The clarification for the rescattering mechanism

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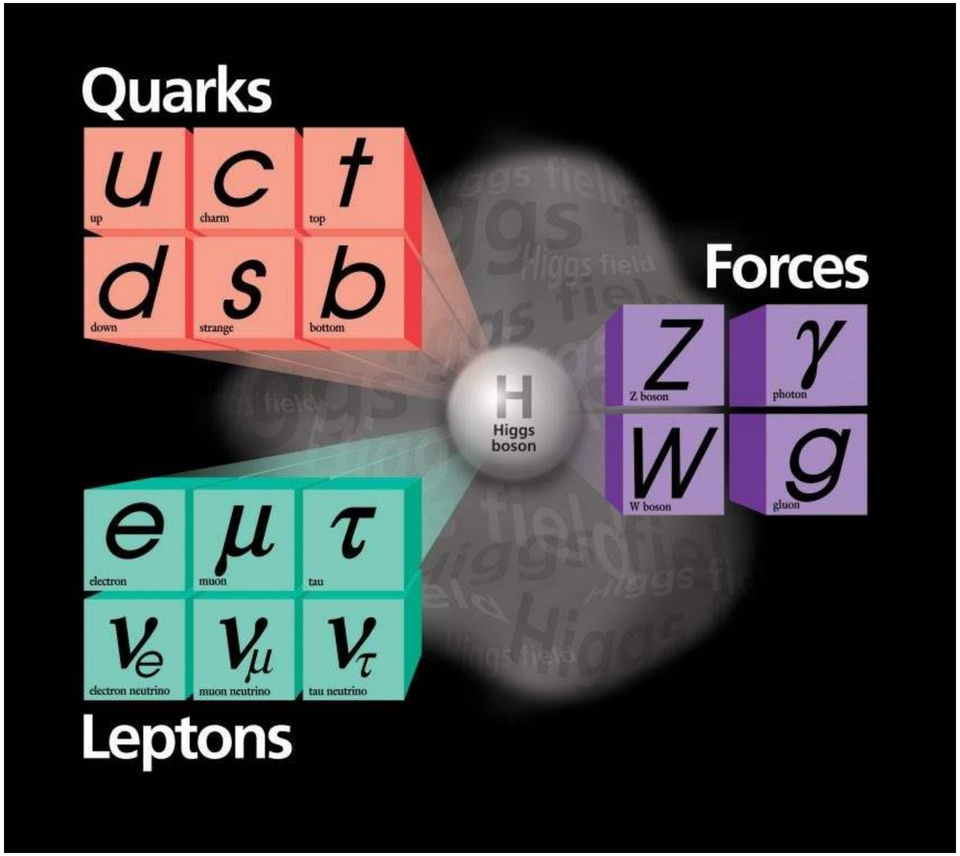
Content

Introduction

- Matter-antimatter asymmetry
- CP-violation

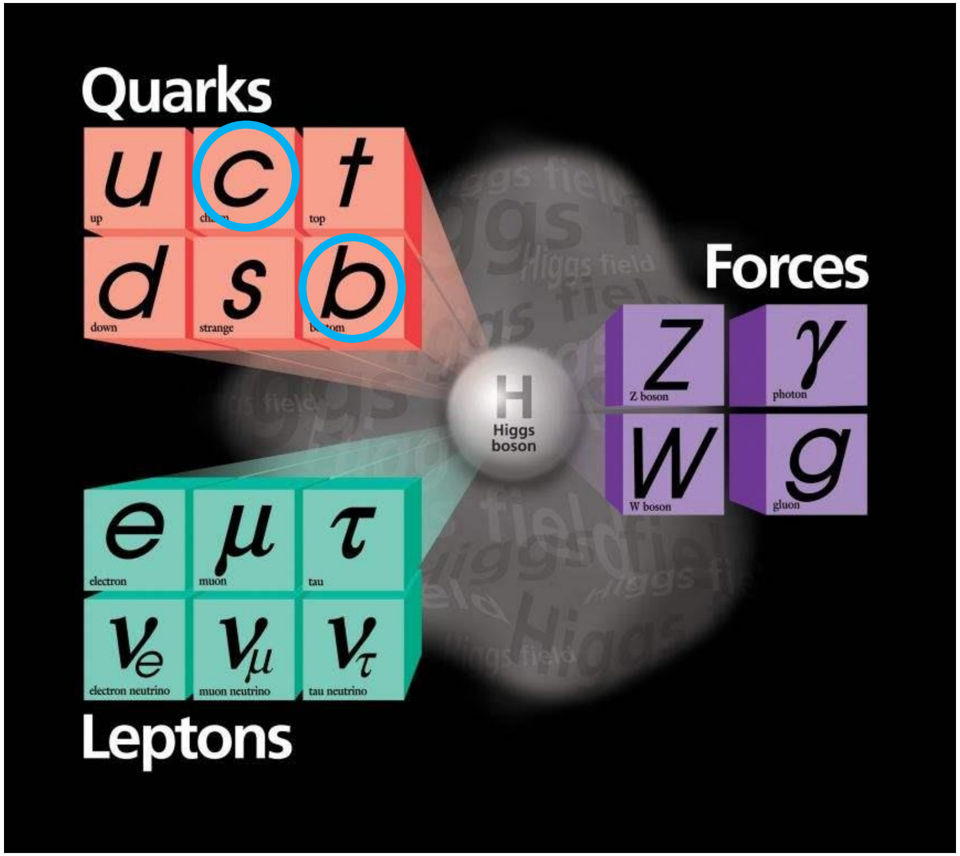
$$\begin{aligned}\Delta A_{CP} &\equiv A_{CP}(D^0 \to K^+K^-) \\ &\quad - A_{CP}(D^0 \to \pi^+\pi^-) \\ &= (-1.54 \pm 0.29) \times 10^{-3}\end{aligned}$$

- How to effectively calculate the decay amplitude of hadron?



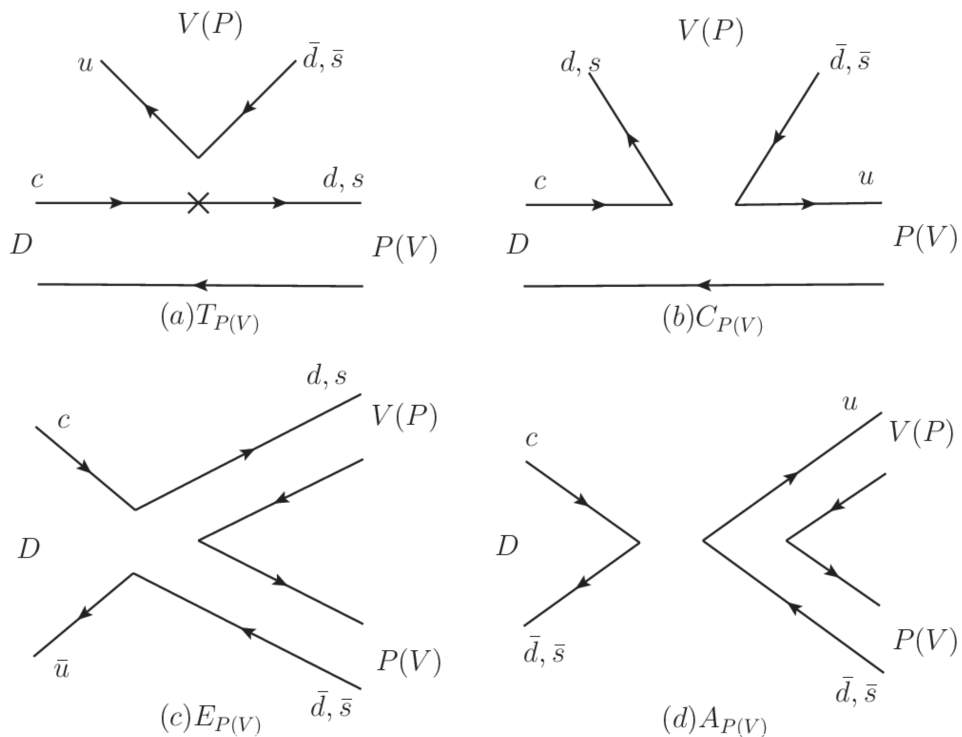
Introduction

- Heavy quark physics
- HQET
- QCDSR, LCSR
- pQCD
- QCDF
- The models



Introduction

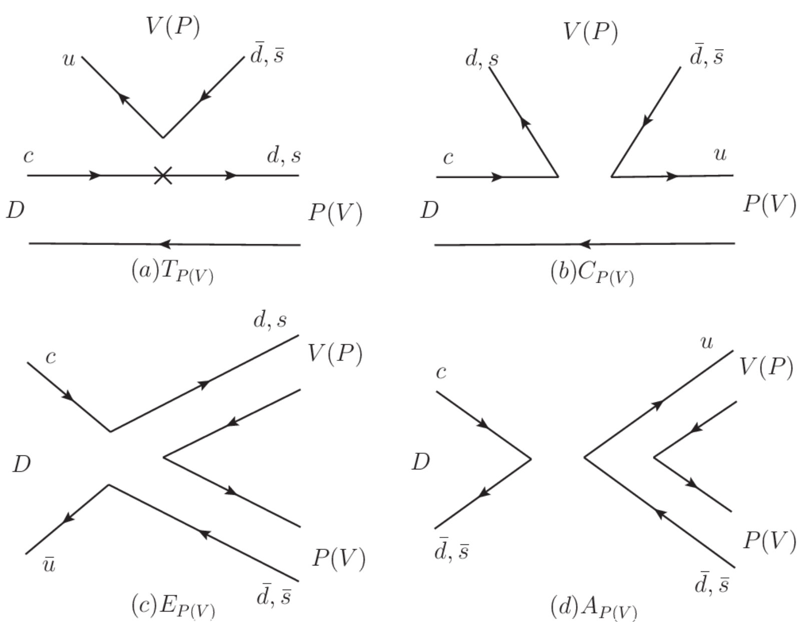
- In general, the study of the b- and c-hadron weak decays based on topology diagram



Which can be calculated by different method, such as: Topology amplitude approach (Hai Yang Cheng et.al), Factorization (BSW), FAT(Fu-Sheng Yu et.al), pQCD(Hsiang-nan Li et.al), Final state interaction (Hai Yang Cheng et.al)

Introduction

- In general, the study of the b- and c-hadron weak decays based on topology diagram



