QCD bound-state effect on dark matter relic abundance

Ellis@CERN&KCL, **Olive**@Minnesota

Evans@KIAS

Liew@TUM

Shirai@IPMU

Fukuda@Berkeley

Zheng@SJTU&Utah

Ellis, FL, Olive, 2015
Ellis, Evans, FL, Olive, 2016
Liew, FL, 2017
Ellis, Evans, FL, Olive, Zheng, 2018
Fukuda, FL, Shirai, 2019

Feng Luo (羅峰) @Sun Yat-sen U (中山大学)

Why are we interested in calculating dark matter relic abundance?

Because

this is the ***only precise quantity*** we know about dark matter.

Therefore, we hope to work out its implications to the underlying particle theory models by carefully calculating it.

$$\boxed{\Omega_{CDM} h^2 = 0.1193 \pm 0.0014 \ \text{(1-}\sigma\text{, Planck 2015)}}$$

We use thermal freeze-out mechanism to calculate the relic abundance of WIMP dark matter.

Why focus on these?

Because

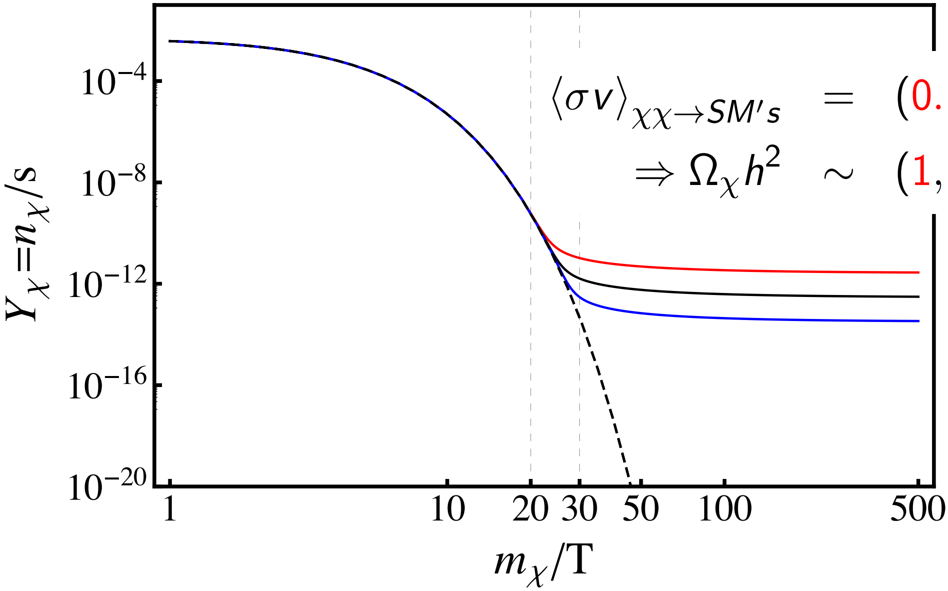
Weakly Interacting Massive Particle (WIMP) is one of the ***best*** candidates for dark matter,

and

thermal freeze-out mechanism is a ***standard mechanism*** to get the dark matter relic abundance.

thermal freeze-out mechanism

$$\boxed{\frac{dn_\chi}{dt} + 3H(T)n_\chi = -\langle \sigma v \rangle_{\chi\chi\to SM's}\left[n_\chi^2 - \left(n_\chi^{eq}\right)^2\right]}$$



$$\langle \sigma v \rangle_{\chi\chi\to SM's} \;=\; (0.1, 1, 10) \times 3 \times 10^{-26}\,\mathrm{cm}^3/\mathrm{s}$$

$$\Rightarrow \Omega_\chi h^2 \;\sim\; (1, 0.1, 0.01)$$

$\langle \sigma v \rangle_{\chi\chi\to SM's} \sim \alpha^2/m_\chi^2$

larger m_χ $\Rightarrow$ smaller $\langle \sigma v \rangle_{\chi\chi\to SM's}$ $\Rightarrow$ larger $\Omega_\chi h^2$,

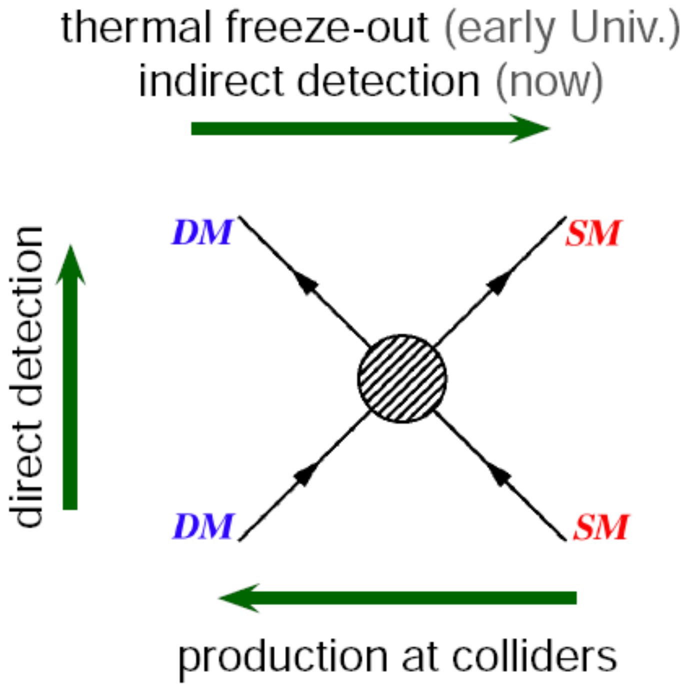
$\Rightarrow$ an upper limit for m_χ

I'm DM.

I'm the expanding Universe.

$$\frac{dn_\chi}{dt} + 3H(T)n_\chi = -\langle \sigma v \rangle_{\chi\chi \to SM's} \left[n_\chi^2 - \left(n_\chi^{eq} \right)^2 \right]$$

No signal yet for WIMP dark matter.



Maybe it is just *too heavy* to be produced in collider?

Maybe its interaction with the Standard Model particles is just *too weak* to give signals for direct/indirect detections?

goal for this project

Considering that the very merits for WIMP being a favored dark matter candidate are its “weak” and “heavy”, and the null result of its searches is directly related to these two features, we want to address:

how weak the interactions of a WIMP could have,

and

how heavy a WIMP could be.

approach for this project

Since so far the only precise quantity we know about dark matter is its relic abundance, we try to address these two questions through calculations of this quantity including the *bound-state effects in coannihilation scenarios*, which have recently been found to play an important role for heavy dark matter.

Literatures on bound-state effects **(INCOMPLETE)**

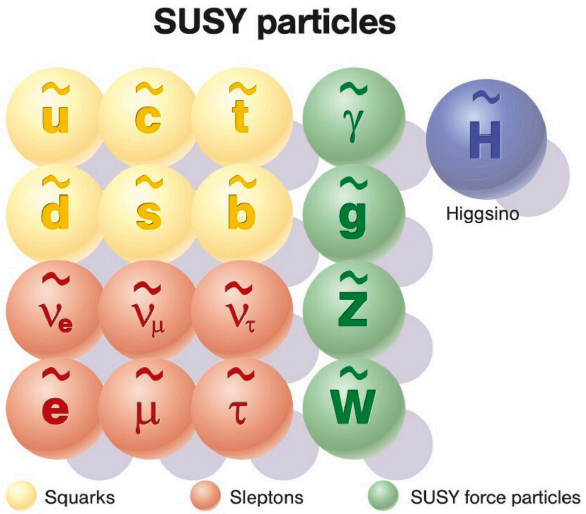
Feng, Kaplinghat, Tu, Yu, 2009;
Kim, Laine, 2016;
Keung, Low, Zhang, 2017;
Harz, Petraki, 2018;
Ko, Matsui, Tang, 2019;
von Harling, Petraki, 2014;
Mitridate, Redi, Smirnov, Strumia, 2017;
An, Wise, Zhang, 2017;
Binder, Covi, Mukaida, 2018;
etc..

I’m a coannihilator.

I’m DM.

I’m the expanding Universe.

