Applications of Higher Order QCD

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SLAC

Physics In Collision 2013
Beijing, 09/04/13
2013 → Higgs physics has moved from discovery to precision stage

Improved theoretical predictions required to search for (small) deviations from Standard Model

Great success of SM so far, but should keep looking everywhere
Any event at the LHC involves QCD, and so does most of our work.
QCD theory needs to keep up with incredible pace of LHC experiments
Event structure at hadron colliders

- How to make predictions for complex events?
- Must account for multiple physics effects at widely different scales

- Key strategy: Factorization of hard and soft QCD effects
  \[
  \sigma_{h_1 h_2 \to X} = \int dx_1 dx_2 \, f_{h_1, i}(x_1, \mu_F^2) f_{h_2, j}(x_2, \mu_F^2) \, \hat{\sigma}_{ij \to X}(x_1 x_2 S, \mu_F^2) + \mathcal{O}(\Lambda_{QCD}/Q)^n
  \]

  - PDFs inherently non-perturbative, but evolution with \( \mu_F \) calculable
  - Universality \( \rightarrow \) Measured in DIS \& fixed-target and applied to LHC
  - Focus of this talk will be calculation of partonic cross sections
Toolkit inventory

- All processes of interest
  - Parton shower Monte Carlo (Herwig, Pythia, Sherpa, ...)
  - Automated tree-level calculations & merging with PS (Alpgen, CompHEP, Helac, MadGraph, Sherpa, ...)
- Available for increasingly complex final states (2→4,5,6)
  - Automated NLO (BlackHat, GoSam, Helac, MadLoop, MadGolem, NJet, OpenLoops, Rocket, ...)
  - Matching to parton shower (aMC@NLO, Herwig, POWHEG Box, Sherpa, ...)
  - Merging at NLO (aMC@NLO, Pythia, Sherpa, ...)
- Available for some processes
  - Inclusive NNLO (W,Z,gg→H,t̅t,jets,H+jet)
  - Fully differential NNLO (FEHiP, FEWZ, HNNLO)
  - NNLO+N^xLL resummation (e⁺e⁻→2/3 jets, pp→H)
Automated NLO calculations

- **NLO subtraction methods**

  \[
  d\hat{\sigma}_{NLO} = \int_{\Phi_n} \left( d\hat{\sigma}^B + d\hat{\sigma}^V + d\hat{\sigma}^{MF} + \int_{\Phi_1} d\hat{\sigma}^S \right) + \int_{\Phi_{n+1}} \left( d\hat{\sigma}^R - d\hat{\sigma}^S \right)
  \]

  finite, compute with MC

- **Universal infrared behaviour of amplitudes**
  - FKS subtraction Frixione,Kunszt,Signer 1995
  - Dipole subtraction Catani,Seymour 1996 +Dittmaier,Trocsanyi 2002
  - Antenna subtraction Kosower 1997

- **Realized in tree-level ME generators & stand-alone codes**
  - Sherpa Gleisberg,Krauss 2007
  - MadDipole Frederix,Greiner,Gehrmann 2008
  - Helac Czakon,Papadopoulos,Worek 2009
  - TeVJet Seymour,Tevlin 2008
  - AutoDipole Hasegawa,Moch,Uwer 2008
  - MadFKS Frederix,Frixione,Maltoni,Stelzer 2009
The NLO revolution

- One-loop amplitudes evaluated by extracting coefficients of box/triangle/bubble/tadpole master integrals
  \[ A = \sum d_i + \sum c_i + \sum b_i + R \]

- “Feynmanian” approach → Improved decomposition & reduction
  Denner, Dittmaier 2005  Binoth, Guillet, Pilon, Heinrich, Schubert 2005

- “Unitarian” approach → Multi-particle cuts & complex momenta
  Bern, Dixon, Dunbar, Kosower 1994  Britto, Cachazo, Feng 2004
  Ossola, Papadopoulos, Pittau 2006  Forde 2007  Ellis, Giele, Kunszt, Melnikov 2008