$(a)T_{P(V)}$ $(b)C_{P(V)}$ $(c)E_{P(V)}$ $(d)A_{P(V)}$

In FAT (Fu-Sheng Yu et.al)

$$T_P(C_P) = \frac{G_F}{\sqrt{2}} V_{CKM} a_1(\mu)\left(a_2^P(\mu)\right) f_V m_V F_1^{DP}(m_V^2) 2(\varepsilon_V \cdot p_D),$$

$$T_V(C_V) = \frac{G_F}{\sqrt{2}} V_{CKM} a_1(\mu)\left(a_2^V(\mu)\right) f_P m_V A_0^{DV}(m_P^2) 2(\varepsilon_V \cdot p_D),$$

$$a_1(\mu) = C_2(\mu) + \frac{C_1(\mu)}{N_C},$$

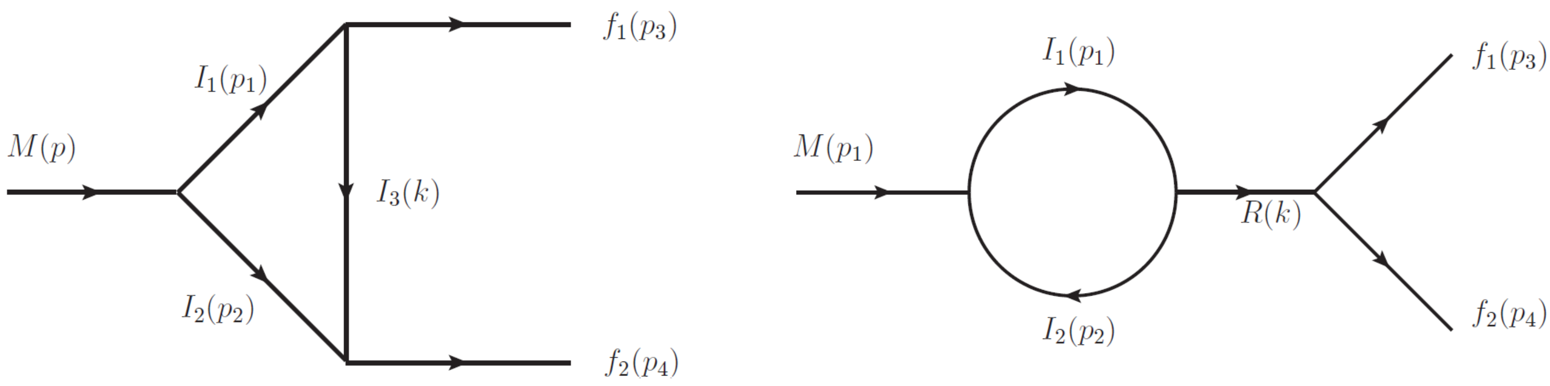
$$a_2^{P(V)}(\mu) = C_1(\mu) + C_2(\mu)\left(\frac{1}{N_C} + \chi^C_{P(V)} e^{i\phi^C_{P(V)}}\right)$$

$$E_{P,V} = \frac{G_F}{\sqrt{2}} V_{CKM} C_2(\mu) \chi^E_{q(s)} e^{i\phi^E_{q(s)}} f_D m_D \frac{f_P}{f_\pi}\frac{f_V}{f_\rho}(\varepsilon_V \cdot p_D),$$

$$A_{P,V} = \frac{G_F}{\sqrt{2}} V_{CKM} C_1(\mu) \chi^A_{q(s)} e^{i\phi^A_{q(s)}} f_D m_D \frac{f_P}{f_\pi}\frac{f_V}{f_\rho}(\varepsilon_V \cdot p_D),$$

Introduction

- The long-distance contributions can not be effectively calculated
- The final state interaction plays an important role
- Which includes the rescattering mechanism and the resonance contributions



Introduction

- Xue-Qian Li, Bing-Song Zou, Phys.Lett. B399 (1997) 297-302
- You-Shan Dai, Dong-Sheng Du, Xue-Qian Li, Zheng-Tao Wei, Bing-Song Zou, Phys.Rev. D60 (1999) 014014
- Medina Ablikim , Dong-Sheng Du, Mao-Zhi Yang, Phys.Lett.B 536 (2002) 34-42
- Hai-Yang Cheng, Chun-Khiang Chua, Amarjit Soni, Phys.Rev.D 71, 014030(2005)
- Fu-Sheng Yu, **Hua-Yu Jiang**, Cai-Dian Lü, Wei Wang, Zhen-Xing Zhao, Chin.Phys. C42 (2018), 051001

Introduction

- $D \to \pi\pi$ decay in rescattering mechanism

[1] Medina Ablikim , Dong-Sheng Du, Mao-Zhi Yang, Phys.Lett.B 536 (2002) 34-42

$$\frac{1}{\sqrt{2}} A\left(D^0 \to \pi^+\pi^-\right) = A\left(D^0 \to \pi^0\pi^0\right) - A\left(D^+ \to \pi^+\pi^0\right)$$

- $B \to D\pi$ decay in rescattering mechanism

[2] Hai-Yang Cheng, Chun-Khiang Chua, Amarjit Soni, Phys.Rev.D 71, 014030(2005)

$$A\left(\overline{B}^0 \to D^+\pi^-\right) = \sqrt{2} A\left(\overline{B}^0 \to D^0\pi^0\right) + A\left(B^- \to D^0\pi^-\right)$$

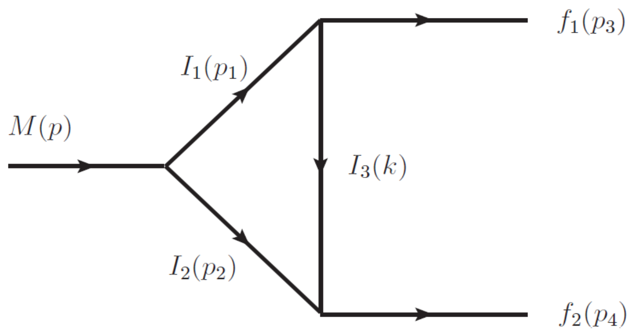
- Where they made a wrong assumption

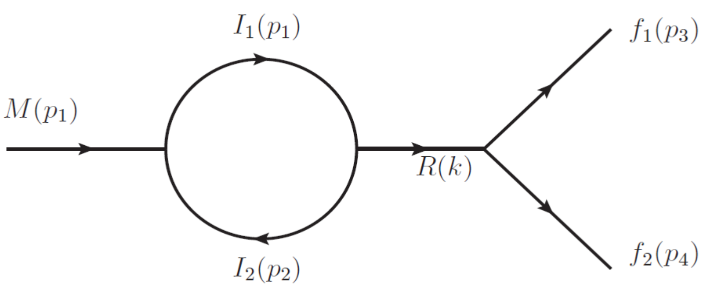
Introduction

- This problem produce some effects, when we use the rescattering mechanism to calculate.
- We will clarify the problem that has been displayed in history in the using of rescattering mechanism.
- And we also will discuss the quark level diagram of the rescattering mechanism.
- Actually, the rescattering process may be as a realistic physical process, that is the exist of triangle singularity.

The rescattering mechanism

- The rescattering mechanism in hadron level





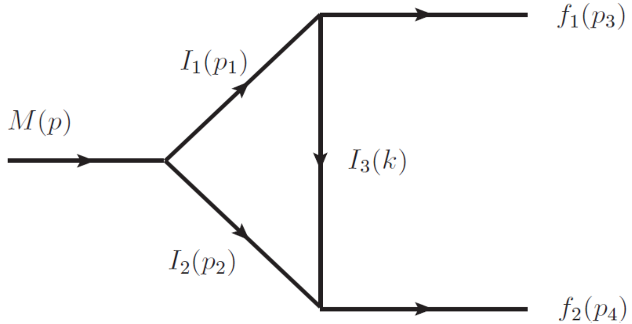
- The effective Lagrangian

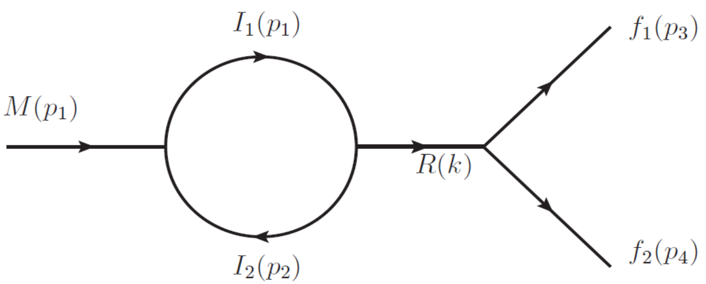
$$\begin{aligned}\mathcal{L}_{eff} &= \mathcal{L}_{\rho\pi\pi} + \mathcal{L}_{D^* D\pi} + \cdots \\ &= -ig_{\rho\pi\pi}\left(\rho^+_\mu \pi^0 \vec{\partial}^\mu \pi^- + \rho^-_\mu \pi^+ \vec{\partial}^\mu \pi^0 + \rho^0_\mu \pi^- \vec{\partial}^\mu \pi^+\right) \\ &\quad -ig_{D^* DP}(D^i \partial^\mu P_{ij} D^{*j\dagger}_\mu - D^{*i}_\mu \partial^\mu P_{ij} D^{j\dagger}) + \cdots\end{aligned}$$

We can get
the Feynman rules
and
the Feynman diagrams
in hadron level

The rescattering mechanism

- The calculation approach of the triangle diagram





- There have two different approaches

 --(a) Directly calculate the triangle loop and add a form factor(cut off) to deal with the divergence

 --(b) By using the optical theorem and Cutkosky cutting rule and also add a form factor

The rescattering mechanism

- (a) Directly calculate the triangle loop

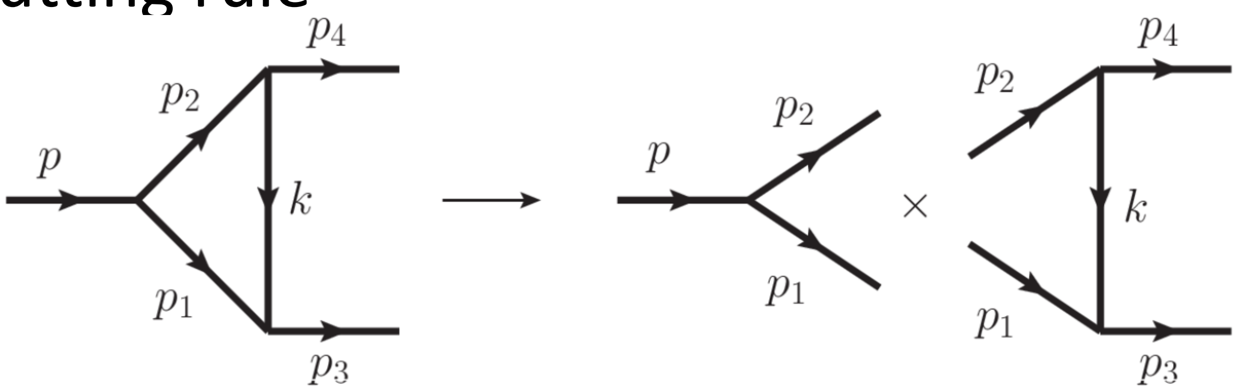
$$\mathcal{M}(I_1,I_2;I_3)=\int\frac{d^4k}{(2\pi)^4}\frac{A(B(p)\to I_1(p_1)I_2(p_2))\,A\Big(I_1(p_1)I_2(p_2)\xrightarrow{I_3(k)}f_1(p_3)f_2(p_4)\Big)}{(p_1^2-m_1^2)(p_2^2-m_2^2)}$$

- The introduced form factor $F(k^2)=\dfrac{\Lambda^2-m_k^2}{\Lambda^2-k^2}$
- The weak decay form factor will introduce some complexity in the loop integration

$$F(q^2)=\frac{F(0)}{\left(1-\alpha\frac{q^2}{M^2}\right)}\quad or\quad F(q^2)=\frac{F(0)}{\left(1-\alpha_1\frac{q^2}{M^2}\right)\left(1-\alpha_2\frac{q^2}{M^2}\right)}$$

The rescattering mechanism

- (b) By using the optical theorem and Cutkosky cutting rule



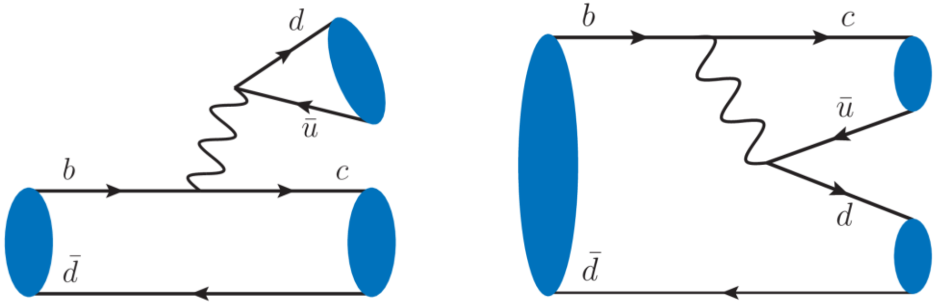
$$\mathrm{Abs}\mathcal{M}(I_1,I_2;I_3)=\int\frac{d^4p_1}{(2\pi)^3 2E_1}\frac{d^4p_2}{(2\pi)^3 2E_2}\delta^4(p-p_1-p_2)A(B(p)\to I_1(p_1)I_2(p_2))$$
$$\times A\Big(I_1(p_1)I_2(p_2)\xrightarrow{I_3(k)}f_1(p_3)f_2(p_4)\Big)$$