We use the neutralino dark matter in supersymmetry as an example to study the bound-state effects.



Why use it?

Because

neutralino in supersymmetry is a typical and well-studied WIMP dark matter.

But, the idea and calculation method are applicable to other models as well.

specify the example

We consider

- The *simplest* version of SUSY --- R-parity conserving MSSM
- The *most studied* DM candidate --- neutralino
- The *standard* mechanism to calculate relic abundance --- freeze-out
- *Coannihilation* between neutralino and some colored particle

conditions for coannihilation to reduce DM relic density

If there is another R-odd species χ_2 almost degenerate in mass with the LSP χ_1,

and if χ_2 has a big annihilation cross section with itself and/or with χ_1,

and if χ_1 can efficiently convert to χ_2,

then χ_1 and χ_2 can freeze out together, resulting in a smaller dark matter abundance than if without the existence of χ_2.

Griest and Seckel, 1991

I’m a coannihilator.

I’m a neutralino.

I’m the expanding Universe.

To get the largest neutralino dark matter mass, we just need to find his fastest running and most muscular friend.

neutralino-gluino coannihilation

$\chi\chi \leftrightarrow SM,\ \chi\tilde{g} \leftrightarrow q\bar{q},\ \tilde{g}\tilde{g} \leftrightarrow q\bar{q}$ or gg,

$\tilde{g}\tilde{g} \leftrightarrow \tilde{R}g, \tilde{R} \leftrightarrow gg$,

$\chi q \leftrightarrow \tilde{g}q,\ \tilde{g} \leftrightarrow \chi q\bar{q}$

neutralino-gluino coannihilation

$$\chi\chi \leftrightarrow SM,\ \chi\tilde{g} \leftrightarrow q\bar{q},\ \tilde{g}\tilde{g} \leftrightarrow q\bar{q} \text{ or } gg$$

(1) Sommerfeld effects for $\tilde{g}\tilde{g} \rightarrow q\bar{q}$ or gg

Explanation:

Depending on the colour configuration of the initial $\tilde{g}\tilde{g}$, the long range Coulomb-like potential between $\tilde{g}\tilde{g}$ can be attractive or repulsive.

⇒ modify the otherwise free initial particle wave function

Baer, Cheung and Gunion, 1999
Profumo and Yaguna, 2004
De Simone, Giudice and Strumia, 2014
Harigaya, Kaneta and Matsumoto, 2014

neutralino-gluino coannihilation

(2) Gluino bound-state effect

$\tilde{g}\tilde{g} \leftrightarrow \tilde{R}g$, $\tilde{R} \leftrightarrow gg$

Coulomb potential	$\sim$	$-\alpha_s/r$
Bohr radius	$\sim$	$(\alpha_s m_{\tilde{g}})^{-1}$
binding energy	$\sim$	$\alpha_s^2 m_{\tilde{g}}$
$\tilde{R}$ annihilation decay rate	$\sim$	$\alpha_s^5 m_{\tilde{g}}$
individual $\tilde{g}$ decay rate	$\sim$	$(m_{\tilde{g}} - m_\chi)^5 m_{\tilde{q}}^{-4}$

Explanation:

- $\tilde{g}\tilde{g}$ can form a positronium-like bound state $\tilde{R}$
- $\tilde{R} \rightarrow gg$ removes two R-odd particles $\Longrightarrow$ decreases the final R-odd particle number density (i.e., DM number density)

neutralino-gluino coannihilation

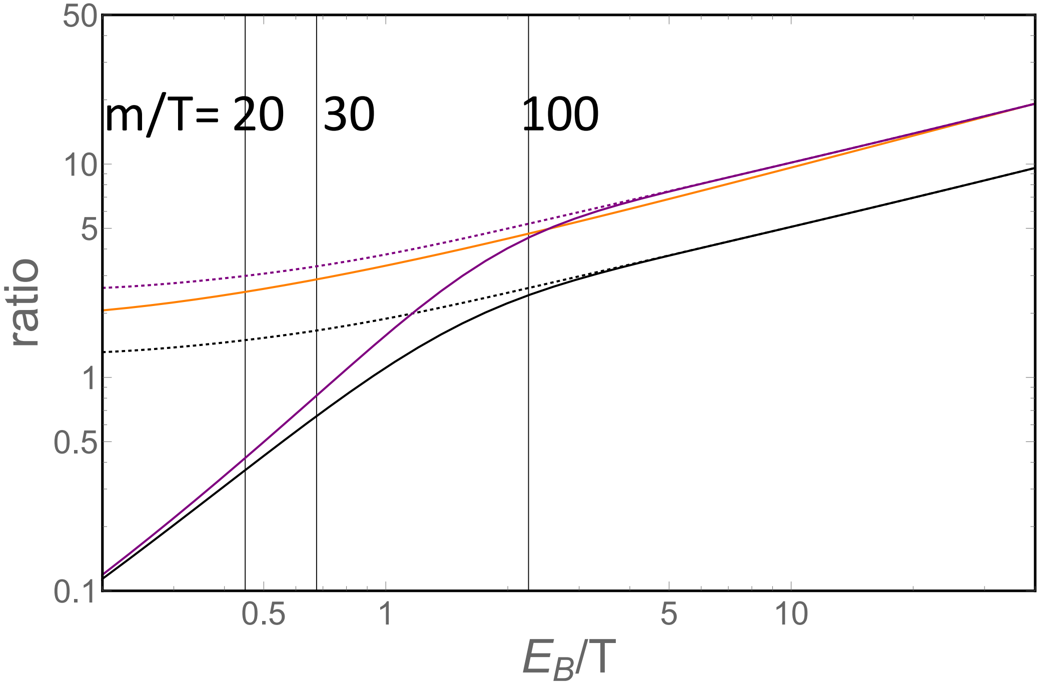
(2) Gluino bound-state effect

$\tilde{g}\tilde{g} \leftrightarrow \tilde{R}g$, $\tilde{R} \leftrightarrow gg$

$$\Rightarrow \frac{dn}{dt} + 3Hn \approx \quad -\sum_{i,j=\chi,\tilde{g}} \langle \sigma v \rangle_{ij\to SM} \left[n_i n_j - n_i^{eq} n_j^{eq} \right]$$

$$-\langle \sigma v \rangle_{\tilde{g}\tilde{g}\to\tilde{R}g} \frac{\langle \Gamma \rangle_{\tilde{R}\to gg}}{\langle \Gamma \rangle_{\tilde{R}\to gg} + \langle \Gamma \rangle_{\tilde{R}g\to\tilde{g}\tilde{g}}} \left[n_{\tilde{g}} n_{\tilde{g}} - n_{\tilde{g}}^{eq} n_{\tilde{g}}^{eq} \right]$$

