EFT Fits for Triple Higgs Couplings at Lepton Colliders

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Takehome messages

To measure Higgs potential deviations:

1. A general approach is possible and needed for unambiguous measure, interpret and sys improvement.  (Deviations mean a new physics.)

2. LEP EWPT precisions are often not good enough.

3. LHC, linear/circular e+e- can all do something good.
Higgs potential

- What we know now:

  \[ V'(h) = 0 \quad \text{at} \quad h = \nu \approx 250 \text{ GeV} \]
  \[ m_h^2 = V''(\nu), \quad m_h \approx 125 \text{ GeV} \]

- Measuring Higgs cubic coupling is the next step in extending our knowledge of the shape of \( V \):

  \[ \lambda_3 = \frac{1}{6} V'''(\nu) \]
Probing EWPhT, hierarchy (and eventually early universe)

Plots from 0711.3018

\[ V = \mu^2 |H|^2 + \lambda |H|^4 + \frac{1}{\Lambda^2} |H|^6 \]
How we usually think about triple Higgs measurement

\[
\frac{\delta \lambda}{\lambda} = 4/\text{ab}
\]

\[
\frac{\delta \lambda}{\lambda} = 100 \text{ TeV, } 30/\text{ab}
\]

\[
\frac{\delta \lambda}{\lambda} = 10 \text{ TeV, } 30/\text{ab}
\]

\[
\frac{\delta \lambda}{\lambda} = 5 \text{ TeV, } 30/\text{ab}
\]

\[
\frac{\delta \lambda}{\lambda} = 2 \text{ TeV, } 30/\text{ab}
\]
Extracting triple Higgs

- These results of Delta lambda might be good enough if the only question is to test the SM.

- If there’s a deviation, there’s a new physics! Not only lambda, but many others will be non-SM.

- To interpret Higgs-potential deviation from the SM, it is needed to separate deviations in the Higgs triple coupling from possible deviations of other SM parameters.
How shall we do?
HEFT as a model-independent framework

• In the HEFT, the deviation of the Higgs potential (triple Higgs coupling in particular) is associated with

\[ \Delta \mathcal{L} = -\frac{c_6 \lambda}{v^2} |\Phi^\dagger \Phi|^3 \]

• However, many other SM and EFT parameters contribute to the same double Higgs observables.
The 10 HEFT ops consist of:

(1) at least one Higgs or EW gauge,
(2) only Higgs, EW gauge and electrons
All 10 ops contribute!

\[ \Delta \mathcal{L} = \frac{c_H}{2v^2} \partial^\mu (\partial_\mu H) - \frac{c_6 \lambda}{v^2} (\Phi^\dagger \Phi)^3 \]

9 EFT ops (+ SM parameters) contribute to Zhh!
1 EFT op indirectly contribute to Zhh.
All 10 ops contribute!

\[ \Delta \mathcal{L} = \frac{c_H}{2v^2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi) + \frac{c_T}{2v^2} (\Phi^\dagger \not{D}^\mu \Phi)(\Phi^\dagger \not{D}_\mu \Phi) - \frac{c_6 \lambda}{v^2} (\Phi^\dagger \Phi)^3 \]

How (well) can they be constrained? Are these constraints good enough? Or, what challenges and what need to be done?

\[ +i \frac{c_{HE}}{v^2} (\Phi^\dagger \not{D}^\mu \Phi)(\bar{e} \gamma_\mu e) . \]

9 EFT ops (+ SM parameters) contribute to Zhh! 1 EFT op indirectly contribute to Zhh.
First of all, $c_6$ is our main parameter for triple Higgs coupling

$$
\Delta \mathcal{L} = \frac{c_H}{2v^2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi) + \frac{c_T}{2v^2} (\Phi^\dagger \stackrel{\leftrightarrow}{D}^\mu \Phi)(\Phi^\dagger \stackrel{\leftrightarrow}{D}_\mu \Phi) - \frac{c_6 \lambda}{v^2} (\Phi^\dagger \Phi)^3
$$

$$
+ \frac{g^2 c_{WW}}{m_W^2} \Phi^\dagger \Phi W^a_{\mu \nu} W^{a,\mu \nu} + \frac{4gg'c_{WB}}{m_W^2} \Phi^\dagger t^a \Phi W^a_{\mu \nu} B^{\mu \nu}
$$