- Plethora of (semi-)automated programs emerged: BlackHat, GoSam, HelacNLO, MadLoop, MadGolem, NJet, OpenLoops, Rocket, . . .
  Badger, Bern, Bevilacqua, Biedermann, Binoth, Cascioli, Cullen, Czakon, Dixon, Ellis, Febres Cordero, Frederix, Frixione, Garzelli, Giele, Goncalves Netto, Greiner, Guffanti, Guillet, van Hameren, Heinrich, Hirschi, Ita, Kardos, Karg, Kauer, Kosower, Lopez-Val, Kunszt, Luisoni, Maierhöfer, Maître, Maltoni, Mastrolia, Mawatari, Melnikov, Ossola, Ozeren, Papadopoulos, Pittau, Plehn, Pozzorini, Reiter, Reuter, Tramontano, Uwer, Wigmore, Worek, Yundin, Zanderighi, Zeppenfeld, ...
### Experimenter's NLO wishlist

<table>
<thead>
<tr>
<th>Process ((V \in {Z, W, \gamma}))</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (pp \to VV) jet</td>
<td>(WW) jet completed by Dittmaier/Kallweit/Uwer; Campbell/Ellis/Zanderighi; (ZZ) jet completed by Binoth/Gleisberg/Karg/Kauer/Sanguinetti; (WZ) jet, (W\gamma) jet completed by Campanario et al.</td>
</tr>
<tr>
<td>2. (pp \to Higgs+2) jets</td>
<td>NLO QCD to the (gg) channel completed by Campbell/Ellis/Zanderighi; NLO QCD+EW to the VBF channel completed by Ciccolini/Denner/Dittmaier; Interference QCD-EW in VBF channel</td>
</tr>
<tr>
<td>3. (pp \to VVV) ZZ (Z) +3 jets</td>
<td>(ZZZ) completed by Lazopoulos/Melnikov/Petriello and (WWZ) by Hankele/Zeppenfeld; see also Binoth/Ossola/Papadopoulos/Pittau; VBFNLO meanwhile also contains (WWW, ZZW, ZZZ, WW, ZZ, WZ, W\gamma, Z\gamma, \gamma\gamma, W\gamma\gamma)</td>
</tr>
<tr>
<td>4. (pp \to t\bar{t}b\bar{b})</td>
<td>relevant for (t\bar{t}H), computed by Bredenstein/Denner/Dittmaier/Pozzorini and Bevilacqua/Czakon/Papadopoulos/Pittau/Worek</td>
</tr>
<tr>
<td>5. (pp \to V+3) jets</td>
<td>(W+3) jets calculated by the Blackhat/Sherpa and Rocket collaborations; (Z+3) jets by Blackhat/Sherpa</td>
</tr>
<tr>
<td>6. (pp \to t\bar{t}+2) jets</td>
<td>relevant for (t\bar{t}H), computed by Bevilacqua/Czakon/Papadopoulos/Worek</td>
</tr>
<tr>
<td>7. (pp \to VV) (b\bar{b})</td>
<td>Pozzorini et al./Bevilacqua et al.</td>
</tr>
<tr>
<td>8. (pp \to VV+2) jets</td>
<td>(W^+W^-+2) jets, (W^+W^-+2) jets, relevant for VBF (H \to VV); VBF contributions by (Bozzi/)Jäger/Oleari/Zeppenfeld</td>
</tr>
<tr>
<td>9. (pp \to b\bar{b}b\bar{b})</td>
<td>Binoth et al.</td>
</tr>
<tr>
<td>10. (pp \to V+4) jets</td>
<td>top pair production, various new physics signatures Blackhat/Sherpa: (W^+4) jets, (Z+4) jets; see also HEJ for (W+n) jets</td>
</tr>
<tr>
<td>11. (pp \to Wb\bar{b}j)</td>
<td>top, new physics signatures, Reina/Scutzmeier</td>
</tr>
<tr>
<td>12. (pp \to t\bar{t}t\bar{t})</td>
<td>various new physics signatures, Bevilacqua/Worek</td>
</tr>
</tbody>
</table>

