Five-body calculation of heavy pentaquark system

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I have been studying the following subjects from viewpoint of few-body problem using my own method called Gaussian Expansion Method.

Few-nucleon system

Hypernucleus = Nucleus + hyperon

Pentaquark

Tetraquark $X(3872)$
Observation of $J/\psi p$ Resonances Consistent with Pentaquark States in $\Lambda_b^0 \rightarrow J/\psi K^- p$ Decays

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(LHCb Collaboration)
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Observations of exotic structures in the $J/\psi p$ channel, which we refer to as charmonium-pentaquark states, in $\Lambda_b^0 \rightarrow J/\psi K^- p$ decays are presented. The data sample corresponds to an integrated luminosity of 3 fb$^{-1}$ acquired with the LHCb detector from 7 and 8 TeV $pp$ collisions. An amplitude analysis of the three-body final state reproduces the two-body mass and angular distributions. To obtain a satisfactory fit of the structures seen in the $J/\psi p$ mass spectrum, it is necessary to include two Breit-Wigner amplitudes that each describe a resonant state. The significance of each of these resonances is more than 9 standard deviations. One has a mass of $4380 \pm 8 \pm 29$ MeV and a width of $205 \pm 18 \pm 86$ MeV, while the second is narrower, with a mass of $4449.8 \pm 1.7 \pm 2.5$ MeV and a width of $39 \pm 5 \pm 19$ MeV. The preferred $J^P$ assignments are of opposite parity, with one state having spin $3/2$ and the other $5/2$.

<table>
<thead>
<tr>
<th>State</th>
<th>Mass (MeV)</th>
<th>Width (MeV)</th>
<th>Fit fraction (%)</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_c(4380)^+$</td>
<td>$4380 \pm 8 \pm 29$</td>
<td>$205 \pm 18 \pm 86$</td>
<td>$8.4 \pm 0.7 \pm 4.2$</td>
<td>$9\sigma$</td>
</tr>
<tr>
<td>$P_c(4450)^+$</td>
<td>$4449.8 \pm 1.7 \pm 2.5$</td>
<td>$39 \pm 5 \pm 19$</td>
<td>$4.1 \pm 0.5 \pm 1.1$</td>
<td>$12\sigma$</td>
</tr>
</tbody>
</table>

- Best fit has $J^P=(3/2^-, 5/2^+)$, also $(3/2^+, 5/2^-)$ & $(5/2^+, 3/2^-)$ are preferred
To describe the data of $Pc(4380)^+$ and $Pc(4459)^+$ state, there are theoretical effort.

- **Cusp?**

- **Meson-Baryon state?**

- **Baryoncharmonnia**

- **Tightly bound pentaquark states**
Motivated by the experimental data of pentaquark system at LHCb, we calculate this system within the framework of non-relativistic constituent quark model.

To describe the experimental data, it is necessary to reproduce the observed threshold. The Hamiltonian is important to reproduce the low-lying energy spectra of meson and baryon system.
Hamiltonian

\[ H = \sum_i \left( m_i + \frac{p_i^2}{2m_i} \right) - T_G + V_{\text{Conf}} + V_{\text{CM}} - \Lambda / r \]

\[ \Lambda = 0.1653 \text{GeV}^2 \]

\[ V_{\text{Conf}} = -\sum_{i<j} \sum_{\alpha=1}^8 \frac{\lambda_i^\alpha \lambda_j^\alpha}{2} \left[ \frac{k}{2} (x_i - x_j) + v_0 \right], \]

\[ V_{\text{CM}} = \sum_{i<j} \sum_{\alpha=1}^8 \frac{\xi_\alpha}{2} \frac{\lambda_i^\alpha \lambda_j^\alpha}{m_i m_j} e^{-\frac{(x_i - x_j)^2}{\beta^2}} \sigma_i \cdot \sigma_j. \]

\[ \xi_\alpha = (2\pi/3) k \quad \beta = A((2m_i m_j)/(m_i + m_j))^{(1/3)} \]

\[ K_p = 1.8609 \quad A = 1.6553 \quad B = 0.2204 \]

\[ m_q = 315 \text{ MeV}, \quad m_c = 1836 \text{ MeV} \]

Calculated energy spectra for meson and baryon systems are in good agreement with the observed data.