- The introduced form factor $F(k^2)=\dfrac{\Lambda^2-m_k^2}{\Lambda^2-k^2}$

The rescattering mechanism

- The short-distance amplitude

$$A(B(p) \to I_1(p_1) I_2(p_2))$$



- We deal with it in naïve factorization approach (BSW), to avoid double counting

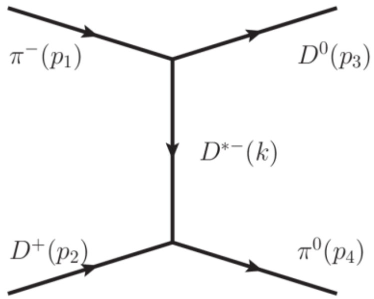
$$\langle \mathcal{B}_c M | \mathcal{H}_{eff} | \mathcal{B}_{cc} \rangle^T_{\rm SD} = \frac{G_F}{\sqrt{2}} V^*_{cq'} V_{uq} a_1(\mu) \langle M | \bar{u} \gamma^\mu (1-\gamma_5) q | 0 \rangle \langle \mathcal{B}_c | \bar{q}' \gamma_\mu (1-\gamma_5) c | \mathcal{B}_{cc} \rangle$$

$$\langle \mathcal{B}_c M | \mathcal{H}_{eff} | \mathcal{B}_{cc} \rangle^C_{\rm SD} = \frac{G_F}{\sqrt{2}} V^*_{cq'} V_{uq} a_2(\mu) \langle M | \bar{q}' \gamma^\mu (1-\gamma_5) q | 0 \rangle \langle \mathcal{B}_c | \bar{u} \gamma_\mu (1-\gamma_5) c | \mathcal{B}_{cc} \rangle$$

The rescattering mechanism

- The hadron-hadron scattering amplitude

$$A\Big(I_1(p_1)\,I_2(p_2)\xrightarrow{I_3(k)} f_1(p_3)\,f_2(p_4)\Big)$$



- We calculate this amplitude based on the hadronic level effective Lagrangian
- Chiral effective theory
- Heavy meson chiral perturbation theory

The rescattering mechanism

- Reliability of the rescattering mechanism

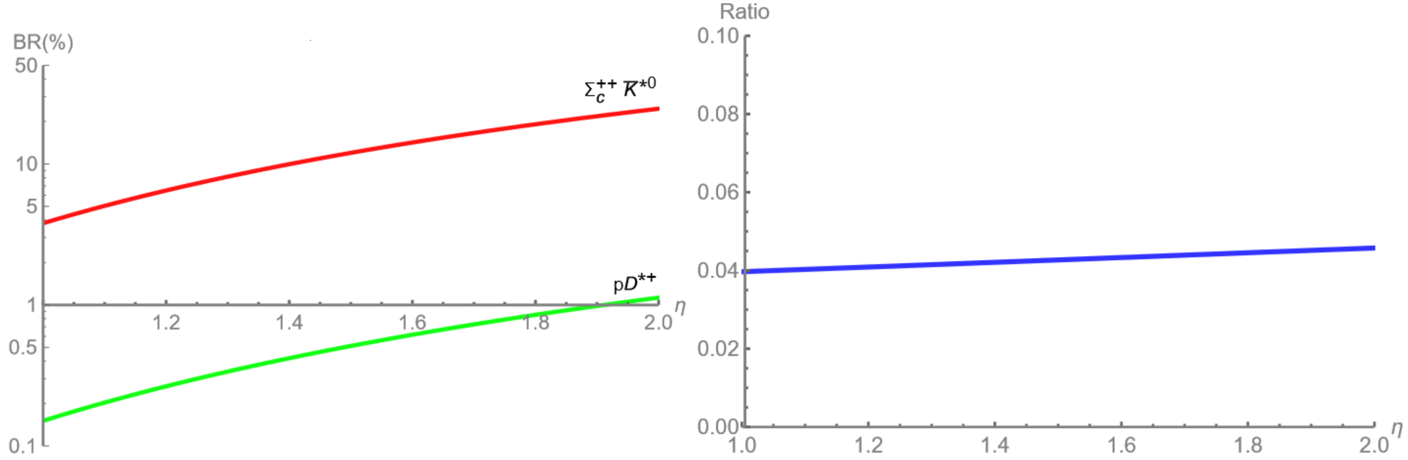


FIG. 3: The absolute branching fractions of $\Xi_{cc}^{++} \to \Sigma_c^{++}\overline{K}^{*0}$ and pD^{*+} (top) and the ratio of $\mathcal{B}(\Xi_{cc}^{++} \to pD^{*+})/\mathcal{B}(\Xi_{cc}^{++} \to \Sigma_c^{++}\overline{K}^{*0})$ (bottom) as functions of η.

[Fu-Sheng Yu, **Hua-Yu Jiang**, Cai-Dian Lü, Wei Wang, Zhen-Xing Zhao, Chin.Phys. C42 (2018), 051001]

The isospin triangle relation

- $SU(3)_F$ flavor symmetry and $SU(2)_I \times SU(2)_U \times SU(2)_V$ symmetry (Isospin, U-spin, V-spin)
- One of the importance of the $SU(3)_F$ relation or isospin relation is to test the self-consistency of the theory
- Usually, the U-spin and V-spin symmetry will have a relatively bigger breaking effect

The isospin triangle relation

- The different definition for the hadron matrix

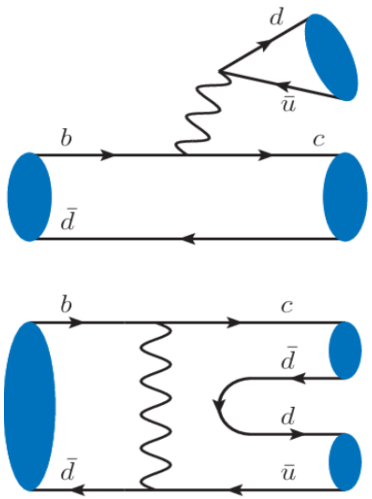
  isospin doublet $\begin{pmatrix} u \\ d \end{pmatrix}$ $\begin{pmatrix} \bar{d} \\ -\bar{u} \end{pmatrix}$ or V-spin doublet $\begin{pmatrix} d \\ s \end{pmatrix}$ $\begin{pmatrix} \bar{s} \\ -\bar{d} \end{pmatrix}$

$$\begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta_8 \end{pmatrix} \quad \begin{pmatrix} \frac{\rho^0}{\sqrt{2}}+\frac{\omega_8}{\sqrt{6}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}}+\frac{\omega_8}{\sqrt{6}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\sqrt{\frac{2}{3}}\omega_8 \end{pmatrix} \quad \pi^0 = \frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$$

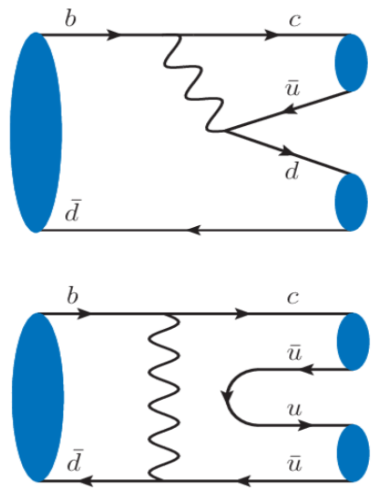
$$\begin{pmatrix} -\frac{\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ -\pi^- & \frac{\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}} & K^0 \\ -K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta_8 \end{pmatrix} \quad \begin{pmatrix} -\frac{\rho^0}{\sqrt{2}}+\frac{\omega_8}{\sqrt{6}} & \rho^+ & K^{*+} \\ -\rho^- & \frac{\rho^0}{\sqrt{2}}+\frac{\omega_8}{\sqrt{6}} & K^{*0} \\ -K^{*-} & \bar{K}^{*0} & -\sqrt{\frac{2}{3}}\omega_8 \end{pmatrix} \quad \pi^0 = \frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$$

The isospin triangle relation

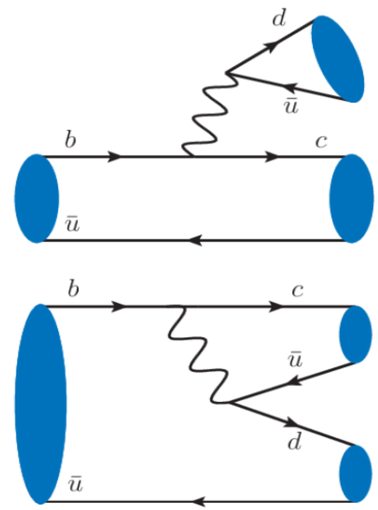
- Take $B \to D\pi$ as an example:
- In the topology amplitude approach



$T + E$



$\frac{1}{\sqrt{2}}(-C + E)$



$T + C$

$$A\left(\overline{B}^0 \to D^+\pi^-\right) = \sqrt{2}\,A\left(\overline{B}^0 \to D^0\pi^0\right) + A\left(B^- \to D^0\pi^-\right)$$

The isospin triangle relation

- We can also directly calculate the relation by the isospin relation

$$\bar{B}^0 = b\bar{d} = \left|\frac{1}{2},\frac{1}{2}\right\rangle, B^- = b\bar{u} = -\left|\frac{1}{2},-\frac{1}{2}\right\rangle, D^+ = c\bar{d} = \left|\frac{1}{2},\frac{1}{2}\right\rangle, D^0 = c\bar{u} = -\left|\frac{1}{2},-\frac{1}{2}\right\rangle$$

$$\pi^0 = \frac{u\bar{u} - d\bar{d}}{\sqrt{2}} = -|1,0\rangle,\ \pi^- = d\bar{u} = -|1,-1\rangle,\ H_{eff} = -|1,-1\rangle,\ b \to cd\bar{u}$$

$$\langle D^0\pi^0|H_{eff}|B^0\rangle = -\frac{\sqrt{2}}{3}A_{\frac{3}{2}} + \frac{\sqrt{2}}{3}A_{\frac{1}{2}},$$

$$\langle D^+\pi^-|H_{eff}|B^0\rangle = \frac{1}{3}A_{\frac{3}{2}} + \frac{2}{3}A_{\frac{1}{2}}$$

$$\langle D^0\pi^-|H_{eff}|B^-\rangle = A_{\frac{3}{2}}$$

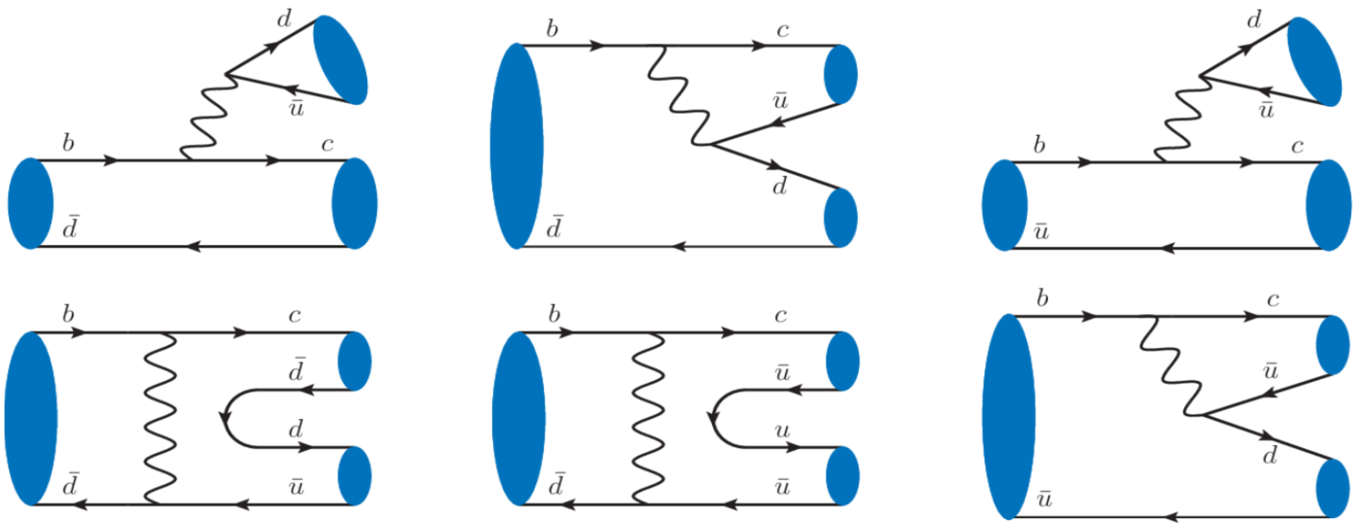
$$A_{\frac{3}{2}} = \left\langle \frac{3}{2},-\frac{1}{2}\middle|\frac{3}{2},-\frac{1}{2}\right\rangle$$

$$A_{\frac{1}{2}} = \left\langle \frac{1}{2},-\frac{1}{2}\middle|\frac{1}{2},-\frac{1}{2}\right\rangle$$

$$\sqrt{2}\,A\left(\bar{B}^0 \to D^0\pi^0\right) + A\left(B^- \to D^0\pi^-\right) = A\left(\bar{B}^0 \to D^+\pi^-\right)$$

