Asymptotic safety meets dark matter

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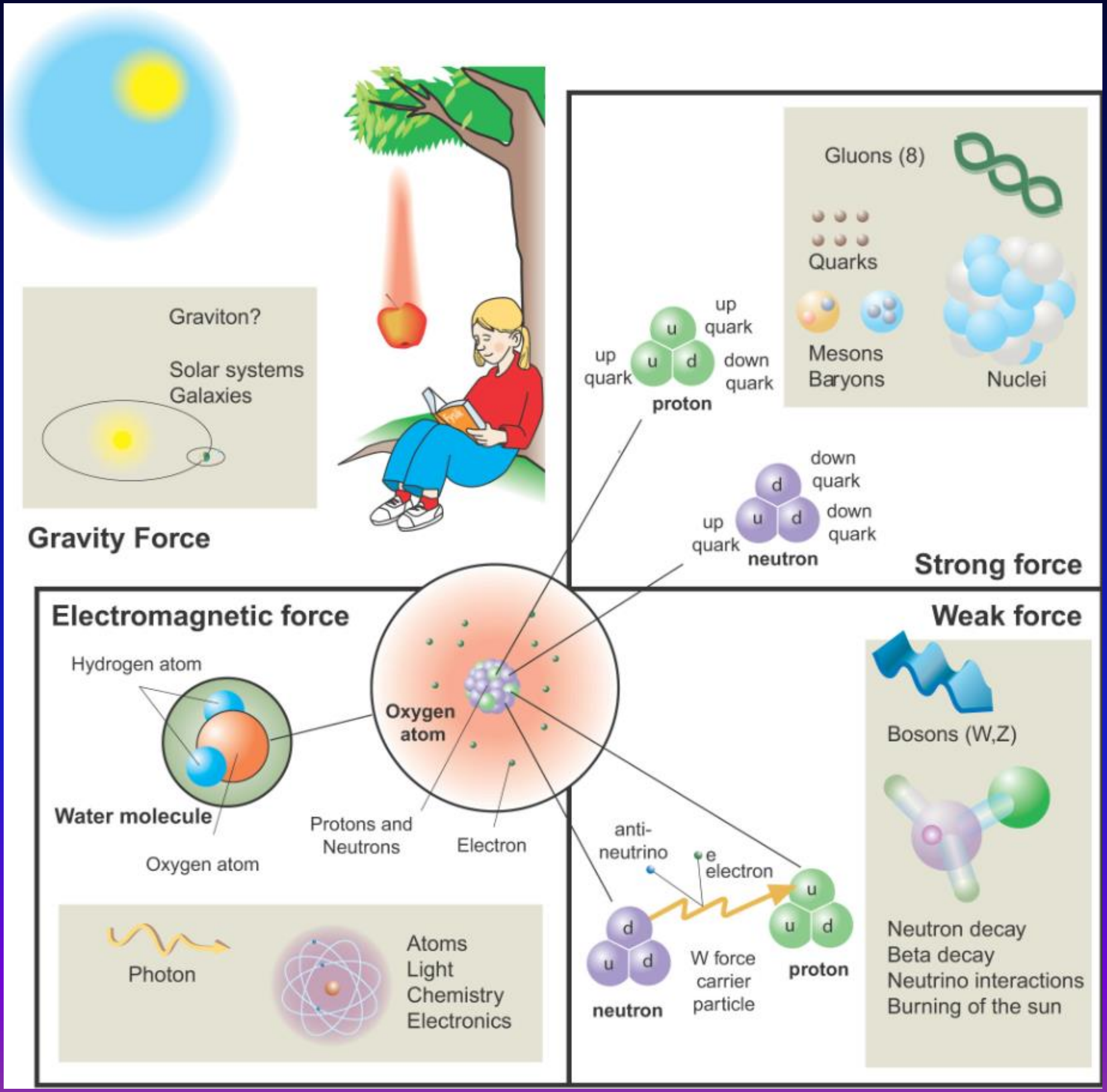
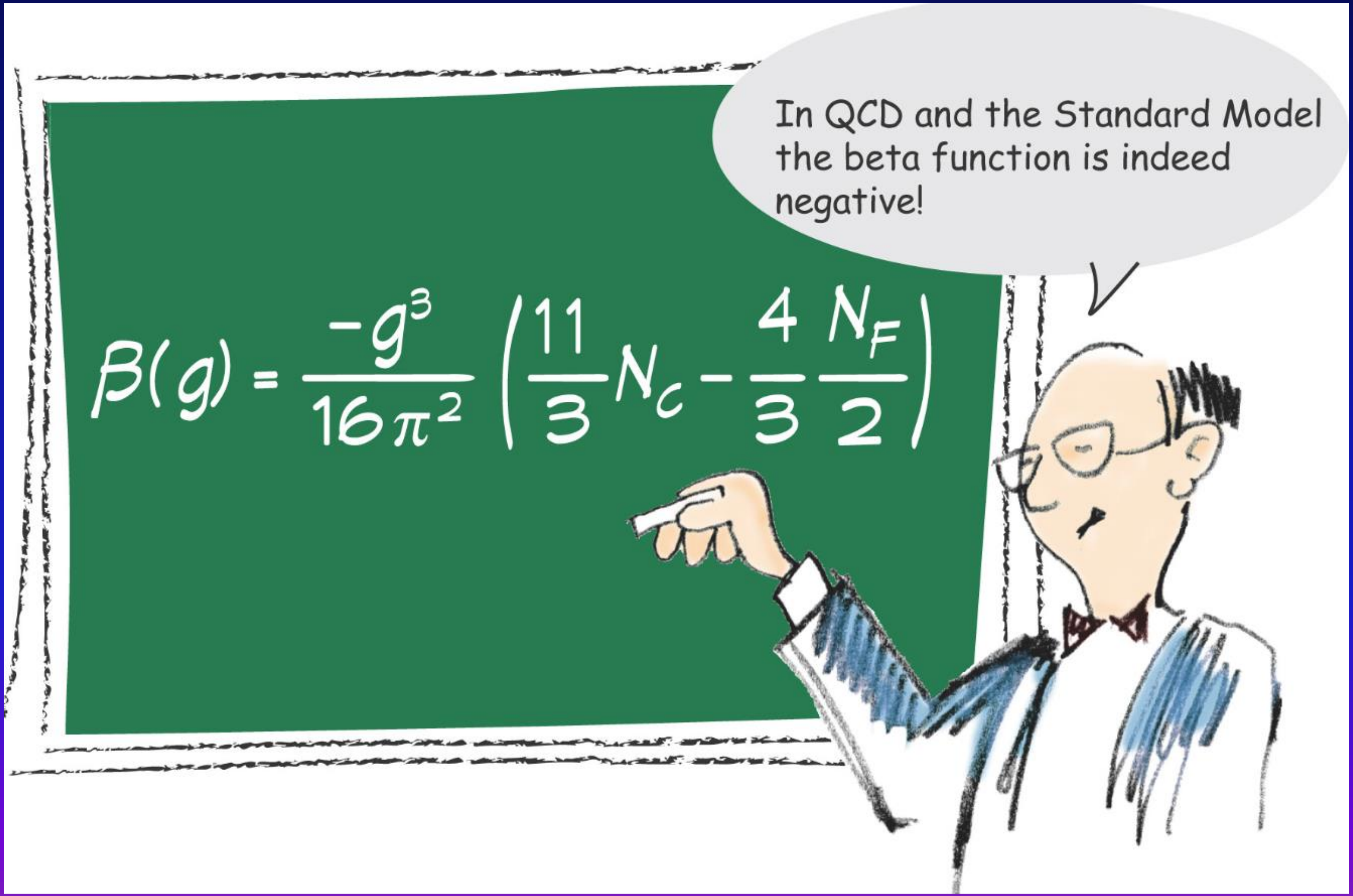
Based on C. Cai, H. H. Zhang, 1905.04227
Phys.Lett. B798 (2019) 134947

Four fundamental forces of Nature



Asymptotic freedom: g decreases with the energy scale increases

g=0 is a UV fixed point. For example: QCD



The Nobel Prize in Physics 2004

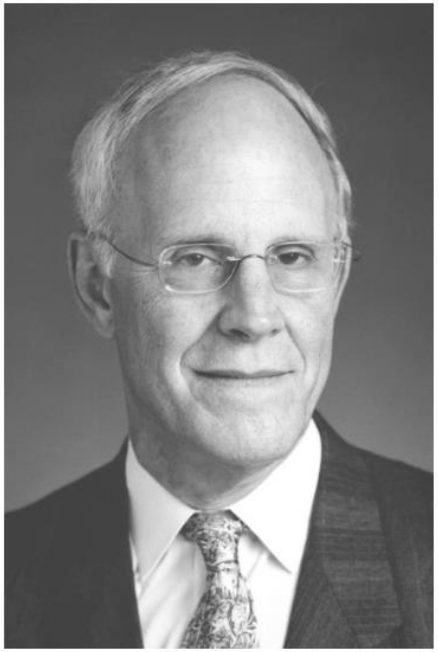


Photo from the Nobel Foundation archive.

