Theoretical review of the muon g-2

International Workshop on $e^+e^-$ collisions from $\phi$ to $\psi$
Hefei
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with great help from Robert Szafron
Outline

Contributions to the muon g-2:
QED
Weak
Hadronic
  Vacuum polarization
  Light-by-light scattering

Other effects?
Relation to other observables
The puzzle of the muon magnetic moment

The 3.6 sigma discrepancy persists,

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 287(80) \times 10^{-11}$$

This is rather large when compared with other bounds on New Physics:

Muon MDM

$$d_\mu \sim \frac{e}{2m_\mu} a_\mu^{\text{NP}} \sim 3 \times 10^{-22} \text{ e \cdot cm}$$

Muon-electron transition moment

$$|d_{\mu\rightarrow e}| < 4 \times 10^{-27} \text{ e \cdot cm}$$

Electron EDM

$$|d_e| < 8.7 \times 10^{-29} \text{ e \cdot cm}$$

Will be probed by Mu2e and COMET (at this meeting: Hai-Bo Li)
How can $g_\mu -2$ be checked?

New experiments at Fermilab

talk by SeungCheon Kim
Also at this meeting:
B. Lee Roberts

and J-PARC (new concept!)

talk by Boris Shwartz

Can we use $g_e -2$?
QED contribution

One-loop: universal for all leptons

Two- and three-loop: known analytically

Four- and five-loop: about 13k diagrams;


Recent: partial analytical four-loop results!

   Kurz, Liu, Marquard, Smirnov, Smirnov, Steinhauser, arXiv:1508.00901
QED contributions

<table>
<thead>
<tr>
<th>number of loops</th>
<th>( C \left( \frac{\alpha}{\pi} \right)^n )</th>
<th>Contribution x ( 10^{11} )</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>Schwinger, 1948</td>
<td>116 140 973.318(77)</td>
</tr>
<tr>
<td>2</td>
<td>Petermann, 1957</td>
<td>413 217.629(09)</td>
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<tr>
<td></td>
<td>Sommerfield, 1958</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>S. Laporta</td>
<td>30 141.9024(4)</td>
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<tr>
<td></td>
<td>E. Remiddi, 1996</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Kinoshita, 1999</td>
<td>381.008(19)</td>
</tr>
<tr>
<td></td>
<td>Kinoshita, Nio 2004</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Ayoama et al. 2012</td>
<td>5.094(07)</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>116 584 718.951(80)</td>
</tr>
</tbody>
</table>

Enhancement specific to the muon g-2, absent for the electron g-2:
Weak contributions to the muon $g-2$

One loop:

- $a_{\mu}^{\text{EW}}(1\text{-loop}) \approx \frac{10 G_F}{3} \frac{m_{\mu}^2}{\sqrt{2}} = 195 \times 10^{-11}$
- No $\ln \frac{M_{W,Z}^2}{m_{\mu}^2}$
- Negligible higgs effect

Two loops:

- Large logs $\text{Kukhto, Kuraev, Schiller, Silagadze}$
- Relative to one-loop: $\frac{\alpha}{\pi} \ln \frac{M_{W,Z}^2}{m_{\mu}^2} \approx 3\%$
- Large coefficient $\approx 7$

$$a_{\mu}^{\text{EW}}(2\text{-loop}) \approx a_{\mu}^{\text{EW}}(1\text{-loop}) \left[ 1 + \left( C_{\text{ferm}} + C_{\text{bos}} \right) \frac{\alpha}{\pi} \right]$$

- Higgs boson important!

After LHC discovery, $m_H$ used by
Gnendinger, Stockinger, Stockinger-Kim
PRD 88 (2013) 053005
Large, universal two-loop electroweak correction

A record QED correction!
Also affects mu -> e gamma

Hadronic vacuum polarization

\[ a_{\mu}^{\text{had}, \ LO} = \left( \frac{\alpha m_{\mu}}{3\pi} \right)^2 \int_{4m_{\pi}^2}^{\infty} ds \frac{R(s)K(s)}{s^2} \]

\[ R(s) = \frac{\sigma_{\text{tot}}(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^-)} \]

**Contributions**

**Errors**

**Great importance of the \( \varphi \) to \( \psi \) region!**

SND: Tatyana Dimova, Konstantin Beloborodov
CMD-3: Gennady Fedotovich
KEDR: Korneliy Todyshev
BaBar: Evgeny Kozyrev
KLOE(2): Veronica De Leo
BESIII: Haiming Ho, Yaqian Wang
Discrepancy of the prediction and the result

\[ a_{\mu}^{\text{had};LO} = (6949 \pm 43) \times 10^{-11} \]

Hagiwara, Liao, Martin, Nomura, Teubner 2011

from Davier
Reconciling $e^+e^-$ with $\tau$: isospin breaking

In order to correctly use tau data, isospin-breaking effects must be included: especially gamma-rho mixing

Jegerlehner & Szafron 2011
Reconciling $e^+e^-$ with $\tau$: isospin breaking

In order to correctly use tau data, isospin-breaking effects must be included: especially gamma-rho mixing

Jegerlehner & Szafron 2011

Note the KLOE-BABAR discrepancy; can BESIII resolve it?
rho-gamma mixing

Interference in the neutral channel; suppressed by $M_W$ in charged;

Tau data must be corrected for the mixing effect.
Other approaches to the vacuum polarization

Effective lagrangians & global fits
Benayoun, David, DelBuono, Jegerlehner

Future possibility: lattice
First results: ETM Collaboration '13
Uncertainties don't seem to be under full control yet

Extraction from Bhabha?
talk by Carlo Carloni Calame

Hadronic light-by-light

For now, cannot be computed from first principles or connected to measurements.

