Glauber-gluon effects and $B \to \pi \pi, K \pi$ puzzles

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April 2016

Presented at Shanghai Jiao Tong University
Outline

☞ Motivation
☞ Factorization formulas
☞ Numerical results and discussions
☞ Summary
Motivation

- Experimentally, the current data on the BR of $B^0 \rightarrow \pi^0\pi^0$

\[
\begin{align*}
(1.83 \pm 0.21 \pm 0.13) \times 10^{-6} \quad &\text{(BaBar),} \\
(0.90 \pm 0.12 \pm 0.10) \times 10^{-6} \quad &\text{(Belle),} \\
(1.17 \pm 0.13) \times 10^{-6} \quad &\text{(HFAG),}
\end{align*}
\]

Hierarchy,

\[
Br (B^+ \rightarrow \pi^+\pi^0) \gtrsim (\sim) \quad Br (B^0 \rightarrow \pi^+\pi^-) \quad > \quad Br (B^0 \rightarrow \pi^0\pi^0)
\]

and on the $\Delta A_{K\pi}$

\[
\Delta A_{K\pi} = 0.119 \pm 0.022
\]
Theoretically, e.g., in the NLO pQCD formalism,

\[ \text{Br} (B^0 \rightarrow \pi^+ \pi^-) > \text{Br} (B^+ \rightarrow \pi^+ \pi^0) \gg \text{Br} (B^0 \rightarrow \pi^0 \pi^0) \]

In particular, in the factorization theorems,

\[ \text{Br} (B^0 \rightarrow \pi^0 \pi^0) \sim (0.2 - 0.3) \times 10^{-6} \]

For \( B \rightarrow K\pi \) decays,

\[ A_{CP}^{\text{dir}}(B^0 \rightarrow K^\pm \pi^\mp) \sim A_{CP}^{\text{dir}}(B^\pm \rightarrow K^\pm \pi^0) \]

The above contradictions between theory and experiment for the \( B \rightarrow \pi\pi \) decay rates and the differences of the \( B \rightarrow K\pi \) direct CP asymmetries have been known as long-standing puzzles.
✿ Color-suppressed tree amplitude $C$ is an important but least-understood quantity in $B$ decays;

\[ \mathcal{B}(B^0 \to \rho^0 \rho^0) = \begin{cases} (0.92 \pm 0.32 \pm 0.14) \times 10^{-6} & \text{(BaBar)}, \\
(1.02 \pm 0.30 \pm 0.15) \times 10^{-6} & \text{(Belle)}, \\
(0.97 \pm 0.24) \times 10^{-6} & \text{(HFAG)}. \end{cases} \]

Phys.Rev.D 83, 034023 (2011), Li and Mishima

Can $C$ be larger? How to reach? Sub-leading corrections or new QCD mechanism?

The $B \to \rho \rho$ data seriously constrained the possibility of resolving the $B \to \pi \pi$ puzzle. (Phys.Rev.D 73, 114014 (2006), Li and Mishima)

Many efforts have been made on this puzzle. But, no satisfactory resolution can be achieved yet. (Naively enhancing the hard spectator amplitudes(HSA) to $C$ !)

To our best knowledge, all the strategies in the literature adopted to resolve this puzzle either evaded the $B^0 \to \rho^0 \rho^0$ constraint or did not survive the constraint in the SM.

A new mechanism is demanded!

Mechanism must differentiate pion from $\rho$ meson!
Have checked the $k_T$ factorization of the spectator nonfactorizable diagrams

Considered the factorization of $M_2$ meson wave function
Observed the existence of Glauber gluons

Leading IR regions

\[
\begin{align*}
\text{Collinear} & : l^+ (Q) \gg l_T (\Lambda_{QCD}) \gg l^- (\Lambda_{QCD}^2 / Q) \\
\text{Soft} & : l^+ (\Lambda_{QCD}) \sim l_T (\Lambda_{QCD}) \sim l^- (\Lambda_{QCD}) \\
\text{Glauber} & : l^+ (\Lambda_{QCD}^2 / Q) \sim l^- (\Lambda_{QCD}^2 / Q) \ll l_T (\Lambda_{QCD})
\end{align*}
\]
Li and Mishima observed the glauber divergences with the NLO spectator amplitudes in the $k_T$ factorization theorem, then gave universal phase factors by all-order summation. 

