k}q_{c2} \cdot k_1 + X_{5c}(E_{9,1,k} - E_{9,2,k})f_3 - (X_{1h} + X_{5c}) \cdot \
(4E_{5,1,k} - 4E_{5,2,k} + E_{9,1,k} - E_{9,2,k})f_4 - X_{2g}(E_{9,3,k} - E_{9,4,k})f_5 + 2((X_{3b} + X_{5b})E_{2,1,k} + X_{5b}(E_{3,1,k} - E_{4,1,k} - E_{5,1,k}(-s_1 + s_b + s_c))) \right), \]
\[
M'_{F4} = \frac{D_1}{2} \left( -2(X_{4a} + X_{5a})E_{1,2,k} + 2(X_{3g} + X_{4a})E_{3,2,k} + 2X_{3h}E_{4,2,k} - 2X_{5c}E_{6,1,k} + 2(X_{5c}E_{6,2,k} + (X_{2a} + X_{2g})E_{6,3,k} - X_{2g}E_{6,4,k} + X_{1c}E_{7,1,k} + X_{2c}E_{7,3,k} + (X_{1e} + X_{1h} + X_{5c})E_{8,1,k} - (X_{1h} + X_{5c})E_{8,2,k}) + X_{5d}(2E_{5,2,k} + E_{9,1,k} + E_{9,2,k}) + 2X_{4g}(E_{2,3,k} - E_{4,3,k} - 2E_{5,3,k}q_{b1} \cdot k_1) - 4X_{4a}E_{5,2,k}q_{c2} \cdot k_2 - X_{5c}(E_{9,1,k}E_{9,2,k})f_3 - (X_{1h} + X_{5c})(4E_{5,2,k} - E_{9,1,k} + E_{9,2,k})f_4 + 4E_{5,1,k}(-X_{5d} + (X_{1h} + X_{5c})f_4) + X_{2g}(E_{9,3,k} - E_{9,4,k})f_5 + 2((X_{3f} + X_{5a})E_{2,2,k} + X_{5a}(E_{3,2,k} - E_{4,2,k} - E_{5,2,k}(-s_2 + s_b + s_c)))) ,
\]

\[
M'_{F5} = D_1 \left( - (X_{5b}E_{1,1,k}) + (X_{4d} + X_{5b})E_{2,1,k} - (X_{3d} + X_{4d})E_{4,1,k} - X_{3h}E_{4,2,k} - X_{2a}(E_{6,3,k} + E_{6,4,k}) - X_{1e}E_{8,1,k} - X_{1f}E_{8,2,k} - X_{5d}(E_{5,1,k} + E_{5,2,k} - E_{9,1,k} - E_{9,2,k}) + X_{4h}(-E_{1,3,k} + E_{3,3,k} - 2E_{5,3,k}q_{b2} \cdot k_1) + X_{4f}(-E_{1,4,k} + E_{3,4,k} - 2E_{5,4,k}q_{b2} \cdot k_2) - 2X_{4d}E_{5,1,k}q_{c1} \cdot k_1 + X_{4b}(E_{2,2,k} - E_{4,2,k} - 2E_{5,2,k}q_{c1} \cdot k_2) - X_{5b}(-E_{3,1,k} + E_{4,1,k} + E_{5,1,k}(-s_1 + s_b + s_c)) - X_{5a}(E_{1,2,k} - E_{2,2,k} - E_{3,2,k} + E_{4,2,k} + E_{5,2,k}(-s_2 + s_b + s_c))) ,
\]
\[ |M^{(\lambda_1, \lambda_4, \lambda_5, \lambda_6)}(q_{b1}, q_{b2}, q_{c1}, q_{c2}, k_1, k_2)|^2 = \frac{4}{27} |8M'_1 - M'_3|^2 + \frac{4}{27} |8M'_2 - M'_4|^2 - \frac{1}{27} |(8M'_1 - M'_3) \cdot (8M'_2 - M'_4)| + \frac{3}{2} |M'_5|^2 - \frac{1}{3} |(8M'_1 + 8M'_2 - M'_3 - M'_4)M'_5| \]