<table>
<thead>
<tr>
<th></th>
<th>Cal.</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Baryon</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N:</td>
<td>953 MeV</td>
<td>939 MeV</td>
</tr>
<tr>
<td>Δ:</td>
<td>1265 MeV</td>
<td>1232</td>
</tr>
<tr>
<td>Λc:</td>
<td>2276 MeV</td>
<td>2286</td>
</tr>
<tr>
<td>Σc:</td>
<td>2451 MeV</td>
<td>2465</td>
</tr>
<tr>
<td>Σc*:</td>
<td>2531 MeV</td>
<td>2545</td>
</tr>
<tr>
<td><strong>Meson</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D:</td>
<td>1862 MeV</td>
<td>1870</td>
</tr>
<tr>
<td>D*:</td>
<td>2016 MeV</td>
<td>2010</td>
</tr>
<tr>
<td>J/Ψ:</td>
<td>3102 MeV</td>
<td>3094</td>
</tr>
<tr>
<td>ηc :</td>
<td>3007 MeV</td>
<td>2984</td>
</tr>
<tr>
<td>( χ_c ) ( l=1,s=0 ):</td>
<td>3462.4 MeV</td>
<td>3525 MeV</td>
</tr>
<tr>
<td>( L=1,S=1 ) :</td>
<td>3486.5 MeV</td>
<td>3530 MeV</td>
</tr>
</tbody>
</table>
In order to solve few-body problem accurately,

**Gaussian Expansion Method (GEM), since 1987**
- A variational method using Gaussian basis functions
- Take all the sets of Jacobi coordinates

Developed by Kyushu Univ. Group, Kamimura and his collaborators.
Review article:

**High-precision calculations** of various 3- and 4-body systems:

- Exotic atoms / molecules,
- 3- and 4-nucleon systems,
- multi-cluster structure of light nuclei,
- Light hypernuclei,
- 3-quark systems,
- $^4$He-atom tetramer
\[ \Psi_{JM}(qqqc) = \Phi_{JM}^{(C=1)} + \Phi_{JM}^{(C=2)} + \Phi_{JM}^{(C=3)} + \Phi_{JM}^{(C=4)} \]

\[ \Phi_{\alpha JM}(qqqc) = A_{qqq} \left\{ (\text{color})^{(c)}_{\alpha} (\text{isospin})^{(c)}_{\alpha} (\text{spin})^{(c)}_{\alpha} (\text{spatial})^{(c)}_{\alpha} \right\}_{JM} \]
Wavefunction of
Color part

Similar for $C=2$

$C=2(\Lambda c + D, \Sigma c + D)$
Confining channels

I take color singlet.
\[ \Psi_{JM}(qqqcc) = \Phi_{JM}^{(C=1)} + \Phi_{JM}^{(C=2)} + \Phi_{JM}^{(c=3)} + \Phi_{JM}^{(C=4)} \]

\[ \Phi_{\alpha JM}(qqqcc) = A_{qqq} \left\{ \left( \text{isospin} \right)^{(c)}_{\alpha} \left( \text{spin} \right)^{(c)}_{\alpha} \left( \text{spatial} \right)^{(c)}_{\alpha} \right\}_{JM} \right\} \]

\[ (\text{spatial})^{(c)}_{\alpha} = \phi_{nl}^{(c)}(r_c) \psi_{\nu \lambda}^{(c)}(\rho_c) \phi^{(c)}_{ki}(s_c) \Phi_{n_R L M}^{(c)}(R_c) \]

\[ \phi_{n_R L c M}(R) = R^{L c} e^{-(R/\tilde{R}_{n R})^2} Y_{L c M}(\hat{R}) \quad \tilde{R}_{n R} = \tilde{R}_1 a^{n_R - 1} \quad (n_R = 1 - n_R^{\text{max}}) \]