\(pp \to W\gamma\gamma\) jet
\(pp \to 4\) jets
Campanario/Englert/Rauch/Zeppenfeld
Blackhat/Sherpa

- Started Les Houches 2005
- Item 9 added in 2007, 10-12 in 2009
- Finally retired in 2012
NLO highlights: $pp \rightarrow W + 5$ jets

Bern, Dixon, Febres Cordero, SH, Ita, Kosower, Maître, Ozeren 2013

First 2 $\rightarrow$ 6 NLO calculation for hadron colliders

- Allows extrapolation of jet rates to higher multiplicity (scaling)

- Flat $K$-factor for $5^{\text{th}}$ jet with suitable scale $\hat{H}'_T = \sum p_{T,j} + E_{T,W}$
NLO highlights: Higgs+3 jets

Cullen, van Deurzen, Greiner, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, Tramontano 2013

- Largely reduced scale dependence
- Can be used to improve prediction of exclusive $H + 2$jets production

\[ \sigma_{2j,\text{excl}} = \sigma_{2j,\text{incl}} - \sigma_{3j,\text{incl}} \]

- Combination of One Loop program (GoSam) and MC (Sherpa, MadEvent)

Using Binoth LesHouches accord Binoth et al. 2010; Alioli et al. 2013
The NNLO frontier

- Structure of the calculation

\[ d\hat{\sigma}_{\text{NNLO}} = \int_{\Phi_{n+2}} (d\hat{\sigma}^{RR} - d\hat{\sigma}^{S}) + \int_{\Phi_{n+1}} (d\hat{\sigma}^{RV} - d\hat{\sigma}^{VS} + d\hat{\sigma}^{MF,1}) \]
  
  \[ + \int_{\Phi_n} (d\hat{\sigma}^{VV} + d\hat{\sigma}^{MF,2}) + \int_{\Phi_{n+1}} d\hat{\sigma}^{VS} + \int_{\Phi_{n+2}} d\hat{\sigma}^{S} \]

- Require three principal ingredients
  - Two-loop matrix elements
    explicit poles from loop integrals
  - One-loop matrix elements
    explicit poles from loop integral
    implicit poles from real emission
  - Tree-level matrix elements
    implicit poles from real emissions

- Challenge: Construction of subtraction methods for RR and RV contribution
Methods for real radiation at NNLO

- **Sector decomposition** Binoth, Heinrich 2004; Anastasiou, Melnikov, Petriello 2004
  - $pp \rightarrow H$, $pp \rightarrow V$ Anastasiou, Melnikov, Petriello; Bühler, Herzog, Lazopoulos, Müller

- **Antenna subtraction** Gehrmann, Gehrmann-DeRidder, Glover
  - $e^+ e^- \rightarrow 3\text{jets}$ Gehrmann, Gehrmann-DeRidder, Glover, Heinrich, Weinzierl
  - $pp \rightarrow 2\text{jets}$ Gehrmann, Gehrmann-DeRidder, Glover, Pires

- **$q_T$ subtraction** Catani, Grazzini 2007
  - $pp \rightarrow H$, $pp \rightarrow V$, $pp \rightarrow VH$, $pp \rightarrow \gamma\gamma$ Catani, Cieri, DeFlorian, Ferrera, Grazzini, Tramontano

- **Sector-improved subtraction** Czakon 2010; Boughezal, Melnikov, Petriello 2011
  - $pp \rightarrow t\bar{t}$ Czakon, Fiedler, Mitov
  - $pp \rightarrow H+\text{jet}$ Boughezal, Caola, Melnikov, Petriello, Schulze
Diphoton production at NNLO