Square of the amplitude:

\[ |M|^2 = \sum_{\lambda_1 = \pm} \sum_{\lambda_4 = \pm} \sum_{\lambda_5 = \pm} \sum_{\lambda_6 = \pm} |M^{(\lambda_1, \lambda_4, \lambda_5, \lambda_6)}(q_{b1}, q_{b2}, q_{c1}, q_{c2}, k_1, k_2)|^2 \]

(F), phase space integration

cross section for the subprocess

\[ d\sigma = \frac{1}{211 \times 3} \frac{(2\pi)^4 |M|^2}{4(k_1 \cdot k_2)} \times d\Phi_3(k_1 + k_2, P, q_{b1}, q_{c2}), \]

two to three body phase space

\[ d\Phi_3(k_1 + k_2, P, q_{b1}, q_{c2}) = \delta^4(k_1 + k_2 - P - q_{b1} - q_{c2}) \frac{d^3P}{(2\pi)^3 2E_P} \frac{d^3q_{b1}}{(2\pi)^3 2E_{q_{b1}}} \frac{d^3q_{c2}}{(2\pi)^3 2E_{q_{c2}}} \]
BEGIN

BASIC INPUT: NEV, NUMBER, ITMX, VEGASOPEN

VEGASOPEN

IT=0; IT=IT+1

NUM=0; NUM=NUM+1

CALL PHPOINT()

CALL AMP2UP()

NUM<=NUMBER

IT<=ITMX

New GRADE

CALL PHPOINT()

CALL AMP2UP()

NZERO=0

GENERAND(X)

PHPOINT(X,WT)

NZERO=NZERO+1

WT=0.0D0; NZERO<1

NZERO=NZERO+1

TOTFUN(X,WT)

CALL AMP2UP()

NZERO=10000

WT=0.0D0; NZERO<1

NZERO=NZERO+1

TOTFUN(X,WT)

CALL AMP2UP()

NZERO=0

GENERAND(X)

PHPOINT(X,WT)

NZERO=NZERO+1

WT=0.0D0; NZERO<1

NZERO=NZERO+1

TOTFUN(X,WT)

CALL AMP2UP()

NZERO=10000

WT=0.0D0; NZERO<1

NZERO=NZERO+1

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NZERO=NZERO+1

WT=0.0D0; NZERO<1

NZERO=NZERO+1

TOTFUN(X,WT)

CALL AMP2UP()

NZERO=10000

WT=0.0D0; NZERO<1

NZERO=NZERO+1

TOTFUN(X,WT)

CALL AMP2UP()
Color flow for different processes

Color-flow decomposition derived by taking the large $N_c$ limit

PYTHIA
LUND Model

F. Maltoni, K. Paul, T. Stelzer, and S. Willenbrock

Color-flow decomposition of QCD amplitudes

$$\text{Color flow}$$

Feynman rules for color-flow
In large $N_c$ limit
Bc-meson color-flow probability

Partial amplitude (or color-ordered amplitude)-----whose square is just the probability for a particular color-flow. It is the same for 3-different decomposition schemes (fundamental-, adjoint-representation, color-flow decomposition )—demonstrated in PRD67,014026(2003)

Color-singlet:

\[ M = (T^a T^b)_{ij} M_1 + (T^b T^a)_{ij} M_2 + (\delta_{ij} Tr[T^a T^b]) M_3, \]

Color-octet:

\[ M = (T^b T^a T^d)_{ij} M_1 + (T^a T^b T^d)_{ij} M_2 + (T^b T^a T^d)_{ij} M_3 + (\delta_{ij} Tr[T^b T^d]) M_4 + (\delta_{ij} Tr[T^a T^d]) M_5 + (T^d T^a T^b)_{ij} M_7 + (T^d T^b T^a)_{ij} M_8 + (T^a T^d T^b)_{ij} M_9 + (T^b T^d T^a)_{ij} M_{10}. \]
For the color-singlet production processes, there are totally three independent color-flows.

\[ gg \rightarrow (c\bar{b})_1 + b + \bar{c} \]

