Two-Photon-Exchange Contribution to Proton Form Factors in Both Space-Like and Time-Like Regions

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1. Introduction

2. Two-photon-exchange contribution to elastic $e^+e^- ightarrow pp$ scattering

3. Two-photon-annihilation contribution to $e^+e^- ightarrow pp$ in hadronic model

4. Summary
Introduction: form factors of proton

One of the main problems in hadronic physics is to extract the elemental non-perturbative physical quantities, such as: quantum number of hadrons, decay constant, form factor, parton distribution, distribution amplitude, GPD, GDA, etc.

The electromagnetic (EM) form factors of proton are two of them. By the symmetry, the non-perturbative EM current matrix element of proton can be decomposed as

\[ < P(p') | J_{\mu}^{EM}(0) | P(p) > = \bar{u}(p') [ F_1(Q^2) \gamma_{\mu} + F_2(Q^2) \frac{i\sigma_{\mu\nu}}{2M} q^\nu ] u(p) \]

\[ Q^2 = -q^2 = -(p' - p)^2 \]
Up to now, two methods are used to extract the EM form factors of proton in the space-like region.

- Rosenbluth method:
  extract the EM form factors from the cross section of un-polarized elastic ep scattering.

- polarization method:
  extract the ratio of EM form factor from the polarization observables in elastic ep scattering.
Introduction: Rosenbluth method

For the un-polarized elastic ep scattering, taking one photon exchange approximation,

\[ e(p_1) \rightarrow e'(p_3) \]
\[ P(p_2) \rightarrow \gamma \rightarrow P'(p_4) \]

the reduced cross section is written as:

\[ d\sigma \propto G_M^2(Q^2) + \frac{\mathcal{E}}{\tau} G_E^2(Q^2) \]
Introduction: Rosenbluth method

\[ d\sigma \propto G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2) \]

\[ \tau \equiv \frac{Q^2}{4M_N^2}, \varepsilon = [1 + 2(\tau + 1)\tan^2(\theta_e/2)]^{-1} \]

\[ G_E = F_1 - \tau F_2, \quad G_M = F_1 + F_2 \]

\( \theta_e \) the scattering angle of electron in the rest frame.

By the measurements of cross sections at fixed \( Q^2 \) and different \( \varepsilon \), the EM ffs can be extracted, and also the ratios of ffs are obtained.
**Introduction: polarization method**

For the polarized ep scattering

\[ \vec{e} + p \rightarrow e + \vec{p} \]

there are polarization observables

- \( P_T \): proton polarization perpendicular to proton momentum in the scattering plane;
- \( P_L \): proton polarization parallel to proton momentum in the scattering plane.
Introduction: polarization method

by the one photon exchange approximation, there is a relation

\[ R \equiv \frac{G_E}{G_M} = -\frac{P_T}{P_L} \frac{E + E'}{2M_N} \tan \frac{\theta_e}{2} \]

\( E, E' \) the energies of initial and final electrons

The measurements of \( P_T, P_L \) at fixed \( Q^2 \) and \( \varepsilon \), can give us the ratio of the EM ffs.
Introduction: results from experiments

Rosenbluth method

polarization method

experimental values of $R$ by Rosenbluth method and polarization method
references in PRL91,142304(2003)

Un-consistent! Which is right?
Introduction: reason-two photon exchange

Such un-consistence is explained by the two-photon-exchange effects (box diagrams of EM radiative corrections) in literatures.
In literatures, four methods are mainly used to calculate the TPE correction, and also some model-independent analysis or fitting are discussed.

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| model-independent analysis or fitting |                                                                                   |
|                                      | M. Vanderhaeghen etc                                                             |
|                                      | E. Tomasi-Gustafsson etc                                                          |
|                                      | S.N.Yang etc                                                                     |
|                                      | ………….                                                                         |
The Born cross section is calculated with the form factors from the polarization transfer experiment.

The reduced cross section $\rightarrow$ ratio $R$
The results show that the TPE corrections to the ratio $R$ by polarization method are positive and as large as +4% for $Q^2 = 3$ and as large as +10% for $Q^2 = 6$ at small $\epsilon$. $\epsilon = 0.1$

Only the N intermediate state is included.

TPE in ep scattering: GDP

Cross section for ep elastic scattering

2-γ corrections to polarization ratio - proton

$\sigma_R / (\mu_p G_{\text{dipole}})^2$ consistent with Ex $P_T^{1\gamma+2\gamma} / P_T^{1\gamma}, \%$

$P_L^{1\gamma+2\gamma} / P_L^{1\gamma}, \%$

-5% $\varepsilon = 0.3$

-7% $\varepsilon = 0.3$

TPE in ep scattering: pQCD

Dash-Blue: $1\gamma$
Dotted black: $1\gamma + 2\gamma$ (COZ)
Solid red: $1\gamma + 2\gamma$ (BLW)
Dash-dotted green: $1\gamma + 2\gamma$ (QCDSF)

consistent with Ex

$-7\%$(COZ) $\epsilon = 0.3$
$-3\%$(BLW)

Nikolai Kivel, Marc Vanderhaeghen
PRL103,092004(2009).
TPE in ep scattering: dispersion method vs. HM

(c) consistent with each other

Only the N intermediate state is included.


