Track Parameter Resolution

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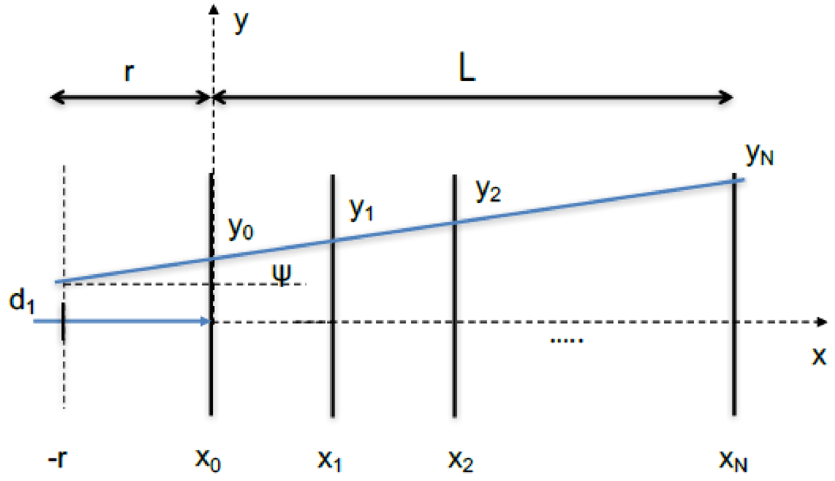
General formulas

Track function:

$$f(x) = \Sigma_{i=0}^{M} a_i g_i(x)$$

a_i are unknown parameters. They are estimated by minimizing χ^2 defined as:

$$\chi^2 = (y - Ga)^T W (y - Ga)$$



In which:

y: measured values.

G: $g_m(x_n)$

W: weight matrix

To minimize χ^2, we have

$$a = (G^T W G)^{-1} G^T W y$$

Now we have relation between a and y. What we want is the relation between covariance matrices C_a and C_y. After some calculation and applying the generalized Gauss-Markov theorem, we have

$$C_a = (G^T C_y^{-1} G)^{-1}$$

Covariance matrix C_y

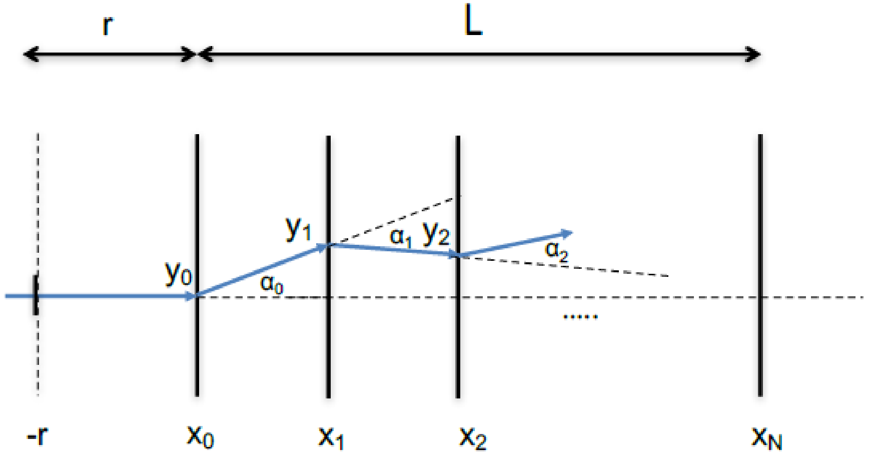
In general, C_y has the form

$$(C_y)_{mn} = \sigma_n^2 \delta_{mn} + \sum_{j=0}^{Min[m,n]-1} \sigma_{\alpha_j}^2 (x_m - x_j)(x_n - x_j)$$

- First term: intrinsic detector resolution, not correlated
- Second term: multiple scattering, highly correlated and energy dependent

We consider them separately for simplicity, and use R_{mn} to denote the first term and M_{mn} to denote the second term. The total resolution is

$$\sigma_{total} = \sigma_{intrinsic} \oplus \sigma_{ms}$$



Calculating σ_{z0}

For simplicity, let's assume the incident angle of the particle is almost perpendicular to the measurement surface. The fitting function is