Catani, Cieri, deFlorian, Ferrera, Grazzini 2011

- Frixione photon isolation criterion
- $q_T$ subtraction for real corrections
- First fully consistent inclusion of box contribution

\[ \sigma_T, \frac{d\sigma}{dp} \]

\[ \gamma \gamma, Ldt = 4.9 \text{ fb} \]

\[ \int \text{Data 2011, DIPHOX+GAMMA2MC (CT10) NNLO (MSTW2008)} \]

\[ \sqrt{s} = 7 \text{ TeV} \]

\[ \text{ATLAS} \]

\[ \text{NNLO, LO, NLO, } \mu_R = \mu_F = M_{\gamma\gamma} \]

\[ \text{LHC 14 TeV} \]

\[ \text{data/DIPHOX} \]

\[ \text{data/2\gammaNNLO (MSTW2008)} \]
Top pair production at NNLO

\[ q\bar{q} \rightarrow t\bar{t} \quad \text{Bärnreuther,Czakon,Mitov 2012} \]
\[ gg \rightarrow t\bar{t} \quad \text{Czakon,Fiedler,Mitov 2013} \]

- Sector-improved subtraction for double real contribution
- First hadron collider calculation at NNLO with more than 2 colored partons
- First NNLO hadron collider calculation with massive fermions
- Point of saturation reached, where uncertainties (scale, PDF, \(\alpha_s\), \(m_t\)) are all of same size
- Already used to constrain PDFs
  Czakon,Mangano,Mitov,Rojo 2013

\[ \sigma_{\text{tot}} \quad [\text{pb}] \]

\[ \sqrt{s} \quad [\text{TeV}] \]

\[ \text{Indep. } \mu_{F,R} \text{ variation} \]

PP \(\rightarrow\) tt+X; \(m_{\text{top}}=173.3\) GeV
MSTW2008(68c.l.) LO; NLO; NNLO

\[ \text{CMS, 7TeV} \]
\[ \text{ATLAS+CMS, 7TeV} \]
\[ \text{ATLAS, 7TeV} \]
\[ \text{CMS, 8TeV} \]
Jet production at NNLO

$pp \rightarrow 2$ jets Gehrmann, Gehrmann-DeRidder, Glover, Pires 2013

- Antenna subtraction in double real and real-virtual contribution
- Calculation implemented in a parton-level event generator
- Leading colour, gluons only very small scale dependence

![Graph showing jet production at different scales.](image)
Higgs+jet production at NNLO

Boughezal, Caola, Melnikov, Petriello, Schulze 2013

- Two independent calculations
- Sector-improved subtraction for double real contribution
- Large $K$-factor, 30% enhancement w.r.t. NLO for $\mu = m_H$
- Gluonic contribution only very small scale dependence 20% at NLO $\rightarrow$ 5% at NNLO
The importance of exclusive calculations

- Higgs measurements in $WW$ channel binned in number of jets to reduce background (top veto)
- Also used to separate gluon fusion from VBF
- Different uncertainties in different jet bins

\[
\begin{align*}
\text{Events} & \quad \text{Data} \oplus \text{SM (sys \& stat)} \\
\text{N}_{\text{jets}} & \quad \text{WW} \quad \text{WZ/ZZ/W} \quad \text{t+1} \quad \text{Single Top} \\
\text{H} [125 \text{ GeV}] \times 10
\end{align*}
\]
Why are exclusive calculations difficult?