For the color-singlet production processes, there are totally three independent color-flows:

\[ c_1 = (\delta^l_{i_2} \delta^p_{i_1} \delta^s_{j_1}), \quad c_2 = (\delta^l_{i_2} \delta^{i_1}_{i_1} \delta^p_{j_2}), \quad c_3 = (\delta^l_{i_2} \delta^{s_2}_{i_1} \delta^{s_1}_{j_2}), \]

\[ c_1 \rightarrow ((T^a T^b)_{i_2 j_1}, \quad c_2 \rightarrow ((T^b T^a)_{i_1 j_2}, \quad c_3 \rightarrow ((\delta_{i_2} T r [T^a T^b])_{j_1 j_2}). \]

**FIG. 1:** Color flow diagrams for the color-singlet case based on the color-flow decomposition[11]. Each pair of indices \( i_k \) and \( j_k \) corresponds to an external gluon, i.e. \( k = 1 \) is for gluon-1 and \( k = 2 \) for gluon-2. \( i \) and \( j \) are the decomposed color indices for the outgoing \( \bar{c} \) and \( b \) respectively.
For the color-octet production processes, there are totally ten independent color-flows

\[
geq g g \rightarrow (c \bar{b})_8 + b + \bar{c}
\]

\[
c_1 = (\delta^b_{i_2} \delta^d_{i_4} \delta^l_{i_1} \delta^l_{i_2})_y, \quad c_2 = (\delta^h_{i_1} \delta^d_{i_2} \delta^l_{i_3} \delta^l_{i_4})_y, \quad c_3 = (\delta^b_{i_1} \delta^d_{i_2} \delta^l_{i_3} \delta^l_{i_4})_y,
\]
\[
c_4 = (\delta^l_{i_2} \delta^l_{i_3} \delta^h_{i_1} \delta^l_{i_4})_y, \quad c_5 = (\delta^l_{i_1} \delta^l_{i_3} \delta^b_{i_2} \delta^l_{i_4})_y, \quad c_6 = (\delta^l_{i_1} \delta^l_{i_3} \delta^b_{i_2} \delta^l_{i_4})_y,
\]
\[
c_7 = (\delta^l_{i_2} \delta^l_{i_3} \delta^h_{i_1} \delta^l_{i_4})_y, \quad c_8 = (\delta^l_{i_1} \delta^l_{i_3} \delta^b_{i_2} \delta^l_{i_4})_y, \quad c_9 = (\delta^l_{i_1} \delta^l_{i_3} \delta^b_{i_2} \delta^l_{i_4})_y, \quad c_{10} = (\delta^l_{i_1} \delta^l_{i_3} \delta^b_{i_2} \delta^l_{i_4})_y,
\]

Equivalent

\[
c_1 \rightarrow (T^b T^a T^d)_y, \quad c_2 \rightarrow (T^a T^b T^d)_y, \quad c_3 \rightarrow (T^b_T^a T^d)_y,
\]
\[
c_4 \rightarrow (T^a T^b T^d)_y, \quad c_5 \rightarrow (\delta_{ij} T^b T^d)_y, \quad c_6 \rightarrow (\delta_{ij} T^b T^a T^d)_y,
\]
\[
c_7 \rightarrow (T^d T^b T^a)_y, \quad c_8 \rightarrow (T^d T^a T^b)_y, \quad c_9 \rightarrow (T^a T^d T^b)_y, \quad c_{10} \rightarrow (T^b T^d T^a)_y.
\]
Exchange the color indices

The cross-terms are suppressed by powers of $N_c$ and can be safely neglected in the large $N_c$ limit.---at least $1/N^2_c$
FIG. 2: (Color on line) Typical color flow for gluon-gluon fusion mechanism (Left), gluon-charm collision mechanism (Middle) and charm-charm collision mechanism (Right), where the thick dashed line shows the corresponding intermediate diquark state \((cc) [^1 S_0]_0\). Three colorful lines are for color flow lines according to PYTHIA naming rules.