TPE in ep scattering: summary of the results

1: the corrections to un-polarized cross sections: consistent.

2: the corrections to ratio R in polarization method: un-consistent at high $Q^2$ and small $\varepsilon$.

- hadronic model: positive and as large as +4% for $Q^2 = 3$ and as large as +10% for $Q^2 = 6$ at $\varepsilon = 0.1.$ (where only N is included)

- GDPs and pQCD: negative about -5% for $Q^2 = 2.5, 5$ at $\varepsilon = 0.3$.

Which is right?

JLab/Hall C exp. E-04-019
TPE in ep: re-discussion of hadronic model

Two problem exists in the calculation of hadronic model when including the Delta(1232) intermediated state:

- vertex relation
- Coulomb term

TPE in ep: re-discussion of hadronic model

vertex relation

\[ \Gamma_{\gamma N \rightarrow \Delta} = \gamma_0 [\Gamma_{\gamma \Delta \rightarrow N}]^+ \gamma_0 \]

We modify it as

\[ \Gamma_{\gamma N \rightarrow \Delta}(p, q) = -\gamma_0 [\Gamma_{\gamma \Delta \rightarrow N}(p, -q)]^+ \gamma_0 \]

p is the momentum of proton, q is the momentum of incoming photon.
The explicit expression

\[
\Gamma_{\gamma\Delta\rightarrow N}^{\mu\alpha} = \frac{-F_\Delta(q_1^2)}{M_N^2} \left[ g_1 (g_\mu^\alpha \hat{k} \hat{q}_1 - k_\mu \gamma^\alpha \hat{q}_1 - \gamma_\mu \gamma^\alpha k \cdot q_1 + \gamma_\mu \hat{k} q_1^\alpha) \\
+ g_2 (k_\mu q_1^\alpha - k \cdot q_1 g_\mu^\alpha) + g_3/M_N (q_1^2 (k_\mu \gamma^\alpha - g_\mu^\alpha \hat{k}) \\
+ q_1 \mu (q_1^\alpha \hat{k} - \gamma^\alpha k \cdot q_1)) \right] \gamma_5 T_3,
\]

\[
\Gamma_{\gamma\rightarrow \bar{N}\Delta}^{\mu\alpha} = \frac{-F_\Delta(q_2^2)}{M_N^2} (k) T_3^+ \gamma_5 [ g_1 (g_\nu^\beta \hat{q}_2 \hat{k} - k_\nu \hat{q}_2 \gamma^\beta - \gamma^\beta \gamma_\nu k \cdot q_2 + \hat{k} \gamma_\nu q_2^\beta) \\
+ g_2 (k_\nu q_2^\beta - k \cdot q_2 g_\nu^\beta) - g_3/M_N (q_2^2 (k_\nu \gamma^\beta - g_\nu^\beta \hat{k}) \\
+ q_2 \nu (q_2^\beta \hat{k} - \gamma^\beta k \cdot q_2))].
\]

Coulomb term

\[ g_3 = g_c = 0, -2 \]

We take

\[ g_3 = g_c = 7.2 \]

Also the form factors \( F_\Delta \) are modified.

By these modification, the new results:

P. G. Blunden, S. Kondratyuk, W. Melnitchouk, J A. Tjon,
PRL95, 172503(2005), PRC75, 038201(2007).
**TPE in ep: results for un-polarized case**

\[ Q^2 = 1 \text{ GeV}^2 \]

\[ \delta_{un,\Delta} \]

\[ \begin{align*}
g_c = 0 & \quad \text{blue} \\
g_c = -2 & \quad \text{green dotted} \\
g_c = 7.2 & \quad \text{red dashed}
\end{align*} \]

\[ Q^2 = 3 \text{ GeV}^2 \]

\[ \delta_{un,\Delta} \]

\[ \begin{align*}
g_c = 0 & \quad \text{blue} \\
g_c = -2 & \quad \text{green dotted} \\
g_c = 7.2 & \quad \text{red dashed}
\end{align*} \]

\[ Q^2 = 5 \text{ GeV}^2 \]

\[ \delta_{un,\Delta} \]

\[ \begin{align*}
g_c = 0 & \quad \text{blue} \\
g_c = -2 & \quad \text{green dotted} \\
g_c = 7.2 & \quad \text{red dashed}
\end{align*} \]

\[ \delta_{un,\Delta} \equiv \frac{\sigma_{un}^{2\gamma(\Delta)}}{\sigma_{un}^{1\gamma}} \]

**using the new vertex relation**
**TPE in ep: results for polarized case**

\[ Q^2 = 1 \text{ GeV}^2 \]

\[ Q^2 = 3 \text{ GeV}^2 \]

\[ \delta_\Delta = \frac{P_T^{1\gamma + 2\gamma (\Delta)}}{P_L^{1\gamma + 2\gamma (\Delta)}} / \frac{P_T^{1\gamma}}{P_L^{1\gamma}} \]

\[ R_{phy} = R_{Exp} / (\delta_N + \delta_\Delta) \]
For high $Q^2$ and large $\varepsilon$, it shows surprising properties: the correction to $P_T$ is very large in this region.