- NLO corrections include virtual and real-emission part

\[
2 \text{Re} \left\{ \text{virtual} \times \text{real} \right\} \times \left( -\frac{1}{\varepsilon_{IR}^2} + \ldots \right) + \left( +\frac{1}{\varepsilon_{IR}^2} - C \log^2 \left( \frac{Q}{p_T,\text{cut}} \right) + \ldots \right)
\]

- In inclusive case, finite correction remains
- In exclusive case, logarithmic dependence on \( p_T,\text{cut} \)
- Higgs production in gluon fusion:
  \[
  -6 \frac{\alpha_s}{\pi} \log^2 \frac{m_h}{p_T,\text{cut}} \rightarrow \text{large!}
  \]
- Negative correction leads to pinch point in scale variation
- Uncertainty estimate requires resummation of log corrections

\[
\begin{align*}
\sigma_0(p_{\text{cut}}^T) \text{ [pb]} & \quad \text{for } gg \rightarrow H + 0 \text{ jet (NNLO)} \\
E_{\text{cm}} & = 7 \text{ TeV} \\
m_H & = 165 \text{ GeV} \\
\mu & = m_H/4 \\
\mu & = m_H \\
\mu & = m_H/2 \\
\mu & = m_H \\
\mu & = m_H/4 \\
\text{combined incl. unc.}
\end{align*}
\]
Higgs production with a jet veto

NLL Banfi,Salam,Zanderighi 2012, NNLL Banfi,Monni,Salam,Zanderighi 2012

- Automated NLL resummation (CESAR)
- Continued to NNLL+NNLO using $q_T$ resummation
- Hadronization and UE corrections found to be small (<1%)
Higgs production with a jet veto

Becher, Neubert 2012

- First all-order factorization theorem for Higgs production with a jet veto
- Resummation now being performed at $N^3\text{LL}$ Becher, Neubert, Rothen

\[
pp \to H + X, \sqrt{s} = 8 \text{ TeV} \\
\text{MSTW2008NNLO} \\
m_H = 125 \text{ GeV} \\
R = 0.4
\]
Higgs production with a jet veto

Tackmann, Walsh, Zuberi 2013

- Large fixed-order uncertainty
  \[ \Delta_{\text{incl}}^2 + \Delta_{\text{\geq 1}}^2 \]
  Stewart, Tackmann 2011
  reduced by SCET NNLL’+NNLO

- Full NNLO calculation of soft function
  for \( H_T \) veto + clustering corrections
  Tackmann, Walsh, Zuberi 2012

\[ \sigma_0(p_{\text{cut}}^T) \] [pb]

\[ m_H = 125 \text{ GeV} \]

\[ gg \rightarrow H \] (8 TeV)

\[ R = 0.4 \]

\[ \text{NNLO} \]

\[ \text{NNLL}' \]

\[ p_{\text{T}} \] +NNLO
Higgs+jet production with a jet veto

Liu, Petriello 2013

- Leading jet with transverse momentum of $O(m_H)$ not uncommon
- Fixed-order uncertainty $\Delta^2 = \Delta^2_{\geq 1} + \Delta^2_{\geq 2}$ large at small $p_T, \text{veto}$ Stewart, Tackmann 2011
- Significant reduction by NLL’ SCET resummation matched to NLO
Parton shower event generators

- PS provides resummation to (N)LL accuracy and realistic final states
- Matching allows for NLO precision in all aspects of experimental analysis

New concepts
- Sector showers
  Larkoski, Peskin
- Antenna showers
  Giele, Gehrmann-DeRidder, Hartgring, Kosower, Laenen, Lopez-Villarejo, Ritzmann, Skands

Extension of older methods
- Dipole showers
  Gieseke, Plätzer
- Full color showers
  Höche, Krauss, Plätzer, Schönherr, Siegert, Sjödahl
Matching NLO calculations and parton showers

- Fixed-order corrections improve high-$p_T$ region
- Parton-shower resums logarithmic corrections at small $p_T$
- Generate particle-level events from NLO calculations
Automated NLO+PS Matching

