Shallow S-wave pion-baryon resonances

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\( \Lambda_{c}^{+}(2595) \)

\[
I(J^{P}) = 0(\frac{1}{2}^{-}) \quad I(J^{P}) = 1(\frac{1}{2}^{+})
\]

\( \Lambda_{c}^{+}(2595) \rightarrow \pi \Sigma_{c}(2455) \)

\( \Lambda_{c}^{+}(2595) \) as an S-wave resonance in \( \pi \Sigma_{c} \) channel

- Width \( \sim 2\, \text{MeV} \) — narrow
- \( \leq 4 \, \text{MeV} \) above threshold — extremely shallow

Other channels
1. can be integrated out, for their threshold being much further away;
2. contribute at subleading orders (quantum-numbers)
S-wave resonances
— from the perspective of potential models

- Motion of poles when potential-strength tuned
- Large value of $a$ — close to threshold (shallow)
- Large value of $r$ — narrow resonance (as opposed to broad resonance)
- Two fine-tunings seem to be needed for a potential model to form a narrow, shallow S-wave resonance.

\[ f^{(0)} = \frac{1}{-\frac{1}{a} + \frac{r}{2}k^2 - ik} \]
Chiral Lagrangian for nuclear physics

Few GeV
~ 1 GeV
~ 100 MeV

Perturbative QCD
lattice QCD

M_{hi}
Nucleon mass \( m_N \)
Non-Goldstone mesons mass \( \sigma \) (?), \( \rho \) ...
Q: Generic external momentum

D.o.f.s
Nucleons
Pions

chiral EFT

Chiral effective field theory: A primer

\[ SU(2)_L \times SU(2)_R \]

\( \epsilon_{abc} \tau_a \pi_b \pi_c \)
\( \frac{g_A}{2f_\pi} \tau_a \vec{\sigma} \cdot \vec{\nabla} \pi_a \)

\[ \mathcal{L}_{int} = N^\dagger \left( -\frac{1}{4f_\pi^2} \epsilon_{abc} \tau_a \pi_b \pi_c - \frac{g_A}{2f_\pi} \tau_a \vec{\sigma} \cdot \vec{\nabla} \pi_a \right) \]

\[ N^\dagger C_S(N^\dagger N)^2 \]

\[ N^\dagger N \]

\[ C_T(N^\dagger \vec{\sigma} N)^2 \]

There are \( \infty \) operators

chiral Lagrangian

(Weinberg '69, '90)
Power counting for pion loops (HBChPT)

- Nucleon propagator — $1/Q$
- Pion propagator — $1/Q^2$
- Loop integral — $Q^4/(16\pi^2)$
- A vertex from $\mathcal{L}^{(\nu)}$ — $Q^\nu$

A pion loop brings a suppression factor of $\left( \frac{Q}{4\pi f_\pi} \right)^2$

- Cutoff independence assumed $\rightarrow$ counting free of $\Lambda$

Two-pion exchanges of nuclear forces
Generated by Weinberg-Tomozawa?

Weinberg-Tomozawa

\[
\frac{i}{f^2_\pi} \sum^{a\dagger} \left( \pi^a \dot{\pi}^b - \pi^b \dot{\pi}^a \right) \Sigma^b
\]

Non-linear realization of chiral symmetry → fixed coupling

❖ S-wave interaction of pi-Sigma system

❖ But a pion loop always suppressed by \( \left( \frac{Q}{4\pi f_\pi} \right)^2 \) (Weinberg, ’79)

❖ It’s unlikely that W.T. alone can generate a S-wave resonance

❖ “Subleading” \( \pi \pi \Sigma \Sigma \) highly enhanced — fine-tuning
What we can do with ChPT then?

- Q: With chiral symmetry, are double fine-tuning still needed as in potential models?
  A: No, a single fine-tuning is sufficient

- Q: mutil pions + $\Sigma_c$?
  A: likely to form more resonances
Explicit field of the excited baryon

\[ \Psi^\dagger (i\partial_0 - \Delta) \Psi + \frac{h}{\sqrt{3} f_\pi} (\Sigma^a \dot{\pi}^a \Psi + h.c.) \]

# of flavor of pions

- \( \Psi \) coupled to the S wave \( \rightarrow \) time derivative on \( \pi \), required by chiral symmetry (crucial)
- Nonrelativistic pion \( \rightarrow \) coupling \( \propto m_\pi \)
- Mass splitting \( \Delta \sim m_\pi \rightarrow \) near threshold
The pion-baryon propagator is counted as $\Sigma_c$.