David J. Gross

Prize share: 1/3

Photo from the Nobel Foundation archive.

H. David Politzer

Prize share: 1/3

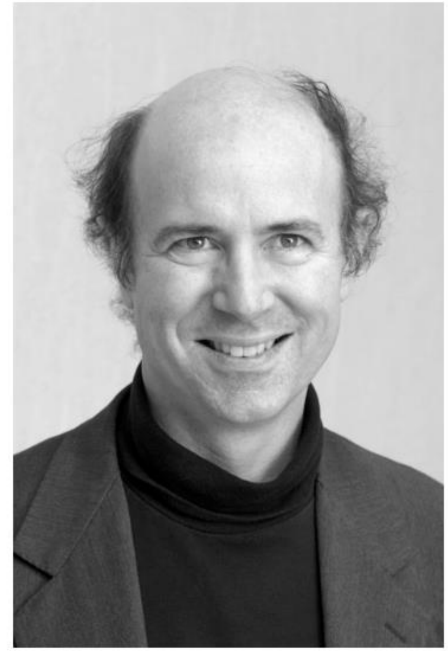


Photo from the Nobel Foundation archive.

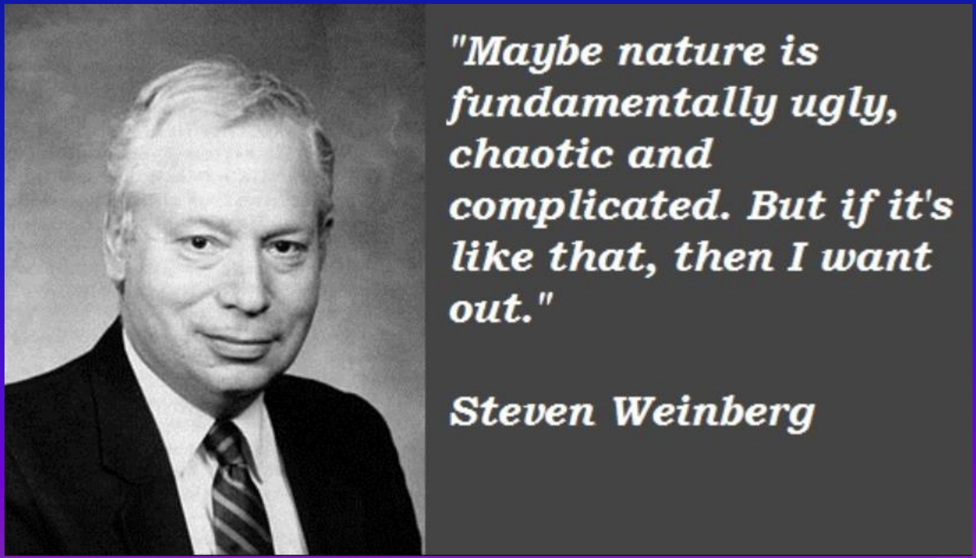
Frank Wilczek

Prize share: 1/3

The prize was awarded to Gross, Politzer and Wilczek for the discovery of asymptotic freedom in the theory of the strong interaction.

Asymptotic safety is an another concept in quantum field theory, which means that there is a nontrivial UV fixed point of the renormalization group flow of the coupling constants in the theory space, and thus physical quantities are safe from divergences.

Although originally proposed by Steven Weinberg in 1976 to find a theory of quantum gravity, the idea of a nontrivial fixed point providing a possible UV completion can be applied also to other field theories.



CRITICAL PHENOMENA FOR FIELD THEORISTS

Steven Weinberg

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1. INTRODUCTION

Many of us who are not habitually concerned with problems in statistical physics have gradually been becoming aware of dramatic progress in that field. The mystery surrounding the phenomenon of second-order phase transitions seems to have lifted, and theorists now seem to be able to explain all sorts of scaling laws associated with these transitions, and even (more or less) to calculate the "critical exponents" of the scaling laws.[1] Furthermore, the methods used to solve these problems appear to have a profound connection with the methods of field theory — one overhears talk of "renormalization group equations", "infrared divergences", "ultraviolet cut-offs", and so on. It is natural to conclude that field theorists have a lot to learn from their statistical brethren.

Weinberg 1976

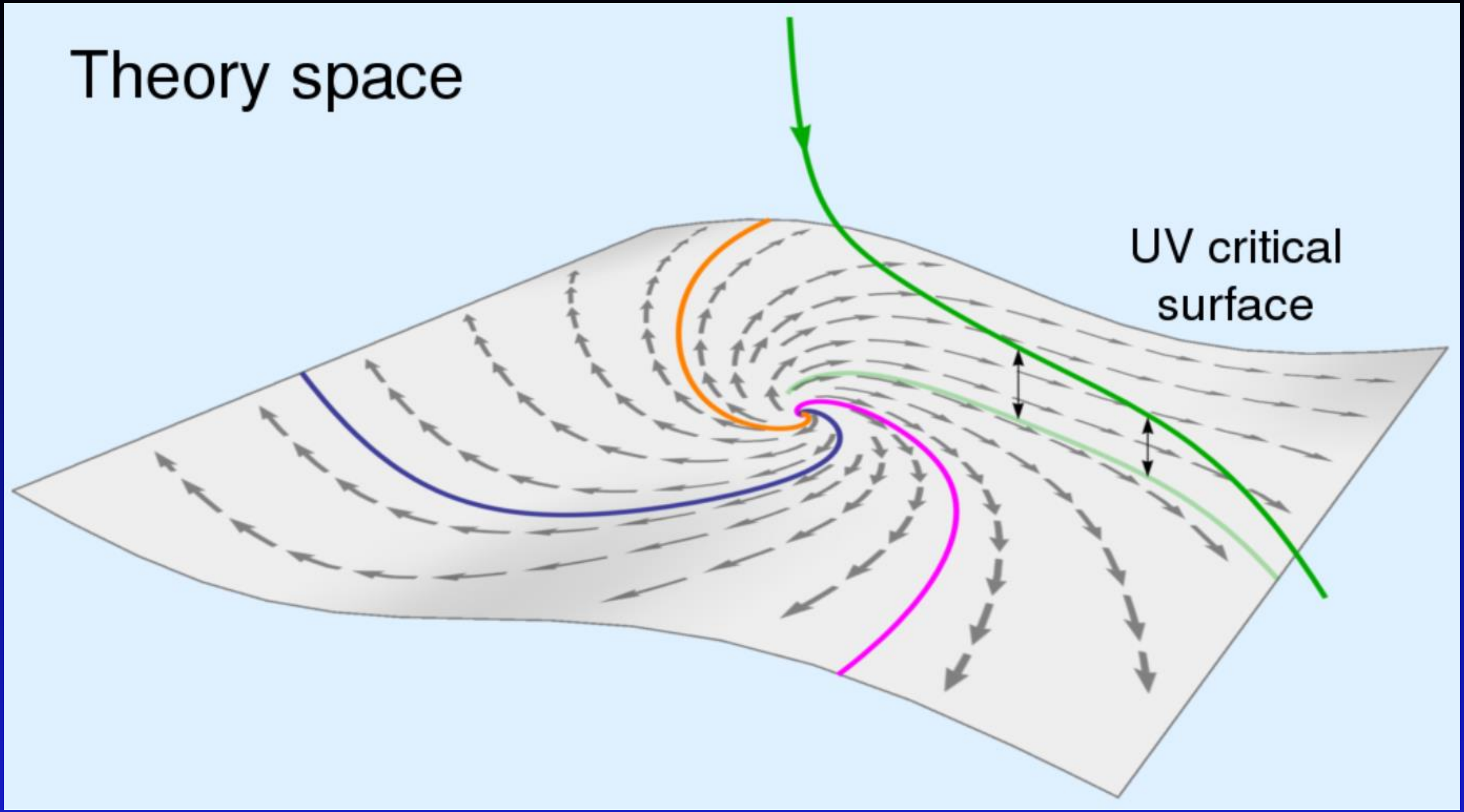


Fig: Trajectories of the renormalization group flow in theory space with arrows pointing from UV to IR scales

The green trajectory does not belong to the theory space of asymptotic safety.