Typical color flow for \(g+g->X_{icc} (3s1)\)

\[
[0, 503] \rightarrow [502, 503] \rightarrow [501, 502] \rightarrow [501, 0] \rightarrow \text{colorless bound state } \Xi_{cc}^{++}/\Omega_{cc}^+ \\
[0, 504] \rightarrow [504, 0] \rightarrow \text{colorless bound state } \Xi_{cc}^{++}/\Omega_{cc}^+ \\
[0, 505] \rightarrow [505, 0] \rightarrow \text{colorless bound state } \Xi_{cc}^{++}/\Omega_{cc}^+ 
\]
PYTHIA8.0 solve the problem of color flow

FIG. 1: (Color on line) Typical color flow for gluon-gluon fusion mechanism (Left), gluon-charm collision mechanism (Middle) and charm-charm collision mechanism (Right), where the thick dashed line shows the corresponding intermediate diquark state \((cc)[^3S_1]|\bar{3}\). Three colorful lines are for color flow lines according to PYTHIA naming rules.

Typical color flow for \(g+g->\Xi_{cc} (3s1)\)

\[
[0, 503] \rightarrow [502, 503] \rightarrow [501, 502] \rightarrow [501, 0] \rightarrow \text{colorless bound state } \Xi_{cc}^{++}/\Omega_{cc}^{++}
\]

\[
[0, 504] \rightarrow [504, 0] \rightarrow \text{colorless bound state } \Xi_{cc}^{++}/\Omega_{cc}^{++}
\]

\[
[0, 505] \rightarrow [505, 0] \rightarrow \text{colorless bound state } \Xi_{cc}^{++}/\Omega_{cc}^{++}
\]
A cross-check

Table 4
Comparison of total cross sections for $gg \to B_c(B_c^*) + b + \bar{c}$ with the corresponding results of Ref. [7]. The input parameters are $m_b = 4.9$ GeV, $m_c = 1.5$ GeV, $M_{B_c^*} = m_b + m_c$, $f_{B_c}$ in parenthesis shows the Monte Carlo uncertainty in the last digit. The cross sections are expressed in nb.

<table>
<thead>
<tr>
<th>$\sqrt{s}$</th>
<th>20 GeV</th>
<th>30 GeV</th>
<th>60 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{B_c}$</td>
<td>$0.6579(5) \times 10^{-2}$</td>
<td>$0.9465(8) \times 10^{-2}$</td>
<td>$0.7872(8) \times 10^{-2}$</td>
</tr>
<tr>
<td>$\sigma_{B_c}$ [7]</td>
<td>$0.661(7) \times 10^{-2}$</td>
<td>$0.949(8) \times 10^{-2}$</td>
<td>$0.782(9) \times 10^{-2}$</td>
</tr>
<tr>
<td>$\sigma_{B_c^*}$</td>
<td>$0.1606(1) \times 10^{-1}$</td>
<td>$0.2460(3) \times 10^{-1}$</td>
<td>$0.2033(2) \times 10^{-1}$</td>
</tr>
<tr>
<td>$\sigma_{B_c^*}$ [7]</td>
<td>$0.160(2) \times 10^{-1}$</td>
<td>$0.244(3) \times 10^{-1}$</td>
<td>$0.203(3) \times 10^{-1}$</td>
</tr>
</tbody>
</table>

Table 5
Comparison of total cross sections for $gg \to B_c + b + \bar{c}$ with the corresponding results of Ref. [8]. The input parameters are $m_b = 3m_c$, $M_{B_c} = 6.30$ GeV, $f_{B_c} = 0.480$ GeV, $\alpha_s = 0.2$. The number in parenthesis shows the Monte Carlo uncertainty in the last digit. The cross sections are expressed in nb.

<table>
<thead>
<tr>
<th>$\sqrt{s}$</th>
<th>20 GeV</th>
<th>30 GeV</th>
<th>60 GeV</th>
<th>80 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{B_c}$</td>
<td>$0.6853(5) \times 10^{-2}$</td>
<td>$0.9731(8) \times 10^{-2}$</td>
<td>$0.7997(9) \times 10^{-2}$</td>
<td>$0.6244(9) \times 10^{-2}$</td>
</tr>
<tr>
<td>$\sigma_{B_c}$ [8]</td>
<td>$0.686(2) \times 10^{-2}$</td>
<td>$0.971(4) \times 10^{-2}$</td>
<td>$0.793(5) \times 10^{-2}$</td>
<td>$0.623(5) \times 10^{-2}$</td>
</tr>
</tbody>
</table>
Improve the efficiency for unweighted events by using BCVEGPY and GENXICC

Weighted events: (IDWTUP=3) – time-saving – no waste events
All partonic events are accepted by PYTHIA with unit weight (100% pass): two ways
A) The phase-space are uniformly generated.
B) VEGAS is adopted to generate sampling importance function to improve its accuracy. The phase-space are generated according to the relative importance of this point. Then, one can use the weight of each points to restore total CS or distributions.