neutralino-gluino coannihilation



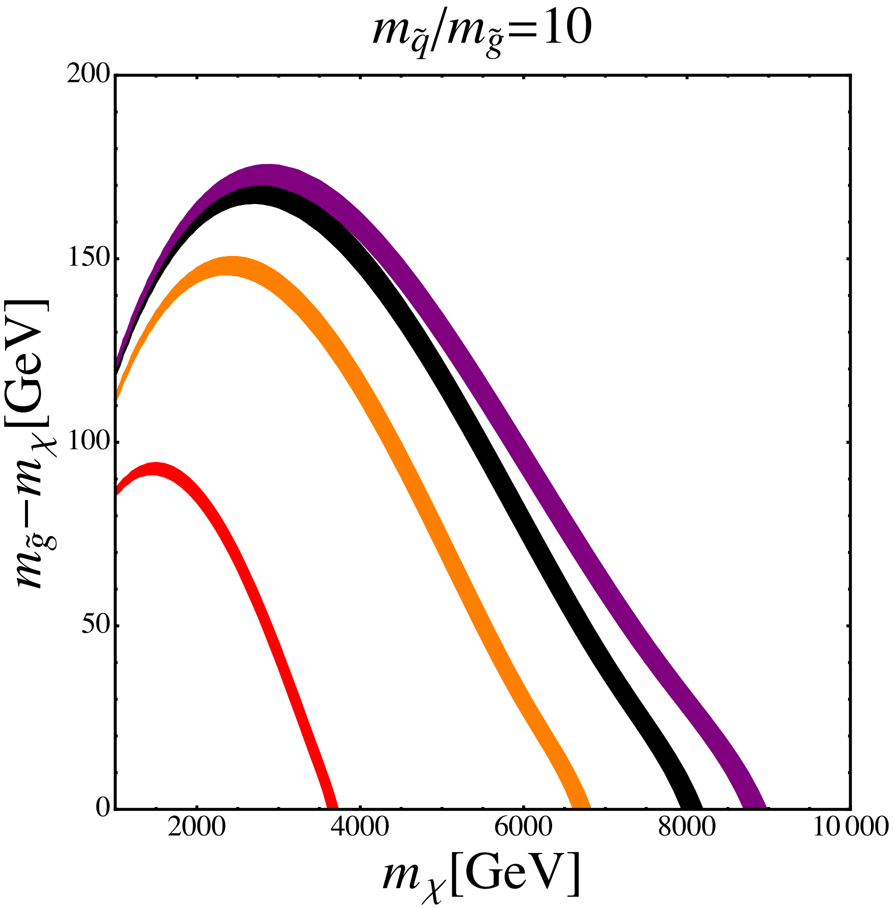
Due to dissociation, bound-state effect catches up Sommerfeld effect after $T \lesssim E_B$

Solid lines: compare Sommerfeld enhancement with bound-state effect

The "ratios" are normalized to the tree-level annihilation cross section.
Purple lines enlarge the bound-state effect by a factor of 2 comparing to black lines.

Dashed lines: if there were no dissociation process

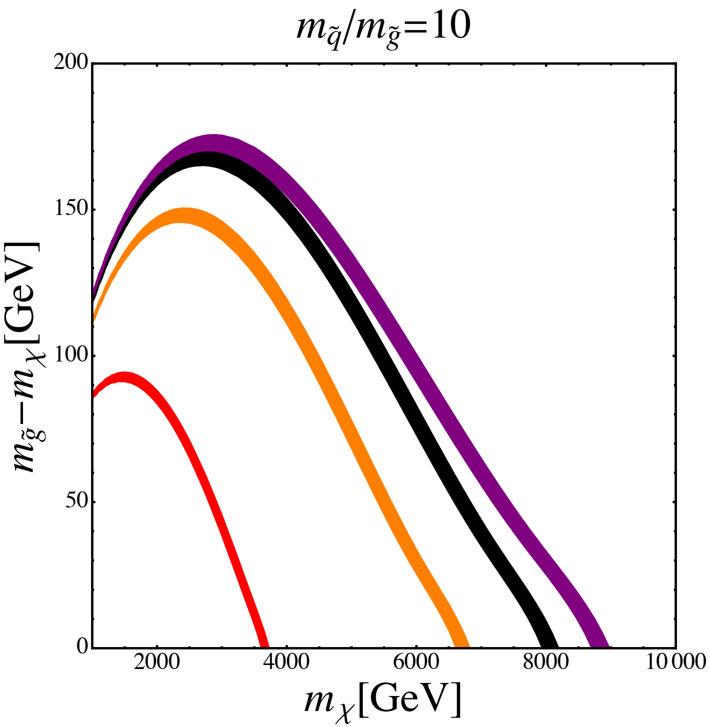
neutralino-gluino coannihilation



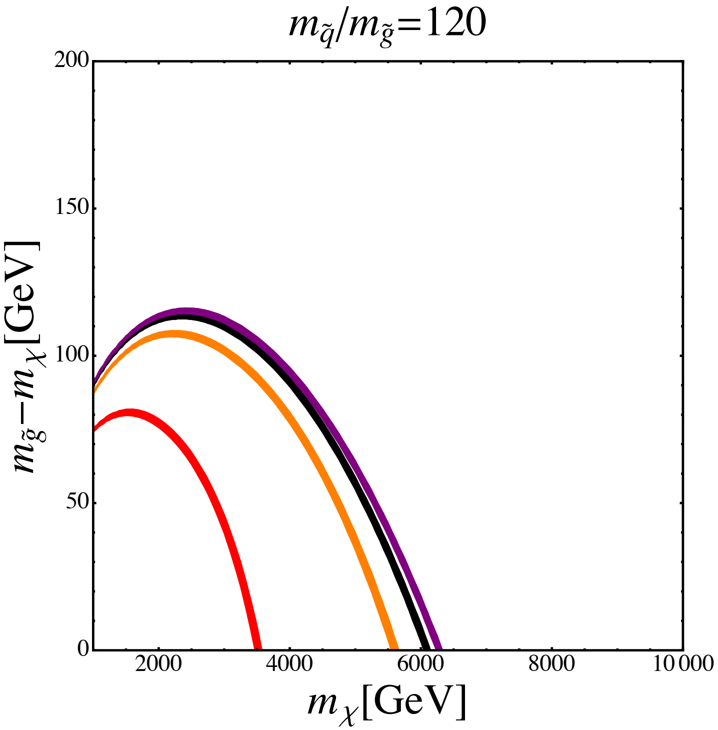
The bands give correct DM relic abundance: $\Omega_\chi h^2 = 0.1193 \pm 0.0042$ (i.e., 3-σ)

red: w/o Sommerfeld and w/o bound-state
orange: w/ Sommerfeld but w/o bound-state
black: w/ Sommerfeld and w/ bound-state
purple: w/ Sommerfeld and w/ 2 times bound-state

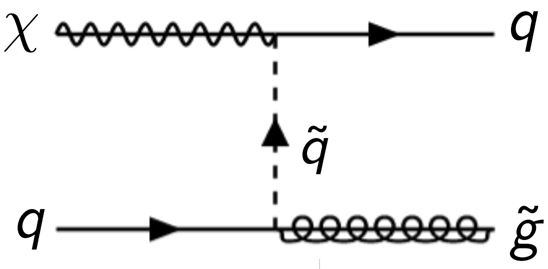
neutralino-gluino coannihilation



$m_{\tilde{q}}/m_{\tilde{g}}=10$



$m_{\tilde{q}}/m_{\tilde{g}}=120$



neutralino-gluino coannihilation

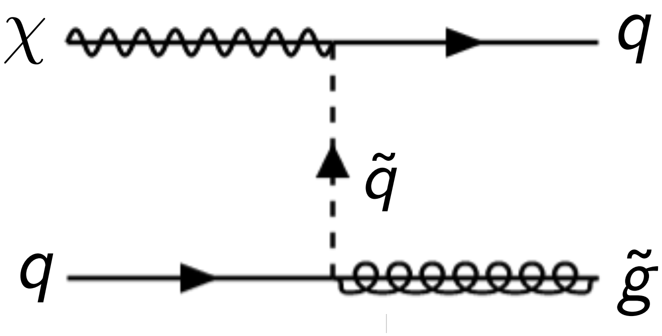
$\chi\chi \leftrightarrow SM$, $\chi\tilde{g} \leftrightarrow q\bar{q}$, $\tilde{g}\tilde{g} \leftrightarrow q\bar{q}$ or gg,
$\tilde{g}\tilde{g} \leftrightarrow \tilde{R}g$, $\tilde{R} \leftrightarrow gg$,
$\chi q \leftrightarrow \tilde{g}q$, $\tilde{g} \leftrightarrow \chi q\bar{q}$

