The Higgs couplings and self-coupling in the EFT framework

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and current work, S. Di Vita, G. Durieux, C. Grojean, JG, Z. Liu, G. Panico,
M. Riembau, T. Vantalon
Introduction

- Higgs and nothing else? What next?

- An $e^+ e^-$ collider is an obvious direction to go.

- Higgs factory ($e^+ e^- \rightarrow hZ$ at 240-250 GeV, $e^+ e^- \rightarrow \nu\bar{\nu} h$ at higher energies), and many more other measurements.

- The scale of new physics $\Lambda$ is large $\Rightarrow$ effective field theory (EFT) is a good description at low energy.

- A global analysis of the Higgs coupling constraints, in the EFT framework. See also e.g.,
  - [arXiv:1510.04561, 1701.04804] Ellis et al.,
  - [arXiv:1708.08912, 1708.09079] Peskin et al. (See Sunghoon’s talk)

- Robust constraints on the triple Higgs coupling at both circular and linear colliders. (current work, to appear soon)
Higgs measurements

- $e^+ e^- \rightarrow hZ$, cross section maximized at around 250 GeV.
- $e^+ e^- \rightarrow \nu \bar{\nu} h$, cross section increases with energy.
- $e^+ e^- \rightarrow t\bar{t}h$, can be measured with $\sqrt{s} \gtrsim 500$ GeV.
- $e^+ e^- \rightarrow Zhh$ and $e^+ e^- \rightarrow \nu \bar{\nu} hh$ (triple Higgs coupling).
Introduction

Global fit in the EFT framework

Results

Conclusion

$k$ framework vs. EFT

From the CEPC preCDR and “Physics Case for the ILC” ([arXiv:1506.05992])

- Conventionally, the constraints on Higgs couplings are obtained from global fits in the so-called “$k$” framework.

$$g_{h}^{SM} \rightarrow \kappa g_{h}^{SM}.$$ 

- Anomalous couplings such as $hZ_{\mu\nu} Z_{\mu\nu}$ or $hZ_{\mu} \partial_{\nu} Z^{\mu\nu}$ are assumed to be zero.

- EFT framework
  - Assuming $\nu \ll \Lambda$, leading contribution from BSM physics are well-parameterized by D6 operators.
  - Gauge invariance is built in the parameterization.

- Lots of parameters! (Is it practical to perform a global fit?)
The “12-parameter” framework in EFT

- Assume the new physics
  - is CP-even,
  - does not generate dipole interaction of fermions,
  - only modifies the diagonal entries of the Yukawa matrix,
  - has no corrections to $Z$-pole observables and $W$ mass (more justified if the machine will run at $Z$-pole).

- Additional measurements
  - Triple gauge couplings from $e^+ e^- \rightarrow WW$. (The LEP constraints will be improved at future colliders.)
  - Angular observables in $e^+ e^- \rightarrow hZ$. (see e.g. [arXiv:1512.06877] N. Craig, JG, Z. Liu, K. Wang)
  - $h \rightarrow Z\gamma$ is also important.

- Only 12 combinations of operators are relevant for the measurements considered (with the inclusion of the Yukawa couplings of $t$, $c$, $b$, $\tau$, $\mu$).

- All 12 EFT parameters can be constrained reasonably well in the global fit!
EFT basis

- We work in the Higgs basis (LHCHXSWG-INT-2015-001, A. Falkowski) with the following 12 parameters,
  
  \[ \delta c_Z, \ c_{\text{ZZ}}, \ c_{\text{Z} \Box}, \ c_{\gamma \gamma}, \ c_{Z \gamma}, \ c_{gg}, \ \delta y_t, \ \delta y_c, \ \delta y_b, \ \delta y_\tau, \ \delta y_\mu, \ \lambda_Z. \]

- The Higgs basis is defined in the broken electroweak phase.
  