$$f(x) = a_0 + a_1 x$$

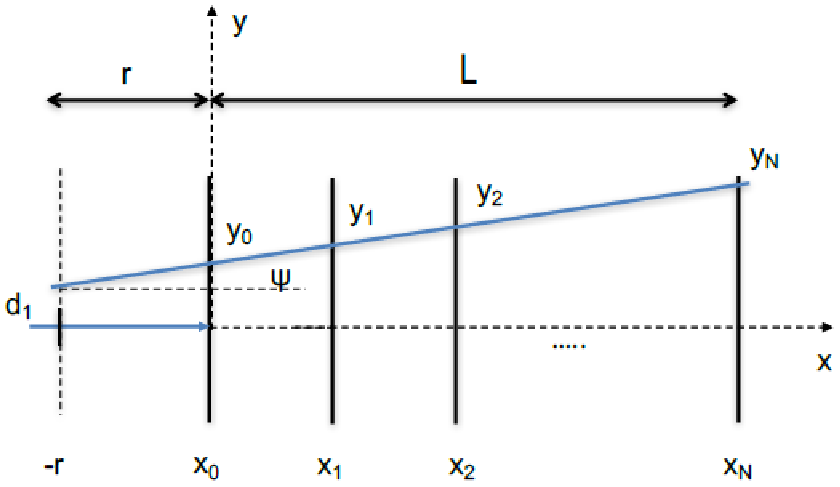
The G matrix is

$$G^T = \begin{pmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ x_0 & x_1 & x_2 & x_3 & \cdots & x_n \end{pmatrix}$$

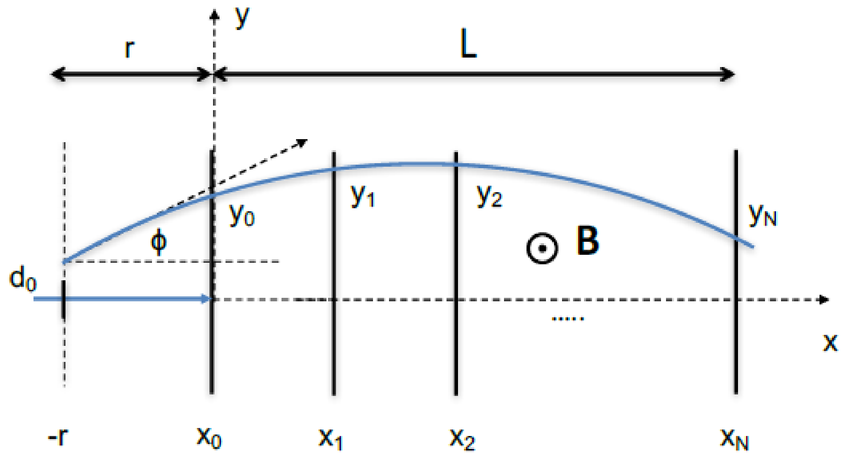
$$C_{a,intrinsic} = (G^T R^{-1} G)^{-1}$$

$$\sigma_{z0,intrinsic} = \left(\Delta f(0)\right)^2 = g(0)^T C_{a,intrinsic} g(0)^T$$

in which $g(x) = (1 \quad x)$. We can do similar calculation for $\sigma_{z0,ms}$.



Calculating σ_{1/p_T}



The circle is approximated by

$$f(x) = a_0 + a_1 x + a_2 x^2/2$$

The G matrix is

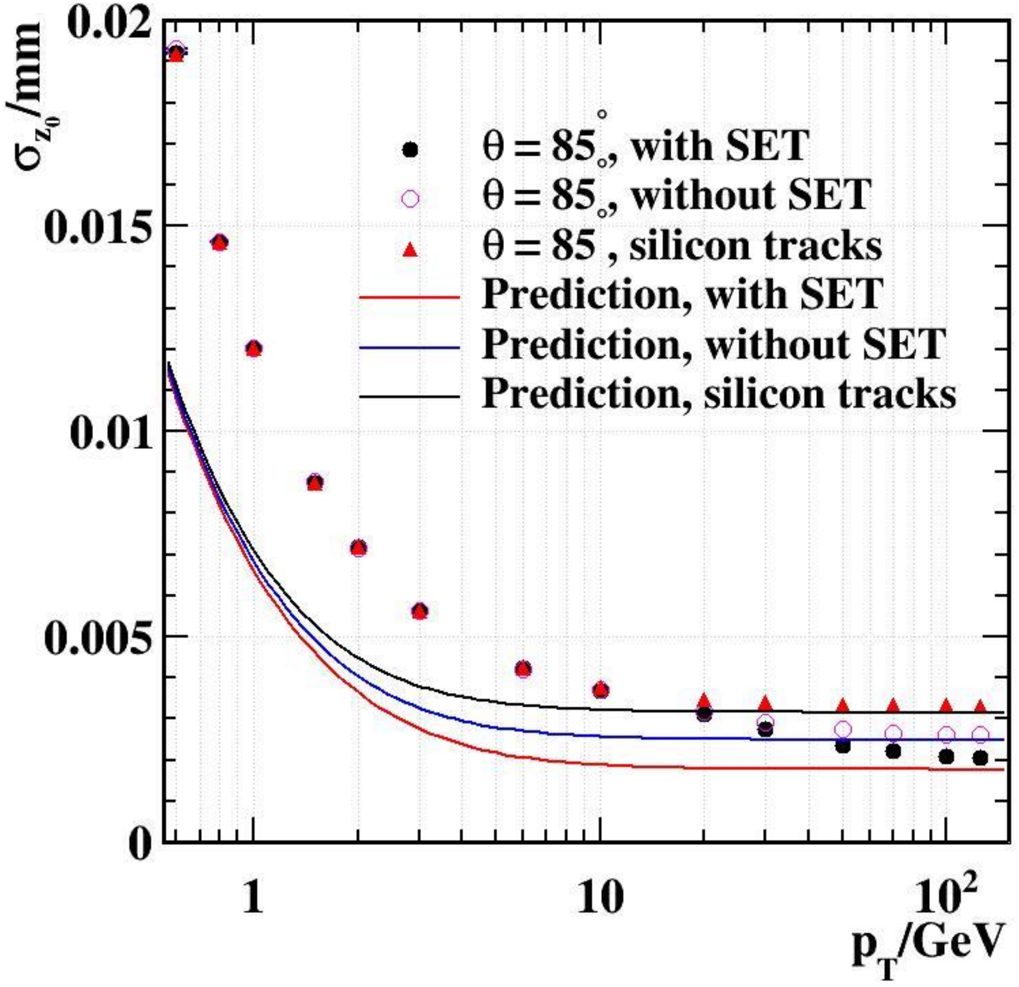
$$G^T = \begin{pmatrix} 1 & 1 & 1 & & 1 \\ x_0 & x_1 & x_2 & & x_n \\ \frac{1}{2}x_0^2 & \frac{1}{2}x_1^2 & \frac{1}{2}x_2^2 & \cdots & \frac{1}{2}x_n^2 \end{pmatrix}$$