Examples of asymptotic safety

Consider a dimensionless coupling $\alpha \equiv g\mu^A$, where the mass dimension of g is $-A$. The β function of α is given by

$$\beta(\alpha) \equiv \partial_t \alpha = A\alpha - B\alpha^2$$

where $t \equiv \ln(\mu/\Lambda)$ denotes the logarithmic RG 'time', μ the RG energy scale, and Λ a characteristic reference scale of the theory.

The linear term in the β function, $A\alpha$, is the tree-level contribution. The quadratic term $-B\alpha^2$, stands for the one-loop contribution.

1406.2337

The RG flow displays two types of fixed points, a trivial one at $\alpha_* = 0$, and a non-trivial one at

$$\alpha_* = A/B$$

In the spirit of perturbation theory, the non-trivial fixed point $\alpha_* = A/B$ is accessible in the domain of validity of the RG flow as long as $\alpha_* \ll 1$.

This can be achieved in two manners,

- either by having $A \ll 1$ for fixed B,
- or by making $1/B \ll 1$ at fixed A.

$$\beta(\alpha) = A\alpha - B\alpha^2 \quad \Rightarrow \quad \beta'(\alpha) = A - 2B\alpha$$
$$\Rightarrow \quad \beta'(0) = A, \quad \beta'(A/B) = -A$$

It implies that $\alpha_* = A/B$ is an UV fixed point provided $A > 0$.

The existence of an interesting UV fixed point is the bare bone of asymptotic safety.

Gravity in 2+ε dimensions

Consider Einstein gravity with action

$$S = \frac{1}{16\pi G_N} \int d^d x \sqrt{-g} R$$

in d dimensions.

$$[S] = 0, \quad [R] = 2, \quad [S] = -[G_N] - d + [R]$$
$$\Rightarrow \quad 0 = -[G_N] - d + 2 \quad \Rightarrow \quad [G_N] = 2 - d$$

Define the dimensionless gravitational coupling of the model as

$$\alpha \equiv G_N(\mu)\mu^{d-2}$$

In $d = 2 + \epsilon$ dimensions,

$$[G_N] = -\epsilon \quad \Rightarrow \quad A = \epsilon \ll 1$$

One-loop calculation gives $B = 50/3$. There exists an UV fixed point in the perturbative regime.

Gross-Neveu model in 2+ε dimensions and in 3 dimensions

Consider a purely fermionic theory of N_F self-coupled massless Dirac fermions with Gross-Neveu interaction $(1/2)g_{GN}(\bar{\psi}\psi)^2$ in d dimensions.

$$\mathcal{L} \supset \bar{\psi} i \partial\!\!\!/ \psi \quad \Rightarrow \quad [\psi] = [\bar{\psi}] = \frac{d-1}{2}$$

$$\mathcal{L} \supset \frac{1}{2} g_{GN} (\bar{\psi}\psi)^2 \quad \Rightarrow \quad d = [g_{GN}] + \frac{d-1}{2} \cdot 4 \quad \Rightarrow \quad [g_{GN}] = 2 - d$$

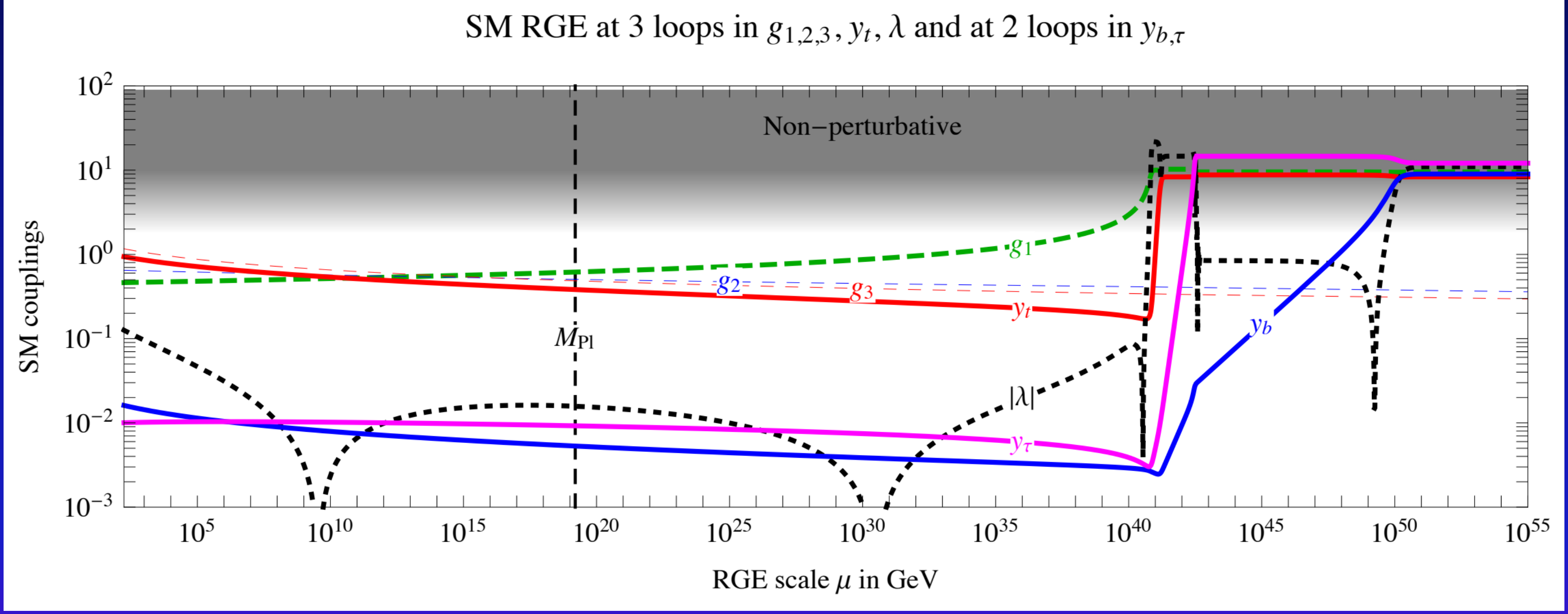
Define the dimensionless coupling as

$$\alpha \equiv \frac{g_{GN}(\mu)}{2\pi N_F} \mu^{d-2}$$

We can compute the $\beta(\alpha) = A\alpha - B\alpha^2$ in $d = 2 + \epsilon$ dimensions. The coefficient A, given by minus the canonical mass dimension, becomes $A = \epsilon \ll 1$. The coefficient B, to leading order in ϵ, is of order one and given by the 1-loop calculation in the two-dimensional theory. Hence, the model has a reliable UV fixed point in the perturbative regime.

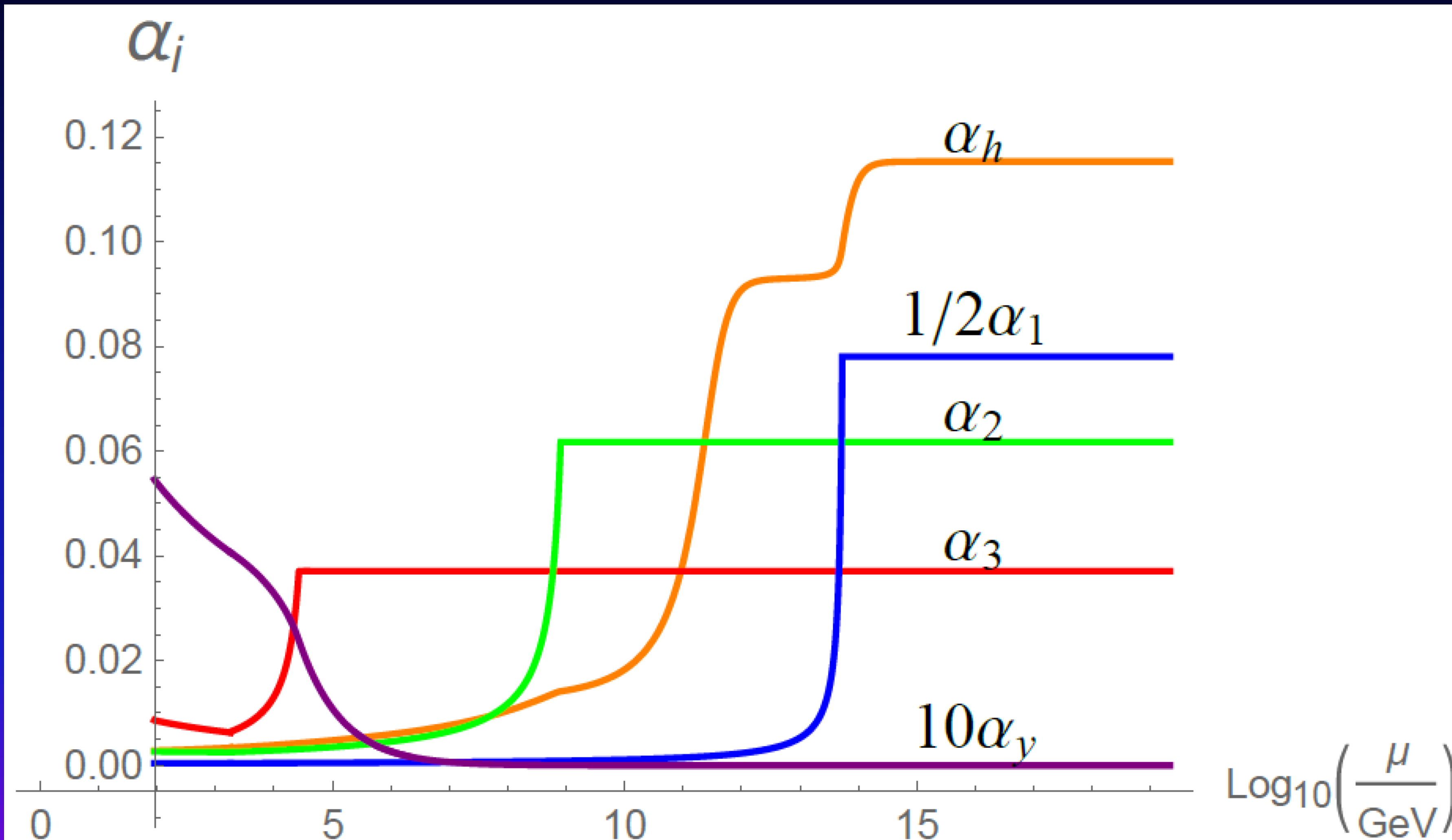
In the large N_F limit and at fixed dimension $d = 3$, one finds that $A \propto 1/N_F \ll 1$ while the coefficient $B > 0$ remains of order unity, leading to the same conclusion.