  - \[ \delta c_Z \leftrightarrow hZ^\mu Z_\mu, \ c_{\text{ZZ}} \leftrightarrow hZ^{\mu \nu} Z_{\mu \nu}, \ c_{\text{Z} \Box} \leftrightarrow hZ_\mu \partial_\nu Z^{\mu \nu}. \]

- Couplings of \( h \) to \( W \) are written in terms of couplings of \( h \) to \( Z \) and \( \gamma \).

- 3 aTGC parameters (\( \delta g_{1,Z}, \delta \kappa_\gamma, \lambda_Z \)), 2 written in terms of Higgs parameters.

- It can be easily mapped to the following basis with D6 operators.

\[
\begin{align*}
\mathcal{O}_H &= \frac{1}{2} (\partial_\mu |H|^2)^2 \\
\mathcal{O}_{WW} &= g^2 |H|^2 W^a_{\mu \nu} W^a_{\mu \nu} \\
\mathcal{O}_{BB} &= g^2 |H|^2 B_{\mu \nu} B^{\mu \nu} \\
\mathcal{O}_{HW} &= ig (D^\mu H)^\dagger \sigma^a (D^\nu H) W^a_{\mu \nu} \\
\mathcal{O}_{HB} &= ig' (D^\mu H)^\dagger (D^\nu H) B_{\mu \nu} \\
\mathcal{O}_{GG} &= g_s^2 |H|^2 G^A_{\mu \nu} G^A_{\mu \nu} \\
\mathcal{O}_{yu} &= y_u |H|^2 \tilde{Q}_L \tilde{H} u_R \\
\mathcal{O}_{yd} &= y_d |H|^2 \tilde{Q}_L H d_R \\
\mathcal{O}_{ye} &= y_e |H|^2 \tilde{L}_L H e_R \\
\mathcal{O}_{3W} &= \frac{1}{3!} g \epsilon_{abc} W^a_{\mu \nu} W^b_{\nu \rho} W^c_{\rho \mu}
\end{align*}
\]
Results of the “12-parameter” fit

Assuming the following run plans  (no official plan for CEPC 350 GeV run)

- CEPC 240 GeV (5/ab) + 350 GeV (200/fb)
- FCC-ee 240 GeV (10/ab) + 350 GeV (2.6/ab)
- ILC 250 GeV (2/ab) + 350 GeV (200/fb) + 500 GeV (4/ab)
- CLIC 350 GeV (500/fb) + 1.4 TeV (1.5/ab) + 3 TeV (2/ab)
Impact of a 350 GeV run

Advantages of the runs at higher energies

- Much better measurement of the $WW$ fusion process ($e^+ e^- \rightarrow \nu \bar{\nu} h$).
- Probing $e^+ e^- \rightarrow hZ$ at different energies.
- Improving constraints on aTGCs ($e^+ e^- \rightarrow WW$).

Very helpful in resolving the degeneracies among parameters!

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Beam polarization helps discriminate different parameters. 

- Two polarization configurations are considered, \( P(e^-, e^+) = (-0.8, +0.3) \) and \( (+0.8, -0.3) \).
- \( F(-+) \) in the range of 0.6-0.8 gives an optimal overall results.

- Runs with different polarizations probe different combinations of EFT parameters in Higgs production.
Triple Higgs coupling in the EFT framework


- **Triple Higgs coupling**
  
  \[ \kappa_\lambda \equiv \frac{\lambda_3}{\lambda_{3,SM}}, \quad \delta \kappa_\lambda \equiv \kappa_\lambda - 1 = c_6 - \frac{3}{2} c_H, \quad \text{with} \quad \mathcal{L} \supset -\frac{c_6 \lambda}{v^2} (H^\dagger H)^3 \]

- **HL-LHC:** \( \sim \mathcal{O}(1) \) determination. (\( \kappa_\lambda \in [-0.8, 7.7] \) at 95% CL from Atlas projection for the \( b\bar{b}\gamma\gamma \) channel, ATL-PHYS-PUB-2017-001)

- **Linear colliders:** direct measurements with \( e^+ e^- \to Zhh, e^+ e^- \to \nu\bar{\nu}hh \).