The isospin triangle relation

- In the topology amplitude approach



$-T-E$ $\qquad$ $\frac{1}{\sqrt{2}}(-C+E)$ $\qquad$ $-T-C$

$$\sqrt{2}\,A\left(\overline{B}^0 \to D^0\pi^0\right) + A\left(\overline{B}^0 \to D^+\pi^-\right) = A\left(B^- \to D^0\pi^-\right)$$

The isospin triangle relation

- We can also directly calculate the relation by the isospin relation

$$\bar{B}^0 = b\bar{d} = \left|\frac{1}{2},\frac{1}{2}\right\rangle, B^- = -b\bar{u} = \left|\frac{1}{2},-\frac{1}{2}\right\rangle, D^+ = c\bar{d} = \left|\frac{1}{2},\frac{1}{2}\right\rangle, D^0 = -c\bar{u} = \left|\frac{1}{2},-\frac{1}{2}\right\rangle$$

$$\pi^0 = \frac{d\bar{d} - u\bar{u}}{\sqrt{2}} = |1,0\rangle,\ \pi^- = -d\bar{u} = |1,-1\rangle,\ H_{eff} = |1,-1\rangle,\ b \to cd\bar{u}$$

$$\langle D^0\pi^0|H_{eff}|\bar{B}^0\rangle = \frac{\sqrt{2}}{3}A_{\frac{3}{2}} - \frac{\sqrt{2}}{3}A_{\frac{1}{2}},$$

$$\langle D^+\pi^-|H_{eff}|\bar{B}^0\rangle = \frac{1}{3}A_{\frac{3}{2}} + \frac{2}{3}A_{\frac{1}{2}}$$

$$\langle D^0\pi^-|H_{eff}|B^-\rangle = A_{\frac{3}{2}}$$

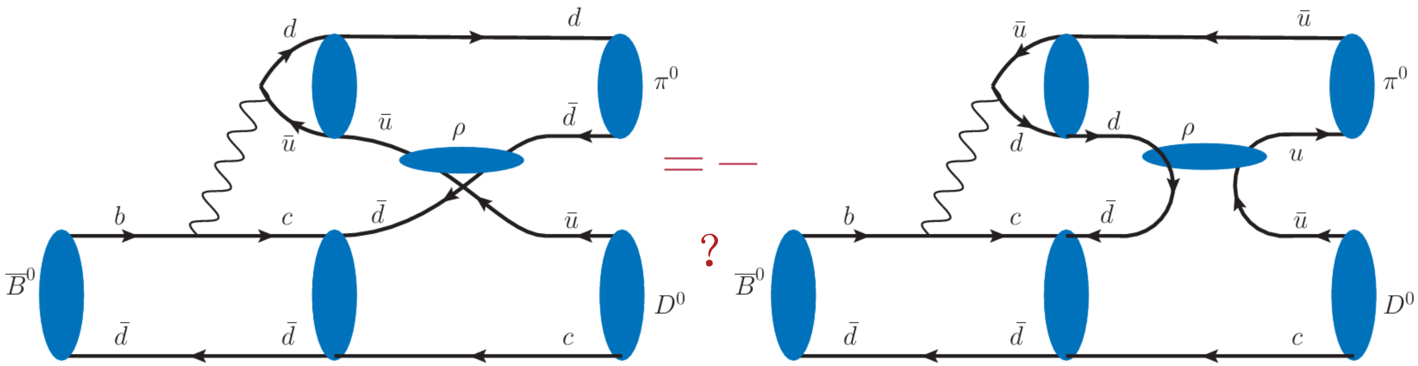
$$A_{\frac{3}{2}} = \left\langle \frac{3}{2},-\frac{1}{2}\middle|\frac{3}{2},-\frac{1}{2}\right\rangle$$

$$A_{\frac{1}{2}} = \left\langle \frac{1}{2},-\frac{1}{2}\middle|\frac{1}{2},-\frac{1}{2}\right\rangle$$

$$\sqrt{2}\,A\left(\bar{B}^0 \to D^0\pi^0\right) + A\left(\bar{B}^0 \to D^+\pi^-\right) = A(B^- \to D^0\pi^-)$$

The ITR in rescattering mechanism

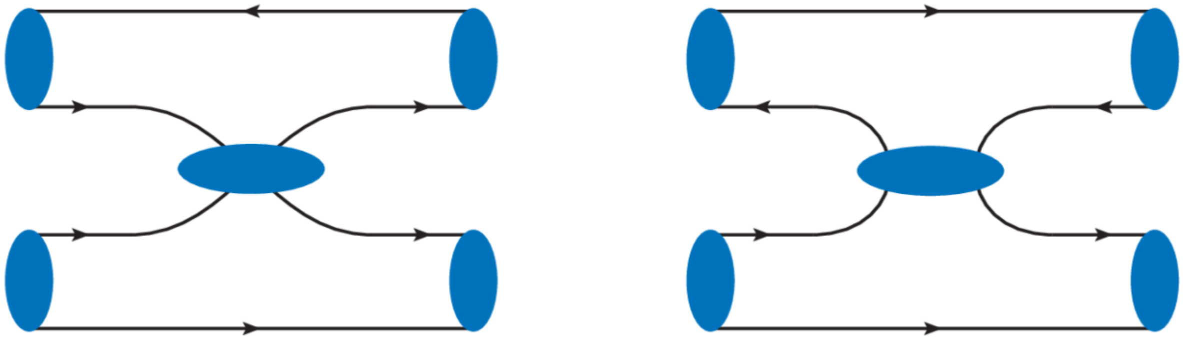
- The isospin triangle relation should also be kept in rescattering mechanism
- To obtain the isospin triangle relation, the assumption had been made[Phys.Rev.D 71, 014030(2005)]



- Since the wave function $\pi^0 = \frac{d\bar{d} - u\bar{u}}{\sqrt{2}}$ or $\frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$

The ITR in rescattering mechanism

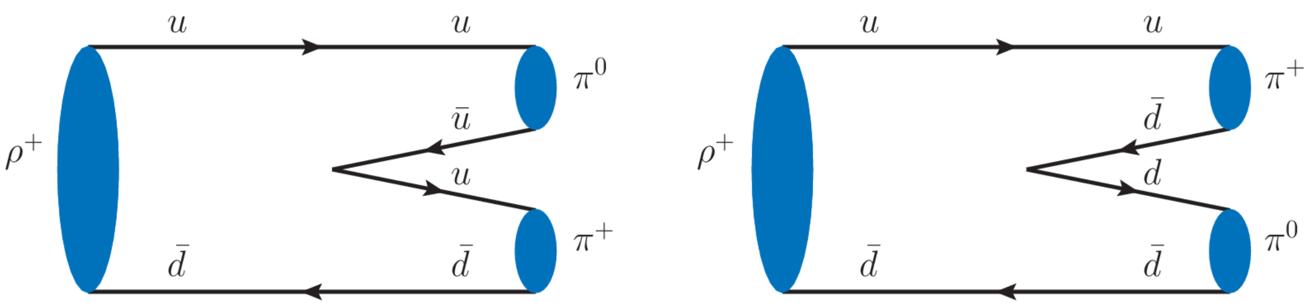
- Consider the next two points
- First, the two diagram can not be canceled



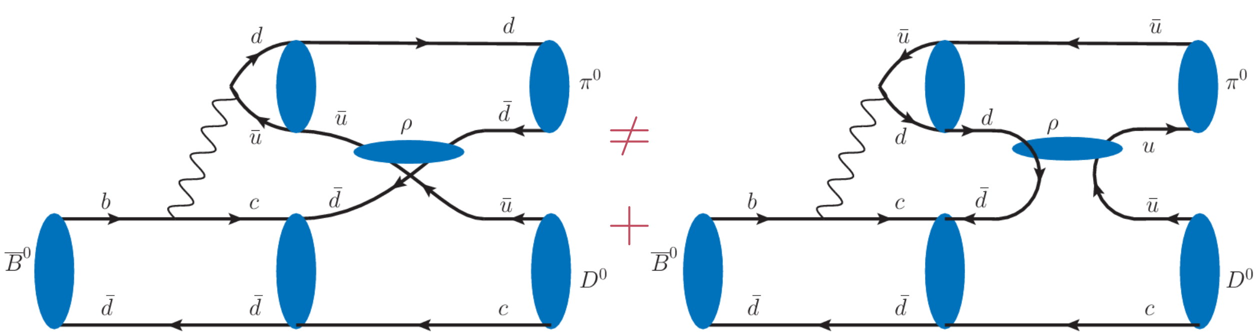
- Since, the two amplitudes aren’t identical
- In Lattice QCD, the first one can be easily calculated

The ITR in rescattering mechanism

- Second, the phase difference should be considered



- Since the exchange of the final state will also produce another relative minus sign



The ITR in rescattering mechanism

- Consider the external *W*-emission diagram ***T*** in the weak vertex, then in isospin symmetry

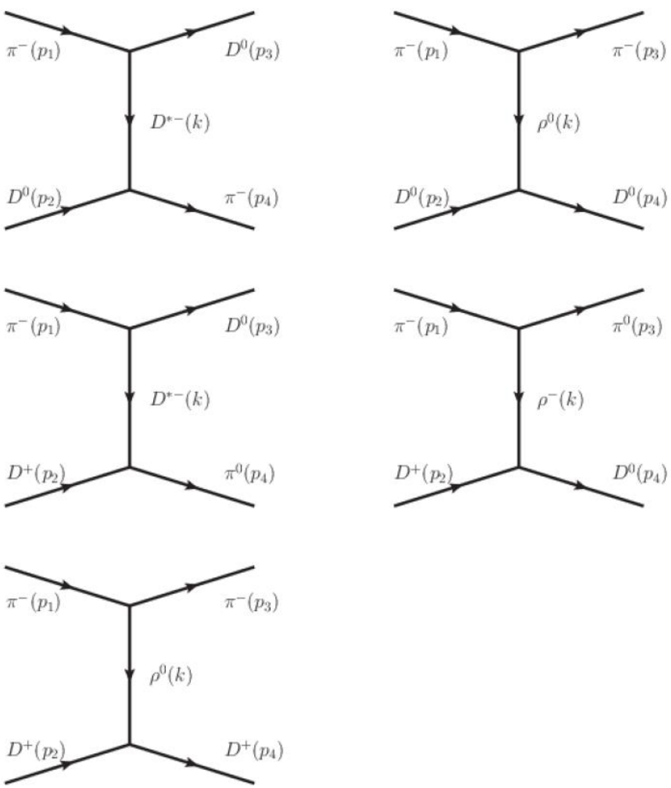
$$A\left(\overline{B}^0 \to D^+\pi^-\right) = \quad T \quad = A(B^- \to D^0\pi^-)$$