Asymptotically safe bahavior of the SM by neglecting b and τ contributions to the 3-loop terms and Yukawa couplings of the 1st, 2nd generations



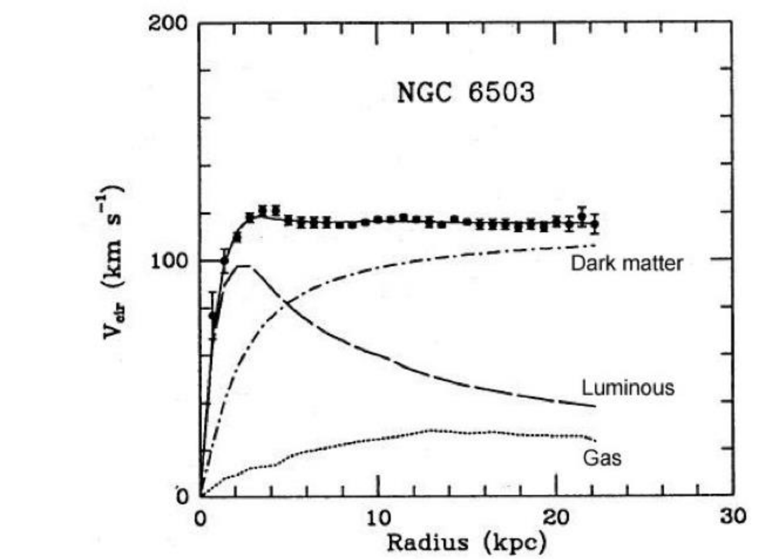
Pelaggi, Sannino, Strumia, Vigiani, 1701.01453

By neglecting top Yukawa coupling, the SM is asymptotically safe:



Mann, Meffe, Sannino, et al. 2017

Evidence of dark matter



K.G. Begeman, A.H. Broels, R.H. Sanders. 1991. Mon.Not.RAS 249, 523.

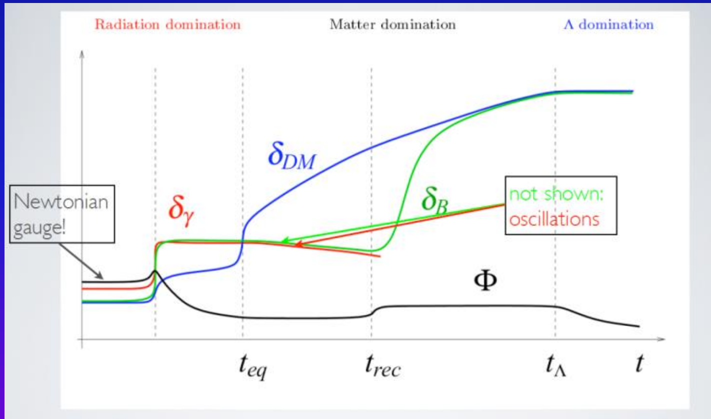
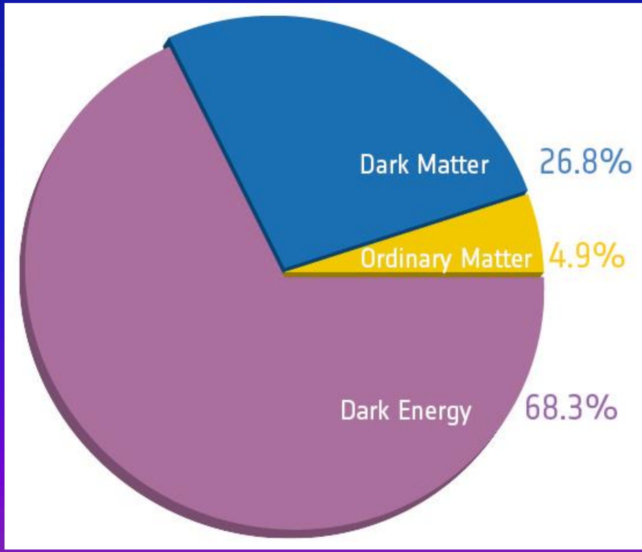


Figure 5: Schematic plot of the evolution of density pertirbations in different components. Here, $\delta = \delta\rho/\rho$, and Φ is the gravitational potential. The left dashed vertical line is the time of horizon crossing of a mode considered.

Rubakov & Vlasov, 1008.1704



WIMP miracle

If dark matter is made of weakly interacting massive particles (WIMPs), what we observe is the relic density of these particles after the cooling of the universe.

Boltzmann equation in the thermal freeze-out mechanism:

$$\dot{n} + 3Hn = -\langle \sigma v \rangle \left[n^2 - (n^{\mathrm{eq}})^2 \right]$$

$$\Rightarrow \quad \Omega h^2 = \frac{s_0}{\rho_c / h^2} \sqrt{\frac{45}{\pi g_*}} \frac{x_f}{M_{\mathrm{pl}}} \frac{1}{\langle \sigma v \rangle}$$

$$\approx \frac{0.1\ \mathrm{pb}}{\langle \sigma v \rangle} \approx 0.1 \left(\frac{m_{\mathrm{WIMP}}}{100\ \mathrm{GeV}} \right)^2$$

Thermal relic → dark matter at weak scale!

A typical WIMP -- minimal dark matter

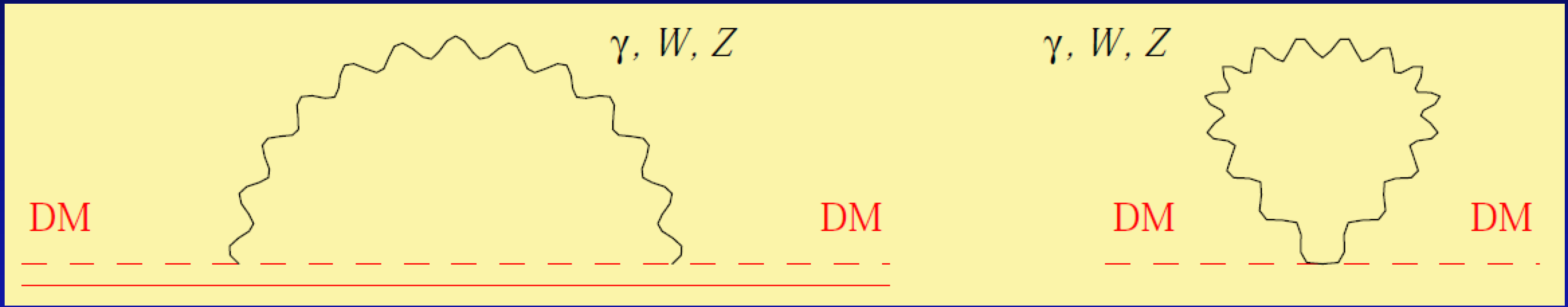
Introduce an extra electroweak multipliet χ to the SM:

$$\mathrm{L} = \mathrm{L}_{\mathrm{SM}} + c \begin{cases} \bar{\chi}(i\not{D} - M)\chi & \text{(fermionic multiplet)} \\ (D_\mu \chi^+)(D^\mu \chi) - M^2 \chi^+ \chi & \text{(scalar multiplet)} \end{cases}$$