- **Circular colliders:** probe indirectly via the loop contribution in \( e^+ e^- \to hZ \). ([arXiv:1312.3322] M. McCullough)
  - TLEP (FCC-ee) 240 GeV: \( |\delta \kappa_\lambda| \lesssim 28\% \) assuming all other Higgs couplings are SM-like.
  - **What if other Higgs couplings are not SM-like?**

- **A global fit with 12+1 parameters!**
Triple Higgs coupling at circular colliders (240 & 350 GeV)

- One loop corrections to all Higgs couplings (production and decay).
- 240 GeV: $hZ$ near threshold (more sensitive to $\delta \kappa_\lambda$)
- at 350 GeV:
  - WW fusion
  - $hZ$ at a different energy
- $h \rightarrow WW^*/ZZ^*$ also have some discriminating power (but turned out to be not enough).
Runs at both 240 GeV and 350 GeV are needed to obtain good constraints on $\delta\kappa_\lambda$!

Bounds are further improved if combined with HL-LHC measurements.

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The Higgs couplings and self-coupling in the EFT framework

Double-Higgs measurements \((e^+e^- \rightarrow Zhh \& e^+e^- \rightarrow \nu \bar{\nu} hh)\)

- Destructive interference in \(e^+e^- \rightarrow \nu \bar{\nu} hh\)! The square term is important.

- \(hh\) invariant mass distribution helps discriminate the “2nd solution.”
Inputs:

- 500 GeV (4 ab$^{-1}$): $\sigma(Zhh)$ measured to 16.8% [C. F. Dürig, PhD thesis, Hamburg U. (2016)]
- 1 TeV (2 ab$^{-1}$): $\sigma(\nu\bar{\nu}hh)$ measured to 2.7$\sigma$ significance $\Rightarrow \sim 37\%$ [talk by Dürig at ALCW15]

Complementarity between the 500 GeV run and the 1 TeV run.

Single Higgs measurements provide non-negligible improvement.

- up to 500 GeV: $[-0.31, +0.28]$ $\rightarrow$ $[-0.26, +0.25]$,
- up to 1 TeV: $[-0.20, +0.23]$ $\rightarrow$ $[-0.18, +0.20]$
\( \chi^2 \text{ vs. } \delta_{K\lambda}, \text{ CLIC} \)

**Input:**
- \( \sigma(\nu \bar{\nu} hh) \) measured to 44% at 1.4 TeV and 20% at 3 TeV (Higgs Physics at the CLIC Electron-Positron Linear Collider [arXiv:1608.07538], Assuming unpolarized beam.)
- \( \sigma(Zhh) \) measured to \( \sim 50\% \) at 1.4 TeV (our own naive estimation).

**The measurement of \( Zhh \) or the \( M_{hh} \) distribution of \( \nu \bar{\nu} hh \) can help resolve the “2nd solution.”**

**The bounds on \( \delta_{K\lambda} \) can be further improved by having a \( hZ \) threshold run (e.g., by combining with CEPC 240 GeV or ILC 250 GeV).**
A summary of the (future) bounds on $\delta \kappa_\lambda$

**Introduction**

**Global fit in the EFT framework**

**Results**

**Conclusion**

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The Higgs couplings and self-coupling in the EFT framework

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**bounds on $\delta \kappa_\lambda$ from EFT global fit**

-2 | -1 | 0 | 1 | 2 | 3

| HL-LHC | | | | | |
|--------|---|---|---|---|
| $-0.62$ | $0.12$ |

-68%,95% CL bounds, lepton collider only

68%,95% CL bounds, combined with HL-LHC

68% CL bounds (combined with HL-LHC)

68%,95% CL bounds, 1h only (w/ HL-LHC 1h)

HL-LHC results from arXiv:1704.01953

| CEPC & FCC-ee | | | | |
|---------------|---|---|---|
| $-0.81$ | $1.04$ |
| $-1.25$ | $1.35$ |
| $-0.39$ | $0.40$ |
| $-0.77$ | $0.80$ |

240GeV(5/ab) only (CEPC)