the phase factors associated with the emitted meson will turn the destructive interference between the LO spectator diagrams into a constructive one, then modify $C$;

\[ I_a \approx \exp(iS_e) \mathcal{M}_a^{(0)}, \quad I_b \approx \exp(-iS_e) \mathcal{M}_b^{(0)}, \]

like \[1 - 1 \Rightarrow e^{iS_e} - e^{-iS_e}, \text{ large imaginary } C \]

the phase factors associated with the recoiled meson will rotate the enhanced $C$ then modify the interference between $C$ and $T$. 

Phys. Rev. D 83, 034023 (2011); ibid. 90, 074018 (2014), Li and Mishima
After treating the glauber phases as free real parameters in the $B \to \pi \pi$ decays, the BR was

$$\text{Br} \ (B^0 \to \pi^0\pi^0) \sim 1.2 \times 10^{-6}$$

the postulation on vanishing of glauber phases in the $B \to \rho \rho$ decays was made.

But, the question should be answered:

Why the color-suppressed tree amplitudes are so different in the $B \to \pi \pi$ and $B \to \rho \rho$ decays?

We attempt to answer this question by quantitatively estimating different glauber effects through convolution in the $B \to \pi \pi$ and $B \to \rho \rho$ decays.
Factorization formulas

- Glauber gluons have been identified from the higher order corrections to the spectator diagrams in $B \to M_1 M_2$ decays.

Phys. Rev. D 83, 034023 (2011); ibid. 90, 074018 (2014), Li and Mishima

NLO spectator diagrams contain the glauber divergences associated with the $M_2$ meson for $B \to M_1 M_2$ decay. Other NLO diagrams with the glauber divergences are referred to Phys.Rev.D 83, 034023(2011).
NLO spectator diagrams contain the glauber divergences associated with the $M_1$ meson for $B \to M_1 M_2$ decay. Other NLO diagrams with the glauber divergences are referred to Phys. Rev. D 90, 074018 (2014).

\[
\int d^2 b_1 d^2 b_2 \int d^2 b_{s1} d^2 b_{s2} \bar{\phi}_B(b_1) \bar{\phi}_1(b_{s1} + b_1, b_{s1}) \times \bar{\phi}_2(b_{s2}, b_{s2} + b_2) \exp \left[ -iS(b_{s1} - b_2) + iS(b_{s2} - b_1) \right] H_a(b_1, b_2)
\]
\[
\int d^2 b_1 d^2 b_2 \int d^2 b_{s1} d^2 b_{s2} \bar{\phi}_B(b_1) \bar{\phi}_1(b_{s1} + b_1, b_{s1})
\times \bar{\phi}_2(b_{s2} + b_2, b_{s2}) \exp \left[ -iS(b_{s1} - b_2) - i\bar{S}(b_{s2} - b_1) \right] H_b(b_1, b_2)
\]

Comments:

- the universal glauber factor associated with \( M_1 \), same for both amplitudes
- the universal glauber factor associated with \( M_2 \), “(-)+” denotes the glauber gluons radiated from valence (anti-)quark
- Though the glauber factor is universal, the glauber effect appears different through the convolution with the TMD wave functions \( \bar{\phi}_1(b_{s1} + b_1, b_{s1}) \) and \( \bar{\phi}_2(b_{s2} + b_2, b_{s2}) \) which contains different intrinsic \( b \) dependences for pion, kaon, and \( \rho \) meson.
Numerical analysis and discussions

Two important elements:

① TMD meson wave function (different intrinsic $k_T$)

② Parameterization of glauber phase factor

TMD meson wave function with the intrinsic $k_T$ part in the Gaussian form

\[
\phi_M(x, k_T) = \frac{\pi}{2\beta_M^2} \exp \left( -\frac{M^2}{8\beta_M^2} \right) \frac{\phi_M(x)}{x(1-x)},
\]

For pion:

\[
M^2 = \frac{k_T^2 + m^2}{x} + \frac{k_T^2 + m^2}{1-x},
\]