(3) Breakdown of coannihilation by large squark masses

$\chi q \leftrightarrow \tilde{g}q$, $\tilde{g} \leftrightarrow \chi q\bar{q}$

Chung, Farrar and Kolb, 1997

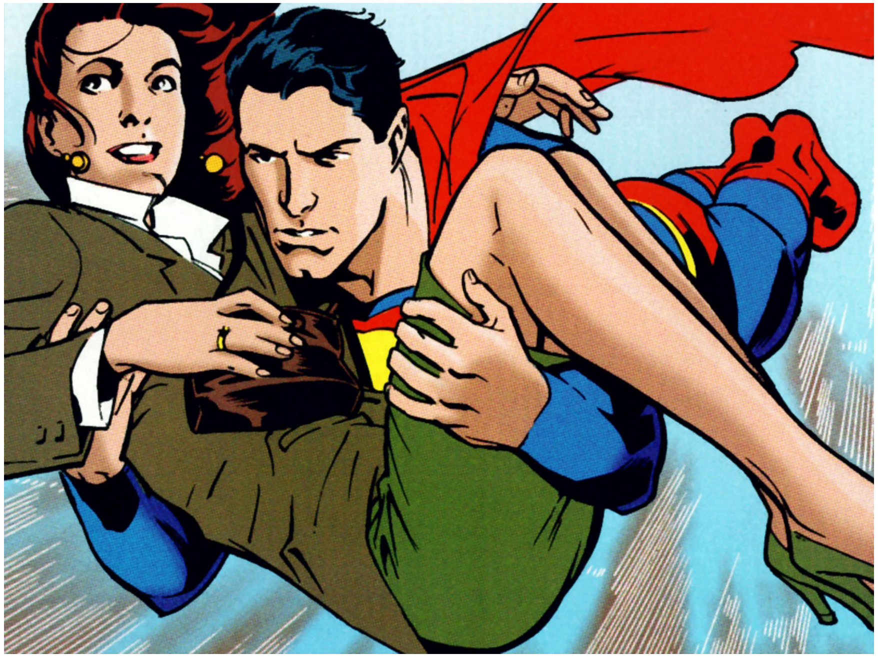
Explanation:



$\Rightarrow$ coannihilation mechanism breaks down, and therefore Sommerfeld enhancement and bound-state effect cannot reduce the χ number density $\boxed{\text{even if}}$ they are large and $\boxed{\text{even if}}$ $\tilde{g}$ and χ are degenerate in mass

coannihilation with Sommerfeld and bound-state effects

I’m a neutralino. I’m a gluino.



I’m the expanding Universe.

coannihilation breaks down

Sorry, squarks are too heavy.
I cannot give you a hand…

Too large squark masses makes the effective coupling between the gluino and neutralino too small, so that coannihilation breaks down.

⇒ *a lower limit of the interaction strength between DM and the SM*

neutralino-squark coannihilation

$$\tilde{q}\tilde{q}^* \leftrightarrow q\bar{q}, gg, W^+W^-, ZZ, \ldots$$
$$\tilde{q}\tilde{q}^* \leftrightarrow \tilde{R}g, \;\; \tilde{q}\tilde{q}^* \leftrightarrow \tilde{R}\gamma$$
$$\tilde{R} \leftrightarrow gg, W^+W^-, ZZ, \ldots$$

New ingredients compared to the gluino case:

✓ squark has electric charge, while gluino does not

- ➢ affect the potential
- ➢ photon emission/absorption processes

✓ squark anti-squark color **potential prior** to forming a bound state is **repulsive**, while the one for gluino pair is attractive

$$\mathbf{3} \otimes \overline{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$

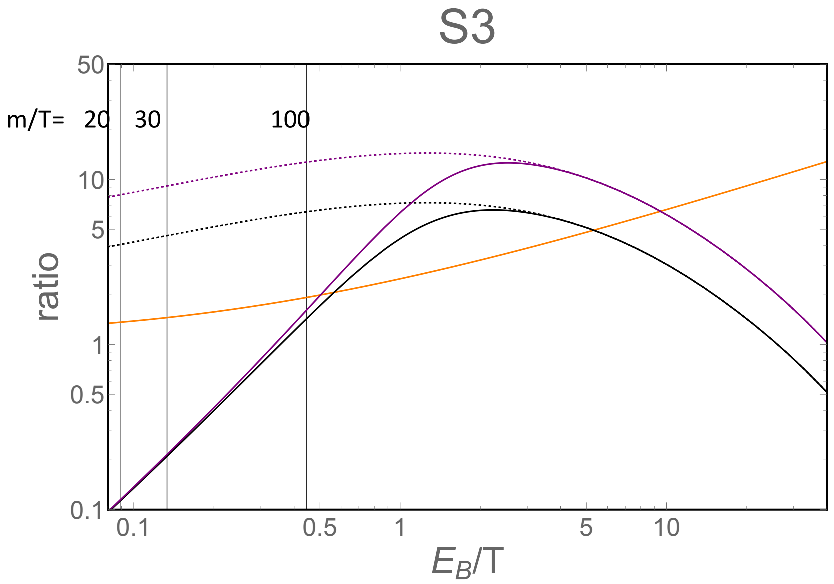
vs. $\mathbf{8} \otimes \mathbf{8} = \mathbf{1}_S \oplus \mathbf{8}_A \oplus \mathbf{8}_S \oplus \mathbf{10}_A \oplus \overline{\mathbf{10}}_A \oplus \mathbf{27}_S$