Unweighted events: (IDWTUP=1) – time-consuming – less waste events more better
A) Using PYTHIA inner hit-and-miss technology (von Neumann algorithm):
   \[ \text{XWGTUP}/\text{XMAXUP} \geq \text{PYR}(0) \] accept
   \[ \text{XWGTUP}/\text{XMAXUP} < \text{PYR}(0) \] reject
B) Using improved hit-and-miss technology:
   MINT, divided into mesh grade, XMAXUP to be a group, pass the criteria much more effectively
Due to its high efficiency, BCVEGPY and GENXICC, are very useful for MC simulation and also for theoretical studies.

Now it has been adopted by ATLAS, CMS, LHCb, CDF and D0 groups respectively.

The coming LHC experiment shall provide a better platform to check all the theoretical predications and to learn the Bc, Xicc, Xibc, Xibb properties in more detail.

The programmed super Z factory, GIGAZ, LEP3, and etc. shall provide other platforms for doubly heavy meson and baryon productions, which are in progress. Especially, a generator BEEC shall be available soon.
Backup slides for BCVEGPY and GENXICCC applications
some results for the S-wave Bc production

Uncertainties

PDF

$Q^2$
Uncertainties

Mechanism
TABLE III: Total cross-section for the hadronic production of $B_c[1^1S_0]$ and $B_c^*[1^3S_1]$ at TEVATRON and at LHC with the leading order (LLO) running $\alpha_s$ and the characteristic energy scale $Q^2 = \hat{s}/4$ or $Q^2 = p_T^2 + m_{B_c}^2$. The cross section is in unit of nb.

<table>
<thead>
<tr>
<th></th>
<th>CTEQ5L</th>
<th>CTEQ6L</th>
<th>GRV98L</th>
<th>MRST2001L</th>
<th>CTEQ5L</th>
<th>CTEQ6L</th>
<th>GRV98L</th>
<th>MRST2001L</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{B_c[1^1S_0]}$</td>
<td>3.12</td>
<td>3.79</td>
<td>3.27</td>
<td>3.40</td>
<td>4.39</td>
<td>5.50</td>
<td>4.54</td>
<td>4.86</td>
</tr>
<tr>
<td>$\sigma_{B_c^*[1^3S_1]}$</td>
<td>7.39</td>
<td>9.07</td>
<td>7.88</td>
<td>8.16</td>
<td>10.7</td>
<td>13.4</td>
<td>11.1</td>
<td>11.9</td>
</tr>
</tbody>
</table>

Cross-section

TABLE VI: The integrated hadronic cross section for TEVATRON at different C.M. energies. The gluon distribution is chosen from CTEQ5L and the characteristic energy scale of the production is chosen as Type A, i.e. $Q^2 = \hat{s}/4$. In addition, a cut for transverse momentum $p_T$ ($p_T < 5$ GeV) and a cut for rapidity $y$ ($|y| > 1.5$) have been imposed.

<table>
<thead>
<tr>
<th>C.M. energy</th>
<th>1.8(TeV)</th>
<th>1.9(TeV)</th>
<th>1.96(TeV)</th>
<th>2.0(TeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_c[1^1S_0]$</td>
<td>0.40</td>
<td>0.44</td>
<td>0.46</td>
<td>0.47</td>
</tr>
<tr>
<td>$B_c^*[1^3S_1]$</td>
<td>1.00</td>
<td>1.09</td>
<td>1.14</td>
<td>1.18</td>
</tr>
</tbody>
</table>
\( P_T \text{ cut} \)

FIG. 6: \( B_c \) differential distributions versus its \( y \) with various \( p_T^{\text{cut}} \) in TEVATRON (left diagram) and in LHC (right diagram). Solid line corresponds to the full production without \( p_T^{\text{cut}} \); dashed line to \( p_T^{\text{cut}} = 5.0 \) GeV; dash-dot line to \( p_T^{\text{cut}} = 20.0 \) GeV; the dashed line to \( p_T^{\text{cut}} = 35.0 \) GeV; the big dotted line to \( p_T^{\text{cut}} = 50.0 \) GeV and the solid line with diamonds to \( p_T^{\text{cut}} = 100 \) GeV.