For kaon:

\[
M^2 = \frac{k_T^2 + m_q^2}{x} + \frac{k_T^2 + m_s^2}{1-x},
\]

Then the modified wave function is expressed as,

\[
\phi_M(x, b', b) \equiv \phi'_M(b', b) \phi_M(x) = \frac{2\beta_M^2}{\pi} \exp \left[ -2\beta_M^2 x b'^2 - 2\beta_M^2 (1 - x) b^2 \right] \phi_M(x)
\]

We simply parameterize the glauber phase \( S(b) \) by a sinusoidal function,

\[
S(b) = r \pi \sin(p \cdot b)
\]

another important procedure— determination of the shape parameters of pion, kaon and \( \rho \) meson

① widely adopted $\beta_\pi \sim 0.4$ GeV for pion in the literature

② $\beta_\rho \sim 1/3 \beta_\pi$ fixed through the $B \to \rho$ form factor

③ $\beta_K \sim 0.25$ GeV fixed through the $B \to K$ form factor

☞ Important implication:

➢ pion ($\rho$ -meson) WF with a weak(strong) falloff in parton TM $k_T$, and kaon WF reveals a stronger(weaker) falloff in $k_T$ compared to the pion ($\rho$ -meson) one, which means pion requires a tighter spatial distribution of its leading Fock state relative to higher Fock states.

➢ consistent with the dual role of pion as a massless NGB and as a qq bound state simultaneously.

☞ two sets of parameters $r$ and $p$ corresponding to largest BR of $B^0 \to \pi^0\pi^0$ decay
- $r \sim 0.47, p \sim -0.632$ GeV

- $r \sim 0.60, p \sim 0.544$ GeV

<table>
<thead>
<tr>
<th>Modes</th>
<th>Data [1, 2]</th>
<th>NLO</th>
<th>NLOG</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0 \rightarrow \pi^+ \pi^-$</td>
<td>$5.10 \pm 0.19$</td>
<td>$6.19^{+2.09}<em>{-1.48}(\omega_B)^{+0.38}</em>{-0.34}(a_2^\pi)$</td>
<td>$5.39^{+1.86}<em>{-1.31}(\omega_B)^{+0.28}</em>{-0.25}(a_2^\pi)$</td>
</tr>
<tr>
<td>$B^+ \rightarrow \pi^+ \pi^0$</td>
<td>$5.48^{+0.35}_{-0.34}$</td>
<td>$3.35^{+1.08}<em>{-0.77}(\omega_B)^{+0.23}</em>{-0.22}(a_2^\pi)$</td>
<td>$4.45^{+1.38}<em>{-0.99}(\omega_B)^{+0.39}</em>{-0.36}(a_2^\pi)$</td>
</tr>
<tr>
<td>$B^0 \rightarrow \pi^0 \pi^0$</td>
<td>$0.90 \pm 0.16$</td>
<td>$0.29^{+0.11}<em>{-0.07}(\omega_B)^{+0.03}</em>{-0.02}(a_2^\pi)$</td>
<td>$0.61^{+0.16}<em>{-0.12}(\omega_B)^{+0.14}</em>{-0.12}(a_2^\pi)$</td>
</tr>
<tr>
<td>$B^0 \rightarrow \rho^0 \rho^0$</td>
<td>$0.97 \pm 0.24$</td>
<td>$1.06^{+0.29}<em>{-0.21}(\omega_B)^{+0.19}</em>{-0.16}(a_2^\rho)$</td>
<td>$0.89^{+0.26}<em>{-0.18}(\omega_B)^{+0.13}</em>{-0.10}(a_2^\rho)$</td>
</tr>
</tbody>
</table>

Phys.Rev.D 91, 114019(2015), Liu, Li, and Xiao
The results obtained with second set of parameters show the preferred pattern: $B^0 \rightarrow \pi^+ \pi^-$ and $B^0 \rightarrow \rho^0 \rho^0$ BRs decrease 13% and 16%, respectively, while $B^+ \rightarrow \pi^+ \pi^0$ and $B^0 \rightarrow \pi^0 \pi^0$ ones increase by 33% and a factor of 2.1, respectively.

