$\Upsilon(nS)$ polarization measurement at LHCb

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on behalf of the LHCb Collaboration
Introduction: *investigate the production mechanism in pp*

The understanding of the production mechanism of quarkonium states improved greatly in the last few years, also thanks to the input of the LHC.

Golden approach: NRQCD models \[ d\sigma_{pp \rightarrow Q+X} = \sum_n d\hat{\sigma}_{pp \rightarrow Q\bar{Q}[n]+X} \langle \mathcal{O}_Q(n) \rangle. \]

A complete theoretical picture of quarkonium production should explain:

- production cross-section measurements
  - see for example Liupan’s talk on central exclusive production
- associative production measurements
  - see for example the talk by Jia-Jia Qin on $J/\psi + \text{jet}$
- polarization of the quarkonium states

Adding measurements can challenge theoretical predictions, and help extending the prediction to low $p_T$ regions.
The LHCb experiment: *designed for heavy flavours*

**pp** collisions at the LHC
A fully instrumented detector in the forward region

**Prompt to detached discrimination**

**Hadron identification (non-dimuon final states)**

**Excellent mass resolution:**
better combinatorial rejection

**Photon detection**
**Electron identification**

**Muon identification**

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*JINST 3 (2008) S08005*  
*IJMPA 30 (2015) 1530022*
**Dataset: data-taking conditions and kinematic range**

Analysis performed on 2011 and 2012 data collected in **pp collisions** at $\sqrt{s} = 7$ TeV [1 fb$^{-1}$] and $\sqrt{s} = 8$ TeV [2 fb$^{-1}$].

Unique momentum and rapidity acceptance

2.2 < $y(\Upsilon)$ < 4.5  \( p_T(\Upsilon) < 30 \) GeV/c

The analysis is performed also in three rapidity bins (but statistics become insufficient for $p_T$ larger than 20 GeV/c).

2.2 < $y(\Upsilon)$ < 3.0  \( p_T(\Upsilon) < 20 \) GeV/c

3.0 < $y(\Upsilon)$ < 3.5  \( p_T(\Upsilon) < 20 \) GeV/c

3.5 < $y(\Upsilon)$ < 4.5  \( p_T(\Upsilon) < 20 \) GeV/c

No overlap between CMS/CDF and LHCb kinematic regions, but mild dependence on $y$ is expected, allowing for the comparison of results.
**Signal selection: clean dimuon final state**

High-momentum muons ($p > 10$ GeV/$c$) from $\Upsilon \rightarrow \mu \mu$ decay can be selected with great purity and high efficiency already at trigger level.

Excellent momentum resolution allows to get **narrow peaks** despite the large energy release in the decay.

**Fit:** 3 Crystal Ball Functions (peaks) + exponential background.

**Constraints:** $m_{\Upsilon(2S)} - m_{\Upsilon(1S)}$ [from PDG]
$m_{\Upsilon(3S)} - m_{\Upsilon(1S)}$ [from PDG]

resolution scaling with the mass shared “tail-shape parameters”
Production: *the cross-section measurement*

Production measurements performed on the same dataset found to disagree with (some) theoretical predictions, with an unexpectedly large increase in the production cross-section from 7 to 8 TeV in the c.m.s.

Polarization became an intriguing measurement.
Polarization: *anisotropic $\Upsilon \rightarrow \mu \mu$ decay*

The $\Upsilon$ (nS) mesons have quantum numbers $J^{PC} = 1^{--}$.

The $\Upsilon$ meson is **polarized** if it is **preferentially produced** in one of the spin states $|J, J_z \rangle = |1, -1 \rangle$ or $|1, 0 \rangle$ or $|1, 1 \rangle$ and the **angular distribution of the produced muons** in the C.M.S. is not isotropic.

A *spin-independent* production mechanisms result into **unpolarized $\Upsilon$ mesons** and into an isotropic angular distribution for the muons.