TABLE V: Values of the ratio \( R_{p_T^{\text{cut}}} \) (see definition in text) for the hadronic production of pseudo-scalar \( B_c \) meson in TEVATRON and LHC.

<table>
<thead>
<tr>
<th>( p_T^{\text{cut}} )</th>
<th>0.0 GeV</th>
<th>5 GeV</th>
<th>20 GeV</th>
<th>35 GeV</th>
<th>50 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y^{\text{cut}} )</td>
<td>1.0 1.5 2.0</td>
<td>1.0 1.5 2.0</td>
<td>1.0 1.5 2.0</td>
<td>1.0 1.5 2.0</td>
<td>1.0 1.5 2.0</td>
</tr>
<tr>
<td>( R_{p_T^{\text{cut}}} ) (TEVATRON)</td>
<td>0.45 0.64 0.79</td>
<td>0.46 0.65 0.80</td>
<td>0.57 0.77 0.91</td>
<td>0.65 0.85 0.95</td>
<td>0.70 0.90 0.98</td>
</tr>
<tr>
<td>( R_{p_T^{\text{cut}}} ) (LHC)</td>
<td>0.31 0.46 0.59</td>
<td>0.32 0.47 0.60</td>
<td>0.38 0.54 0.69</td>
<td>0.42 0.60 0.74</td>
<td>0.45 0.64 0.79</td>
</tr>
</tbody>
</table>
Results for the P-wave $B_c$ states

<table>
<thead>
<tr>
<th>C.M. energy (GeV)</th>
<th>20GeV</th>
<th>40GeV</th>
<th>60GeV</th>
<th>80GeV</th>
<th>100GeV</th>
<th>200GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(^1P_1)(pb)$</td>
<td>0.184</td>
<td>0.743</td>
<td>0.657</td>
<td>0.538</td>
<td>0.439</td>
<td>0.195</td>
</tr>
<tr>
<td>$\sigma(^3P_0)(pb)$</td>
<td>0.367</td>
<td>0.207</td>
<td>0.175</td>
<td>0.141</td>
<td>0.114</td>
<td>0.0496</td>
</tr>
<tr>
<td>$\sigma(^3P_1)(pb)$</td>
<td>0.346</td>
<td>0.598</td>
<td>0.503</td>
<td>0.402</td>
<td>0.324</td>
<td>0.139</td>
</tr>
<tr>
<td>$\sigma(^3P_2)(pb)$</td>
<td>0.721</td>
<td>1.49</td>
<td>1.31</td>
<td>1.06</td>
<td>0.862</td>
<td>0.374</td>
</tr>
</tbody>
</table>

Good check between each other !!!

100GeV

subprocess

A.V. Berezhnoy.

totally numerically
total hadronic production

\( \frac{d\sigma}{dp_t} (\text{nb/GeV}) \) vs. \( p_t(\text{GeV}) \)

LHC

TEVATRON
### Cross-section

<table>
<thead>
<tr>
<th>$Q^2$</th>
<th>LHC ($\sqrt{s} = 14$ TeV)</th>
<th>TEVATRON ($\sqrt{s} = 1.96$ TeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A$</td>
<td>$B$</td>
</tr>
<tr>
<td>$\sigma(^1P_1)(nb)$</td>
<td>4.738</td>
<td>9.123</td>
</tr>
<tr>
<td>$\sigma(^3P_0)(nb)$</td>
<td>1.910</td>
<td>3.288</td>
</tr>
<tr>
<td>$\sigma(^3P_1)(nb)$</td>
<td>4.117</td>
<td>7.382</td>
</tr>
<tr>
<td>$\sigma(^3P_2)(nb)$</td>
<td>10.18</td>
<td>20.40</td>
</tr>
</tbody>
</table>