240GeV(5/ab)+350GeV(200/fb)

240GeV(5/ab)+350GeV(1.5/ab) (FCC-ee)

FCC-ee with zero aTGCs

| ILC | | | | |
|-----|---|---|---|
| $-0.85$ | $1.13$ |
| $-1.59$ | $1.94$ |
| $-0.72$ | $0.83$ |
| $-1.37$ | $1.83$ |

250GeV(2/ab) only

250GeV(2/ab)+350GeV(200/fb)

above + 500GeV(4/ab)

above + 1TeV(2/ab)

| CLIC | | | | |
|------|---|---|---|
| $-0.21$ | $0.33$ |
| $-0.39$ | $1.56$ |
| $-0.21$ | $0.32$ |
| $-0.39$ | $1.44$ |
| $-0.18$ | $0.28$ |
| $-0.32$ | $1.07$ |

350GeV(500/fb)+1.4TeV(1.5/ab)+3TeV(2/ab)

+ Zh at 1.4 TeV

binned $M_{hh}$ in $\nu \bar{\nu} hh$ (4 bins)

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$\delta \kappa_\lambda \equiv \frac{\lambda_3}{\Lambda_{\text{SM}}^3} - 1$
Conclusion

- Lepton colliders are great for Higgs precision measurements!

- It makes sense to go beyond the “$\kappa$” frame and study Higgs physics in the EFT framework.

- We can obtain strong and robust constraints on the coefficients of the relevant dimension-6 operators!
  - If discrepancy is observed, the EFT global fit can help identify the underlying new physics!

- We can obtain robust constraints on the triple Higgs coupling!
backup slides
Global Determinant Parameter (GDP ≡ $\frac{2^n}{\sqrt{\det \sigma^2}}$).

- Ratios of GDPs are basis-independent.
- Smaller GDP $\rightarrow$ better precision!
The Higgs couplings and self-coupling in the EFT framework

If you don’t like the Higgs basis...

Results in the SILH’(-like) basis ($O_{W,B} \rightarrow O_{WW, WB}$)

$$\mathcal{L}_{D6} = \frac{c_H}{v^2} O_H + \frac{\kappa_{WW}}{m_W^2} O_{WW} + \frac{\kappa_{BB}}{m_W^2} O_{BB} + \frac{\kappa_{HW}}{m_W^2} O_{HW} + \frac{\kappa_{HB}}{m_W^2} O_{HB}$$

$$+ \frac{\kappa_{GG}}{m_W^2} O_{GG} + \frac{\kappa_{3W}}{m_W^2} O_{3W} + \sum_{f=t,c,b,\tau,\mu} \frac{c_y_f}{v^2} O_{y_f}.$$
angular observables in $e^+ e^- \rightarrow hZ$

- Angular distributions in $e^+ e^- \rightarrow hZ$ can provide information in addition to the rate measurement alone.

- Previous studies

- 6 independent asymmetry observables from 3 angles

$$A_{\theta_1}, A_{\phi}^{(1)}, A_{\phi}^{(2)}, A_{\phi}^{(3)}, A_{\phi}^{(4)}, A_{c\theta_1,c\theta_2}.$$

- Focusing on leptonic decays of $Z$ (good resolution, small background, statistical uncertainty dominates).
The importance of combining all measurements

- The results are much worse if we only include the rates of Higgs measurements alone!
- There is some overlap in the information from different measurements.
- Measurements at different energies can be very helpful.
The precision reach of $\delta_{K\lambda}$ at the low-energy ILC

- $e^+ e^- \rightarrow hZ$ is more sensitive to $\delta_{K\lambda}$ near the threshold (240 GeV vs. 250 GeV).