<table>
<thead>
<tr>
<th>Model</th>
<th>HLbL contribution x 10^{10}</th>
</tr>
</thead>
<tbody>
<tr>
<td>VMD (Hayakawa)</td>
<td>8.96(1.54)</td>
</tr>
<tr>
<td>ENJL (Bijnens)</td>
<td>8.3(3.2)</td>
</tr>
<tr>
<td>LMD+V (Knecht)</td>
<td>8.0(4.0)</td>
</tr>
<tr>
<td>Q-box (Pivovarov)</td>
<td>14.05</td>
</tr>
<tr>
<td>LENJL (Bartos)</td>
<td>10.77(1.68)</td>
</tr>
<tr>
<td>(LMD+V)’ (Melnikov)</td>
<td>13.6(0.25)</td>
</tr>
<tr>
<td>oLMDV (Nyffeler)</td>
<td>11.6(0.4)</td>
</tr>
<tr>
<td>DS (Goecke)</td>
<td>18.8(0.4)</td>
</tr>
<tr>
<td>NxQM (Dorokhov)</td>
<td>16.8(1.25)</td>
</tr>
<tr>
<td>CxQM (Greynat)</td>
<td>15.0(0.3)</td>
</tr>
</tbody>
</table>

New dispersive approach, model-independent!
Colangelo, Hoferichter, Procura and Stoffer 2014

see talks by Alexey Zhevlakov and Rafael Escribano

experimental input: talks by Paolo Gauuzzi, Qingnian Xu, Christoph Redmer
How do we actually determine $g-2$?

Measure

$\omega_a = \frac{g - 2}{2} \frac{e}{m_\mu} B$

$B$ from NMR

$\omega_p = \frac{2\mu_p B}{\hbar}$

$e/m_\mu$ from

$\mu_\mu = g \frac{e\hbar}{4m_\mu}$

Master formula

$\frac{g - 2}{2} = \frac{\omega_\mu/\omega_p}{\mu_\mu/\mu_p - \omega_\mu/\omega_p}$

Measured by E821

From muonium
Muonium spectrum determines $\mu_\mu/\mu_p$

Measured to relative 120ppb (like $15 \cdot 10^{-11}$ in $a_\mu$)
Will need improvement for the new $g$-2 results
How is the muon magnetic moment extracted?

\[ H = -(\vec{\mu}_e + \vec{\mu}_\mu) \cdot \vec{B} + c\vec{I} \cdot \vec{J} \]

Caveat: the magnetic moment in muonium differs from that of a free muon (slightly). Theory input needed!
Binding corrections to $g$-2

\[ g = 2 - \frac{2(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + \ldots \]

\[ + \frac{\alpha}{\pi} \left[ 1 + \frac{(Z\alpha)^2}{6} + (Z\alpha)^4 (a_{41} \ln Z\alpha + a_{40}) + \ldots \right] \]

\[ + \left( \frac{\alpha}{\pi} \right)^2 \left[ -0.65\ldots \left( 1 + \frac{(Z\alpha)^2}{6} \right) + (Z\alpha)^4 (b_{41} \ln Z\alpha + b_{40}) + \ldots \right] \]

**two-loop corrections**

\[ b_{41} = \frac{28}{9} \]

\[ b_{40} = -16.4 \]

Pachucki, AC
Jentschura, Yerokhin
Binding corrections to $g-2$

$$g = 2 - \frac{2(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + \ldots$$

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**two-loop corrections**

$$b_{41} = \frac{28}{9}$$

$$b_{40} = -16.4$$

This has another application: to the determination of the electron mass and, indirectly, of alpha.
How to use $g_e^{-2}$ to check $g_\mu^{-2}$?

The second best determination of alpha: from atomic spectroscopy

$$R_\infty = \frac{m_e c \alpha^2}{2\hbar}$$

Needed precision:

$$14 \cdot 10^{-11}$$

$7 \cdot 10^{-12}$

New Nature 2014 Sturm et al

$8 \cdot 10^{-11}$

for Rb (better for He)

$12 \cdot 10^{-11}$

improvement needed by factor $\sim 10$

$124 \cdot 10^{-11}$

$\alpha(Rb) = 1/137.035999049(90) \ [66 \cdot 10^{-11}]$

PRL 106, 080801 (2011)

Also at this meeting: Massimo Passera
The gyromagnetic ratios of an electron bound in muonium, \( g_J \), and of a muon in muonium, \( g'_\mu \), differ from the free values, \( g_e \) and \( g_\mu \), by binding corrections [2]

\[
g_J = g_e \left(1 - \frac{\alpha^2}{3} + \frac{\alpha^2}{2} m_e \mu + \frac{\alpha^3}{4\pi}\right),
\]

\[
g'_\mu = g_\mu \left(1 - \frac{\alpha^2}{3} + \frac{\alpha^2}{2} m_e \mu \right).
\]
Conclusions

QED part well-known. Anticipating analytical results: will improve confidence (even more important for the electron $g-2$ and alpha)

Hadronic VP: New data needed to reduce errors

Tau data now agree with $e^+e^-$

Lattice: part of the future solution?

LbL poorly known; may soon dominate the error; model-independent evaluations needed.

The “other part” of $g-2$ should be scrutinized.