A.V. Berezhnoy,
LHC

\[ \mu_R^2 \]

\[ \mu_F^2 \]

TEVATRON
For experimental usage

$y$ cut

$P_T$ cut
Results for the color-octet Bc states

TABLE I: Total cross-section (in unit of nb) for the hadronic production of the (c\bar{b}) meson at LHC (14.0 TeV) and TEVATRON (1.96 TeV), where for short the |(^1S_0)\rangle denotes (c\bar{b}) state in color-singlet (1S_0) configuration, and so forth. Here \( m_b = 4.90 \) GeV, \( m_c = 1.50 \) GeV and \( M = 6.40 \) GeV. For the color-octet matrix elements, we take \( \Delta \phi(v) \in (0.10, \ 0.30) \).

|       | |\(^1S_0\rangle\rangle | |\(^3S_1\rangle\rangle | |\(^1S_0\rangle g\rangle | |\(^3S_1\rangle g\rangle | |\(^1P_1\rangle\rangle | |\(^3P_0\rangle\rangle | |\(^3P_1\rangle\rangle | |\(^3P_2\rangle\rangle |
|-------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| LHC   | 71.1           | 177.           | (0.357, 3.21)  | (1.58, 14.2)   | 9.12           | 3.29           | 7.38           | 20.4           |
| TEVATRON | 5.50          | 13.4           | (0.0284, 0.256)| (0.129, 1.16)  | 0.655          | 0.256          | 0.560          | 1.35           |

3-20%     60%
Results for Xicc, Xibc and Xibb Production

FIG. 5 (color online). The energy dependence of the integrated partonic cross-section for the production of the baryons via the heavy diquarks in terms of the gluon-gluon fusion mechanism. The dotted line, solid line, dashed line and dash-dot line stand for those via the diquarks \((cc)_{3}^{[3]S_{1}}\), \((bc)_{3}^{[3]S_{1}}\), \((bc)_{3}^{[1]S_{0}}\) and \((bb)_{3}^{[3]S_{1}}\) respectively. The curves for \(X_{cc}\) and \(X_{bb}\) both are divided by 2.

TABLE II. Cross sections \((\sigma)\) for the hadronic production of \(X_{cc}\) at colliders TEVATRON and LHC, where the \((cc)\)-diquark is in \((cc)_{3}^{[3]S_{1}}\) or \((cc)_{6}^{[1]S_{0}}\), and the symbol \(g + c\) means \(g + c \rightarrow X_{cc} + \bar{c}\) and etc. In the calculations, cuts \(p_{t} \geq 4\) GeV and \(|y| \leq 1.5\) are taken at LHC, while at TEVATRON cuts \(p_{t} \geq 4\) GeV, \(|y| \leq 0.6\) instead.

<table>
<thead>
<tr>
<th></th>
<th>(\sqrt{s} = 1.96) TeV</th>
<th>(\sqrt{s} = 14.0) TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>((cc)<em>{3}^{[3]S</em>{1}})</td>
<td>((cc)<em>{6}^{[1]S</em>{0}})</td>
</tr>
<tr>
<td>(\sigma_{g+g} (nb))</td>
<td>1.61</td>
<td>0.392</td>
</tr>
<tr>
<td>(\sigma_{c+g} (nb))</td>
<td>2.29</td>
<td>0.360</td>
</tr>
<tr>
<td>(\sigma_{c+c} (nb))</td>
<td>0.751</td>
<td>0.0431</td>
</tr>
</tbody>
</table>