- We calculate ***T*** in naïve factorization approach

$$T\left(\overline{B}^0 \to D^+\pi^-\right) = i\frac{G_F}{\sqrt{2}}V_{cb}V_{ud}^*\boxed{a_1(\mu)}(m_B^2 - m_D^2)f_\pi F_0^{BD}(m_\pi^2)$$

The ITR in rescattering mechanism

- The relation only depends on the scattering process
- And the corresponding Feynman rules



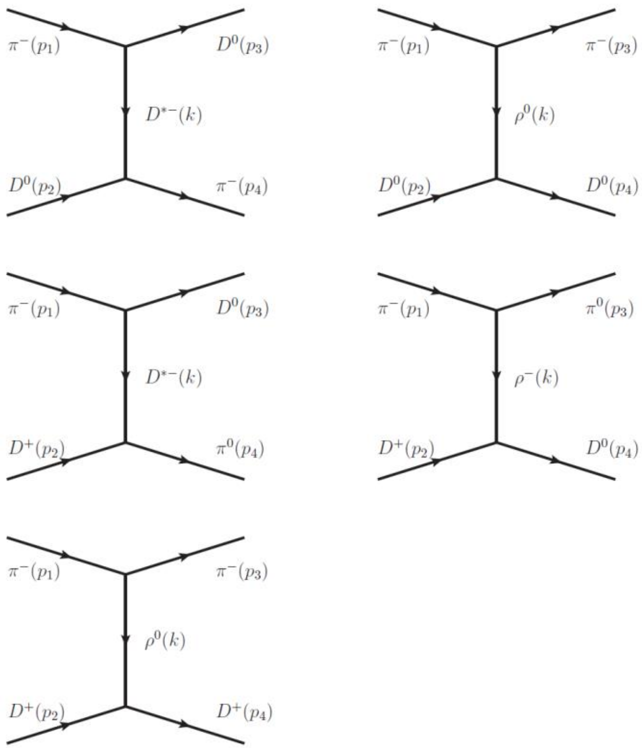
$$\langle \rho^0(k)\pi^-(p_3)|\pi^-\rangle = ig_{\rho\pi\pi}\epsilon^*_\mu(k)\,(p_1+p_3)^\mu$$
$$\langle \rho^-(k)\pi^0(p_3)|\pi^-\rangle = -ig_{\rho\pi\pi}\epsilon^*_\mu(k)\,(p_1+p_3)^\mu$$

$$\langle \pi^-(p_4)|D^0(p_2)D^{*-}(k)\rangle = ig_{D^*D\pi}\epsilon_\mu(k)\,(ip_4)$$
$$\langle \pi^0(p_4)|D^+(p_2)D^{*-}(k)\rangle = -\frac{i}{\sqrt{2}}g_{D^*D\pi}\epsilon_\mu(k)\,(ip_4^\mu)$$

$$\langle D^0(p_4)|\rho^0(k)D^0(p_2)\rangle = -\frac{i}{\sqrt{2}}g_{\rho DD}\epsilon_\mu(k)\,(p_2+p_4)^\mu$$
$$\langle D^0(p_4)|\rho^-(k)D^+(p_2)\rangle = -ig_{\rho DD}\epsilon_\mu(k)\,(p_2+p_4)^\mu$$
$$\langle D^+p_4|\rho^0(k)D^+(p_2)\rangle = \frac{i}{\sqrt{2}}g_{\rho DD}\epsilon_\mu(k)\,(p_2+p_4)^\mu$$

The ITR in rescattering mechanism

- The relation only depends on the scattering process



$$A(D^0\pi^- \to D^0\pi^-) = \boxed{-}\frac{g_{D^*D\pi}g_{D^*D\pi}p_1^\mu p_4^\nu(-g_{\mu\nu}+\frac{k_\mu k_\nu}{m_{D^*}^2})}{k^2-m_{D^*}^2}$$
$$\boxed{+}\frac{1}{\sqrt{2}}\frac{g_{\rho\pi\pi}g_{\rho DD}(p_1+p_3)^\mu(p_2+p_4)^\nu(-g_{\mu\nu}+\frac{k_\mu k_\nu}{m_\rho^2})}{k^2-m_\rho^2}$$

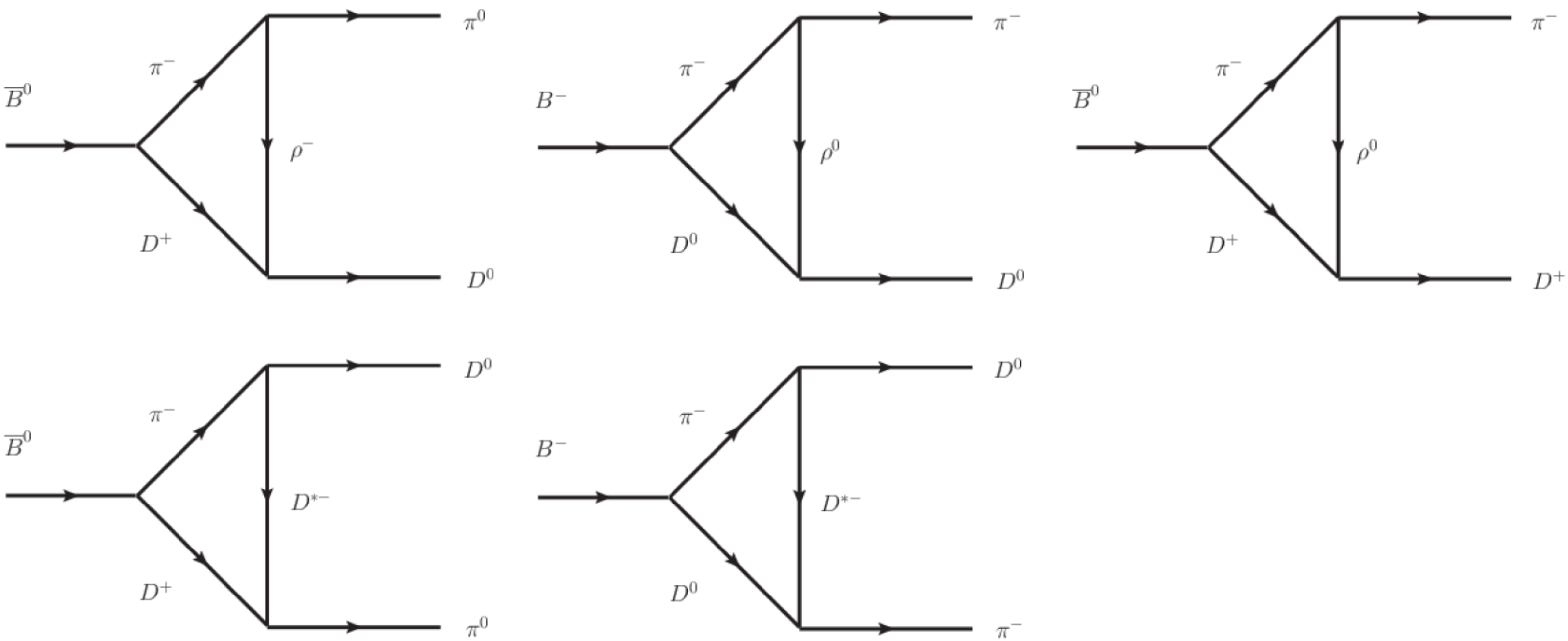
$$A(D^+\pi^- \to D^0\pi^0) = \boxed{+\frac{1}{\sqrt{2}}}\frac{g_{D^*D\pi}g_{D^*D\pi}p_1^\mu p_4^\nu(-g_{\mu\nu}+\frac{k_\mu k_\nu}{m_{D^*}^2})}{k^2-m_{D^*}^2}$$
$$\boxed{-}\frac{g_{\rho\pi\pi}g_{\rho DD}(p_1+p_3)^\mu(p_2+p_4)^\nu(-g_{\mu\nu}+\frac{k_\mu k_\nu}{m_\rho^2})}{k^2-m_\rho^2}$$

$$A(D^+\pi^- \to D^+\pi^-)$$
$$= \boxed{-\frac{1}{\sqrt{2}}}\frac{g_{\rho\pi\pi}g_{\rho DD}(p_1+p_3)^\mu(p_2+p_4)^\nu(-g_{\mu\nu}+\frac{k_\mu k_\nu}{m_\rho^2})}{k^2-m_\rho^2}$$

$$\sqrt{2}\,A(D^+\pi^- \to D^0\pi^0) + A(D^0\pi^- \to D^0\pi^-) = A(D^+\pi^- \to D^+\pi^-)$$

The ITR in rescattering mechanism

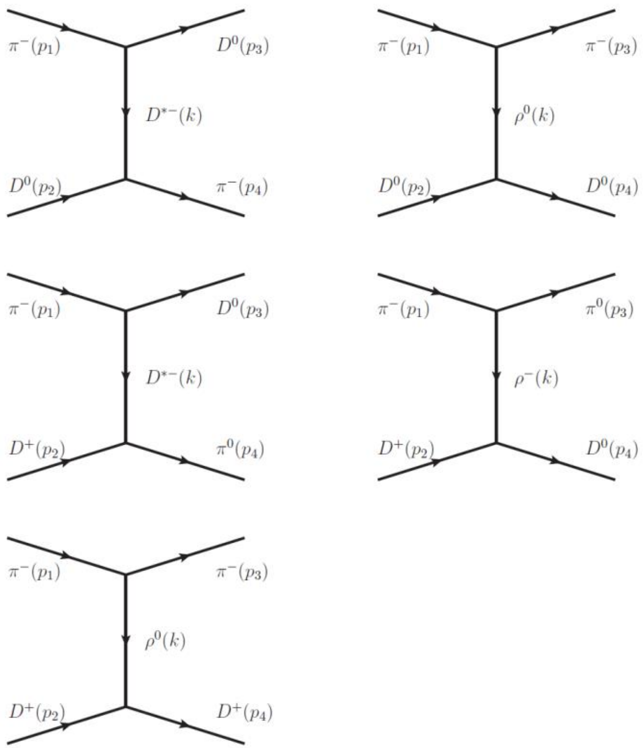
- We can simply obtain the ITR of $B \to D\pi$



$$\sqrt{2}\,A\left(\overline{B}^0 \to D^0\pi^0\right) + A(B^- \to D^0\pi^-) = A\left(\overline{B}^0 \to D^+\pi^-\right)$$

The ITR in rescattering mechanism

- The relation only depends on the scattering process



$$A(D^0\pi^- \to D^0\pi^-) = \boxed{-}\frac{g_{D^*D\pi}g_{D^*D\pi}p_1^\mu p_4^\nu(-g_{\mu\nu}+\frac{k_\mu k_\nu}{m_{D^*}^2})}{k^2-m_{D^*}^2}$$

$$\boxed{+}\frac{1}{\sqrt{2}}\frac{g_{\rho\pi\pi}g_{\rho DD}(p_1+p_3)^\mu(p_2+p_4)^\nu(-g_{\mu\nu}+\frac{k_\mu k_\nu}{m_\rho^2})}{k^2-m_\rho^2}$$

$$A(D^+\pi^- \to D^0\pi^0) = \boxed{-\frac{1}{\sqrt{2}}}\frac{g_{D^*D\pi}g_{D^*D\pi}p_1^\mu p_4^\nu(-g_{\mu\nu}+\frac{k_\mu k_\nu}{m_{D^*}^2})}{k^2-m_{D^*}^2}$$

$$\boxed{+}\frac{g_{\rho\pi\pi}g_{\rho DD}(p_1+p_3)^\mu(p_2+p_4)^\nu(-g_{\mu\nu}+\frac{k_\mu k_\nu}{m_\rho^2})}{k^2-m_\rho^2}$$