The agreement between the theoretical predictions and the data for all the $B \rightarrow \pi \pi$ and $B^0 \rightarrow \rho^0 \rho^0$ BRs is improved simultaneously.

The ratio of enhancement factor of $B^0 \rightarrow \pi^0 \pi^0$ over reduction factor of $B^0 \rightarrow \rho^0 \rho^0$ is about 2.5, close to 3 observed in Phys.Rev.D 90, 074018 (2014).
To clarify the glauber effect, the results (in units of $10^{-2}$ GeV$^3$) of the nonfactorizable spectator amplitudes from operator $O_2$ without and with glauber phase are

\[
A_{a,b}(B^0 \to \pi^0\pi^0) = \begin{cases} 
11.86 - i9.04, \\
10.80 - i7.25,
\end{cases} \quad (NLO),
\]

\[
-7.13 + i6.18, \\
7.67 - i3.42,
\]

\[
A_{a,b}(B^0 \to \rho^0\rho^0) = \begin{cases} 
-42.44 + i24.42, \\
-5.78 + i4.32,
\end{cases} \quad (NLO),
\]

\[
28.88 - i18.07, \\
-3.61 - i3.23,
\]

It is possible to resolve the $B \to \pi\pi$ puzzle!
Glauber-gluon effects modify the branching ratios moderately with around 10% reduction;
Dominance of the penguin contributions make the values be insensitive to amplitude $C$.

**Direct CP asymmetries of $B \to K \pi$ decays,**

Glauber-gluon effects modified the DCP of $B^+ \to K^+ \pi$ mode significantly with sign flipping attributed to its sensitivity to amplitude $C$.
CP-averaged branching ratios of $B \to KK$ decays,

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>$B^\pm \to K^\pm \bar{K}^0$</td>
<td>$1.52 \pm 0.22^a$</td>
<td>$2.45^{+0.83}<em>{-0.58} (\omega_B)^{+0.17}</em>{-0.17} (a^K)$</td>
<td>$2.27^{+0.79}<em>{-0.54} (\omega_B)^{+0.17}</em>{-0.14} (a^K)$</td>
</tr>
<tr>
<td>$B^0 \to K^0 \bar{K}^0$</td>
<td>$1.21 \pm 0.16$</td>
<td>$2.19^{+0.77}<em>{-0.54} (\omega_B)^{+0.09}</em>{-0.09} (a^K)$</td>
<td>$2.02^{+0.72}<em>{-0.50} (\omega_B)^{+0.08}</em>{-0.08} (a^K)$</td>
</tr>
</tbody>
</table>

$^a$This is the very recent measurement reported by the LHCb Collaboration [23], which is comparable with $1.64 \pm 0.45$ by the BABAR Collaboration [24] and a bit larger than $1.11 \pm 0.20$ by the Belle Collaboration [25].

Direct CP asymmetries of $B \to KK$ decays,

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<th>NLOG</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^\pm \to K^\pm \bar{K}^0_S$</td>
<td>$-0.21 \pm 0.14$</td>
<td>$-0.03^{+0.01}<em>{-0.01} (\omega_B)^{+0.02}</em>{-0.02} (a^K)$</td>
<td>$-0.03^{+0.01}<em>{-0.01} (\omega_B)^{+0.02}</em>{-0.02} (a^K)$</td>
</tr>
<tr>
<td>$B^0 \to K^0_S \bar{K}^0_S$</td>
<td>$0.0 \pm 0.4$</td>
<td>$-0.09^{+0.00}<em>{-0.00} (\omega_B)^{+0.01}</em>{-0.01} (a^K)$</td>
<td>$-0.09^{+0.00}<em>{-0.00} (\omega_B)^{+0.00}</em>{-0.00} (a^K)$</td>
</tr>
</tbody>
</table>

No amplitude $C$; only the spectator amplitudes induced by penguin operators;