FIG. 9 (color online). The \(p_{t}\)-distribution for the hadroproduction of \(X_{cc}\) at TEVATRON (left) and at LHC (right), where \(|y| \leq 1.5\) at LHC and \(|y| \leq 0.6\) at TEVATRON are adopted. The dotted line and the solid line are for gluon-gluon fusion mechanism, the triangle line and the diamond line are for \(g + c \rightarrow X_{cc} + \bar{c}\), the dashed line and the dash-dot line are for \(c + c \rightarrow X_{cc} + \text{‘}g\text{‘}\), where the upper lines of each mechanism are for \((cc)_{3}^{[3]S_{1}}\) and the lower lines are for \((cc)_{6}^{[1]S_{0}}\), respectively.
FIG. 10 (color online). The $p_t$-distributions for the hadroproduction of $\Xi_{cc}$ at SELEX. The dotted line and the solid line are for gluon-gluon fusion mechanism, the dashed line and the dash-dot line are for $g + c \rightarrow \Xi_{cc} + \bar{c}$, the triangle line and the diamond line are for $c + c \rightarrow \Xi_{cc} + 'g'$, where the upper lines of each mechanism are for $(cc)^H[3 S_1]$ and the lower lines are for $(cc)^L[1 S_0]$, respectively.

FIG. 11 (color online). The energy scale dependence of the $p_t$-distributions for each mechanism at SELEX, where the contributions from $(cc)^H[3 S_1]$ and $(cc)^L[1 S_0]$ are summed up. The upper band is for the mechanism $g + c \rightarrow \Xi_{cc}$, the middle band is for gluon-gluon fusion mechanism and the lower band is for $c + c \rightarrow \Xi_{cc}$ mechanism, where the solid line in each band corresponds to $\mu = M_t$, the upper edge of the band is for $\mu = M_t/2$ and the lower edge is for $\mu = 2M_t$, respectively.

TABLE IV. $R$ values, which is defined in Eq. (10), for the hadronic production of $\Xi_{cc}$.

<table>
<thead>
<tr>
<th></th>
<th>SELEX</th>
<th>TEVATRON</th>
<th>LHC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p_t &gt; 0.2$ GeV</td>
<td>$p_t \geq 4$ GeV, $</td>
<td>y</td>
</tr>
<tr>
<td>$R$</td>
<td>29</td>
<td>3.4</td>
<td>2.8</td>
</tr>
</tbody>
</table>
**Figure 3.** The $p_t$-distributions (left) and $y$-distributions (right) for the hadroproduction of $\Xi_{cc}$ at SELEX with different values of $A_{in}$. The dotted, the dashed and the dash-dotted lines are for $A_{in} = 0.1\%$, $0.3\%$ and $1\%$, respectively. The result with CTEQ6HQ, i.e., $A_{in} = 0$ is shown by a solid line (the lowest one).

**Table 1.** The contribution of $\sigma_{ab}$ from different sub-processes initialized by the partons $ab$ to the total cross section (in pb) for the $\Xi_{cc}$ hadronic production at SELEX with the cut $p_t > 0.2$ GeV.

<table>
<thead>
<tr>
<th></th>
<th>CTEQ6HQ ($A_{in} = 0$)</th>
<th>$A_{in} = 1%$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_{gg}$</td>
<td>$\sigma_{cc}$</td>
</tr>
<tr>
<td>(cc)$_3^1$</td>
<td>4.03</td>
<td>$1.02 \times 10^{-3}$</td>
</tr>
<tr>
<td>(cc)$_6^1$</td>
<td>0.754</td>
<td>$4.15 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

**Table 2.** The contribution rates of the sub-process $gc \rightarrow \Xi_{cc}$ in the different $x_c$ region in the charm quark PDFs with $A_{in} = 1\%$ and $p_t > 0.2$ GeV.

<table>
<thead>
<tr>
<th>$x_c$ region</th>
<th>25%</th>
<th>50%</th>
<th>22%</th>
<th>3%</th>
<th>~0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.0 \leq x_c \leq 0.2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.2 \leq x_c \leq 0.4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.4 \leq x_c \leq 0.6$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.6 \leq x_c \leq 0.8$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.8 \leq x_c \leq 1.0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 3.** The $R$ values for SELEX with the cut $p_t > 0.2$ GeV.

<table>
<thead>
<tr>
<th>CTEQ6HQ ($A_{in} = 0$)</th>
<th>$A_{in} = 0.1%$</th>
<th>$A_{in} = 0.3%$</th>
<th>$A_{in} = 1%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>29.3</td>
<td>36.6</td>
<td>51.3</td>
</tr>
</tbody>
</table>
LHC, TEVATRON can not see the difference between the cases of with or with intrinsic charm.