Review of crosstalk between beam-beam interaction and lattice nonlinearity in e+e- colliders

ZHANG Yuan(IHEP), ZHOU Demin(KEK)
Outline

- DAFNE
- DAFNE upgrade
- KEKB
- Super-KEKB
- BEPCII
• DAFNE
DAFNE: Cubic lattice nonlinearity
Only one IP

\[ |C_{11}| < 200 \]

Figure 3. Beam-beam blow up and tail growth as a function of the cubic lattice nonlinearity (numerical simulations). Equilibrium density contour plots in the space of normalised betatron amplitudes are shown.

\[ \Delta v_x = 2c_{11} J_x \]

M. Zobov, DAFNE Technical Note G-57, 2001
DAFNE: Cubic lattice nonlinearity
One IP + 2 nearest PC

\[ c_{11} = -350 \]

Figure 4. Equilibrium density contour plots taking into account 2 Parasitic Crossings and lattice nonlinearities.

M. Zobov, DAFNE Technical Note G-57, 2001
• DAFNE-Upgrade
Crab Waist in 3 Steps

1. Large Piwinski’s angle $\Phi = \tan(\theta/2)\sigma_z/\sigma_x$

2. Vertical beta comparable with overlap area $\beta_y \approx 2\sigma_x/\theta$

3. Crab waist transformation $y = xy'/\theta$
1. Large Piwinski’s angle

\[ \Phi = \tan\left(\frac{\theta}{2}\right) \frac{\sigma_z}{\sigma_x} \]

2. Vertical beta comparable with overlap area

\[ \beta_y \approx 2\sigma_x/\theta \]

3. Crabbed waist transformation

\[ y = xy'/\theta \]

M. Zobov, C. Milardi, BB’2013

- **Crabbed Waist Advantages**
  - a) Luminosity gain with N
  - b) Very low horizontal tune shift
  - c) Vertical tune shift decreases with oscillation amplitude

- a) Geometric luminosity gain
  - b) Lower vertical tune shift
  - c) Suppression of vertical synchro-betatron resonances

- a) Geometric luminosity gain
  - b) Suppression of X-Y betatron and synchro-betatron resonances
X-Y Resonance Suppression

**Typical case (KEKB, DAΦNE etc.):**
1. low Piwinski angle $\Phi < 1$
2. $\beta_y$ comparable with $\sigma_z$

**Crab Waist On:**
1. large Piwinski angle $\Phi >> 1$
2. $\beta_y$ comparable with $\sigma_x/\theta$

*M.Zobov, C.Milardi, BB’2013*
Frequency Map Analysis of Beam-Beam Interaction

D. Shatilov, E. Levichev, E. Simonov and M. Zobov
DAΦNE Peak Luminosity

- KLOE
- DEAR
- FINUDA
- SIDDHARTA

Design Goal

NEW COLLISION SCHEME

M.Zobov, C.Milardi, BB’2013
Crabbed Waist Scheme

Sextupole

\[ \beta_x, \beta_y \]

\[ \Delta \mu_x = \pi \]

\[ \Delta \mu_y = \frac{\pi}{2} \]

Sextupole strength

\[ K = \frac{1}{2\theta} \frac{1}{\beta_y \beta_y} \sqrt{\frac{\beta_x^*}{\beta_x}} \]

\[ \beta_y = \beta_y^* + \frac{(s - x/\theta)^2}{\beta_y^*} \]

IP

\[ \beta_x^*, \beta_y^* \]

\[ \Delta \mu_x = \pi \]

\[ \Delta \mu_y = \frac{\pi}{2} \]

(Anti)sextupole

\[ \beta_x, \beta_y \]

Sextupole strength

Equivalent Hamiltonian

\[ H = H_0 + \frac{1}{2\theta} x p_y^2 \]

M.Zobov, C.Milardi, BB’2013
Logarithm of the bunch density at IP (z=0).
The scales are ±10 sigma for X and Y.
Normal form analysis of crabed-wasit transformtation

• One-turn map with beam-beam

\[
\exp(-axp_y^2) \exp(H_{bb}) \exp(axp_y^2) \exp(H_{arc})
\]

• One-turn map without beam-beam at IP

\[
\exp(axp_y^2) \exp(H_{arc}) \exp(-axp_y^2) \\
\exp(f_2(X)) \exp(f_3(X)) \exp(f_4(X))
\]

\[
f_3 = \exp(-f_2(X)) : bxp_y^2 - bxp_y^2
\]

\[
= b(\cos \mu_xx - \sin \mu_xp_x)(\sin \mu_yy + \cos \mu_yy^2) - bxp_y^2
\]

\[
f_4 = -\frac{1}{2} : \exp(-f_2(X)) : bxp_y^2 : bxp_y^2
\]