Glauber-gluon effects decrease the branching ratios by only a few percent; the DCP remains unchanged.
for pion emission from the weak vertex with operator $O_2$,

$$A_{a,b}(B^\pm \to \pi^0 K^\pm)$$

$$= \begin{cases} 
-16.71 + i13.71, & 10.85 - i9.96, \quad \text{(NLO)}, \\
-12.57 + i10.80, & -9.96 + i5.85, \quad \text{(NLOG)}, 
\end{cases}$$

for kaon emission from the weak vertex with operator $O_1$,

$$A_{a,b}(B^\pm \to K^\pm \pi^0)$$

$$= \begin{cases} 
4.55 - i3.98, & -3.37 + i2.25, \quad \text{(NLO)}, \\
3.59 - i3.23, & 3.14 - i0.90, \quad \text{(NLOG)}, 
\end{cases}$$

$$6.96e^{i2.57} (2.09e^{-i0.97}) \times 10^{-2} \text{ GeV}^3$$

$$28.01e^{i2.51} (7.90e^{-i0.55}) \times 10^{-2} \text{ GeV}^3$$
to examine the similarity between pion and kaon from Glauber gluons

\[ R_\pi \equiv \frac{|A_a(\pi^0 K^\pm) + A_b(\pi^0 K^\pm)|_{\text{NLOG-S}_{e2}}}{|A_a(\pi^0 K^\pm) + A_b(\pi^0 K^\pm)|_{\text{NLO}}} \approx 4.69, \]

\[ R_K \equiv \frac{|A_a(K^\pm \pi^0) + A_b(K^\pm \pi^0)|_{\text{NLOG-S}_{e2}}}{|A_a(K^\pm \pi^0) + A_b(K^\pm \pi^0)|_{\text{NLO}}} \approx 3.90, \]

both pion and kaon are pseudo-NGBs, but the latter one with non-negligible SU(3) symmetry breaking effect, which resulting in the weaker Glauber-gluon effects than the pion through the convolution with the TMD WF.
A plausible explanation to the dynamical origin of glauber phase:

the overlap between the $k_T$ distributions of the leading Fock state in a meson and the glauber gluons in a decay process

It is stressed that the $B \rightarrow \pi\pi$, $K\pi$ puzzles must be resolved by resorting to a mechanism and that the glauber gluons should be one of the most crucial mechanisms.
Conclusions and Summary

纪律 The model estimate of the glauber effect has been performed and the convoluted factorization formulas have been obtained.

纪律 A weak falloff in $k_T$ of pion WF is consistent with the dual role of the pion as a massless NGB and as a $qq$ bound state.

纪律 The universal glauber factors make distinct impacts on the $B^0 \rightarrow \pi^0\pi^0$ and $B^0 \rightarrow \rho^0\rho^0$ BRs through convolution with TMD WF and reasonable parameterization of glauber phase.
The more significant glauber effect from pion was observed and the consistency between th. and ex. for all the modes was improved simultaneously.

The glauber gluons should be one of the most crucial mechanisms to resolve the long-standing \( B \rightarrow \pi \pi, K \pi \) puzzles.
BACKUP SLIDES
Contributing to glauber phase $S_2$
Contributing to glauber phase $S_1$

boson [30]: the valence quark and antiquark of the pion are separated by a short distance, like those of the $\rho$ meson, in order to reduce the confinement potential energy. The multiparton states of the pion spread over a huge space-time in order to meet the role of a massless NG boson, which result in a strong Glauber effect.
Nambu-Goldstone boson

- Pion as a $q\bar{q}$ bound state and as a massless Nambu-Goldstone boson?
- Massless boson => huge spacetime => large separation between $q\bar{q}$ => high mass under confinement => contradiction!
- Reconciliation: leading $q\bar{q}$ state is tight, higher Fock state gives soft cloud (Lepage, Brodsky 79; Nussinov, Shrock 08; Duraisamy, Kagan 08)
  Pion is unique.
- Glauber factor for pion corresponds to this soft cloud: 3 partons in $k_T$ space
Loop momentum cutoff: \( \Lambda \sim 0.5 \text{ GeV} \), collecting soft contributions, roughly yielding the period \( p \) of the oscillatory parameterization;

Gluon mass: \( m_g \), together with the coefficient in the associated loop corrections such as strong coupling, controlling the magnitude \( r \) of the oscillation;

\( b \rightarrow 0 \) limit, corresponding to the integration over the transverse momentum in the collinear factorization theorem;