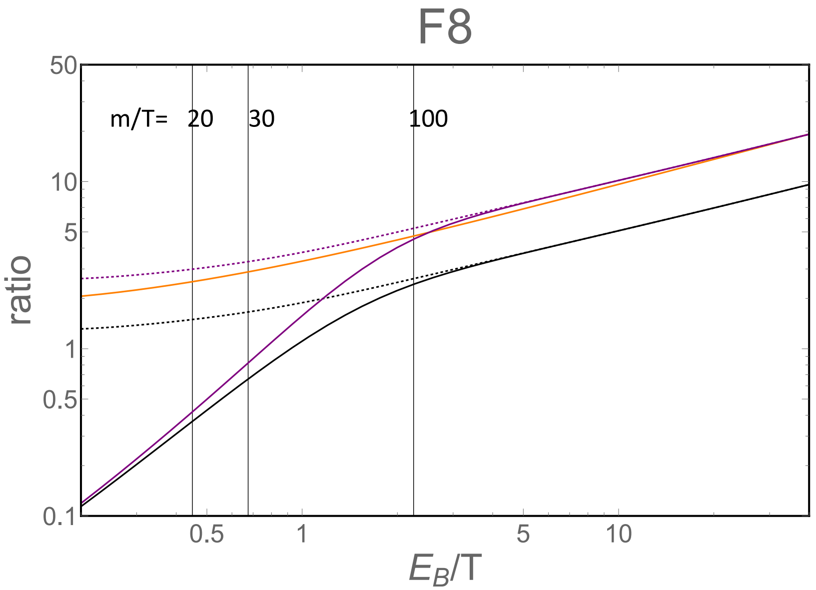
squark is a scalar triplet

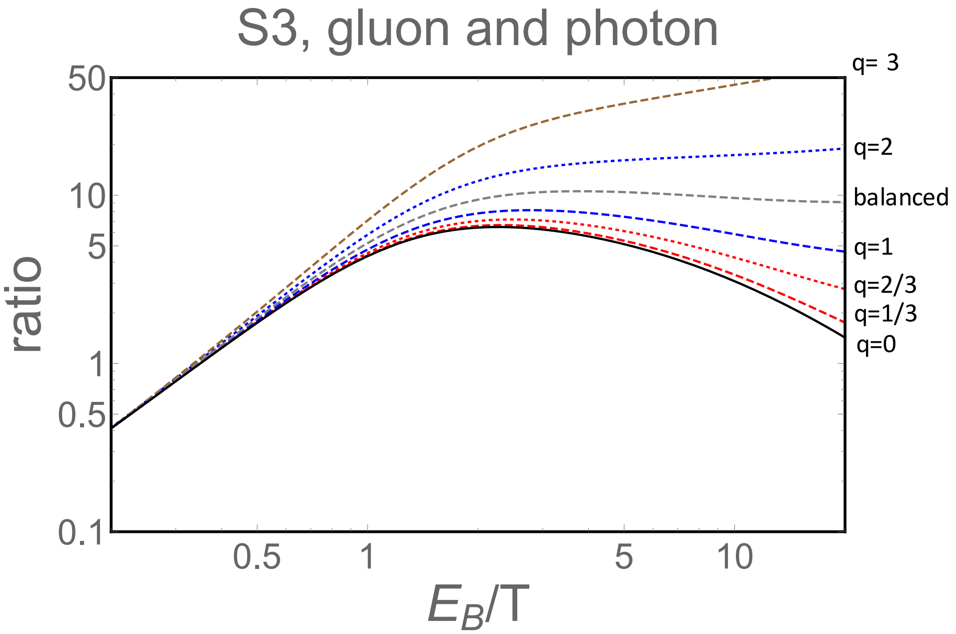
gluino is a fermion octet

squark = S3
gluino = F8

neutralino-squark coannihilation



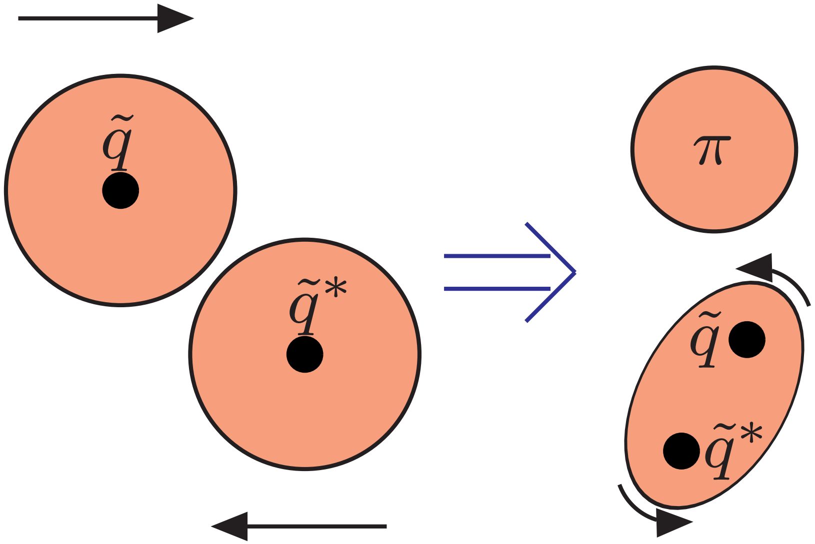




a second round of bound-state formation

If a significant amount of NLSP survive till the era of QCD phase transition, then a second round of bound-state formation can happen, and the effective annihilation cross section drastically increases.

a second round of bound-state formation



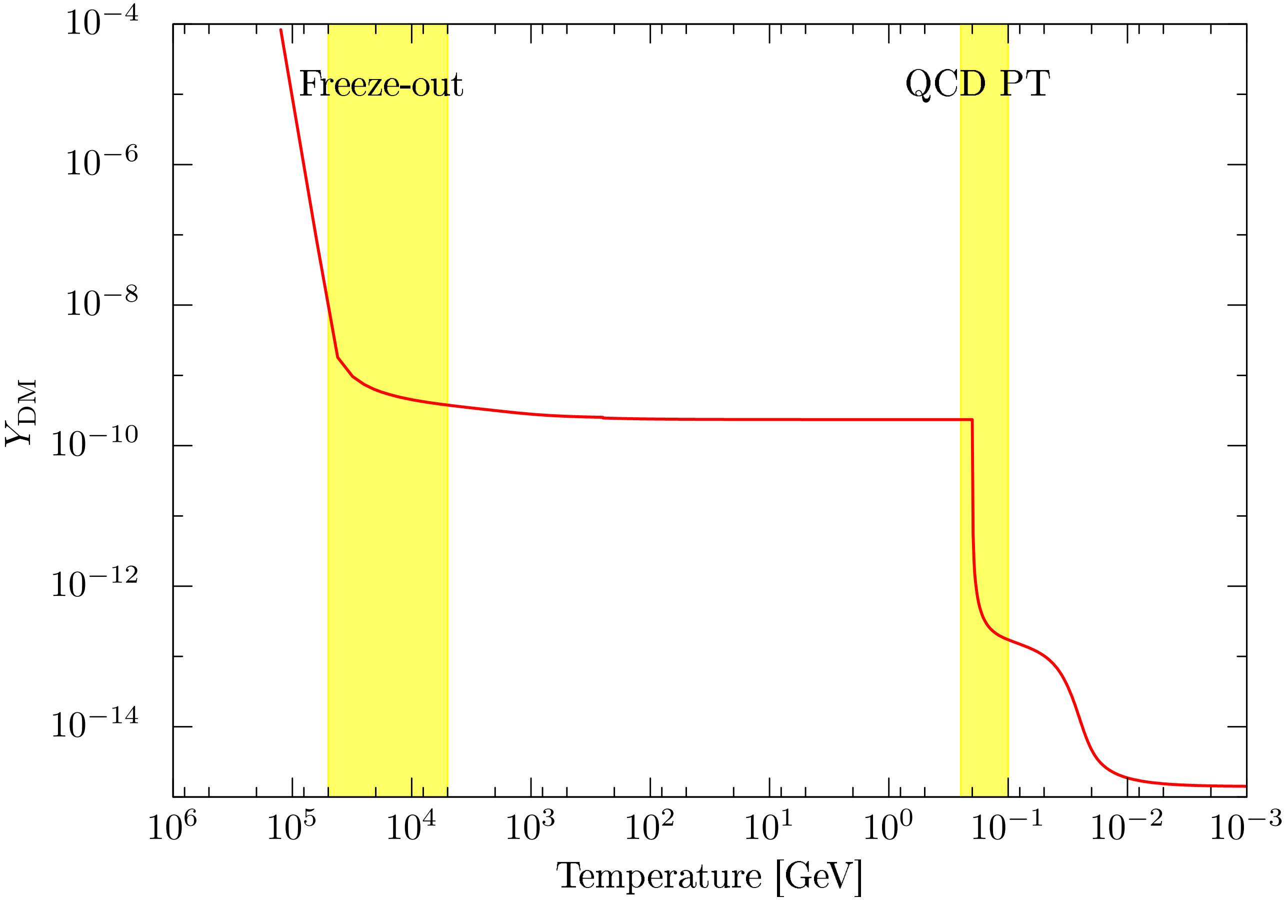
Step 1: A squark becomes a SUSY hadron by combining with a quark/gluon.

Step 2: Two SUSY hadrons meet and form a bound state with a large angular momentum.

Step 3: The bound state de-excites into the ground state.

Step 4: The ground state annihilates into quarks and gluons.

a second round of bound-state formation

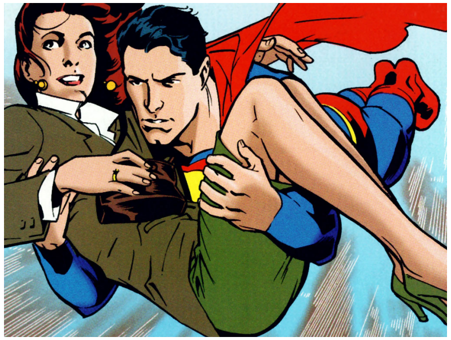


The cosmological evolution of the yield for a 1 PeV neutralino dark matter, having zero mass difference with the right-handed up scalar quark NLSP.

Summary

- Bound-state effect in the perturbative regime can significantly enhance the DM effective annihilation cross section.

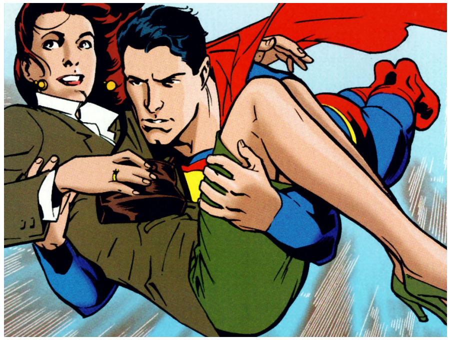
  - ✓ *It's size is **comparable** to the Sommerfeld effect.*
  - ✓ *The potential before forming a bound state can be **either attractive or repulsive**.*
  - ✓ *Too large squark masses can **break down** the neutralino-gluino coannihilation mechanism.*



Summary

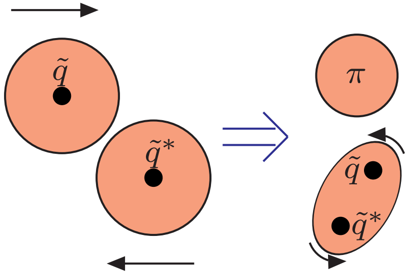
- Bound-state effect in the perturbative regime can significantly enhance the DM effective annihilation cross section.