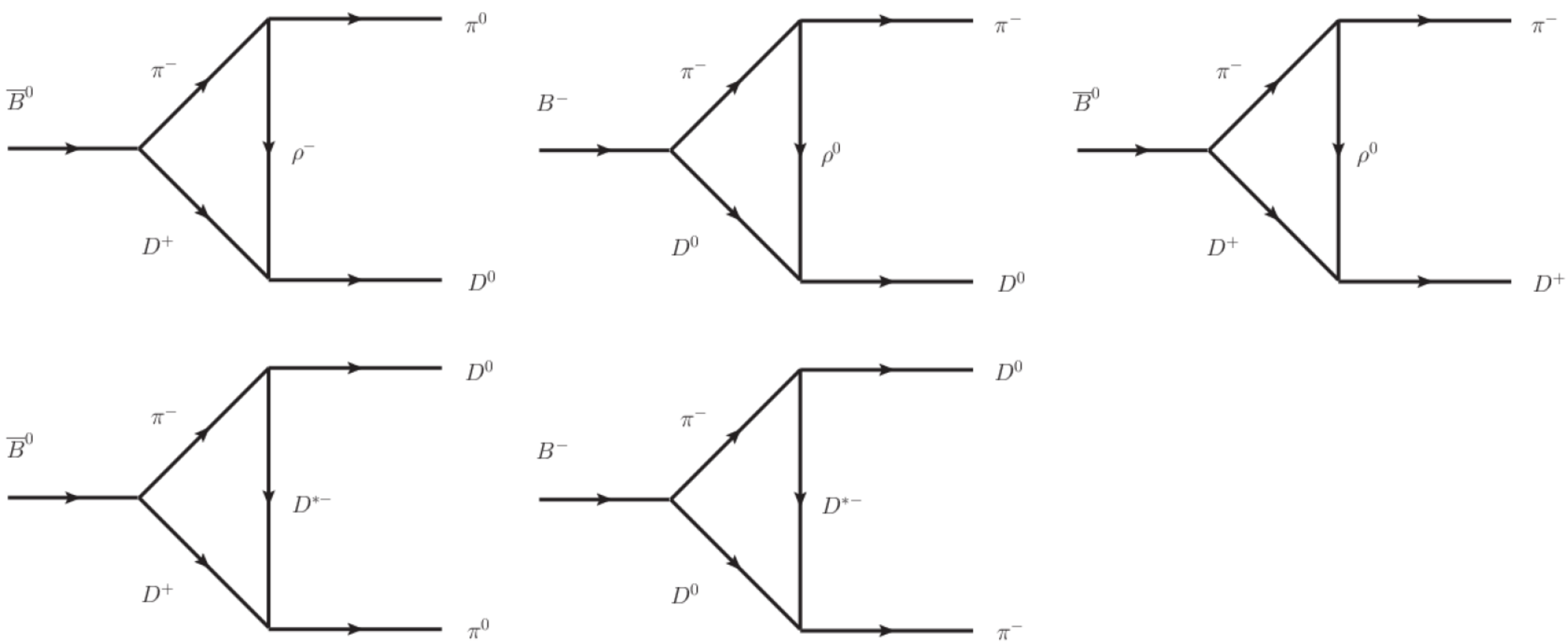
$$A(D^+\pi^- \to D^+\pi^-)$$

$$= \boxed{-\frac{1}{\sqrt{2}}}\frac{g_{\rho\pi\pi}g_{\rho DD}(p_1+p_3)^\mu(p_2+p_4)^\nu(-g_{\mu\nu}+\frac{k_\mu k_\nu}{m_\rho^2})}{k^2-m_\rho^2}$$

$$\sqrt{2}\,A(D^+\pi^- \to D^0\pi^0) + A(D^+\pi^- \to D^+\pi^-) = A(D^0\pi^- \to D^0\pi^-)$$

The ITR in rescattering mechanism

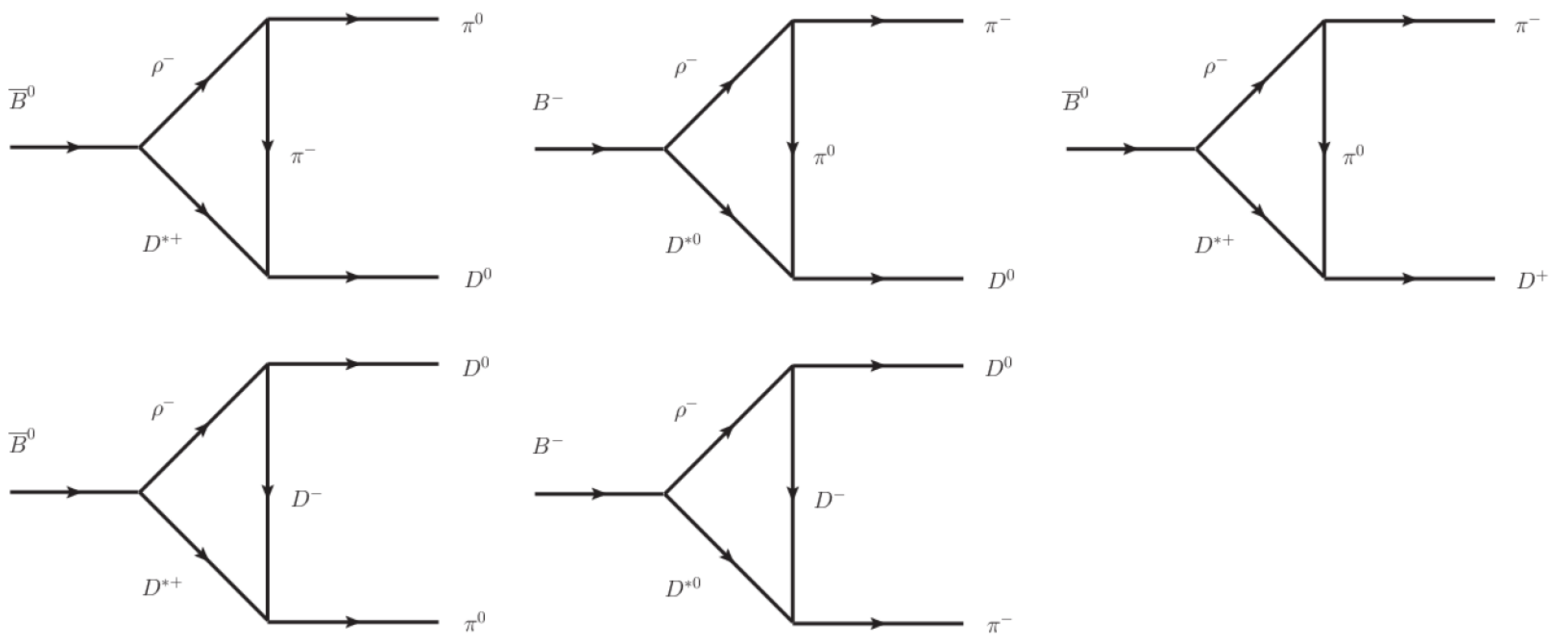
- We can simply obtain the ITR of $B \to D\pi$



$$\sqrt{2}\,A\left(\overline{B}^0 \to D^0\pi^0\right) + A\left(\overline{B}^0 \to D^+\pi^-\right) = A(B^- \to D^0\pi^-)$$

The ITR in rescattering mechanism

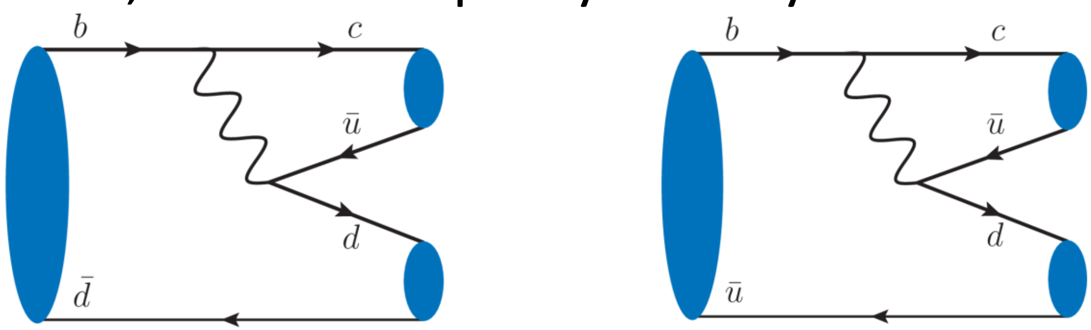
- For different intermediate hadron states or different kinds of effective hadron coupling vertex



$$\sqrt{2}\,A\left(\overline{B}^0 \to D^0\pi^0\right) + A\left(\overline{B}^0 \to D^+\pi^-\right) = A(B^- \to D^0\pi^-)$$

The ITR in rescattering mechanism

- And also, if consider the internal *W*-emission diagram ***C*** in the weak vertex, then in isospin symmetry



$$A\left(\overline{B}^0 \to D^0\pi^0\right) = \quad C \quad = A(B^- \to D^0\pi^-)$$

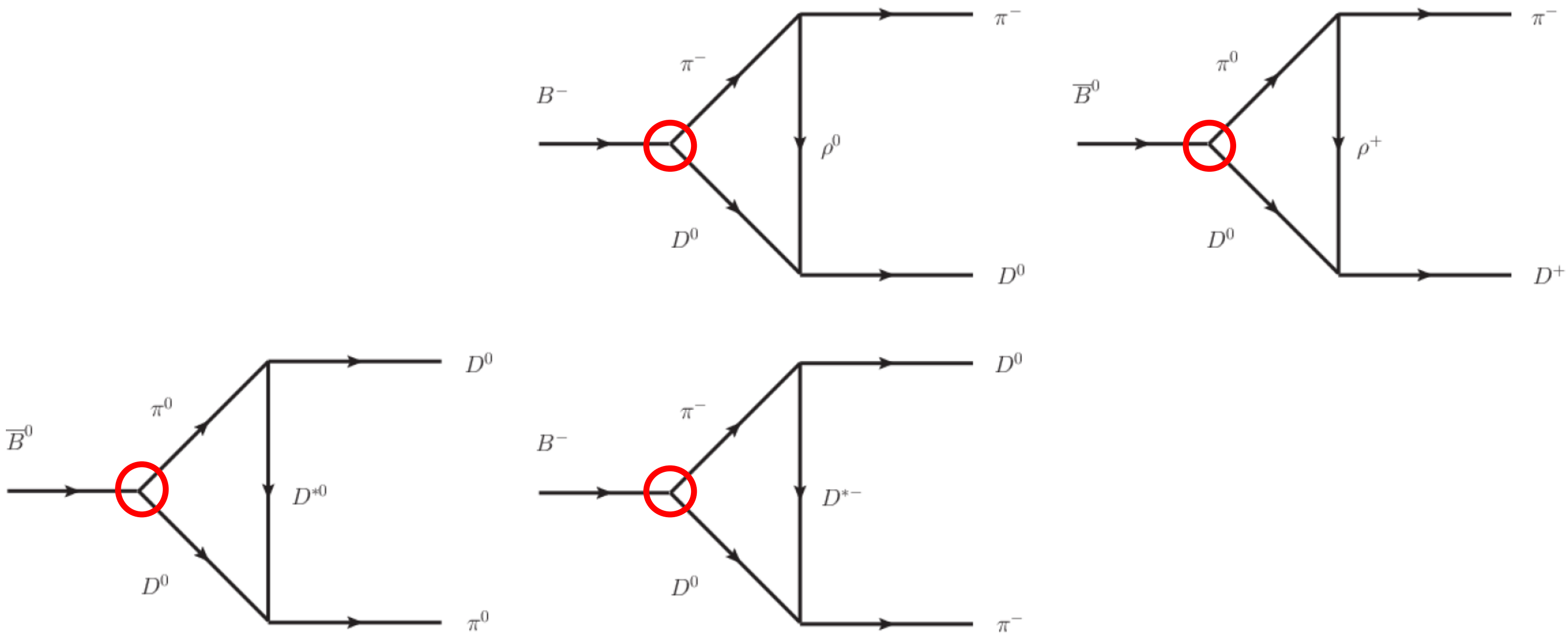
- We also calculate the short-distance contributions of ***C*** in naïve factorization approach (BSW)

$$C\left(\overline{B}^0 \to D^+\pi^-\right) = i\frac{G_F}{\sqrt{2}}V_{cb}V_{ud}^*\boxed{a_2(\mu)}\,(m_B^2 - m_D^2)f_\pi F_0^{BD}(m_\pi^2)$$