$\begin{align*}
a &= \frac{\hbar^2 m^2_\pi}{4\pi f^2_\pi} \frac{1}{m_\pi - \Delta} \sim \left(\frac{140\text{MeV}}{328\text{MeV}}\right)^2 \frac{1}{4\text{MeV}} \\
r &= -\frac{4\pi f^2_\pi}{\hbar^2 m^3_\pi} \sim \left(\frac{328\text{MeV}}{140\text{MeV}}\right)^2 \frac{1}{140\text{MeV}}
\end{align*}$

$f^{(0)} = \frac{1}{-\frac{1}{a} + \frac{r}{2} k^2 - ik}$

$h = 0.65$

$r = -19 \text{ fm}$

$a = -10 \text{ fm}$

- $r$ can be quite large when $\Delta \ll \sqrt{4\pi f_\pi} = 328\text{MeV}$
- a single fine-tuning $\Delta - m_\pi \to 0$ makes both $a$ and $r$ large

$\rightarrow$ Chiral symmetry helps $\Lambda_c^+(2595)$ remain narrow
Breakdown of universality

- **Universality** : observables expected to scale w/ $m_\pi^* - m_\pi \to 0$

- Additional large length scale of $r \to$ universality relations break down sooner than expected

E.g., binding energy when $m_\pi > m_\pi^*$

$$B_0(\delta; m_\pi) = \frac{h^4}{2} \epsilon^2 m_\pi \left( \sqrt{1 - \frac{2\delta}{h^4 \epsilon^2 m_\pi}} - 1 \right)^2$$

$$\delta = m_\pi^* - m_\pi \quad \epsilon = \left( \frac{m_\pi}{\sqrt{4\pi f_\pi}} \right)^2$$

Universality recovered only in a tiny window

$$B = \frac{\delta^2}{h^4 \epsilon^2 m_\pi} \left[ 1 + \mathcal{O} \left( \frac{\delta}{h^2 \epsilon^2 m_\pi} \right) \right] \quad \text{for} \quad \left| \frac{m_\pi - m_\pi^*}{m_\pi} \right| \ll \left( \frac{m_\pi^*}{328\text{MeV}} \right)^4$$
Unlike the phase shifts, the binding energy is more directly linked to lattice calculations, are more favored than two-body interactions alone to generate a near-threshold resonance. The range is more likely naturally sized; therefore, other mechanisms, like three-body decays, only when corrective to the scattering length of contributes a two-loop correction to the self-energy, and it will appear as a subleading term in the theoretical description of the scattering process.

Moreover, Eq. (8) shows that chiral symmetry facilitates the resonance to be near threshold when the pion is weakly coupled to other decay channels. (For instance, in many of its important other decays are. In the particular case of \( \eta \) decays into, among others, X, an insight obtained by accounting for the fact that the pion is \([9, 10, 17–19]\).) The construction in the present paper differs from the cases of the two-body resonances and \( \eta \) pole.

In region I, the phase shifts are dominated by the shallow bound state pole. In region II, the phase shifts are dominated by the virtual state pole. The inflection point on the line corresponding to \( \tilde{\delta} = 3 \) is marked out with a diamond. The dashed lines separate the three different regions defined in the text: the boundary between “I” and “II” is the line \( \epsilon = \left( m_\pi \right)^2 \), with various values of \( \tilde{\delta} \), and it indicates that a shallow two-body resonance is equally possible, with a small tweak of the parameters.

The phase shifts

From top down

\[
\tilde{\delta} = \left( \frac{\sqrt{4\pi f_\pi}}{h m_\pi} \right)^4 (m_\pi^* - m_\pi)
\]

\[
\tilde{\delta} = -0.2, 0.2, \text{ and } 3
\]
Two pions + $\Sigma_c$

- $\pi\pi$ interactions are subleading

- $\pi\Sigma_c$ “potential” is energy-dependent $\rightarrow$ total binding energy is not simple addition of individual pions

- Searching 3-body states by finding poles of $\pi\Lambda_c^+(2595)$ scattering amplitude
\[ \pi \Lambda_c^+(2595) \text{ scattering amplitude} \]

\[ t(q; E, B_2) = \frac{8\pi}{3} \frac{|r|}{q^2 + 2m_\pi B_2} \]
\[ + \frac{2}{3\pi} \int dl \frac{l^2}{l^2 + q^2 - 2m_\pi E - i0} \left( \frac{1}{a} - \frac{|r|}{2} (2m_\pi E - l^2) + \sqrt{l^2 - 2m_\pi E} - i0 \right) \]

q : pion momentum

\[ B_2 : \pi \Sigma_c \text{ binding energy} \]

E : total energy

\[ \Lambda_c^+(2765) \ ? \]

quantum numbers uncertain
Summary

- Chiral symmetry ensures an S-wave pion-baryon to remain narrow close to threshold

- Large value taken by the effective range, ruining universality relations

- 2 pions + $\Sigma_c$