- Polarization doesn’t help too much here...
Impact of $\delta K_\lambda$ on the other parameters

- Adding one more parameter could worsen the bounds on others.
- The effect is under control if the degeneracies are well-resolved.
- The HL-LHC bounds on $\delta K_\lambda$ can also help.
**Impact of the single Higgs measurements**

- What if the single Higgs measurements are much better or much worse?
  - Much better: can further improve the bounds on $\delta \kappa_\lambda$ from double-Higgs measurements.
  - Much worse: can significantly worsen the bounds on $\delta \kappa_\lambda$ from double-Higgs measurements.
Impact of the Higher energy runs

**precision reach at CEPC with different luminosities at 350 GeV**
- CEPC 240GeV (5ab) only
- CEPC 240GeV (5ab) + 350GeV (200/fb)
- CEPC 240GeV (5ab) + 350GeV (500/fb)
- CEPC 240GeV (5ab) + 350GeV (1/ab)
- CEPC 240GeV (5ab) + 350GeV (2/ab)

**precision reach at FCC-ee with different luminosities at 350 GeV**
- FCC-ee 240GeV (10/ab) only
- FCC-ee 240GeV (10/ab) + 350GeV (500/fb)
- FCC-ee 240GeV (10/ab) + 350GeV (2.6/ab)
- FCC-ee 240GeV (10/ab) + 350GeV (5/ab)
- FCC-ee 240GeV (10/ab) + 350GeV (5/ab)

**precision reach at ILC with different run scenarios at 1 TeV**
- ILC 250GeV (2ab; 2 polarizations)
- ILC 250GeV (2ab; 1 polarization) + 350GeV (200/fb)
- ILC 250GeV (2ab; 2 polarizations) + 350GeV (200/fb)
- ILC 250GeV (2ab; 1 polarization) + 350GeV (200/fb) + 500GeV (4/ab)
- ILC 250GeV (2ab; 2 polarizations) + 350GeV (200/fb) + 500GeV (4/ab)

Light shades for columns 2&3: $e^+e^-\to WW$ measurements at 1 TeV not included.
\[ e^+ e^- \rightarrow \nu \bar{\nu} h \]

- It is hard to separate the WW fusion process from \( e^+ e^- \rightarrow hZ, Z \rightarrow \nu \bar{\nu} \) at 240 GeV.
- It is not consistent to focus on one process and treat the other one as SM-like!
- For CEPC/FCC-ee 240 GeV, we analyze the combined \( e^+ e^- \rightarrow \nu \bar{\nu} h \) process, assuming new physics can contribute to both processes.
$e^+e^- \rightarrow WW$

- $e^+e^- \rightarrow WW$ offers a great way to probe the anomalous triple gauge couplings (aTGCs, parameterized by $\delta g_{1,Z}$, $\delta \kappa_\gamma$, $\lambda_Z$).
- $\delta g_{1,Z}$ and $\delta \kappa_\gamma$ are related to Higgs observables.
- CEPC with $5\text{ ab}^{-1}$ data at 240 GeV can produce $\sim 9 \times 10^7 \ e^+e^- \rightarrow WW$ events.
- With such large statistics, the aTGCs can be very well constrained ([1507.02238] Bian, Shu, Zhang), but with two potential issues:
  - Systematic uncertainties can be important!
  - If $e^+e^- \rightarrow WW$ is measured more precisely than the $Z$-pole measurements, is it still ok to assume the fermion gauge couplings are SM-like?
The interplay between Higgs and TGC

- \( \delta g_{1,Z} , \delta \kappa_{\gamma} \leftrightarrow c_{ZZ} , c_{Z\Box} , c_{\gamma\gamma} , c_{Z\gamma} \)

- We try different assumptions on the systematic uncertainties (in each bin with the differential distribution divided into 20 bins).

- Detailed study of \( e^+ e^- \rightarrow WW \) required to estimate the systematic uncertainties!
Asymmetry observables

\[ A_{\theta_1} = \frac{1}{\sigma} \int_{-1}^{1} d\cos \theta_1 \ sgn(\cos(2\theta_1)) \ \frac{d\sigma}{d\cos \theta_1}, \]

\[ A_{\phi}^{(1)} = \frac{1}{\sigma} \int_{0}^{2\pi} d\phi \ sgn(\sin \phi) \ \frac{d\sigma}{d\phi}, \]

\[ A_{\phi}^{(2)} = \frac{1}{\sigma} \int_{0}^{2\pi} d\phi \ sgn(\sin(2\phi)) \ \frac{d\sigma}{d\phi}, \]

\[ A_{\phi}^{(3)} = \frac{1}{\sigma} \int_{0}^{2\pi} d\phi \ sgn(\cos \phi) \ \frac{d\sigma}{d\phi}, \]

\[ A_{\phi}^{(4)} = \frac{1}{\sigma} \int_{0}^{2\pi} d\phi \ sgn(\cos(2\phi)) \ \frac{d\sigma}{d\phi}, \]