In analogy with electromagnetic radiation:

A transverse polarization and a longitudinal polarization.
Polarization: *the arbitrary choice of the quantization axis*

**y-axis**: normal to the production plane.

Three possible choices for the **z-axis**:

- Helicity frame (HX)  
  *Ann. Phys. 7 (1959) 404*

- Collins-Soper frame (CS)  
  *PRD 16 (1977) 2219*

- Gottfried-Jackson frame (GJ)  
  *Nuovo Cim. 33 (1964) 309*

The **x-axis** completes the right-handed coordinate system.

Angular distribution is parametrized through $\lambda_\phi$, $\lambda_\theta$ and $\lambda_{\theta\phi}$.

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega} = \frac{3}{4\pi} \frac{1}{3 + \lambda_\theta} \left[ 1 + \lambda_\theta \cos^2 \theta + \lambda_{\theta\phi} \sin 2\theta \cos \phi + \lambda_\phi \sin^2 \theta \cos 2\phi \right]$$
Polarization: *frame-independent polarization*

The parameters $\lambda_\phi$, $\lambda_\theta$ and $\lambda_{\theta\phi}$ depend on the choice of the frame.

A frame-independent parameter is defined

$$\tilde{\lambda} = \frac{\lambda_\theta + 3\lambda_\phi}{1 - \lambda_\phi}$$

A deviation of $\tilde{\lambda}$ from 0 means **polarized $\gamma$ state**.
**Statistical background subtraction: sPlot technique**

sPlot technique [1] used to statistically subtract the angular distributions of the **sidebands** from the sample, to obtain (and fit) signal distributions.

**A weight** $w_i^\gamma \in \mathbb{R}$ **is assigned to each candidate**, the weighted distributions are, by construction, background-subtracted.

**Example of application:** Studying the $\theta_{GJ}$ distribution, a peaking background structure was found, and rejected by the criterion

$$|\cos \theta_{GJ}| > 0.8$$

Simulation: validation and correction

The polarization measurement relies on different angular distributions between a simulated unpolarized $\Upsilon$ dataset and the collected data.

The simulation of the angle-dependent efficiencies has to be accurate

Corrections

- Geometrical acceptance: simulated. Border-effects limited by fiducial volume.
- Tracking efficiency: simulated and corrected using abundant $J/\psi \to \mu\mu$ decays.
- Trigger efficiency: simulated and cross-checked with events triggered by single-$\mu$
- Muon identification efficiency: taken from data (again $J/\psi \to \mu\mu$ decays).

Validation:
Background subtracted distributions of corrected simulation and real data are compared for fully reconstructed $B^+ \to J/\psi K^+$ decays (polarized $J/\psi$) to validate the procedure (though the kinematic range is a bit different)
The polarization fit: the likelihood function

Angular model:

\[ \mathcal{P}(\Omega_i | \lambda) \equiv 1 + \lambda_\theta \cos^2 \theta_i + \lambda_{\theta\phi} \sin 2\theta_i \cos \phi_i + \lambda_\phi \sin^2 \theta_i \cos 2\phi_i \]

Likelihood function:

\[
\log \mathcal{L}^\gamma (\lambda) = s_w \sum_i w^\gamma_i \log \left[ \frac{\mathcal{P}(\Omega_i | \lambda) \varepsilon(\Omega_i)}{\mathcal{N}(\lambda)} \right]
\]

Scale factor (sFit)

\[ s_w \equiv \frac{\sum_i w^\gamma_i}{\left( \sum_i (w^\gamma_i)^2 \right)} \]

Weight from the sPlot

\[ = s_w \sum_i w^\gamma_i \log \left[ \frac{\mathcal{P}(\Omega_i | \lambda)}{\mathcal{N}(\lambda)} \right] + s_w \sum_i w^\gamma_i \log [\varepsilon(\Omega_i)] , \]