- Cosmologically stable due to accidental symmetry
- Only one parameter: M, which is fixed by relic density
- Lightest component is neutral
- Allowed by WIMP DM searches

Cirelli, Fornengo, Strumia, 2005

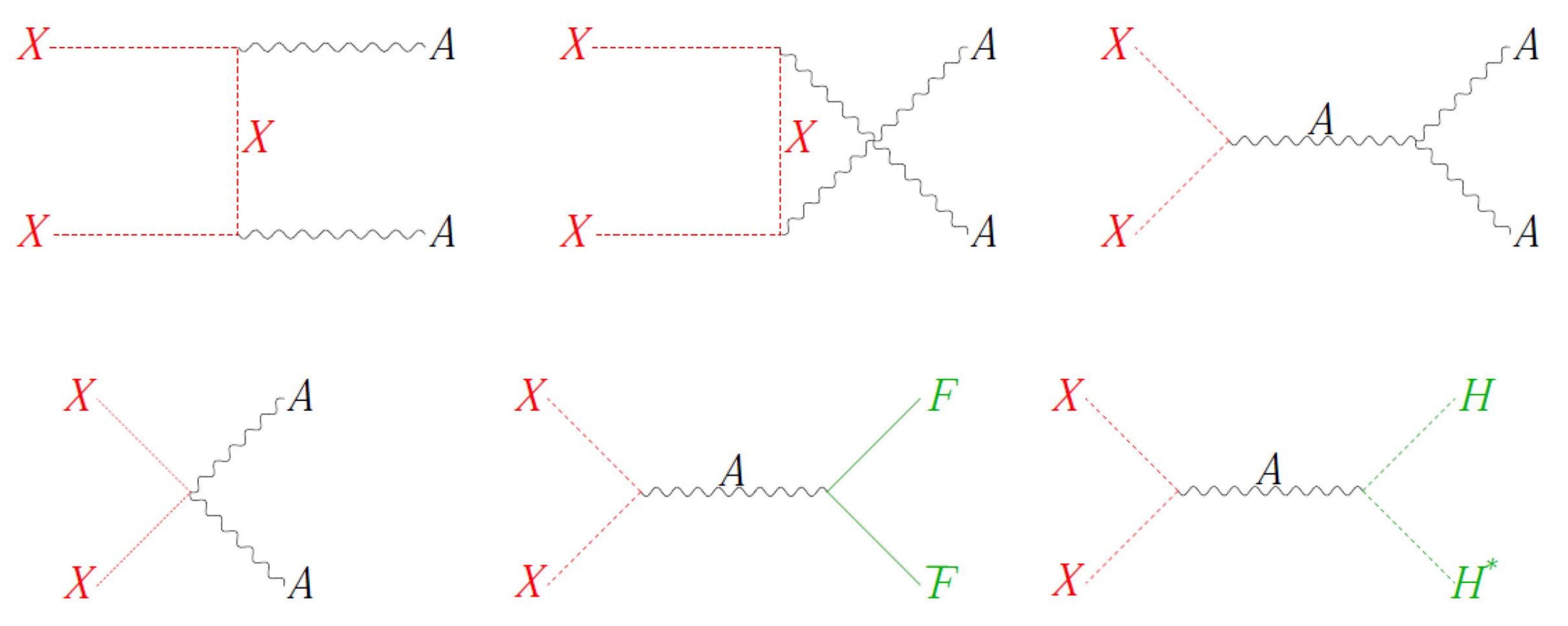
Mass splitting



$$M_Q - M_0 \simeq Q\left(Q + \frac{2Y}{\cos\theta_W}\right)\Delta M\,,$$

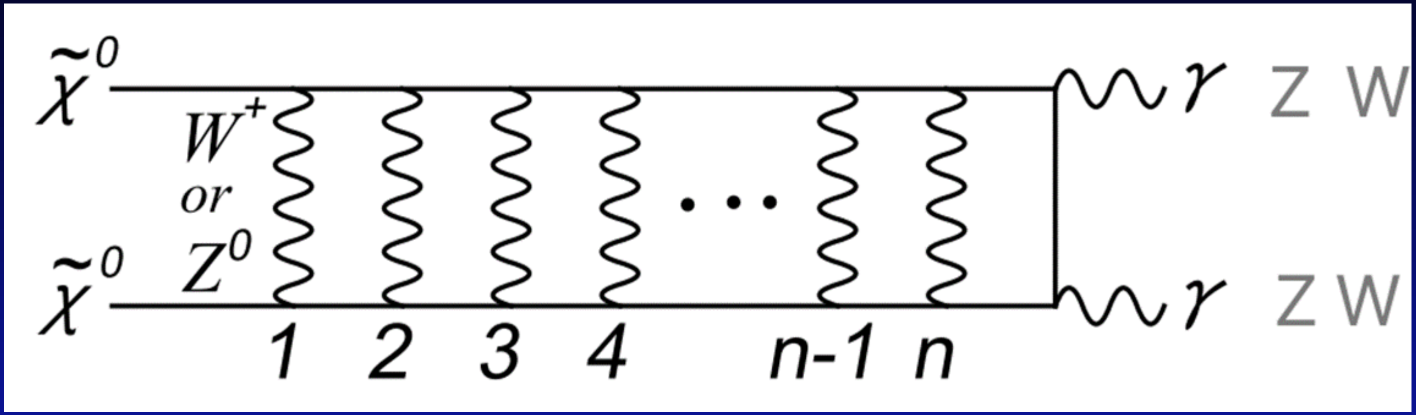
$$\Delta M = \alpha_2 M_W \sin^2\frac{\theta_W}{2} \simeq 166\ \text{MeV} \qquad (\text{for } M \gg M_W, M_Z)$$

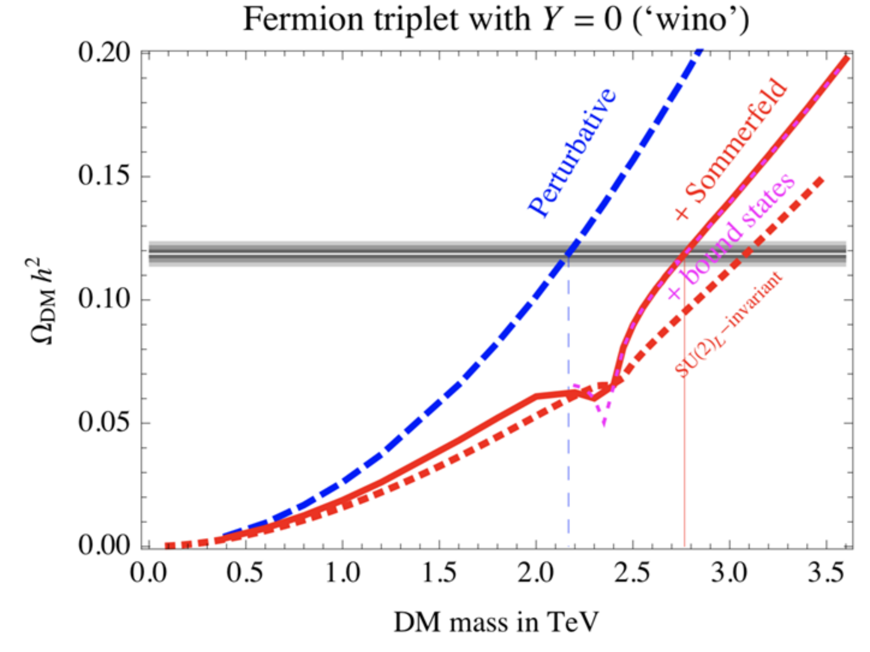
DM annihilation: perturbative calculation