Same procedure is taken for r, p, and s.
For the Pc(4380) and (4450), we consider the following 9 candidates states,

Total orbital angular momentum: \( L=0, 1, 2 \)
Total Spin : \( S=1/2, 3/2, 5/2 \)

For example, in the case of total orbital angular momentum \( L=0, S=1/2, 3/2, 5/2 \), we take s-waves for all coordinates.
\[(H-E)\Psi=0\]

By the diagonalization of Hamiltonian, we obtain \(N\) eigenstates for each \(J^{\pi}\).

Here, we use about 40,000 basis functions. Then, we obtained 40,000 eigenfunction for each \(J^{\pi}\). Here, we investigate \(J=1/2^\text{-}\), namely, \(L\) (total angular momentum)=0, \(S\) (total spin)=1/2.

\[L=0, S=1/2 \text{ for example}\]
First, we take two channels.
Confining channels

\[ 3 = 8 \oplus 1 \]

I take color singlet.
Next, we take two scattering channels.
Results before doing the scattering calculation

Do these states correspond to resonance states or discrete non-resonance continuum states?
useful method: real scaling method
often used in atomic physics

In this method, we artificially scale the range parameters of our Gaussian basis functions by multiplying a factor $\alpha$: 

$$r_n \rightarrow \alpha r_n \text{ in } r \exp\left(-\frac{r}{r_n}\right)^2 \text{ for example } 0.8 < \alpha < 1.5$$

and repeat the diagonalization of Hamiltonian for many value of $\alpha$.

What is the result in our pentaquark calculation?
Resonance state lifetimes from stabilization graphs

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The stabilization method (SM) pioneered by Taylor and co-workers has proven to be a valuable tool for estimating the energies of long-lived metastable states of electron-atom, electron-molecule, and atom-diatom complexes. In implementing the SM one searches for eigenvalues arising from a matrix representation of the relevant Hamiltonian $\mathbf{H}$ which are "stable" as the basis set used to construct $\mathbf{H}$ is varied.

To obtain lifetimes of metastable states, one can choose from among a variety of techniques (e.g., phase shift analysis, Fockhach projection "golden rule" formulas, Siegel-Weinstein methods, and complex coordinate scaling methods), many of which use the stabilized eigenvector as starting information. Here we demonstrate that one can obtain an estimate of the desired lifetime directly from the stabilization graph in a manner which makes a close connection with the complex coordinate rotation method (CRM) for which a satisfactory mathematical basis exists.

The starting point of our development is the observation that both the stable eigenvalue ($E_s$) and the eigenvalue(s) ($E_i$) which come from above and cross $E_s$ (see Fig. 1 and Refs. 9-11 and 12) vary in a nearly linear manner (with $\alpha$) near their avoided crossing point. This observation leads us to propose that the two eigenvalues arising in such an avoided crossing can be thought of as arising from two "uncoupled" states having energies $E_s(\alpha) = E_s(\alpha = \alpha_s)$ and $E_i(\alpha) = E_i(\alpha = \alpha_i)$, where $\alpha_s$ and $\alpha_i$ are the slopes of the linear parts of the stable and "continuum" eigenvalues, respectively. $\alpha_s$ is the value of $\alpha$ at which these two straight lines would intersect, and $\alpha$ is their common value at $\alpha = \alpha_s$. This modeling of $E_s$ and $E_i$ is simply based upon the observa-

![Stabilization graph for the $\frac{1}{2}$ state resonance state of LH$^+$ (Ref. 9).](image-url)
Example of real scaling
Not result of penta quark system