    - ✓ *It's size is **comparable** to the Sommerfeld effect.*
    - ✓ *The potential before forming a bound state can be **either attractive or repulsive**.*
    - ✓ *Too large squark masses can **break down** the neutralino-gluino coannihilation mechanism.*



- If a second round of bound-state formation can happen after the QCD phase transition, then

    non-perturbative strong interaction ⇒ effective annihilation cross section drastically increases ⇒ a second freeze-out ⇒ allowing even a PeV scale DM



back to our goal

how weak the interactions of a WIMP could have,
and
how heavy a WIMP could be.

- In the coannihilation scenario, bound-state effect is needed to be taken into account in answering how heavy a WIMP could be.

- To ensure efficient conversions between DM and the coannihilator (so that coannihilation can happen) gives a lower limit of the interaction strength between DM and the SM sector.

back to our goal

how weak the interactions of a WIMP could have,
and
how heavy a WIMP could be.

- In the coannihilation scenario, bound-state effect is needed to be taken into account in answering how heavy a WIMP could be.

- To ensure efficient conversions between DM and the coannihilator (so that coannihilation can happen) gives a lower limit of the interaction strength between DM and the SM sector.

Thank you for your attention!

Conditions for coannihilation to reduce DM relic density

Define $n \equiv n_1 + n_2$ and $n_{eq} \equiv n_1^{eq} + n_2^{eq}$,

$$\frac{dn}{dt} + 3Hn = -\sum_{i,j=1}^{2} \langle \sigma v \rangle_{ij \to SM} \frac{n_i^{eq} n_j^{eq}}{n_{eq}^2} \left[n^2 - n_{eq}^2 \right]$$

(Recall w/o coannihilation: $\boxed{\frac{dn_\chi}{dt} + 3H(T)n_\chi = -\langle \sigma v \rangle_{\chi\chi \to SM's} \left[n_\chi^2 - (n_\chi^{eq})^2 \right]}$)

Note that $n_i^{eq} = g_i \left(\frac{m_i T}{2\pi} \right)^{3/2} e^{-m_i/T}$ for $T \ll m_i$

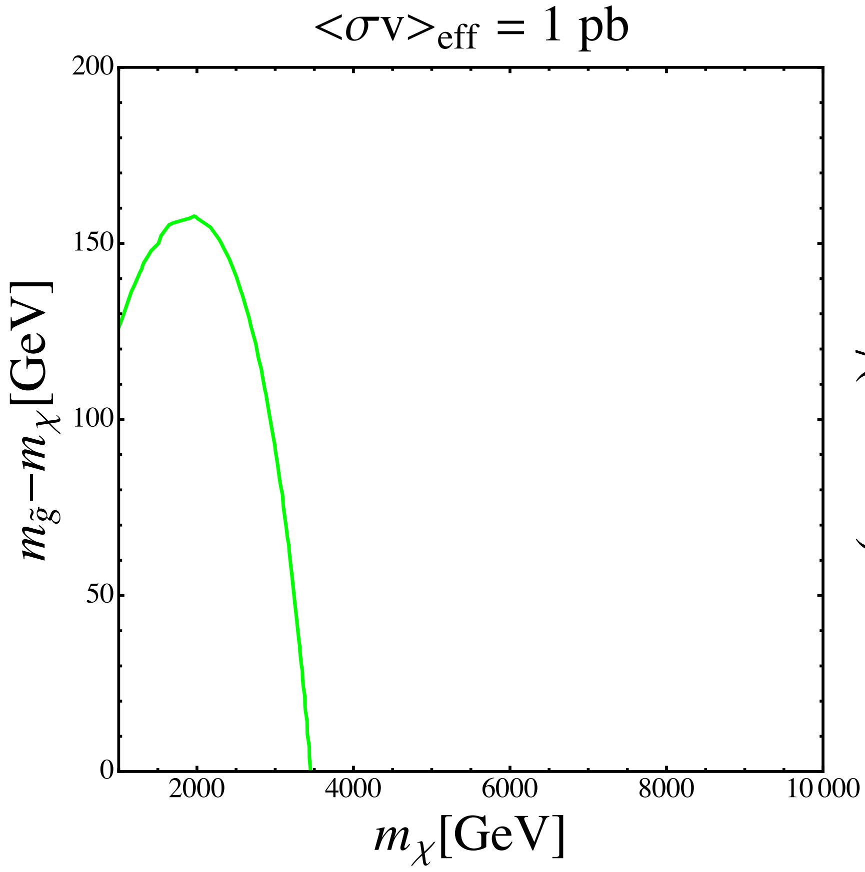
- if $m_2 \gg m_1$, then $n_{eq} \approx n_1^{eq}$, ●● $\approx \langle \sigma v \rangle_{11 \to SM}$

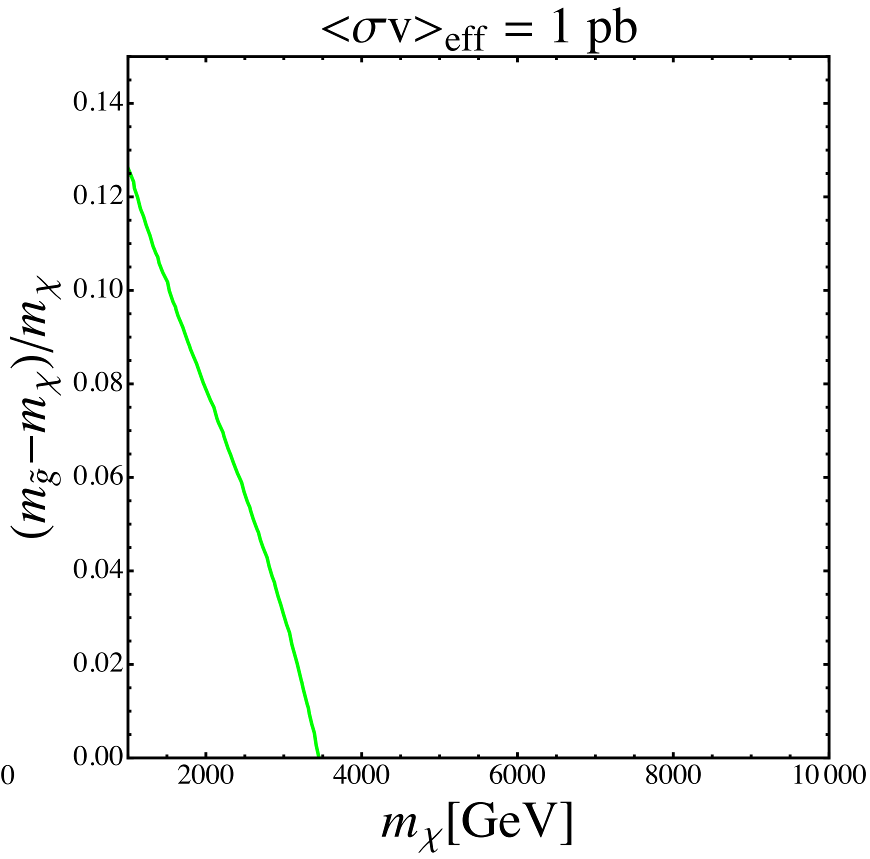
  i.e., no coannihilation

- if $m_2 = m_1$, then ●● $= \frac{g_1^2 \langle \sigma v \rangle_{11 \to SM} + g_2^2 \langle \sigma v \rangle_{22 \to SM} + 2 g_1 g_2 \langle \sigma v \rangle_{12 \to SM}}{(g_1+g_2)^2}$

  if the middle term dominates, then ●● $\boxed{\approx \left(\frac{g_2}{g_1+g_2} \right)^2 \langle \sigma v \rangle_{22 \to SM}}$