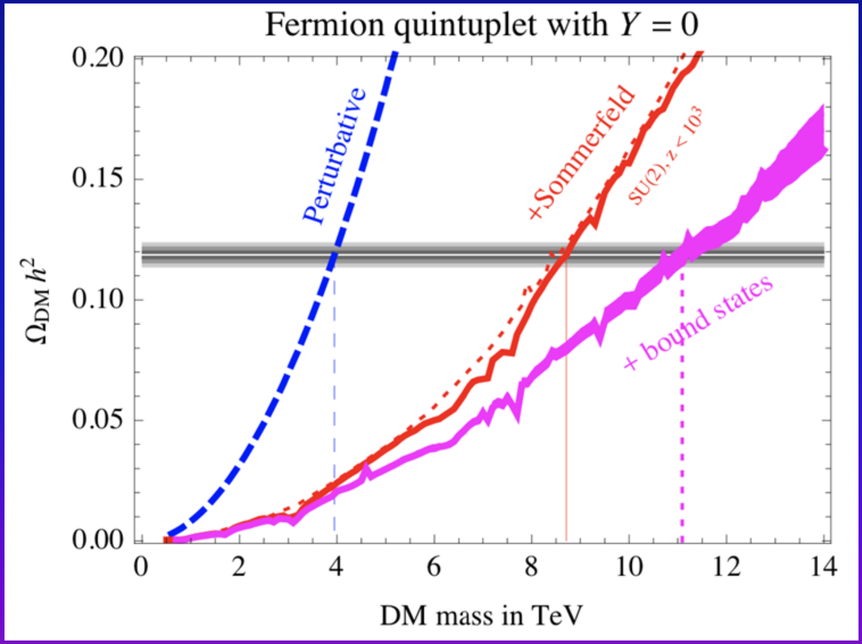


$$\langle \sigma_A v \rangle \simeq \begin{cases} \dfrac{g_2^4\ (3 - 4n^2 + n^4) + \mathcal{O}(g_2^2 g_Y^2, g_Y^4)}{64\pi\ M^2\ \mathrm{dof}_{\mathcal{X}}} & \text{if } \mathcal{X} \text{ is a scalar} \\ \dfrac{g_2^4\ (n^4 + 9n^2 - 10) + \mathcal{O}(g_2^2 g_Y^2, g_Y^4)}{64\pi\ M^2\ \mathrm{dof}_{\mathcal{X}}} & \text{if } \mathcal{X} \text{ is a fermion} \end{cases}$$

Non-perturbative Sommerfeld corrections







1702.01141

Sommerfeld enhancement (SE) effect[JCAP 05 (2017) 006]:

$$\sigma v \to S \cdot \sigma v, \quad S \overset{v\to 0}{\simeq} \frac{2\pi^2 \alpha_{\rm eff} M_\chi}{\kappa M_V} \left(1 - \cos 2\pi \sqrt{\frac{\alpha_{\rm eff} M_\chi}{\kappa M_V}}\right)^{-1}$$

when $\sqrt{\alpha_{\rm eff} M_\chi / \kappa M_V} \to 1, 2, 3...$, S becomes very large.

Direct detection

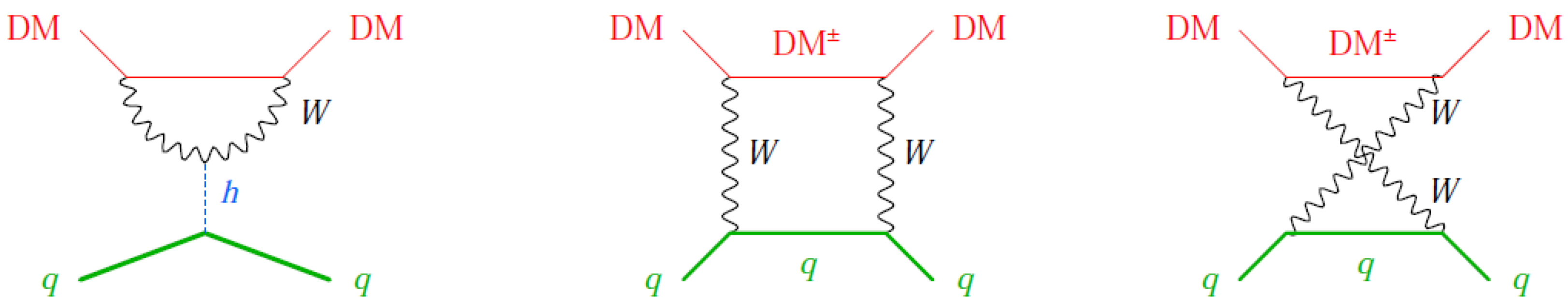
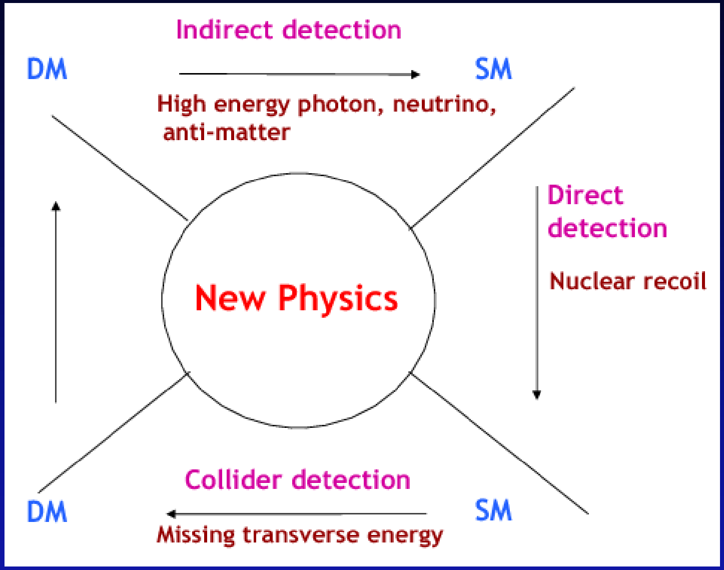
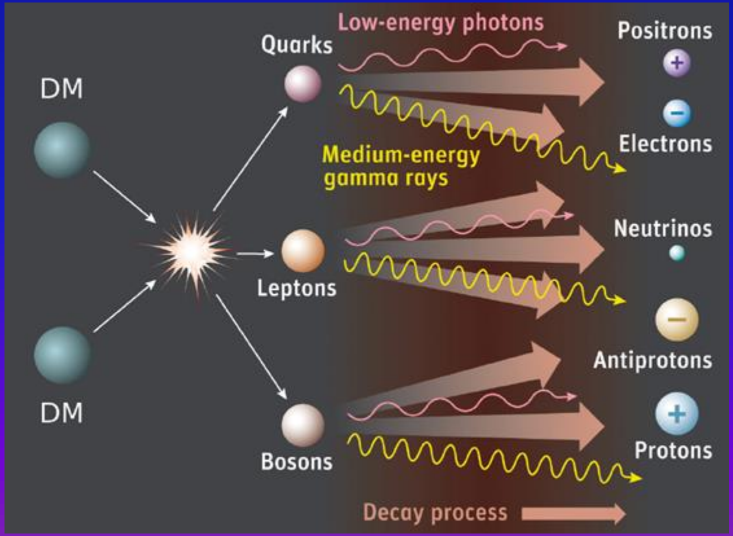


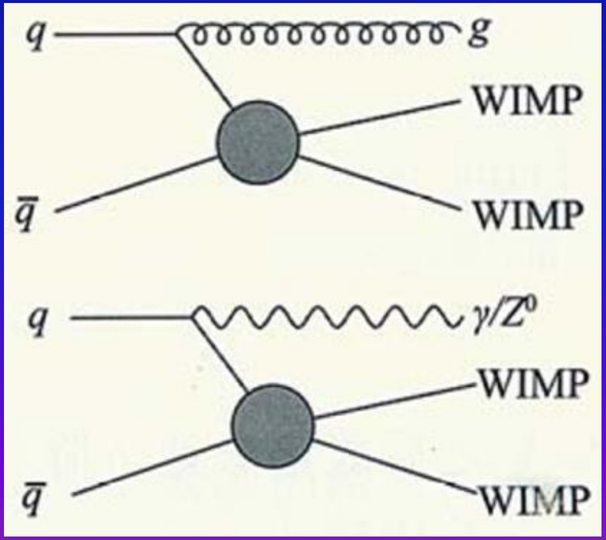
Figure 1: *One loop DM/quark scattering for fermionic MDM with $Y = 0$. Two extra graphs involving the four particle vertex exist in the case of scalar MDM.*

$$\sigma_{\rm SI}(\mathrm{DM}\mathcal{N} \to \mathrm{DM}\mathcal{N}) = (n^2-1)^2 \frac{\pi \alpha_2^4 M_{\mathcal{N}}^4 f^2}{64 M_W^2} \left(\frac{1}{M_W^2} + \frac{1}{m_h^2}\right)^2$$