- $a_1 = 1.07, a_2 = 0.017$

The ITR in rescattering mechanism

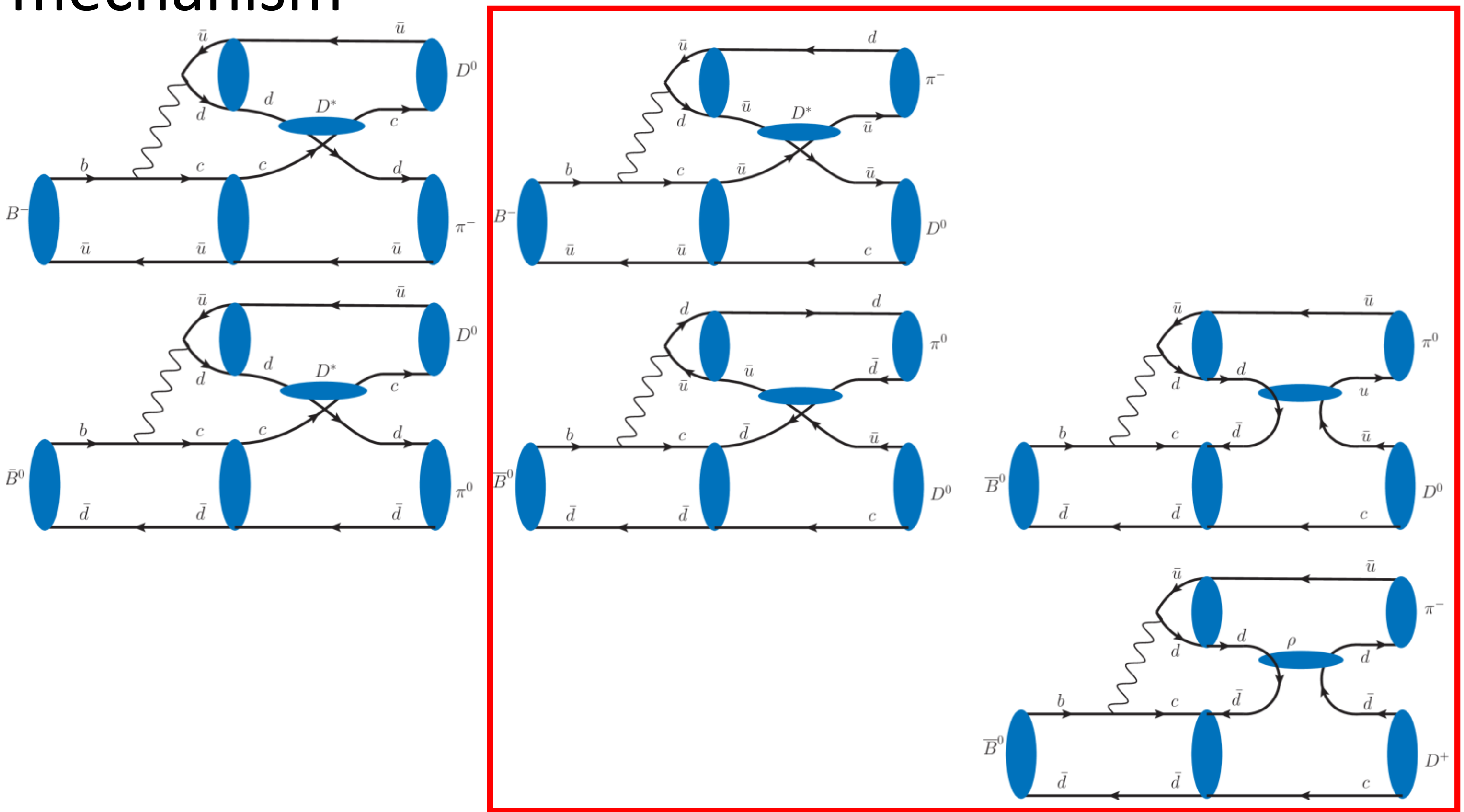
- If the weak vertex is ***C*** diagram



$$\sqrt{2}\,A\left(\overline{B}^0 \to D^0\pi^0\right) + A\left(\overline{B}^0 \to D^+\pi^-\right) = A(B^- \to D^0\pi^-)$$

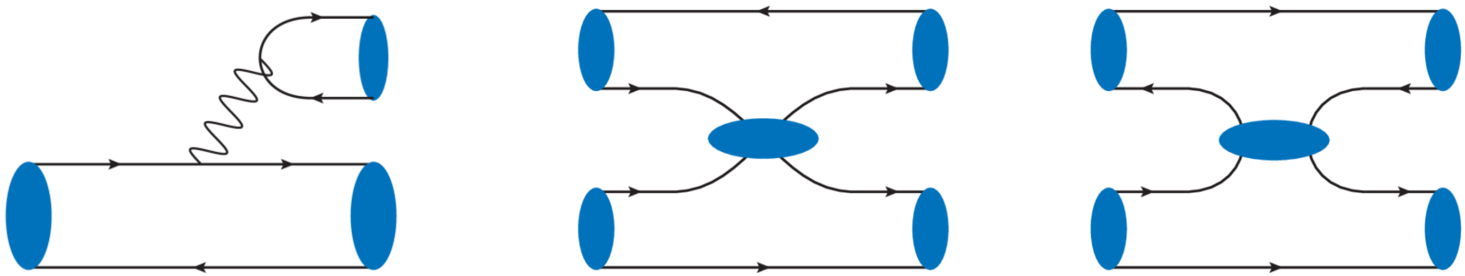
The quark diagram of rescattering mechanism

- The quark level diagram of $B \to D\pi$ in rescattering mechanism

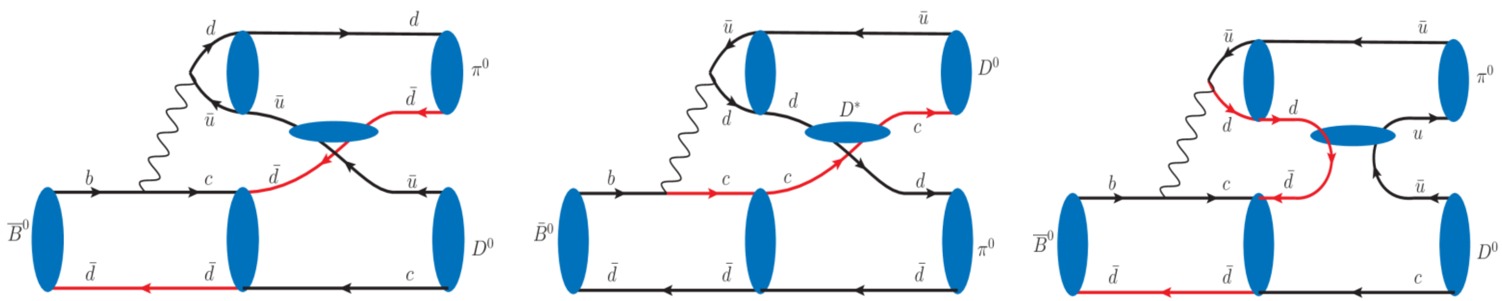


How to get the quark diagram

- We combine the topology diagram of weak vertex with quark level meson-meson scattering diagram