$$C_{a,intrinsic} = (G^T R^{-1} G)^{-1}$$

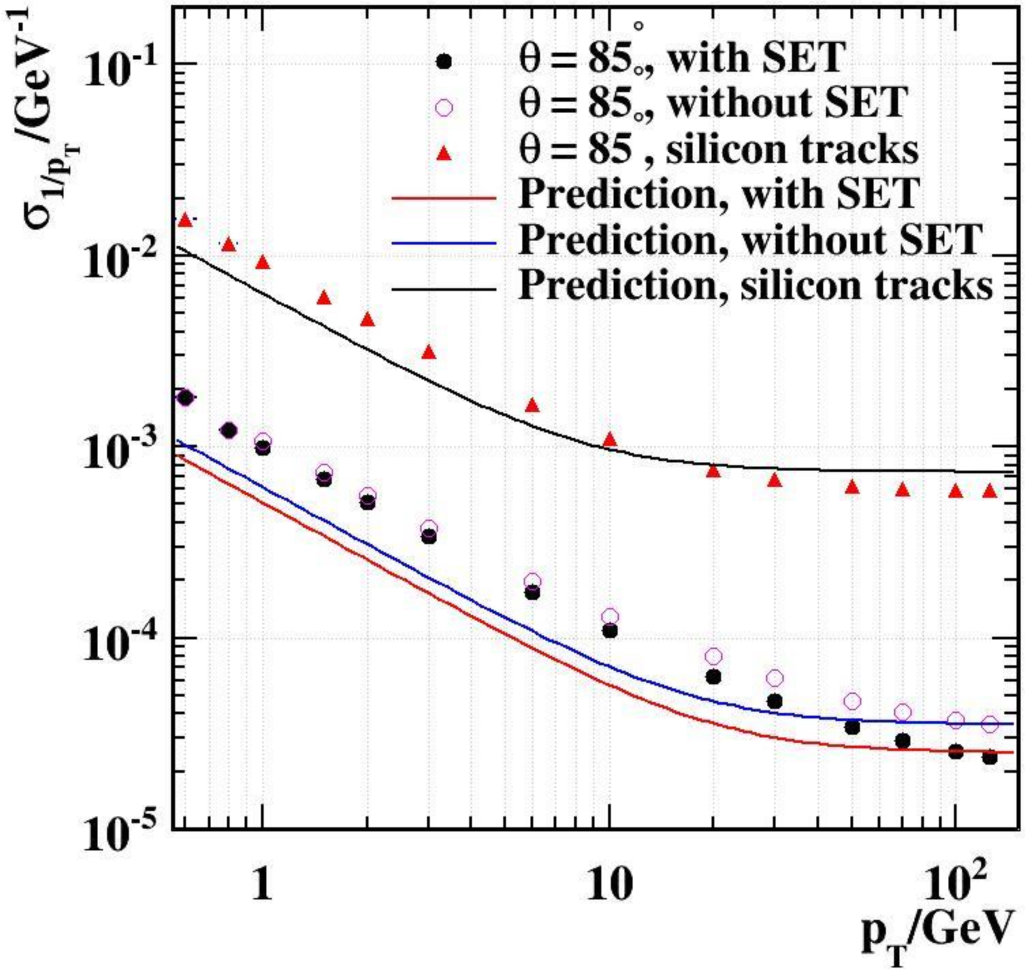
$$\sigma_{1/p_T,intrinsic} = \frac{\Delta a_2}{0.3B} = \frac{\sqrt{(C_a)_{22}}}{0.3B}$$

We can do similar calculation for $\sigma_{1/p_T,ms}$.

Results



σ_{z0}



σ_{1/p_T}