$$
+ \frac{g'^2 c_{BB}}{m_W^2} \Phi^\dagger \Phi B_{\mu \nu} B^{\mu \nu} + \frac{g^3 c_{3W}}{m_W^2} \epsilon_{abc} W^a_{\mu \nu} W^{b \rho} \rho W^{c \mu
u}
$$

$$
+ i \frac{c_{HL}}{v^2} (\Phi^\dagger \stackrel{\leftrightarrow}{D}^\mu \Phi)(\overline{L} \gamma_\mu L) + 4i \frac{c'_{HL}}{v^2} (\Phi^\dagger t^a \stackrel{\leftrightarrow}{D}^\mu \Phi)(\overline{L} \gamma_\mu t^a L)
$$

$$
+ i \frac{c_{HE}}{v^2} (\Phi^\dagger \stackrel{\leftrightarrow}{D}^\mu \Phi)(\overline{e} \gamma_\mu e).
$$
But both $c_6$ and $c_H$ shape triple Higgs (and Higgs potential) $Z_{hh}$ alone cannot distinguish them.

$$\Delta \mathcal{L} = \frac{c_H}{2v^2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi) + \frac{c_T}{2v^2} (\Phi^\dagger \bar{D}_\mu \Phi) (\Phi^\dagger \bar{D}_\mu \Phi) - \frac{c_6 \lambda}{v^2} (\Phi^\dagger \Phi)^3$$

$$+ \frac{g^2 c_{WW}}{m_W^2} \Phi^\dagger \Phi W_{\mu\nu}^a W^{a\mu\nu} + \frac{4gg' c_{WB}}{m_W^2} \Phi^\dagger t^a \Phi W_{\mu\nu}^a B^{\mu\nu}$$

$$+ \frac{g'^2 c_{BB}}{m_W^2} \Phi^\dagger \Phi B_{\mu\nu} B^{\mu\nu} + \frac{g^3 c_{3W}}{m_W^2} \epsilon_{abc} W_{\mu\nu}^a W_{\rho\nu}^b W^{c\rho\mu}$$

$$+ i \frac{c_{HL}}{v^2} (\Phi^\dagger \bar{D}_\mu \Phi) (\bar{L} \gamma_\mu L) + 4i \frac{c'_{HL}}{v^2} (\Phi^\dagger t^a \bar{D}_\mu \Phi) (\bar{L} \gamma_\mu t^a L)$$

$$+ i \frac{c_{HE}}{v^2} (\Phi^\dagger \bar{D}_\mu \Phi) (\bar{e} \gamma_\mu e).$$
But $cH$ renormalizes Higgs field.
Thus single Higgs measurements can be relevant.

$$
\Delta \mathcal{L} = \frac{c_H}{2v^2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi) + \frac{c_T}{2v^2} (\Phi^\dagger \not{D} \mu \Phi)(\Phi^\dagger \not{D}_\mu \Phi) - \frac{c_6 \lambda}{v^2} (\Phi^\dagger \Phi)^3
$$

$$
+ \frac{g^2 c_{WW}}{m_W^2} \Phi^\dagger \Phi W^a_{\mu\nu} W^{a\mu\nu} + \frac{4gg'c_{WB}}{m_W^2} \Phi^\dagger t^a \Phi W^a_{\mu\nu} B^{\mu\nu}
$$

$$
+ \frac{g'^2 c_{BB}}{m_W^2} \Phi^\dagger \Phi B_{\mu\nu} B^{\mu\nu} + \frac{g^3 c_{3W}}{m_W^2} \epsilon_{abc} W^a_{\mu\nu} W^{b\nu}_\rho W^{c\rho\mu}
$$

$$
+ i \frac{c_{HL}}{v^2} (\Phi^\dagger \not{D} \mu \Phi)(\overline{L} \gamma_\mu L) + 4i \frac{c'_{HL}}{v^2} (\Phi^\dagger t^a \not{D} \mu \Phi)(\overline{L} \gamma_\mu t^a L)
$$

$$
+ i \frac{c_{HE}}{v^2} (\Phi^\dagger \not{D} \mu \Phi) (\overline{e} \gamma_\mu e).
$$
cT shifts hZZ coupling and famously mZ.