Un-physical?
TPE in ep: modify?

Problem?

Is hadronic model not reasonable when including \( \text{Delta}(1232) \) at such \( Q^2 \) and \( \varepsilon \)? What is the valid region?

\[
Q^2 = 3, 5? \\
\varepsilon = 0.6, 0.8, 0.9?
\]

or modify?
to regular the behavior when including $\text{Delta}(1232)$, we add a factor by hand in the loop:

\[
f(p_2 + k) = \frac{\Lambda^4}{((p_2 + k)^2 - M^2_{\Delta})^2 + \Lambda^4}
\]
TPE in ep: results for un-polarized case

Hints the valid region of hadronic model? \( \epsilon < 0.6 \)
TPE in ep: results for polarized case

\[ Q^2 = 1 \text{ GeV}^2 \]

\[ Q^2 = 3 \text{ GeV}^2 \]

\[ Q^2 = 5 \text{ GeV}^2 \]

Does not change the total correction to ratio in polarization methods:
\[ N+D: \text{ positive} \]

\[ \epsilon < 0.6 \]

Hints the valid region of hadronic model?
1. The experiment (JLab/Hall C exp. E-04-019) maybe distinguish which model is reasonable for the TPE correction in polarization methods: positive or negative?

2. How to combine those methods is still a problem (the valid region of different methods, not only for the TPE, but also the for $\gamma Z$ exchange).

TPE in time-like region

We simply apply the hadronic model to $e^+e^- \rightarrow p\bar{p}$ to give an estimate of two-photon-annihilation correction
TPE in time-like region: observables

Un-polarized cross section

\[
\left( \frac{d\sigma}{d\Omega} \right)_{CM} = \frac{\alpha^2 \sqrt{1 - 4M_N^2/q^2}}{4q^2} \times \left( |G_M|^2 (1 + \cos^2 \theta) + \frac{1}{\tau} |G_E|^2 \sin^2 \theta \right)
\]
Consider the un-polarized incoming positron, longitudinally polarized incoming electron, and the polarized antiproton in the final state, the cross section

\[
\frac{d\sigma}{d\Omega} = \frac{d\sigma_{un}}{d\Omega} \left[ 1 + P_y \xi_y + \lambda_e P_x \xi_x + \lambda_e P_z \xi_z \right].
\]

double spin polarization observables $P_x$ and $P_z$

\[
P_x = -\frac{2 \sin \theta}{D \sqrt{\tau}} \left\{ \text{Re}[\tilde{G}_M \tilde{G}_E^*] + \text{Re}[\tilde{G}_M \tilde{F}_3^*] \sqrt{\tau (\tau - 1) \cos \theta} \right\}
\]

\[
P_z = \frac{2}{D} \left\{ |\tilde{G}_M|^2 \cos \theta - \text{Re}[\tilde{G}_M \tilde{F}_3^*] \sqrt{\tau (\tau - 1) \sin^2 \theta} \right\}
\]
TPE time-like: results for un-polarized case

properties:

1: odd function of $\cos \theta$

2: contributions from $N$ and Delta(1232) intermediate state are opposite.

3: total TPE contributions are small;

$$s = 4 GeV^2$$

$$\delta_{2\gamma} = 2 \frac{Re\{M_{2\gamma}M_0^\dagger\}}{|M_0|^2}$$

**TPE time-like: results for $P_x$**

properties:

1: odd function of $\cos \theta$

2: main contribution from $N$ and the Delta(1232) contribution is very small.

3: relatively larger at $\cos \theta = \pm 1$

4: the absolute contributions are small.

$$s = 4GeV^2$$

$$\delta(P_x) = \frac{P_x^1\gamma \otimes 2\gamma}{P_x^1\gamma}$$
TPE time-like: results for $P_z$

$s = 4 GeV^2$

properties:

1: odd function of $\cos \theta$

2: $P_{z\gamma}^{1\gamma}(\pi/2) = 0$
   
   $P_{z\gamma}^{1\gamma\otimes2\gamma}(\pi/2) \neq 0$

3: non-zero of $P_z$ at $\pi/2$

reveal TPE and the large correction suggests it may deserve to be considered in the experiment near $\pi/2$.

$$\delta(P_z) = \frac{P_{z\gamma}^{1\gamma\otimes2\gamma}}{P_{z\gamma}^{1\gamma}}$$
Summary

- TPE in ep scattering played important roles while its corrections to polarization observables are not clear now.

- How to combine the four methods is still a problem.

- TPE contributions in $e^+e^- \rightarrow p\bar{p}$ at small $s$ are usually small and is relative larger to polarization observable $P_z$. This may be a considerable quantity to see TPE directly.

- Other methods to estimate the TPE contribution in time-like region?