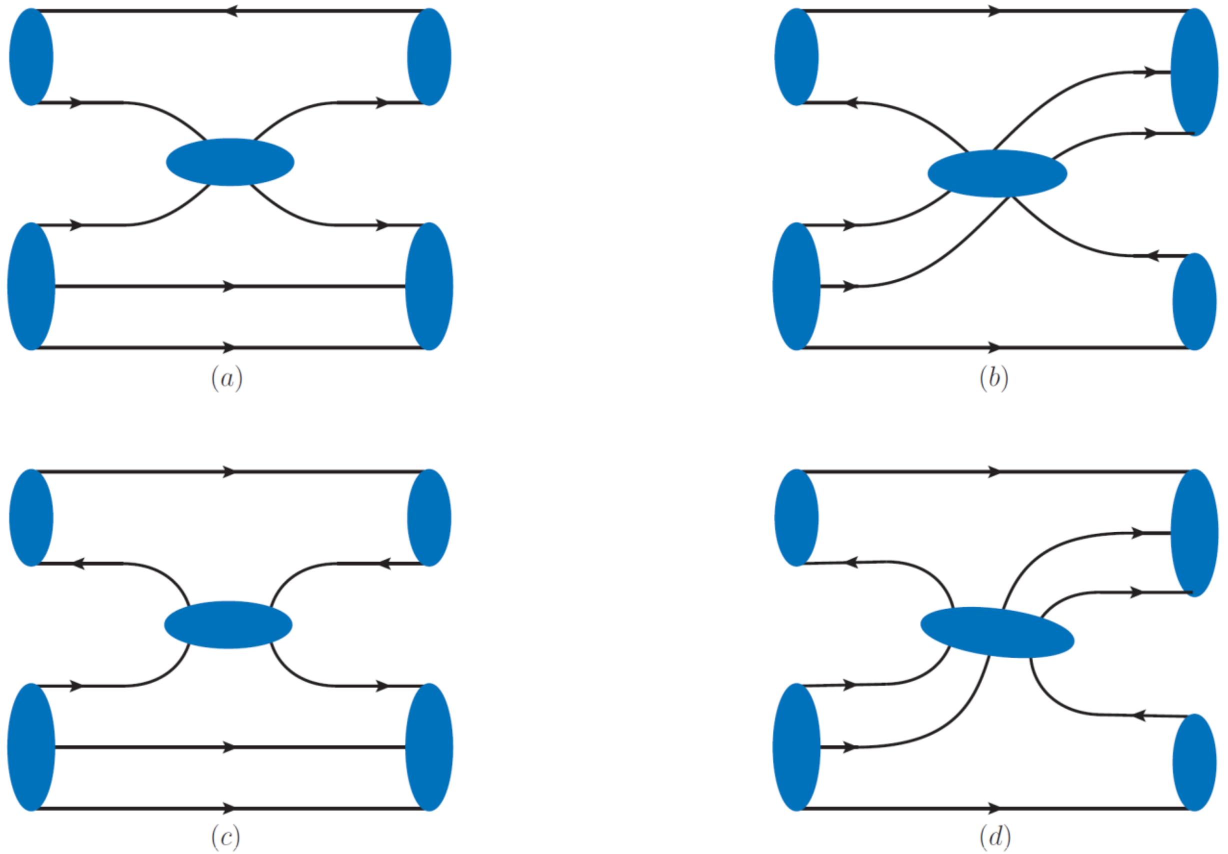


- There have several different conbination



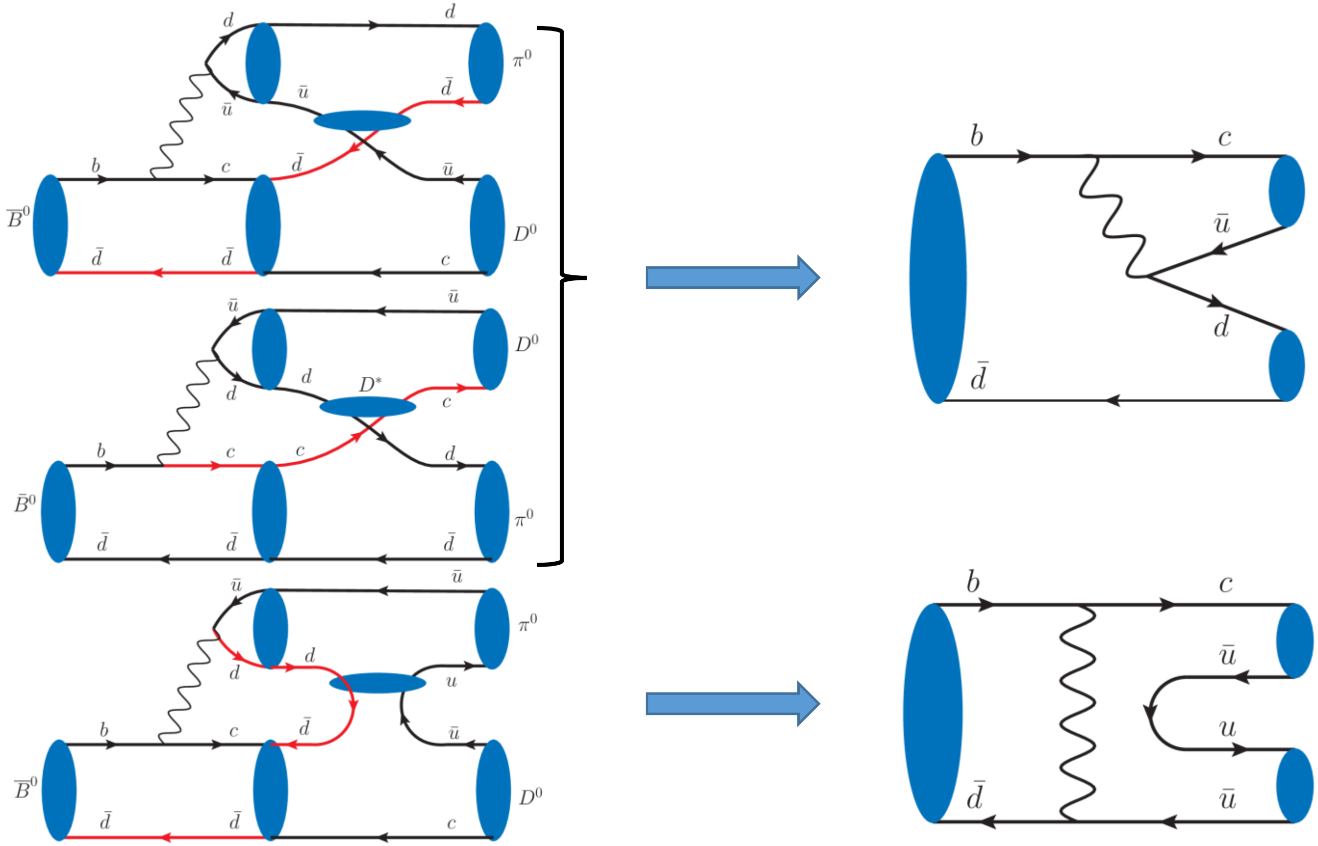
How to get the quark diagram

- For baryon decay, this will be more complicated

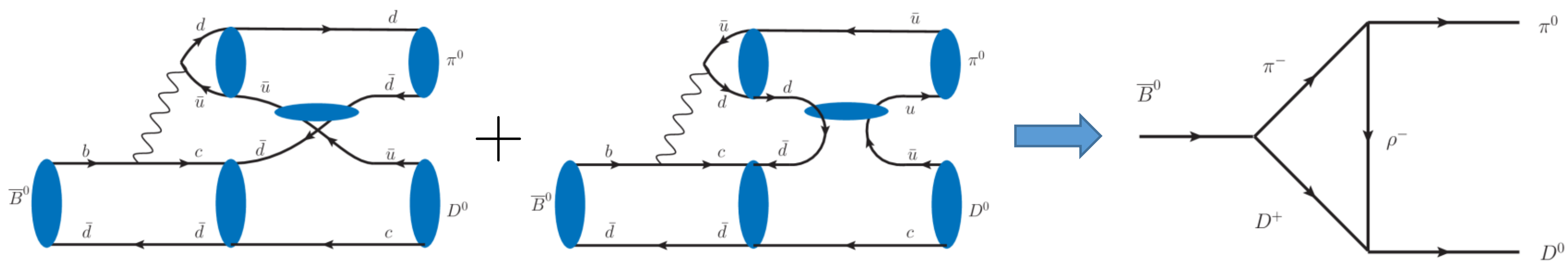


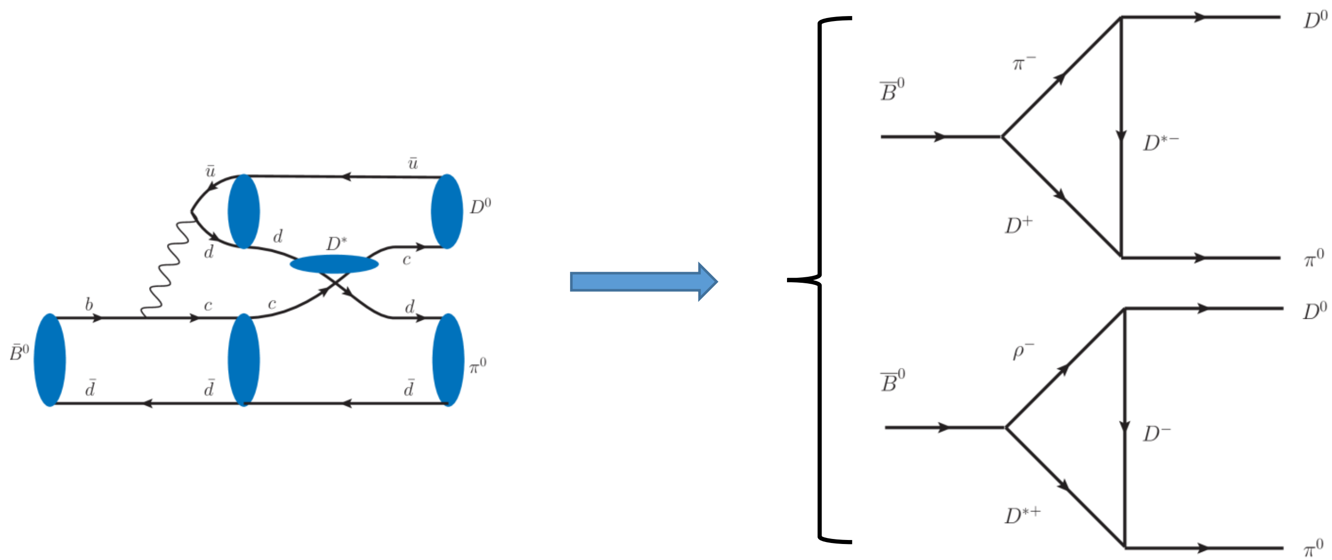
The corresponding between quark diagram and topology diagram

- The quark diagram contribute to the topology



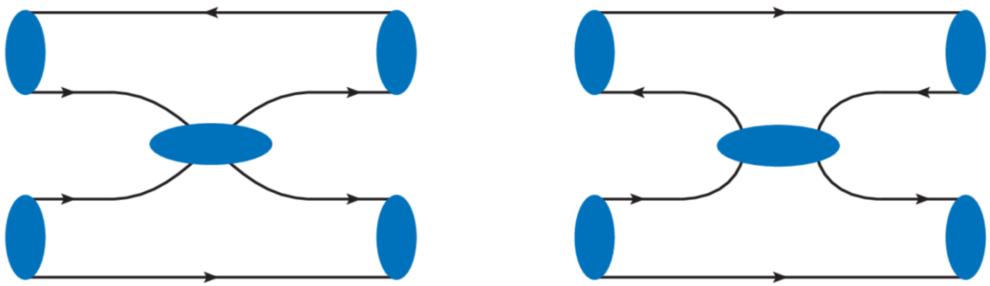
The corresponding between quark diagram and hadron diagram





Meson-meson scattering amplitude

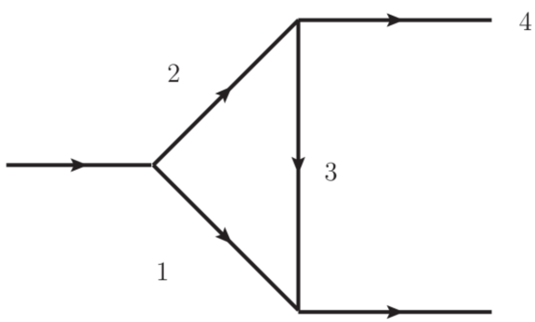
- If we deeply understand the quark diagram of rescattering mechanism, we can extract the meson-meson scattering amplitude

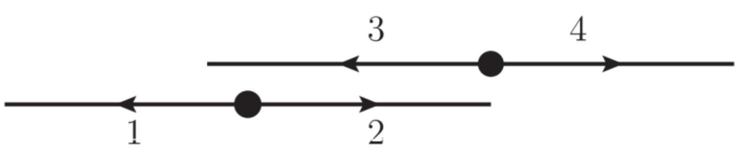


- We can’t calculate these amplitude, except for the Lattice QCD, actually only the first one.
- We can use the Cutkosky rule, the initial and final state particles will be on shell in the scattering.

Triangle singularity

- If all the three intermediate particles are its mass shell in the meantime, i.e., the rescattering process being a realistic physical process





- In this case, the loop-integral will be divergence, that is the triangle singularity.

Triangle singularity

- The criteria for the triangle singularity

$$\frac{m_1 m_4^2 + m_3 M^2}{m_1 + m_3} - m_1 m_2 \leqslant \; m_2^2 \; \leqslant (M - m_1)^2$$

- We can deal with the triangle singularity by substitute the propagator of particle two with an Breit-Wigner propagator, where the width of the particle two are considered

$$\frac{i}{p_2^2 - m_2^2} \;\Rightarrow\; \frac{i}{p_2^2 - m_2^2 + i m_2 \Gamma}$$

An introduction for our group

- We are studying the decay of doubly heavy baryons in rescattering mechanism, including doubly charmed baryon, *bc*-baryon and *bb*-baryon.

  Fu-Sheng Yu, Hua-Yu Jiang et.al,

  Rui-Hui Li et.al,

  Zhen-Jun Xiao et.al

- We are also planning to study the singly charmed baryon decays in rescattering mechanism.

Summary

- We introduce the rescattering mechanism in heavy hadron decay.
- The wrong assumption was made in history to obtain the isospin triangle relation.
- We clarify the error, and the ITR can be obtained without any assumptions.
- We also simply discuss the quark diagram of the rescattering mechanism.
- And the triangle singularity was simply introduced.

Summary

- We introduce the rescattering mechanism in heavy hadron decay.
- The wrong assumption was made in history to obtain the isospin triangle relation.
- We clarify the error, and the ITR can be obtained without any assumptions.
- We also simply discuss the quark diagram of the rescattering mechanism.
- And the triangle singularity was simply introduced.

Thanks for your attention!