(1)

\[ A_{c\theta_1, c\theta_2} = \frac{1}{\sigma} \int_{-1}^{1} d\cos \theta_1 \ sgn(\cos \theta_1) \int_{-1}^{1} d\cos \theta_2 \ sgn(\cos \theta_2) \ \frac{d^2\sigma}{d\cos \theta_1 \ d\cos \theta_2}, \]

(2)
The “12-parameter” framework in the Higgs basis

- The relevant terms in the EFT Lagrangian are

\[ \mathcal{L} \supset \mathcal{L}_{hVV} + \mathcal{L}_{hff} + \mathcal{L}_{tgc} , \]  

(3)

- the Higgs couplings with a pair of gauge bosons

\[
\mathcal{L}_{hVV} = \frac{h}{v} \left[ (1 + \delta c_W) \frac{g^2 v^2}{2} W^+_{\mu} W^-_{\mu} + (1 + \delta c_Z) \frac{(g^2 + g'^2) v^2}{4} Z\mu Z_{\mu} \right. \\
+ c_{WW} \frac{g^2}{2} W_{\mu\nu}^+ W_{\mu\nu}^- + c_{W\Box} g^2 (W_{\mu\nu}^- \partial_\nu W_{\mu\nu}^+ + \text{h.c.}) \\
+ c_{gg} \frac{g_s^2}{4} G_{\mu\nu}^a G_{\mu\nu}^a + c_{\gamma\gamma} \frac{e^2}{4} A_{\mu\nu} A_{\mu\nu} + c_{Z\gamma} \frac{e \sqrt{g^2 + g'^2}}{2} Z_{\mu\nu} A_{\mu\nu} \\
+ c_{ZZ} \frac{g^2 + g'^2}{4} Z_{\mu\nu} Z_{\mu\nu} + c_{Z\Box} g^2 Z_{\mu} \partial_\nu Z_{\mu\nu} + \left. c_{\gamma\Box} g g' Z_{\mu} \partial_\nu A_{\mu\nu} \right] .
\]  

(4)
The “12-parameter” framework in the Higgs basis

Not all the couplings are independent, for instance one could write the following couplings as

\[ \delta c_W = \delta c_Z + 4\delta m, \]
\[ c_{WW} = c_{ZZ} + 2s_{\theta_W}^2 c_{Z\gamma} + s_{\theta_W}^4 c_{\gamma\gamma}, \]
\[ c_{W\Box} = \frac{1}{g^2 - g'^2} \left[ g^2 c_{Z\Box} + g'^2 c_{ZZ} - e^2 s_{\theta_W}^2 c_{\gamma\gamma} - (g^2 - g'^2)s_{\theta_W}^2 c_{Z\gamma} \right], \]
\[ c_{\gamma\Box} = \frac{1}{g^2 - g'^2} \left[ 2g^2 c_{Z\Box} + (g^2 + g'^2) c_{ZZ} - e^2 c_{\gamma\gamma} - (g^2 - g'^2)c_{Z\gamma} \right], \quad (5) \]

we only consider the diagonal elements in the Yukawa matrices relevant for the measurements considered,

\[ \mathcal{L}_{hff} = -\frac{h}{v} \sum_{f=t,c,b,\tau,\mu} m_f (1 + \delta y_f) \bar{f}_R f_L + \text{h.c.}. \quad (6) \]
TGC