There only exist 3rd order generating function:

\[
F_3 = (-235.7 + 3.200564813177209 \times 10^{-15}i)e^{-i\phi_x} \sqrt{A_xA_y} \\
- (235.7 + 3.200564813177209 \times 10^{-15}i)e^{i\phi_x} \sqrt{A_xA_y} \\
+ (117.85 + 4.947431353485854 \times 10^{-15}i)e^{-2i\phi_y} \sqrt{A_xA_y} \\
+ (117.85 + 2.779027008514845 \times 10^{-15}i)e^{2i\phi_y} \sqrt{A_xA_y} \\
+ (117.85 - 2.779027008514845 \times 10^{-15}i)e^{-i\phi_x+2i\phi_y} \sqrt{A_xA_y} \\
+ (117.85 - 4.947431353485854 \times 10^{-15}i)e^{i\phi_x+2i\phi_y} \sqrt{A_xA_y}
\]

以工作点 \(Q_x/Q_y = 0.51/0.58, \beta_x/\beta_y = 1.0/0.015, a = 10\) 为例，
• KEKB
Motivation of crab cavity at KEKB

First proposed by R. B. Palmer in 1988 for linear colliders.

- Crab Crossing can boost the beam-beam parameter higher than 0.15! (K. Ohmi)

\[ \nu_x = 0.508 \]

\[ \xi_y \sim 0.15 \]

Luminosity would be doubled with crab cavities!!!

- After this simulation appeared, the development of crab cavities was revitalized.
Single Crab Cavity Scheme

Beam tilts all around the ring.

**z-dependent horizontal closed orbit.**

**tilt at the IP:**

\[
\frac{\theta_x}{2} = \frac{\sqrt{\beta_x^C \beta_x^*} \cos(\psi_x^C - \mu_x/2)}{2 \sin(\mu_x/2)} \frac{V_C \omega_{rf}}{E_c}
\]

Table 1: Typical parameters for the crab crossing.

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<th>Ring</th>
<th>LER</th>
<th>HER</th>
<th>mrad</th>
<th>cm</th>
<th>m</th>
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<td>80</td>
<td>0.505</td>
<td>0.511</td>
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<tr>
<td>(\beta_x^*)</td>
<td>73</td>
<td>162</td>
<td></td>
<td></td>
<td></td>
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<td>(\beta_x^C)</td>
<td>0.25</td>
<td>0.25</td>
<td>0.95</td>
<td>1.45</td>
<td></td>
</tr>
<tr>
<td>(\omega_{rf}/2\pi)</td>
<td>509</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* 1 crab cavity per ring.

* saves the cost of the cavity and cryogenics.

* avoids synchrotron radiation hitting the cavity.
Luminosity of KEKB
Oct. 1999 - June 2010

Y. Funakoshi, Beam-Beam Workshop, CERN, 2013

Skew-sextupoles

Beam lifetime problem
General Chromaticity

The chromaticities of Twiss parameters and X-Y couplings

The $\delta$-dependent transverse matrix can be split into the product of two matrices. All the chromatic dependences are lumped into $M_H(\delta)$

Generating function $F_2$ is used to represent the transformation of $M_H(\delta)$. The generating function guarantees the 6D symplectic condition. Hamiltonian which expresses generalized chromaticity is given by

Alternative way is the direct map for the betatron variables $x = (x, p_x, y, p_y)^T$ and $z$ as

\[ x(s + L) = M_4(\delta)x(s). \]
\[ z(s + L) = z(s) + x^tM_4^t(\delta)S_4\delta_8M_4(\delta)x/2. \]
Measurement of chromatic coupling

**FIG. 3.** (Color) Measured chromatic $X$-$Y$ coupling at IP in HER. The blue plots indicate those before and the red plots indicate those after the skew sextupole correction. The dashed line indicates the natural chromatic $X$-$Y$ coupling estimated using the model lattice by SAD.

**FIG. 4.** (Color) Measured chromatic $X$-$Y$ coupling at IP in LER. The blue plots indicate those before and the red plots indicate those after the skew sextupole correction. The dashed line indicates the natural chromatic $X$-$Y$ coupling estimated using the model lattice by SAD.