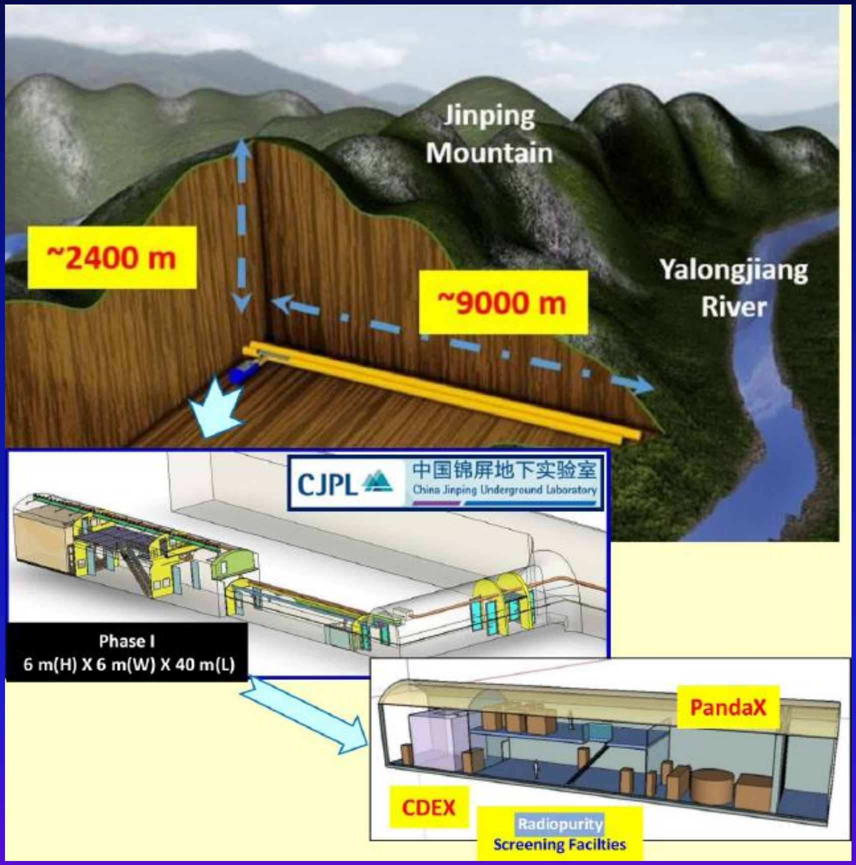
Detection of WIMP dark matter





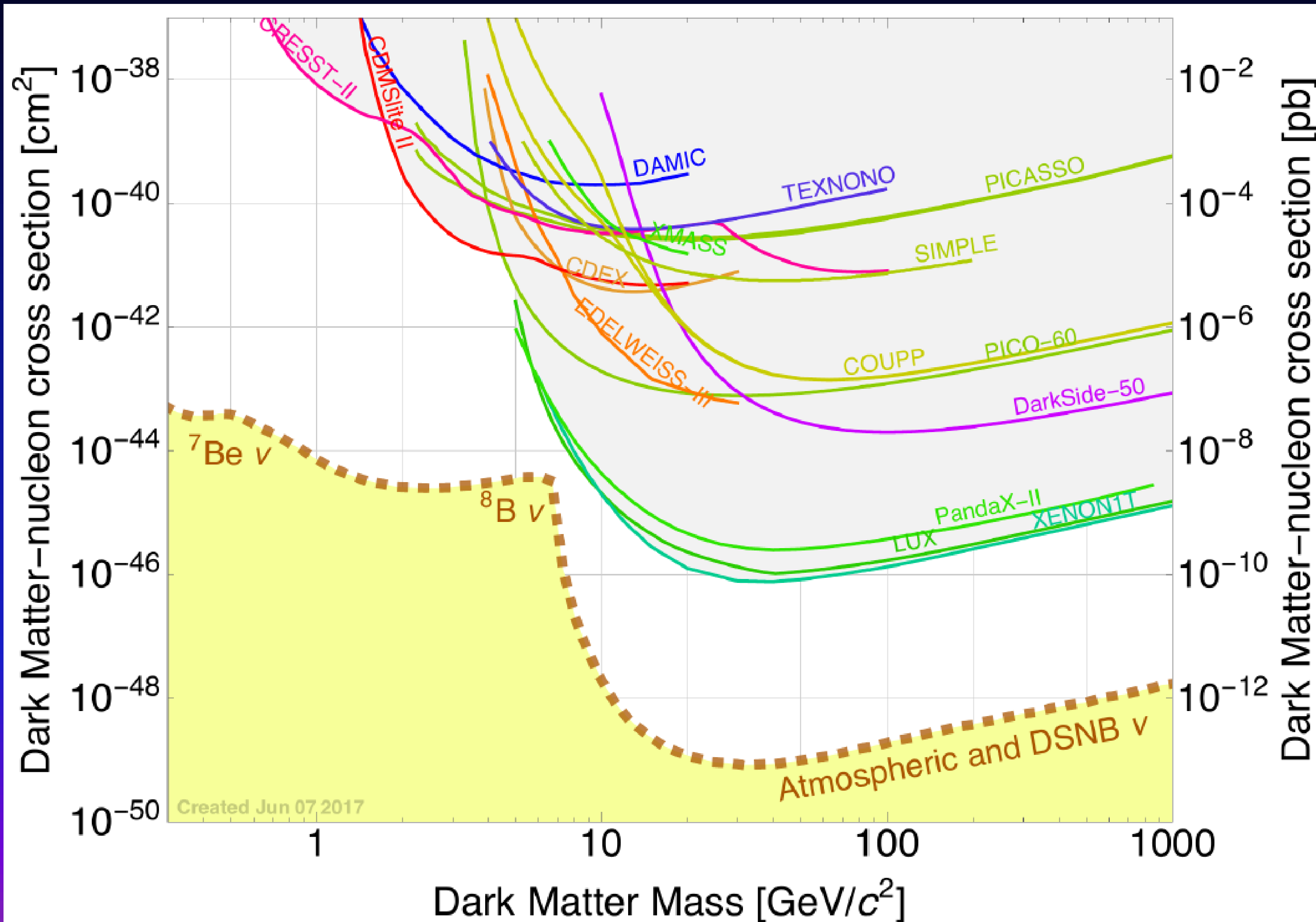


China JinPing Underground Laboratory (CJPL)

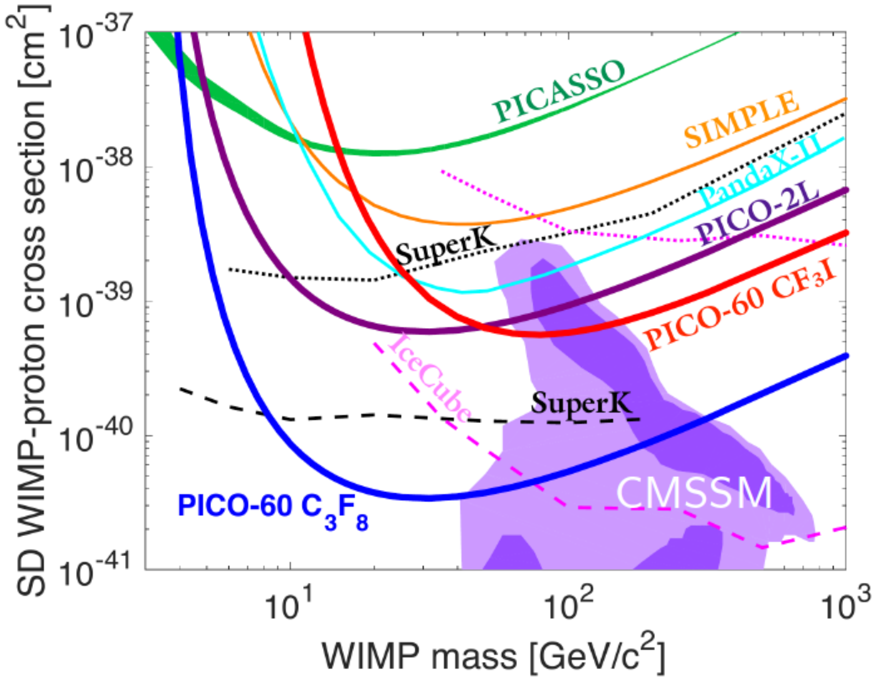


Experiments: CDEX, PandaX

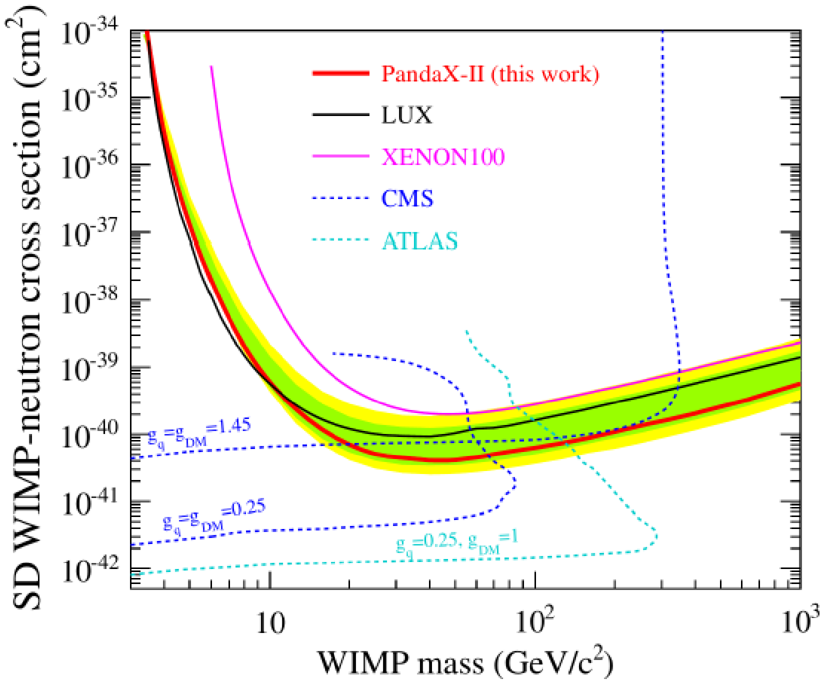
Exclusion Limits for Spin-independent (SI) Scattering