- Methods: MC@NLO Frixione,Webber 2002 and POWHEG Nason 2004
- Public frameworks: POWHEG Box Alioli,Nason,Oleari,Re 2010 and Sherpa SH,Krauss,Schönherr,Siegert 2012
- aMC@NLO → full automation using MadLoop/MadDipole/MadFKS Frederix, Frixione,Hirschi,Maltoni,Pittau,Torrielli 2011
- Most challenging processes so far:
  \(W + 3\text{jets},\ Z + 2\text{jets},\ t\bar{t} + 1\text{jet},\ t\bar{t} + h/W/Z\)
  \(pp \rightarrow 2\text{jets}\)
Multi-scale improved NLO (MINLO)

- Interpret NLO event in terms of QCD branchings, much like a parton-shower
- Assign transverse momentum scales $q$ to splittings, evaluate $\alpha_s$ at these scales
- Multiply with Sudakov factors, but subtract first-order expansion (already included in NLO calculation)
NNLO+PS matching

- Supplement MINLO with known NLL coefficients in Sudakovs
- Can perform NLO calculation for $h+\text{jet}$ in region where $p_{Tj} \to 0$
- Reweight with NNLO prediction → NNLO+PS matched result
Multi-jet merging at next-to-leading order
Evolution of matching and merging methods

- Color coherent evolution: Marchesini, Webber
- Initial-state shower: Sjöstrand
- ME correction: Bengtsson, Sjöstrand
- ME correction: Seymour
- ME re-clustering: André, Sjöstrand
- MLM merging: Mangano, Moretti, Pitala, Puhlhofer
- CKKW merging: Catani, Krauss, Kuhn, Webber
- POWhEG: Nason
- CKKW-L@NLO (NL^3SP): Lavesson, Lönnblad
- MENLOPS: Hamilton, Nason
- MEPS@NLO: Gehrmann, SH, Krauss, Schönher, Siegert
- MEPS: Frixione, Frederix, Löffler, Schönher, Siegert
- NNLOPS: Hamilton, Nason, Re, Zanderighi
Multi-jet merging at next-to-leading order

- Three different methods, implemented in Pythia, Sherpa and aMC@NLO
  - Lavesson, Lönnblad 2008
  - Lönnblad, Prestel 2012,
- Allows inclusive particle-level predictions with uncertainty estimates

\[
s(W + \sigma_n^{10}) \geq (W + \text{jets}) \rightarrow W + \nu + \text{jets} \quad \text{[pb]}
\]

\[
\int \text{Ldt}=36 \text{ pb}^{-1}
\]

\[
\text{anti-k}_\perp, R=0.4
\]

\[
p_{\text{jet}} > 20 \text{ GeV}, |y_{\text{jet}}| < 4.4
\]

ATLAS 2012

Inclusive Jet Multiplicity, \(N_{\text{jet}}\)

SH, Krauss, Schönherr, Siegert 2012
Multi-jet merging at next-to-leading order

- Merging of different NLO processes introduces higher-order corrections
- Typically changes overall cross section → “Unitarity violation”
- Can be avoided using explicit subtraction of excess → UNLOPS

![Graph showing multi-jet merging](image-url)
Higgs backgrounds in jet bins with ME+PS@NLO

Cascioli, SH, Krauss, Maierhöfer, Pozzorini, Siegent 2013

- $p p \rightarrow 4\ell + 0,1$jet at NLO with OpenLoops Cascioli, Maierhöfer, Pozzorini 2012
  - Including squared quark-loop contributions up to one extra jet
- Matched to Sherpa PS and merged (first merging of loop$^2$ contribution)
- Sensible perturbative uncertainties in jet bins due to PS resummation
Top quark forward-backward asymmetry

\[ A_{FB}(p_T, t\bar{t}) \]

Reconstructed top

- Combined 0+1-jet NLO prediction with merging cut at 7GeV
- Large effect of color coherent emission in MC@NLO
- Large dependence on functional form of scale
- NLO-accurate prediction of \( A_{FB}(p_T) \) except for first bin