Scan of first-order chromatic coupling (WS, Crab on) D. Zhou, et al., PRST--AB 13, 021001 (2010).

\[
\left( \begin{array}{cc}
\frac{r_{1N}}{r_{3N}} & \frac{r_{2N}}{r_{4N}} \\
\frac{r_{3N}^*}{r_{1N}^*} & \frac{r_{4N}^*}{r_{2N}^*}
\end{array} \right)
\]

G. 8. (Color) Scan of first-order chromaticity of X-Y couplings at the IP.
Ohmi et al. showed that the linear chromaticity of x-y coupling parameters at IP could degrade the luminosity, if the residual values, which depend on machine errors, are large.

To control the chromaticity, skew sextupole magnets were installed during winter shutdown 2009.

The skew sextuples are very effective to increase the luminosity at KEKB.

The gain of the luminosity by these magnets is ~15%.
Experimental observations

The first scans of chromatic coupling at IP during the KEKB operation:

- Lifetime
- Beam size
- Luminosity

D. Zhou, 2011
Experimental observations (cont’d)

Skew-sextupole tuning was very effective w/ crab on ...
Experimental observations (cont’d)

Specific lum. w/ crab on and w/ skew-sext. tuning optimized
Lum. gain from crab cavities: ~20%, Lum. gain from skew-sext.: ~15%

constant beam-beam parameter: $\xi_y (\text{HER}) = 0.09 \left( \frac{I_{\text{LER}}}{I_{\text{HER}}} = 8/5 \right)$
Experimental observations (cont’d)

Skew-sextupole tuning was very effective even w/ crab off. This was a surprise...

Red: w/ skew-sextuples
Blue: w/o skew-sextuples

99 bunches + pilot bunch /beam
June 25 ~28, 2010

$\beta_x^* = 1.2 \text{m}$
• Super-KEKB
Weak-strong Simulation for LER lattice

➤ Even low current, luminosity loss ~20% is seen.
➤ 30% loss at the design current.
➤ Chromatic effect can not explain the lum. Loss.

K. Ohmi, D. Zhou
SuperKEKB MAC2014, Mar 3-4, 2014
LER: Simplified IR

- Simplified lattice by H. Sugimoto
- Sler_simple001.sad: no solenoid but preserve main optics parameters
- No significant luminosity degradation at low current
- Solenoid is the main source of lattice nonlinearity?

D. Zhou and Y. Zhang (IHEP), SuperKEKB optics meeting, Apr. 17, 2014
Lattice nonlinearity from turn-by-turn data

- Initial coordinates \((x_0, 0, 0, 0, 0, 0)\);
- \(x_0\) changes from 0 to \(5\sigma_x\);
- Watch point is at IP, beam-beam is off

sher-5767 vs ler-1689 in X direction
Lattice nonlinearity from turn-by-turn data (Cont.)

- Evidence of nonlinear X-Y coupling
- COD in Y direction as function of X offset

`scher-5767 vs ller-1689` in Y direction
Frequency Analysis

• Linear Normalized Coordinate

\[ \hat{x} = \frac{x}{\sqrt{\beta_x}}, \quad \hat{p}_x = p_x \ast \sqrt{\beta_x} \]

\[ \hat{y} = \frac{y}{\sqrt{\beta_y}}, \quad \hat{p}_y = p_y \ast \sqrt{\beta_y}, \]

• Turn-by-Turn data could be represented by (with first order approximation)

\[ \hat{x}(m) - i\hat{p}_x(m) = \sqrt{2A_x}e^{i(m\mu_x + \phi_x, 0)} \]

\[ - \sum_{abcd} 2iaf_{abcd}^{(3)}(2A_x)^{a+b-1} (2A_y)^{c+d} e^{i(b-a+1)(m\mu_x + \phi_x, 0)} e^{i(d-c)(m\mu_y + \phi_y, 0)} \]

\[ \hat{y}(m) - i\hat{p}_y(m) = \sqrt{2A_y}e^{i(m\mu_y + \phi_y, 0)} \]

\[ - \sum_{abcd} 2icf_{abcd}^{(3)}(2A_x)^{a+b} (2A_y)^{c+d-1} e^{i(b-a)(m\mu_x + \phi_x, 0)} e^{i(d-c+1)(m\mu_y + \phi_y, 0)} \]

• FFT with

\[ \hat{x} - i\hat{p}_x \text{ in x direction} \]

\[ \hat{y} - i\hat{p}_y \text{ in y direction} \]
Frequency Analysis (cont.)