\[
\Delta \mathcal{L} = \frac{c_H}{2v^2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi) + \frac{c_T}{2v^2} (\Phi^\dagger \not{D^\mu} \Phi)(\Phi^\dagger \not{D_\mu} \Phi) - \frac{c_6 \lambda}{v^2} (\Phi^\dagger \Phi)^3
\]

\[
+ \frac{g^2 c_{WW}}{m_W^2} \Phi^\dagger \Phi W^a_{\mu \nu} W^{a \mu \nu} + \frac{4gg' c_{WB}}{m_W^2} \Phi^\dagger t^a \Phi W^a_{\mu \nu} B^{\mu \nu}
\]

\[
+ \frac{g'^2 c_{BB}}{m_W^2} \Phi^\dagger \Phi B_{\mu \nu} B^{\mu \nu} + \frac{g^3 c_{3W}}{m_W^2} \epsilon_{abc} W^a_{\mu \nu} W^{b \nu} \rho W^{c \rho \mu}
\]

\[
+i \frac{c_{HL}}{v^2} (\Phi^\dagger \not{D^\mu} \Phi)(\bar{L} \gamma_\mu L) + 4i \frac{c'_{HL}}{v^2} (\Phi^\dagger t^a \not{D^\mu} \Phi)(\bar{L} \gamma_\mu t^a L)
\]

\[
+i \frac{c_{HE}}{v^2} (\Phi^\dagger \not{D^\mu} \Phi)(\bar{e} \gamma_\mu e) .
\]
cWW, cBB, cWB
renormalize gauge boson interactions and masses
and induce hVV, hhVV interactions

\[ \Delta \mathcal{L} = \frac{c_H}{2v^2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi) + \frac{c_T}{2v^2} (\Phi^\dagger \not\!D^\mu \Phi) (\Phi^\dagger \not\!D_\mu \Phi) - \frac{c_6 \lambda}{v^2} (\Phi^\dagger \Phi)^3 \]

\[ + \frac{g^2 c_{WW}}{m_W^2} \Phi^\dagger \Phi W^a_{\mu \nu} W^{a \mu \nu} + \frac{4gg'c_{WB}}{m_W^2} \Phi^\dagger t^a \Phi W^a_{\mu \nu} B^{\mu \nu} \]

\[ + \frac{g'^2 c_{BB}}{m_W^2} \Phi^\dagger \Phi B_{\mu \nu} B^{\mu \nu} + \frac{g^3 c_{3W}}{m_W^2} \epsilon_{abc} W^a_{\mu \nu} W^{b \nu} W^{c \rho \mu} \]

\[ + i \frac{c_{HL}}{v^2} (\Phi^\dagger \not\!D^\mu \Phi)(\overline{L} \gamma_\mu L) + 4i \frac{c'_{HL}}{v^2} (\Phi^\dagger t^a \not\!D^\mu \Phi)(\overline{L} \gamma_\mu t^a L) \]

\[ + i \frac{c_{HE}}{v^2} (\Phi^\dagger \not\!D^\mu \Phi)(\overline{e} \gamma_\mu e) . \]
cHL, cHL’, cHE modifies Zee, Zheee, Zhhee couplings

\[ \Delta \mathcal{L} = \frac{c_H}{2v^2} \partial_\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi) + \frac{c_T}{2v^2} (\Phi^\dagger \not{D} \mu \Phi)(\Phi^\dagger \not{D}_\mu \Phi) - \frac{c_6 \lambda}{v^2} (\Phi^\dagger \Phi)^3 \\
+ \frac{g^2 c_{WW}}{m_W^2} \Phi^\dagger \Phi W^{a \mu \nu} W^{a \mu \nu} + \frac{4gg' c_{WB}}{m_W^2} \Phi^\dagger t^a \Phi W^{a \mu \nu} B^{\mu \nu} \\
+ \frac{g'^2 c_{BB}}{m_W^2} \Phi^\dagger \Phi B_{\mu \nu} B^{\mu \nu} + \frac{g^3 c_{3W}}{m_W^2} \epsilon_{abc} W^{a \mu \nu} W^{b \nu \rho} W^{c \rho \mu} \\
+ i \frac{c_{HL}}{v^2} (\Phi^\dagger \not{D} \mu \Phi) (\bar{L} \gamma_\mu L) + 4i \frac{c'_{HL}}{v^2} (\Phi^\dagger t^a \not{D} \mu \Phi) (\bar{L} \gamma_\mu t^a L) \\
+ i \frac{c_{HE}}{v^2} (\Phi^\dagger \not{D} \mu \Phi) (\bar{e} \gamma_\mu e) . \]
Lastly, although c3W doesn’t directly contribute to Zhh, it affects TGC measurements that determine other ops.