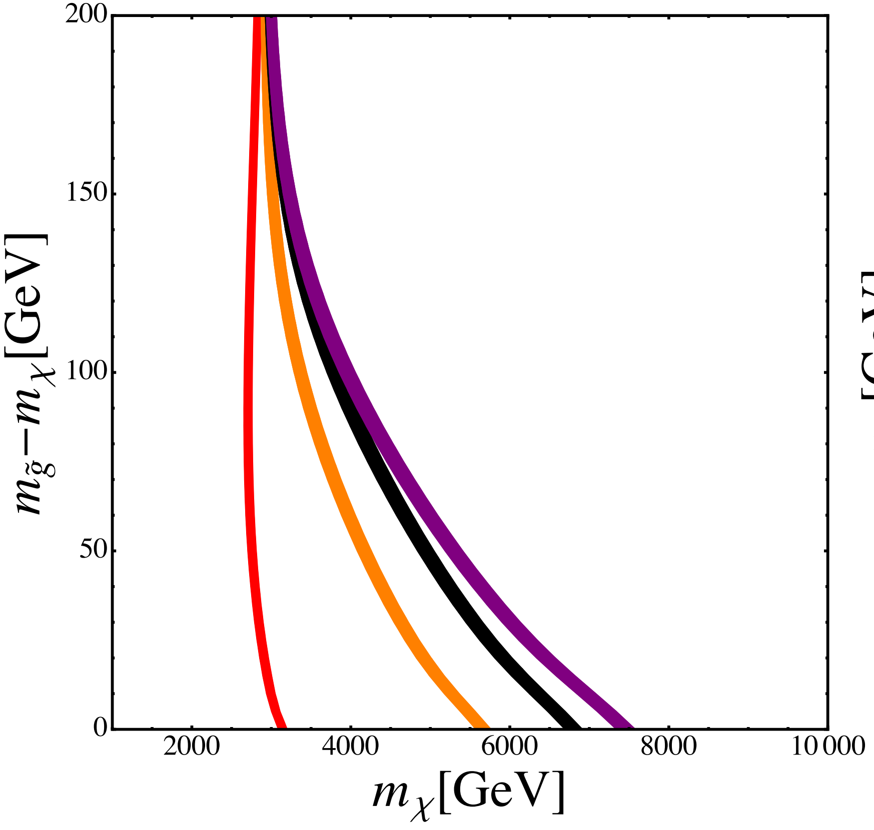
backup: the reason why the Δm vs. m_χ plot has the shape