\begin{align}
L_{\text{tgc}} &= ig s_{\theta_W} A^\mu (W^{-\nu} W^{+\mu}_{\mu\nu} - W^{+\nu} W^-_{\mu\nu}) \\
&+ ig (1 + \delta g^Z_1) c_{\theta_W} Z^\mu (W^{-\nu} W^{+\mu}_{\mu\nu} - W^{+\nu} W^-_{\mu\nu}) \\
&+ ig \left[ (1 + \delta K_Z) c_{\theta_W} Z^\mu + (1 + \delta K_\gamma) s_{\theta_W} A^\mu_{\mu\nu} \right] W^-_{\mu\nu} W^+_{\nu} \\
&+ \frac{ig}{m_W^2} (\lambda_Z c_{\theta_W} Z^\mu + \lambda_\gamma s_{\theta_W} A^\mu_{\mu\nu}) W^-_\nu W^+_{\rho\mu} ,
\end{align}

(7)

- \( V_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu \) for \( V = W^\pm, Z, A_\perp \). Imposing Gauge invariance one obtains \( \delta K_Z = \delta g_{1,Z} - t^2_{\theta_W} \delta K_\gamma \) and \( \lambda_Z = \lambda_\gamma \).

- 3 aTGCs parameters \( \delta g_{1,Z}, \delta K_\gamma \) and \( \lambda_Z \), 2 of them related to Higgs observables by

\begin{align}
\delta g_{1,Z} &= \frac{1}{2(g^2 - g'^2)} \left[ -g^2 (g^2 + g'^2) c_{Z\Box} - g'^2 (g^2 + g'^2) c_{ZZ} + e^2 g'^2 c_{\gamma \gamma} + g'^2 (g^2 - g'^2) c_{Z\gamma} \right] \\
\delta K_\gamma &= - \frac{g^2}{2} \left( c_{\gamma \gamma} \frac{e^2}{g^2 + g'^2} + c_{Z\gamma} \frac{g^2 - g'^2}{g^2 + g'^2} - c_{ZZ} \right) .
\end{align}

(8)
**CEPC/FCC-ee Higgs rate measurements**

<table>
<thead>
<tr>
<th>Production</th>
<th>CEPC</th>
<th>FCC-ee</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[240 GeV, 5 ab$^{-1}$]</td>
<td>[350 GeV, 200 fb$^{-1}$]</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.50%  -</td>
<td>2.4% -</td>
</tr>
<tr>
<td>$h \to b\bar{b}$</td>
<td>0.21% $\star$ 0.39% $\diamondsuit$</td>
<td>2.0% 2.6%</td>
</tr>
<tr>
<td>$h \to c\bar{c}$</td>
<td>2.5% -</td>
<td>15% 26%</td>
</tr>
<tr>
<td>$h \to gg$</td>
<td>1.2% -</td>
<td>11% 17%</td>
</tr>
<tr>
<td>$h \to \tau\tau$</td>
<td>1.0% -</td>
<td>5.3% 37%</td>
</tr>
<tr>
<td>$h \to WW^*$</td>
<td>1.0% -</td>
<td>10% 9.8%</td>
</tr>
<tr>
<td>$h \to ZZ^*$</td>
<td>4.3% -</td>
<td>33% 33%</td>
</tr>
<tr>
<td>$h \to \gamma\gamma$</td>
<td>9.0% -</td>
<td>51% 77%</td>
</tr>
<tr>
<td>$h \to \mu\mu$</td>
<td>12% -</td>
<td>115% 275%</td>
</tr>
<tr>
<td>$h \to Z\gamma$</td>
<td>25% -</td>
<td>144% -</td>
</tr>
</tbody>
</table>

**Table:** For $e^+ e^- \to \nu\bar{\nu} h$, the precisions marked with a diamond $\diamondsuit$ are normalized to the cross section of the inclusive channel which includes both the $WW$ fusion and $e^+ e^- \to hZ, Z \to \nu\bar{\nu}$, while the unmarked ones include $WW$ fusion only.
## ILC Higgs rate measurements