\[
\Delta \mathcal{L} = \frac{c_H}{2v^2} \partial_\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi) + \frac{c_T}{2v^2} (\Phi^\dagger \not{D}^\mu \Phi)(\Phi^\dagger \not{D}_\mu \Phi) - \frac{c_6\lambda}{v^2} (\Phi^\dagger \Phi)^3 \\
+ \frac{g^2 c_{WW}}{m_W^2} \Phi^\dagger \Phi W^a_{\mu\nu} W^{a\mu\nu} + \frac{4gg'c_{WB}}{m_W^2} \Phi^\dagger t^a \Phi W^a_{\mu\nu} B^{\mu\nu} \\
+ \frac{g'^2 c_{BB}}{m_W^2} \Phi^\dagger \Phi B_{\mu\nu} B^{\mu\nu} + \frac{g^3 c_{3W}}{m_W^2} \epsilon_{\alpha\beta\gamma} W^a_{\mu\nu} W^{b\nu}_{\rho\gamma} W^{c\rho\mu} \\
+ \frac{i c_{HL}}{v^2} (\Phi^\dagger \not{D}^\mu \Phi)(\overline{L} \gamma_\mu L) + 4i \frac{c'_{HL}}{v^2} (\Phi^\dagger t^a \not{D}^\mu \Phi)(\overline{L} \gamma_\mu t^a L) \\
+ \frac{i c_{HE}}{v^2} (\Phi^\dagger \not{D}^\mu \Phi)(\overline{e} \gamma_\mu e). 
\]
\[ \Delta \mathcal{L} = \frac{c_H}{2v^2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi) + \frac{c_T}{2v^2} (\Phi^\dagger \gamma^\mu \Phi) (\Phi^\dagger \gamma^\mu \Phi) - \frac{c_6 \lambda}{v^2} (\Phi^\dagger \Phi)^3 \\
+ \frac{g^2}{m_W^2} \Phi^\dagger \Phi B_{\mu \nu} B^{\mu \nu} + \frac{g^3 c_3 W}{m_W^2} \epsilon_{abc} W^a_{\mu \nu} W^b_{\rho \nu} W^c_{\rho \mu} \\
+ i \frac{c_{HL}}{v^2} (\Phi^\dagger \gamma^\mu \Phi) (\bar{L} \gamma_\mu L) + 4i \frac{c'_{HL}}{v^2} (\Phi^\dagger t^a \gamma^\mu \Phi) (\bar{L} \gamma_\mu t^a L) \\
+ i \frac{c_{HE}}{v^2} (\Phi^\dagger \gamma^\mu \Phi) (\bar{e} \gamma_\mu e) . \]
## EWPT (LEP) + mh

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Measured</th>
<th>$\sigma$</th>
<th>PDG SM fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha^{-1}(m_Z)$</td>
<td>128.9220</td>
<td>(78)</td>
<td>same</td>
</tr>
<tr>
<td>$G_F$</td>
<td>1.1663787</td>
<td>(6)</td>
<td>same</td>
</tr>
<tr>
<td>$m_Z$</td>
<td>91.1876</td>
<td>(21)</td>
<td>91.1880</td>
</tr>
<tr>
<td>$m_W$</td>
<td>80.385</td>
<td>(15)</td>
<td>80.361</td>
</tr>
<tr>
<td>$m_h$</td>
<td>125.09</td>
<td>(24)</td>
<td>same</td>
</tr>
<tr>
<td>$A_\ell$</td>
<td>0.1470</td>
<td>(13)</td>
<td>0.1480</td>
</tr>
<tr>
<td>$\Gamma(Z \rightarrow \ell^+\ell^-)$</td>
<td>83.385</td>
<td>(15)</td>
<td>83.995</td>
</tr>
</tbody>
</table>

Using these inputs, we can obtain a covariance matrix for 7 of our coefficients

$$\frac{\delta g}{g}, \frac{\delta g'}{g'}, \frac{\delta v}{v}, \frac{\delta \lambda}{\lambda}, c_T, c_{HL}, c_{HE}$$

with errors on single parameters at the $10^{-3}$ level.