What is the result of our pentaquark calculation?
\[ \phi_{n_R L_c M}(\mathbf{R}) = R^{L_c} e^{-(R/\bar{R}_{n_R})^2} Y_{L_c M}(\mathbf{\hat{R}}) \]

\[ \bar{R}_{n_R} = \bar{R}_1 a^{n_R-1} \quad (n_R = 1 - n_{R}^{\text{max}}) \]

\[ R_{n_R} \rightarrow \alpha R_{n_R} \]
Results before doing the scattering calculation

Bound-state approximation

All states are melted into each meson-baryon continuum decaying state. Then, there is no resonant state between 4000 MeV to 4600 MeV.
One resonance at 4690 MeV

Much higher than the observed data

Why we have a resonance state at such higher energy?
This corresponds to resonant state, like a Feshbach resonant state. It is considered that other states are melted into various threshold.

For example, let us consider this state.

Confining channels
Conjecture: 4119 MeV can be describe as $\eta c+N$ like structure. However, due the restriction of the configurations, namely, by only C=4 and 5 channels, the mass energy is up than the $\eta c+N$ by about 200 MeV. In order to investigate this conjecture, we solve scattering states including $\eta c+N$ channel only with real scaling method. If 4119 MeV is $\eta c+N$ like structure, this state should be melted into $\eta c+N$ threshold.
$J/\Psi+N(4040)$

$\Lambda c+D(4171)$

$\Lambda c+D^{*}(4323)$

$\Sigma c+D(4353)$

$\Sigma c+D^{*}(4505)$

$\eta c+N(3900)$

$N+J/\Psi^{*}(4584)$

$N+\eta c^{*}(4544)$

$Melted into \eta c+N$ threshold

$4119$ MeV is $\eta c+N$ like structure!
L=0, S=1/2

J/Ψ+N channel

\[ \Sigma_c^*+D^*(4587) \]
\[ N+J/Ψ^*(4584) \]
\[ N+\eta_c^*(4544) \]
\[ \Sigma_c+D^*(4505) \]

Pc(4450)

\[ \Sigma_c+D(4353) \]
\[ \Lambda_c+D^*(4323) \]

\[ \Lambda_c+D(4171) \]
\[ J/Ψ+N(4040) \]
\[ \eta_c+N(3900) \]
$J^\pi = 1/2^-$

$L=0, S=1/2$

$\Sigma_c^*+D^*(4587)$
$N+J/\Psi^*(4584)$
$N+\eta_c^* (4544)$
$\Sigma_c+D^*(4505)$

$P_c(4450)$

$\Sigma_c+D(4353)$
$\Lambda_c+D^*(4323)$

$J/\Psi+N$ like structure

Melted into $J/\psi+N$
$J/\Psi+N(4040)$

$\Lambda_c+D(4171)$

$\Lambda_c+D^*(4323)$

$\Sigma_c+D(4353)$

$\Sigma_c+D^*(4505)$

$N^+\eta_c^*(4544)$

$N^+J/\Psi^*(4584)$

$\Sigma_c^*+D^*(4587)$

$\Lambda_c+D,\Lambda_c+D^*$

$P_c(4450)$

$\eta_c^*+N(3900)$

$L=0, S=1/2$

No coupled with any threshold then, exist as a resonant state

$J/\Psi^*+N$ like

Mixture of $\eta_c^*+N,\Lambda_c+D^*,\Sigma_c+D$

$\eta_c^*+N$ like structure

$J/\Psi+N$ like structure

$\Lambda_c+D(4171)$

$\eta_c+N$ like structure

$J/\Psi+N(4040)$

$\eta_c+N(3900)$
Summary

Motivated by the observed Pc(4380) and Pc(4450) systems at LHCb, we calculated energy spectra of $qqqcc^-$ system using non-relativistic constituent quark model. To obtain resonant states, we also use real scaling method.

Currently, we find no penta-quark like resonant states with $L=0, S=1/2$ at observed energy region. However, we have one resonant state at 4690 MeV. This can be penta-quark state.

Future work
To investigate the structure, now I am calculating density distribution. Also, the calculation with $L=0, S=3/2$ is still on going.
Thank you!