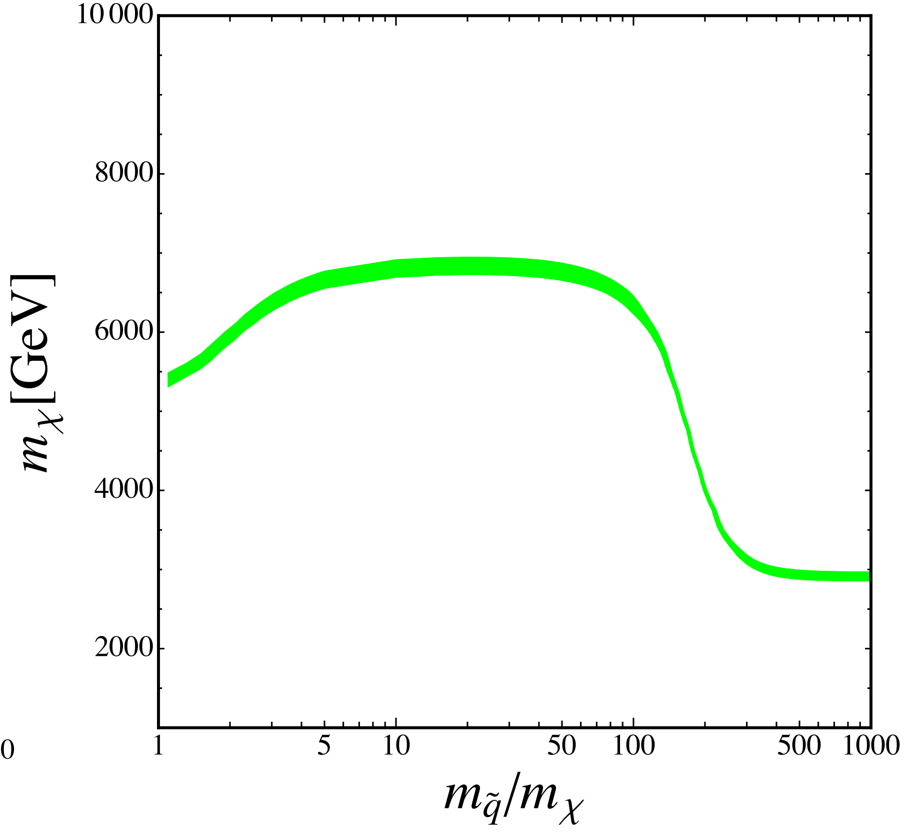




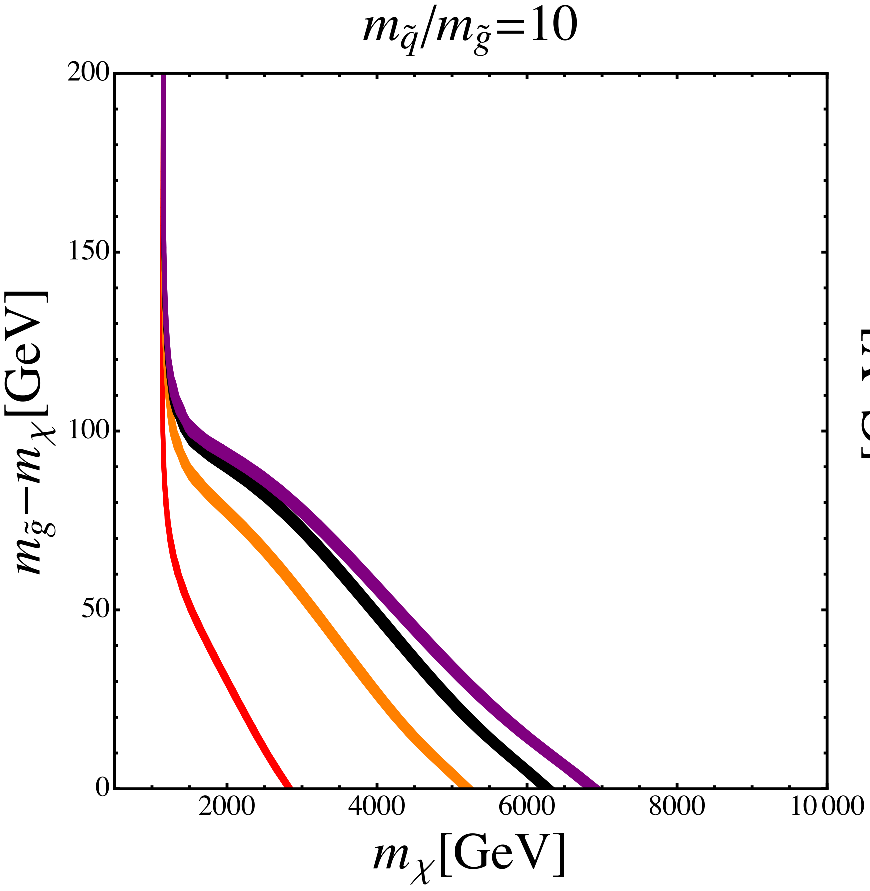
Wino-gluino coannihilation

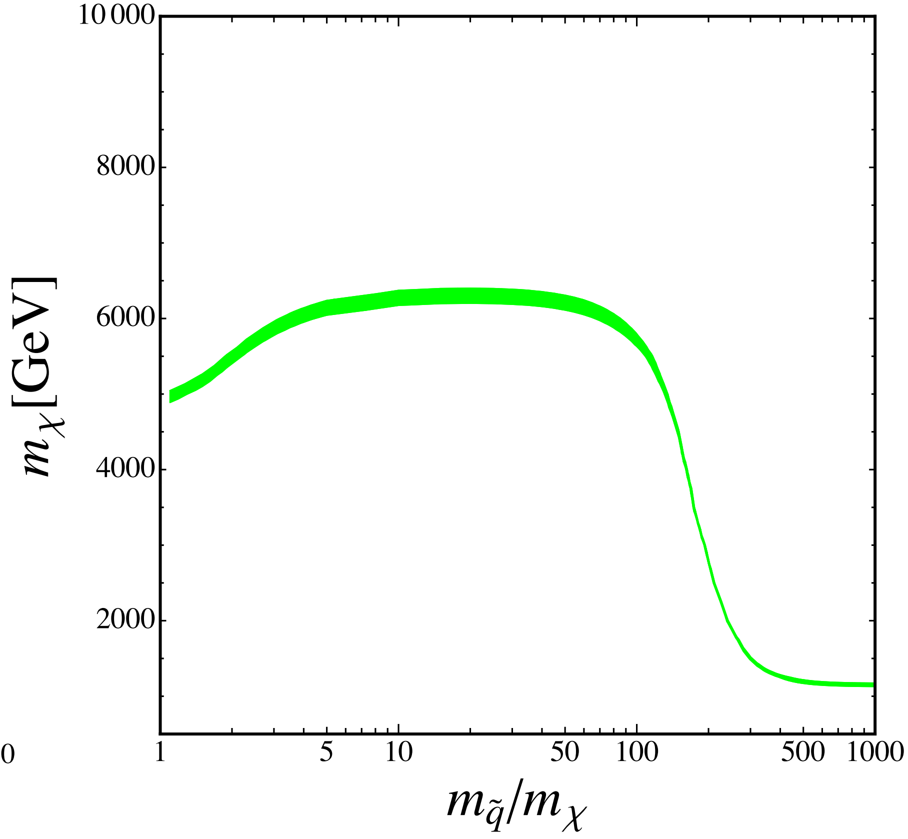
$m_{\tilde{q}}/m_{\tilde{g}}=10$





Higgsino-gluino coannihilation





A remark

> Why the maximum LSP mass is smaller for a Wino ($\sim$ 7 TeV) or a Higgsino ($\sim$ 6 TeV) compared to a Bino ($\sim$ 8 TeV)?
>
> Because there are more *inert* degrees of freedom for Wino (=6) or Higgsino (=8) compared to Bino (=2) at large mass when $\chi\chi$ and $\chi\tilde{g}$ (co)annihilation cross sections are much smaller than $\tilde{g}\tilde{g}$ annihilation cross section.

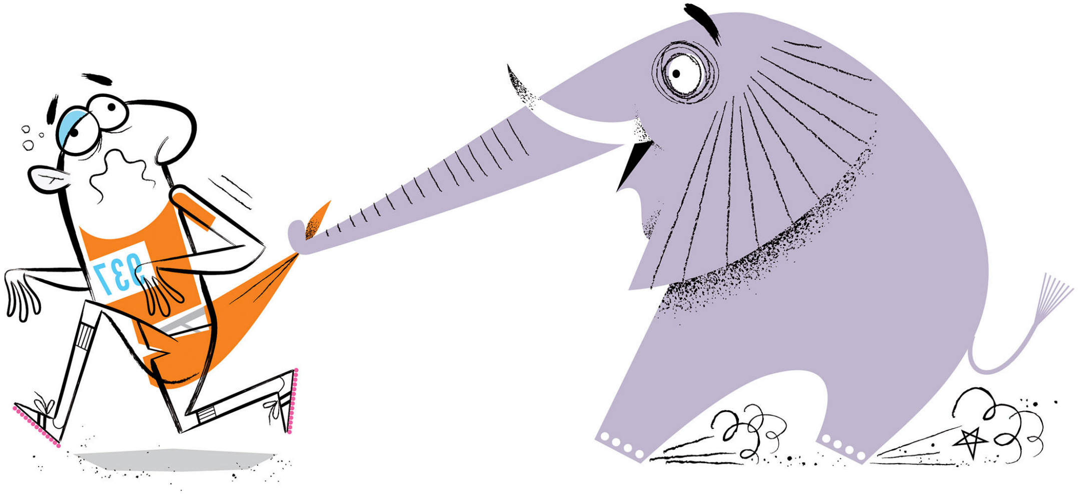
$$\frac{dn}{dt} + 3Hn = -\sum_{i,j=1}^{2} \langle \sigma v \rangle_{ij\to SM} \frac{n_i^{eq} n_j^{eq}}{n_{eq}^2} \left[n^2 - n_{eq}^2\right]$$

- if $m_2 = m_1$, then ●● $= \frac{g_1^2 \langle \sigma v \rangle_{11\to SM} + g_2^2 \langle \sigma v \rangle_{22\to SM} + 2 g_1 g_2 \langle \sigma v \rangle_{12\to SM}}{(g_1+g_2)^2}$

  if the middle term dominates, then ●● $\approx (\frac{g_2}{g_1+g_2})^2 \langle \sigma v \rangle_{22\to SM}$

I’m a gluino…

I’m a Wino (Higgsino).



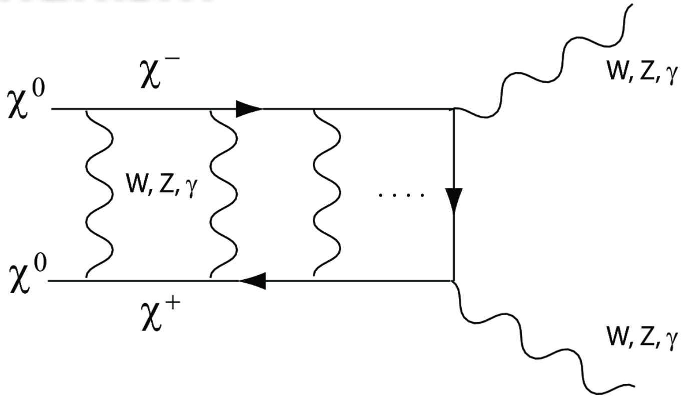
I’m the expanding
Universe.

coannihilation mechanism

3 TeV Wino ✓

1 TeV Higgsino ✓

Here coannihilation is an *unavoidable* add-on.

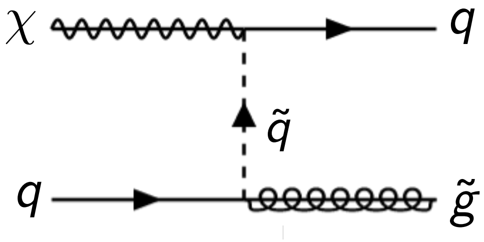


also note the Sommerfeld effect

Hisano, Matsumoto, Nagai, Saito, Senami, 2007

Bino?

- Bino couples to slepton, squark and Higgsino, but not to another Bino.
- Therefore, it usually requires some coannihilation (e.g., with a stau or a stop) to reduce the relic abundance for a Bino of TeV scale.
- Bino-gluino coannihilation is possible by the help of a squark.



probe strongly interacting particle coannihilation scenarios in colliders

- ✓ monojet searches (Low & Wang, 1404.0682)

coannihilator	bkgd. syst.	14 TeV		100 TeV	
		95% limit	5σ discovery	95% limit	5σ discovery
gluino	1%	1.1 TeV	950 GeV	6.2 TeV	5.2 TeV
	2%	1.0 TeV	850 GeV	5.8 TeV	4.8 TeV
stop	1%	530 GeV	420 GeV	2.8 TeV	2.1 TeV
	2%	470 GeV	330 GeV	2.4 TeV	1.7 TeV
squark	1%	740 GeV	600 GeV	4.0 TeV	3.0 TeV
	2%	630 GeV	495 GeV	3.5 TeV	2.6 TeV

- ✓ long-lived colored particles with displaced vertices

(Nagata, Otono & Shirai, 1504.00504)

$$c\tau_{\tilde{g}} = \mathcal{O}(1) \times \left(\frac{\Delta M}{100\,\mathrm{GeV}}\right)^{-5} \left(\frac{m_{\tilde{q}}}{100\,\mathrm{TeV}}\right)^{4} \mathrm{cm}$$

- ✓ squark-gluino associated production (S. Ellis & B. Zheng, 1506.02644)