NB: Interestingly, cWW and cBB cancel in these EWPT obs.
$e^+e^- \rightarrow WW$ (TGC)

$$\Delta \mathcal{L}_{TGC} = ig_V \left\{ g_{1V} V^\mu (\hat{W}_{\mu\nu} W^{+\nu} - \hat{W}_{\mu\nu} W^{-\nu}) + \kappa_{V} W^{+}_\mu W^{-}_\nu \hat{V}^{\mu\nu} \right\} + \frac{\lambda_{V}}{m_W^2} \hat{W}^-_{\mu} \hat{W}^\rho_{\nu} \hat{V}^{\mu\nu},$$

$e^+e^- \rightarrow WW$ physics is described by 3 independent coeffs, constraining 3 additional HEFT ops ($c_{WB}$,$c_{HL'}$,$c_{3W}$).

$$g_{1Z} = 1 + \frac{1}{c_0^2 - s_0^2} \left( -8 \frac{s_0^2}{c_0^2} c_{WB} + \frac{1}{2} c_T - c'_{HL} \right),$$

$$\kappa_Z = g_{1Z} - 8 \frac{s_0^2}{c_0^2} c_{WB}, \quad \kappa_A = g_{1A}$$

$$\lambda_Z = x c_{3W}, \quad \lambda_A = x c_{3W}$$

NB: Interestingly, $c_{WW}$ and $c_{BB}$ cancel out again!

Marchesini 2011

$$\begin{pmatrix}
7.7 & 5.6 & 3.0 \\
5.6 & 7.6 & 2.8 \\
3.0 & 2.8 & 15.6
\end{pmatrix} \times 10^{-4}$$
LHC Single Higgs

\[ \Gamma(h \rightarrow \gamma\gamma) = \Gamma(h \rightarrow \gamma\gamma)_0 (1 + 528s_w^2 (8c_{WW} - 2(8c_{WB}) + 8c_{BB}) + \cdots) \]

\[ \Gamma(h \rightarrow \gamma Z) = \Gamma(h \rightarrow \gamma Z)_0 (1 + 290s_w c_w (8c_{WW} - (1 - t_w^2)(8c_{WB}) - t_w^2 8c_{BB}) + \cdots) \]

The ratios of BRs including gamma gamma, gamma Z, ZZ can be best measured at \( O(1-10)\% \) at LHC.

(\( \rightarrow \) 2 more constraints: cWW and cBB finally.)

More precise and direct width measurements can be possible if combined with lepton collider total width.
After all, $\sigma(e^+e^- \rightarrow Zh)$ is another function of $c_H, c_{WW}, c_{BB}$.

By combining the two single Higgs measurements, the three coefficients can be constrained to $O(0.1\%)$ except for $c_H \sim O(1)\%$ (see later).
Finally, $e^+e^- \rightarrow Zhh$
Finally, $e^+e^- \rightarrow Zhh$

$$\sigma(e^+e^- \rightarrow Zhh)_{SM} = 1 + 0.056c_6 - 4.15c_H + 15.1(8c_{WW}) + \cdots + 62.1(c_{HL} + c'_{HL}) - 53.5c_{HE}$$

Surprisingly, in addition to the well-known $c_6$ dependence, there are several contributions with large coefficients!
Finally, $e^+e^- \rightarrow Zhh$

\[
\frac{\sigma(e^+e^- \rightarrow Zhh)}{SM} = 1 + 0.056c_6 - 4.15c_H + 15.1(8c_{WW}) + \cdots + 62.1(c_{HL} + c'_{HL}) - 53.5c_{HE}
\]

<table>
<thead>
<tr>
<th>$A$</th>
<th>$[&lt; A^2 &gt;]^{1/2}$</th>
<th>$[&lt; A^2 &gt;]^{1/2}$</th>
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<tbody>
<tr>
<td>$c_H$</td>
<td>4.8</td>
<td>$(c_{HL} + c'_{HL})$</td>
</tr>
<tr>
<td>$(8c_{WW})$</td>
<td>0.11</td>
<td>$c_{HE}$</td>
</tr>
<tr>
<td>$(-4.15c_H + 15.1(8c_{WW}))$</td>
<td>21</td>
<td>$62.1(c_{HL} + c'<em>{HL}) - 53.5c</em>{HE}$</td>
</tr>
</tbody>
</table>

After all, only $c_6 \sim 28\%$ is possible
(e.g. ILC 500 2/ab).
Challenge 1: s/mZ^2 enhancement

cHE, cHL, cHL’ give contact-interaction contributions enhanced by s/mz^2 ~ 50 at 500 GeV.