<table>
<thead>
<tr>
<th>Production</th>
<th>ILC</th>
<th>(\times BR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Zh)</td>
<td>([250,\text{GeV}, , 2,\text{ab}^{-1}])</td>
<td>([350,\text{GeV}, , 200,\text{fb}^{-1}])</td>
</tr>
<tr>
<td>(\nu\bar{\nu}h)</td>
<td>(0.71%)</td>
<td>(2.1%)</td>
</tr>
<tr>
<td>(\sigma)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(h \to bb)</td>
<td>(0.42%)</td>
<td>(3.7%)</td>
</tr>
<tr>
<td>(h \to c\bar{c})</td>
<td>(2.9%)</td>
<td>-</td>
</tr>
<tr>
<td>(h \to gg)</td>
<td>(2.5%)</td>
<td>-</td>
</tr>
<tr>
<td>(h \to \tau\bar{\tau})</td>
<td>(1.1%)</td>
<td>-</td>
</tr>
<tr>
<td>(h \to WW^*)</td>
<td>(2.3%)</td>
<td>-</td>
</tr>
<tr>
<td>(h \to ZZ^*)</td>
<td>(6.7%)</td>
<td>-</td>
</tr>
<tr>
<td>(h \to \gamma\gamma)</td>
<td>(12%)</td>
<td>-</td>
</tr>
<tr>
<td>(h \to \mu\mu)</td>
<td>(25%)</td>
<td>-</td>
</tr>
<tr>
<td>(h \to Z\gamma)</td>
<td>(34%)</td>
<td>-</td>
</tr>
</tbody>
</table>
CLIC Higgs rate measurements

| Production | CLIC | \[
\begin{array}{ccc}
\text{ZH} & \nu \bar{\nu} h & \nu \bar{\nu} h \\
\sigma & 1.6\% & - \\
\end{array}
\]

<table>
<thead>
<tr>
<th>[350 \text{ GeV, 500 fb}^{-1}]</th>
<th>[1.4 \text{ TeV, 1.5 ab}^{-1}]</th>
<th>[3 \text{ TeV, 2 ab}^{-1}]</th>
<th>[\nu \bar{\nu} h]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>$h \rightarrow bb$</strong></td>
<td>0.84%</td>
<td>0.4%</td>
<td>0.3%</td>
</tr>
<tr>
<td><strong>$h \rightarrow c\bar{c}$</strong></td>
<td>10.3%</td>
<td>6.1%</td>
<td>6.9%</td>
</tr>
<tr>
<td><strong>$h \rightarrow gg$</strong></td>
<td>4.5%</td>
<td>5.0%</td>
<td>4.3%</td>
</tr>
<tr>
<td><strong>$h \rightarrow \tau\tau$</strong></td>
<td>6.2%</td>
<td>4.2%</td>
<td>4.4%</td>
</tr>
<tr>
<td><strong>$h \rightarrow WW^*$</strong></td>
<td>5.1%</td>
<td>1.0%</td>
<td>0.7%</td>
</tr>
<tr>
<td><strong>$h \rightarrow ZZ^*$</strong></td>
<td>-</td>
<td>5.6%</td>
<td>3.9%</td>
</tr>
<tr>
<td><strong>$h \rightarrow \gamma\gamma$</strong></td>
<td>-</td>
<td>15%</td>
<td>10%</td>
</tr>
<tr>
<td><strong>$h \rightarrow \mu\mu$</strong></td>
<td>-</td>
<td>38%</td>
<td>25%</td>
</tr>
<tr>
<td><strong>$h \rightarrow Z\gamma$</strong></td>
<td>-</td>
<td>42%</td>
<td>30%</td>
</tr>
</tbody>
</table>

**Table:** We also include the estimations for $\sigma(hZ) \times BR(h \rightarrow b\bar{b})$ at high energies in [arXiv:1701.04804] (Ellis et al.), which are 3.3\% (6.8\%) at 1.4 TeV (3 TeV). For simplicity, the measurements of $ZZ$ fusion ($e^+e^- \rightarrow e^+e^- h$) are not included in our analysis.