Constant, neglected term

Normalization:

\[ \mathcal{N}(\lambda) \equiv \int d\Omega(\Omega | \lambda) \varepsilon(\Omega) \propto \sum \varepsilon^{\mu^+\mu^-} \mathcal{P}(\Omega_i | \lambda) \]

Efficiency correction

Efficiency

Sum over selected simulated events
Systematic uncertainties: overview

_fit model_

✓ signal parametrization taken from simulation
✓ background parametrization fitted with \( \exp(a \times m(\mu \mu)) \times \text{polynomial} \)
✓ In every bin, uncertainty due to the fit model < 10% of statistical uncertainty

_trigger efficiency determination_

✓ comparison of efficiency in data and MC shows discrepancy of order 2%

\[
\varepsilon \left[ \text{Trig}(\mu \mu) \& \text{Trig}(\mu) \middle| \text{Trig}(\mu) \right]_{\text{Data}} \quad \varepsilon \left[ \text{Trig}(\mu \mu) \& \text{Trig}(\mu) \middle| \text{Trig}(\mu) \right]_{\text{Simulation}}
\]

_tracking and muon identification efficiency determination_

✓ uncertainty propagated with toys from statistical uncertainty on the calibration samples.

_finite simulated samples_

✓ due to \( Y \) simulated softer than produced, high-\( p_T \) bins are dominated by statistical uncertainty on MC
At low- and mid-\(p_T\) the measurement is statistically limited.

At high-\(p_T\) statistical and systematic uncertainties are competitive.

### Systematic uncertainties: results

<table>
<thead>
<tr>
<th>Source</th>
<th>(\sigma_{\lambda_\theta} [10^{-3}])</th>
<th>(\sigma_{\lambda_\phi} [10^{-3}])</th>
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<th>(\sigma_\lambda [10^{-3}])</th>
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</thead>
<tbody>
<tr>
<td><strong>(\Upsilon(1S))</strong></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Dimuon mass fit</td>
<td>1.0 – 12</td>
<td>0.2 – 10</td>
<td>0.1 – 7</td>
<td>1.8 – 20</td>
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<tr>
<td>Efficiency calculation</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>muon identification</td>
<td>0.2 – 10</td>
<td>0.1 – 7</td>
<td>0.1 – 6</td>
<td>0.2 – 17</td>
</tr>
<tr>
<td>correction factors for (\varepsilon^{\mu^+\mu^-})</td>
<td>0.7 – 12</td>
<td>0.4 – 5</td>
<td>0.1 – 4</td>
<td>2.1 – 14</td>
</tr>
<tr>
<td>trigger</td>
<td>0.1 – 18</td>
<td>0.1 – 8</td>
<td>0.1 – 5</td>
<td>0.3 – 19</td>
</tr>
<tr>
<td>Finite size of simulated samples</td>
<td>6.0 – 82</td>
<td>1.3 – 29</td>
<td>0.9 – 35</td>
<td>6.9 – 95</td>
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<tr>
<td><strong>(\Upsilon(2S))</strong></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Dimuon mass fit</td>
<td>0.6 – 37</td>
<td>0.2 – 19</td>
<td>0.3 – 16</td>
<td>4.6 – 53</td>
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<tr>
<td>muon identification</td>
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<td>0.1 – 6</td>
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<td>0.2 – 13</td>
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<td>0.1 – 7</td>
<td>0.1 – 5</td>
<td>0.3 – 18</td>
</tr>
<tr>
<td>Finite size of simulated samples</td>
<td>9.8 – 210</td>
<td>2.5 – 98</td>
<td>1.5 – 120</td>
<td>14 – 320</td>
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<tr>
<td><strong>(\Upsilon(3S))</strong></td>
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<td></td>
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<td>Dimuon mass fit model</td>
<td>1.4 – 72</td>
<td>0.2 – 24</td>
<td>0.5 – 21</td>
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</tr>
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<td>0.2 – 17</td>
<td>0.1 – 8</td>
<td>0.1 – 4</td>
<td>0.3 – 19</td>
</tr>
<tr>
<td>Finite size of simulated samples</td>
<td>12 – 280</td>
<td>3.5 – 100</td>
<td>2.1 – 110</td>
<td>16 – 350</td>
</tr>
</tbody>
</table>
Polarization fit: results