To measure c6 at 1% level, these ops shall be measured at 0.01% level which is only marginally achieved at LEP EWPT.
Challenge 2: cH measurements from Zh

The $e^+e^- \rightarrow Zh$, constraining $c_H$, also suffers from the same $s/m_Z^2$ enhancement. LEP precisions on $c_{HL}$ etc leads to poor $c_H \sim 1\%$

$$
\frac{(s - m_Z^2)}{2m_Z^2(1/2 - s_w^2)}(c_{HL} + c'_HL) - \frac{(s - m_Z^2)}{2m_Z^2(s_w^2)}c_{HE}.
$$
Fortunately, we have other single Higgs obs

\[ \delta \Gamma(h \rightarrow b \bar{b}) = 1 - c_H + 2c_b \Phi \]

Many more observables (particularly from LHC and 500 e+e-). They depend on additional HEFT parameters as well as the same enhancement.

But many observables with different dependences.
Also, the enhancements at 250 GeV are less severe

<table>
<thead>
<tr>
<th>$c_I$</th>
<th>prec. EW</th>
<th>+ Zh</th>
<th>$c_I$</th>
<th>prec. EW</th>
<th>+ Zh</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_T$</td>
<td>0.011</td>
<td>0.041</td>
<td>$c_T$</td>
<td>0.011</td>
<td>0.048</td>
</tr>
<tr>
<td>$c_{HE}$</td>
<td>0.043</td>
<td>0.040</td>
<td>$c_{HE}$</td>
<td>0.043</td>
<td>0.047</td>
</tr>
<tr>
<td>$c_{HL}$</td>
<td>0.042</td>
<td>0.027</td>
<td>$c_{HL}$</td>
<td>0.042</td>
<td>0.032</td>
</tr>
<tr>
<td>$c'_{HL}$</td>
<td>−</td>
<td>0.026</td>
<td>$c'_{HL}$</td>
<td>−</td>
<td>0.028</td>
</tr>
<tr>
<td>$8c_{WB}$</td>
<td>−</td>
<td>0.067</td>
<td>$8c_{WB}$</td>
<td>−</td>
<td>0.076</td>
</tr>
<tr>
<td>$8c_{BB}$</td>
<td>−</td>
<td>0.15</td>
<td>$8c_{BB}$</td>
<td>−</td>
<td>0.16</td>
</tr>
<tr>
<td>$8c_{WW}$</td>
<td>−</td>
<td>0.11</td>
<td>$8c_{WW}$</td>
<td>−</td>
<td>0.13</td>
</tr>
<tr>
<td>$c_H$</td>
<td>−</td>
<td>4.78</td>
<td>$c_H$</td>
<td>−</td>
<td>1.12</td>
</tr>
</tbody>
</table>

\[ - \frac{(s - m_Z^2)}{2m_Z^2(s_w^2)} c_{HE} \]
Combining all systematically, 5% measurements of $c_6$ is possible!

(e.g. ILC 250 + 500 + LHC)

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_H$</td>
<td>0.65</td>
<td>$(c_{HL} + c'_{HL})$</td>
<td>0.014</td>
</tr>
<tr>
<td>$(8c_{WW})$</td>
<td>0.039</td>
<td>$c_{HE}$</td>
<td>0.009</td>
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<tr>
<td>$(-4.15c_H + 15.1(8c_{WW}))$</td>
<td>2.8</td>
<td>$62.1(c_{HL} + c'<em>{HL}) - 53.5c</em>{HE}$</td>
<td>0.85</td>
</tr>
</tbody>
</table>


What have we combined?

- Beam polarization: cHE vs. cHL, cHL’, finer handle.
- Luminosity: \( \sqrt{N} \) improvement.
- Energy: Bigger sensitivity to \( s/mz^2 \) enhancement, and new channels such as VBF.
- LHC: many single Higgs observables, available early on.

Each lead to similar deg of resolution to the \( s/mZ^2 \) issue.
All have pros and cons. No single one is best.
Might be better ideas too.
General approach gives more than just adding all.

Considering all ops is not just adding all the small errors propagated.

If the results turn out to be not good enough,

we can systematically identify which parameter and which observables to be foremost importantly improved.
Takehome messages

0. Deviations from the SM Higgs potential means a new physics.

1. Only a general approach allows unambiguous measure, interpret and sys improvement.

2. LEP EWPT observables are often not good enough.

3. LHC, ILC/CEPC can all do something good.
Thank you