Spectrum ($x_0 = 3\sigma_x$)

Power spectrum analysis (LER)

Why?
• There exist very strong ‘oscillation’ at 0, 2Qx, -2Qx for LER
• It is suspected the cause is
  \( f_{1110} \rightarrow 0 \) in vertical direction, the amplitude is proportional to \((2A_x)\)
  \( f_{0210} \rightarrow 2Qx \) in vertical direction, the amplitude is proportional to \((2A_x)\)
  \( f_{2010} \rightarrow -2Qx \) in vertical direction, the amplitude is proportional to \((2A_x)\)
All these terms may come from a skew sextupole like magnet.
\( H \sim 3x^2y - y^3 \)
Compensation with a skew-sext map

• Test by inserting a map of $H=K^*x^2y$ into the LER lattice
• COD and oscillation amplitude in $y$ are well suppressed as expected
Compensation with a skew-sext map (Cont.)

➢ Skew-sext. map:
  • to cancel the nonlinear term from solenoid
  • work well at both low and high currents
  • interplay of SC and lattice nonlin. also mitigated partially
Compensation with a skew-sext map (Cont.)

**Skew-sext. map:**
- cause loss in DA and lifetime (to be understood)

*From H. Sugimoto*
• BEPCII
2014 上半年，开始投入使用 1W1。Q 铁强度重新匹配，但六极铁保持不变。经过连续几个班的优化，亮度仍然偏低（3.4e32@430mA）

检查色品；优化六极铁，水平色品由 0.1 增加到 0.8

经过几个 run 的优化，亮度迅速提高至 3.8e32，亮度提升超 10%。
实际调束，新的六极铁搭配（1w12-offline）亮度高，但模拟结果反而低。不稳定性的贡献？

模拟：真实 lattice

模拟：只考虑色品
新的亮度在线调节方法 - chromaticity knob

第一次投入使用 @2014-05-09n，亮度提升超 10%！

之前：最高亮度 4.6e32@450mA*450mA

之后：最高亮度 5.2e32@450mA*450mA

模拟：真实 lattice

模拟：只考虑色品
亮度的高低主要来自水平方向的尺寸差别，即水平方向的色品导致了水平和纵向的共振。

模拟：真实 lattice

模拟：只考虑色品
Fringe effect in BEPCII （using SAD）

\[ K(s) = K_0(s) + k(s) \]

二极铁
原始定义，例子:
R4IMB02 = (L= 1.414956 ANGLE= 0.15468506 E1= 0.5 E2= 0.5 K1= 0. F1=0.177 FRINGE=1 )

修改为：
R4IMB02 = (L= 1.414956 ANGLE= 0.15468506 E1= 0.5 E2= 0.5 K1= 0. F1=0.177 FRINGE=1 )

四极铁
原始定义，例子:
R3IQ02 = (L= 0.5480000000000004 K1= -6.1350440677200004E-002 )
修改为:
R3IQ02 = (L= 0.5480000000000004 K1= -6.1350440677200004E-002 F1 = .133 FRINGE =3 )

超导四极铁
原始定义，例子:
SSCQ011 = (L= 0.025 K1=-0.000370337)
修改为:
SSCQ011 = (L= 0.025 K1=-0.000370337 F1 = .025 FRINGE =3)

螺线管场
原始定义:
ESOL001 = (BZ= 0)
改为:
ESOL001 = (BZ= 0 F1=0.02)

D. Zhou(KEK), 2014
### Linear optics parameters

使用 SAD(Ver.1.0.10.5.7a3) 计算 BPR(Ver. bpr7-sol.sad) 的线性 optics 参数 (@IP):

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<th>Fringe type</th>
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<th>B</th>
<th>Q</th>
<th>SCQ</th>
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<td>1.151</td>
<td>1.144</td>
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原始模型，+边缘场，+LOCO校正

D. Zhou(KEK), 2014
原始模型，+边缘场，+LOCO校正（cont.）
亮度：原始模型 vs 边缘场+LOCO校正
loss~15%
Beam tail

➤ bpr7-sol:

Np=0.01E10  Np=1E10  Np=2E10

Np=3E10  Np=4E10  Np=5E10
Beam tail

\[ bpr7-sol-AllFringeRMsol: \]

- \( N_p = 0.01 \times 10 \)
- \( N_p = 1 \times 10 \)
- \( N_p = 2 \times 10 \)
- \( N_p = 3 \times 10 \)
- \( N_p = 4 \times 10 \)
- \( N_p = 5 \times 10 \)
Summary

所有的非线性都已经在“实际”机器中被发现对亮度产生影响：

• Detuning
• Choromaticity （tune/twiss parameters/coupling）
• noraml/skew multipole magnet