CDF data


Stefan Höche
Applications of Higher Order QCD
Jet ratio scaling patterns

- Consider cross section ratios in $X + n$ jets

\[ R_{n+1}/n = \frac{\sigma_{n+1}^{\text{excl}}}{\sigma_n^{\text{excl}}} \]

\[ \sim \text{stable against QCD corrections} \]

Gerwick, Plehn, Schumann, Schichtel 2012
Can be computed using NLL jet rates Gerwick, Schumann, Gripaios, Webber 2012
Helpful to determine many-jet backgrounds in searches

- **Staircase Scaling:**

\[ R_{n+1}/n = \text{const} \left( \sigma_n = \sigma_0 \, R^n \right) \]

- First predicted for $W/Z + \text{jets}$
  Berends, Giele, Kuijf 1989
- Induced by democratic jet cuts

- **Poisson Scaling:**

\[ R_{n+1}/n = \frac{\bar{n}}{n + 1} \left( \sigma_n = \frac{\bar{n}^n e^{-\bar{n}}}{n!} \right) \]

- Independent emission picture
  (like soft $\gamma$ radiation in QED)
- Driven by large emission probability
- Induced by presence of hard jet
Testing scaling with NLO calculations

Bern,Dixon,Febres Cordero,SH,Ita,Kosower,Maître,Ozeren 2013

- \( W + \) jets at 7 TeV, \( E_T^e > 20 \text{ GeV} \), \( |\eta^e| < 2.5 \), \( E_T > 20 \text{ GeV} \)
  \( p_T^j > 25 \text{ GeV} \), \( |\eta^j| < 3 \), \( M_T^W > 20 \text{ GeV} \)

<table>
<thead>
<tr>
<th>Jets</th>
<th>( \frac{W^- + (n+1)}{W^- + n} ) LO</th>
<th>NLO</th>
<th>( \frac{W^+ + (n+1)}{W^+ + n} ) LO</th>
<th>NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2949(0.0003)</td>
<td>0.238(0.001)</td>
<td>0.3119(0.0005)</td>
<td>0.242(0.002)</td>
</tr>
<tr>
<td>2</td>
<td>0.2511(0.0005)</td>
<td>0.220(0.001)</td>
<td>0.2671(0.0004)</td>
<td>0.235(0.002)</td>
</tr>
<tr>
<td>3</td>
<td>0.2345(0.0008)</td>
<td>0.211(0.003)</td>
<td>0.2490(0.0005)</td>
<td>0.225(0.003)</td>
</tr>
<tr>
<td>4</td>
<td>0.218(0.001)</td>
<td>0.200(0.006)</td>
<td>0.2319(0.0008)</td>
<td>0.218(0.006)</td>
</tr>
</tbody>
</table>

- Fit to straight line for \( W + n \) jets gives \( n \geq 2 \)
  \( R_{n/(n-1)}^{\text{NLO}, W^-} = 0.248 \pm 0.008 - (0.009 \pm 0.002) n \)
  \( R_{n/(n-1)}^{\text{NLO}, W^+} = 0.263 \pm 0.009 - (0.009 \pm 0.003) n \)

- Extrapolation to six jets
  - \( W^- + 6 \) jets : \( 0.15 \pm 0.01 \text{ pb} \)
  - \( W^+ + 6 \) jets : \( 0.30 \pm 0.03 \text{ pb} \)
Summary

- QCD NLO calculations fully automated
  Limited only by final-state multiplicity
- NLO precision for multiple jets in event generators
  Meaningful uncertainty bands for the first time
- NNLO is the new frontier, with lots of progress
  \((pp \rightarrow t\bar{t}, pp \rightarrow \text{jets}, pp \rightarrow H+\text{jet} \text{ at parton level})\)
- NNLO+NNLL resummation for exclusive observables
  \((pp \rightarrow H+0\text{jets}, \text{also } pp \rightarrow H+1\text{jets} \text{ at NLO+NLL})\)
- NNLO+PS matching on the horizon