The results on the frame-independent parameter $\tilde{\lambda}$ are shown as “sum-check” as obtained from fits in the three frames:
+ Helicity frame (HX)
+ Collins-Soper frame (CS)
+ Gottfried-Jackson frame (GJ)

No large polarization effect observed (neither longitudinal nor transverse).
**Positivity test: spin-1 density matrix**

Density matrix of the $\Upsilon$ decay in the $\Upsilon$ rest frame:

\[
\begin{pmatrix}
\frac{1 - \lambda_\theta}{2} & \lambda_{\theta\phi} & 0 \\
\lambda_{\theta\phi} & \frac{1 + \lambda_\theta - 2\lambda_\phi}{2} & 0 \\
0 & 0 & \frac{1 + \lambda_\theta + 2\lambda_\phi}{2}
\end{pmatrix}
\]

The density matrix is positive by construction, implying six constraints (translated into admitted regions in the parameter space)

\[0 \leq C_1 = 1 - |\lambda_\theta|\]
\[0 \leq C_2 = 1 + \lambda_\theta - 2|\lambda_\phi|\]
\[0 \leq C_3 = (1 - \lambda_\theta)(1 + \lambda_\theta - 2\lambda_\phi) - 4\lambda_{\theta\phi}^2\]
\[0 \leq C_4 = (1 - \lambda_\theta)(1 + \lambda_\theta + 2\lambda_\phi)\]
\[0 \leq C_5 = (1 + \lambda_\theta)^2 - 4\lambda_{\phi}^2\]
\[0 \leq C_6 = (1 + \lambda_\theta + 2\lambda_\phi)\left((1 - \lambda_\theta)(1 + \lambda_\theta - 2\lambda_\phi) - 4\lambda_{\theta\phi}^2\right)\]

**All positivity constraints are satisfied in all kinematic bins**
A word on theoretical predictions

Theoretical predictions are based on NRQCD with a factorization approach

\[ d\sigma_{pp \rightarrow \mathcal{Q} + X} = \sum_n d\sigma_{pp \rightarrow Q\bar{Q}[n] + X} \langle \mathcal{O}_\mathcal{Q}(n) \rangle. \]

Perturbative part (QCD scattering)

Non perturbative Long Distance Matrix Elements (LDME) (from fit to data)

An important unknown in the computation comes from the contribution from $\chi_b \rightarrow Y\gamma$ feed-down.

It was measured for $\chi_b(1P)$ and assumed for the other states, leading to important uncertainties.
Comparison with NRQCD

\begin{align*}
\gamma(nS) & \text{ polarization measurement} \\
\text{Predictions: } & \text{PRL 112, 032001 (2014)} \\
\text{CMS data: } & \text{PRL 110 081802 (2013)} \\
\end{align*}
Comparisons with CMS and CDF data

Expecting small dependency of polarization on $\eta$, LHCb results (forward) are compared to CMS and CDF results. **In the HX frame, no good agreement with CMS.**
Comparisons with CMS and CDF data

In Collins-Soper the agreement is good with both CMS and CDF.
Summary and outlook

LHCb measured the polarization of $Y$ mesons through decays $Y \rightarrow \mu^+ \mu^-$. Polarization parameters in different frames are measured in bins of $p_T(Y)$ and $\eta(Y)$. No large longitudinal or transversal polarization is observed in the accessible space domain. The polarization parameters for $\sqrt{s} = 7$ and 8 TeV are in good agreement. The results are in agreement with those of CMS and CDF.

LHCb will keep contributing to production and polarization measurements also thanks to the forthcoming upgrade that will allow higher efficiency on purely hadronic decays.