Exclusion Limits for Spin-dependent (SD) Scattering



[PICO coll., arXiv:1702.07666, PRL]

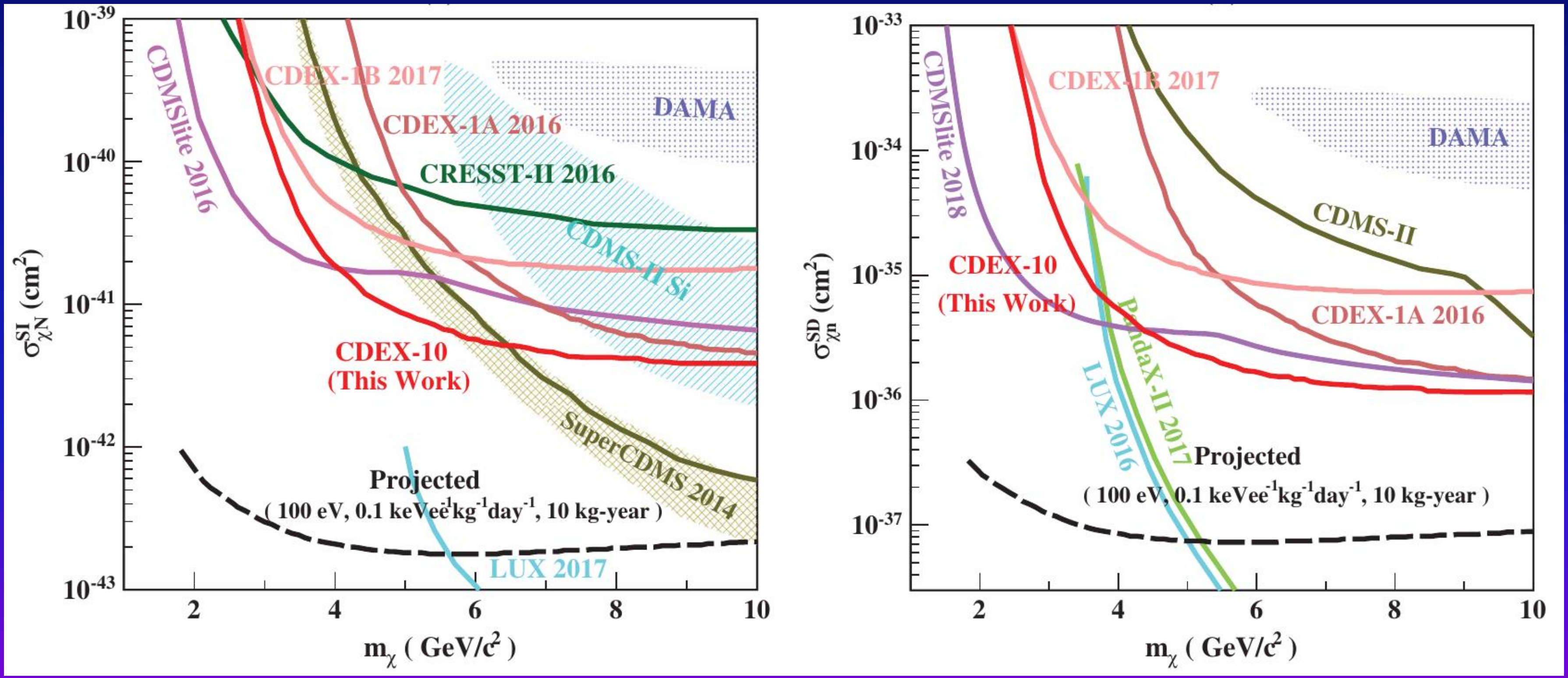


[PandaX-II coll., arXiv:1611.06553, PRL]

CDEX-10 experiment: 102.8 kg·day data

Leading limits for 4-5 GeV WIMP direct detection

SI **SD**



CDEX Collaboration, PRL 120, 241301 (2018)

Minimal asymptotically safe dark matter

Introducing N_F Weyl spinors of $SU(2)_L$ n-plets, with $n = 2k+1$, $k = 1,2,3...$ and $Y = 0$.
Lagrangian:

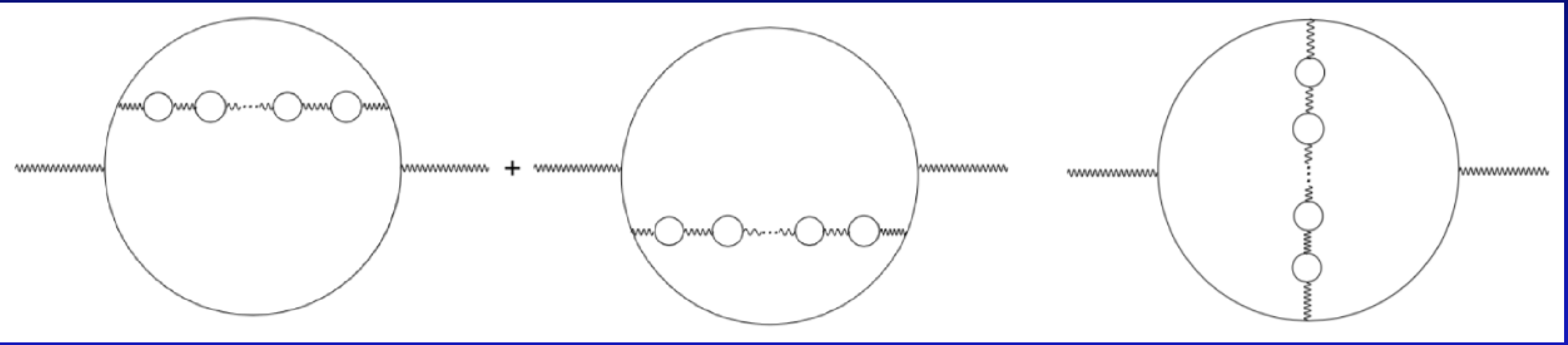
$$\mathcal{L}_{dark} = (\Psi_i^I)^\dagger i\overline{\sigma}^\mu D_\mu \Psi_i^I - \frac{1}{2} M_{DM}[\Psi_i^I \Psi_i^I + h.c.],$$

where $i = 1,2,...,n$; $I = 1,2,...,N_F$.
Benefits of N_F:

- Large N_F cases have asymptotic safety.
- Global $O(N_F)$ symmetry prevents DM from decay.
- Comparing to MDM, DM candidate has a smaller mass. Signals in collider are amplified by an N_F factor. Therefore it is easier to be detected by colliders.
- Annihilation rates in galaxies and dwarf galaxies are suppressed by $1/N_F$. Less tension with current observation bounds.

C.Cai, H.H.Zhang, 1905.04227

For fixed $\alpha_2 * N_F$, one can sum up the leading $1/N_F$ order self-energy contributions:



Beta function in the large N_F limit

$$\beta_{\alpha_2} \approx \frac{\alpha_2^2}{2\pi}(-\frac{19}{6} + \Delta b_2) + \frac{\alpha_2^2}{3\pi} F_2\left(\Delta b_2 \frac{\alpha_2}{4\pi}\right),$$

$$\Delta b_2 = \frac{2}{3} T(R_f) N_F, \quad F_2(A) \equiv \int_0^A I_1(x) I_2(x) dx,$$

$$I_1(x) \equiv \frac{(1+x)(2x-1)^2(2x-3)^2 \sin^3(\pi x)\Gamma(x-1)^2\Gamma(-2x)}{\pi^3(x-2)},$$

$$I_2(x) \equiv \frac{3}{4} + \frac{(20 - 43x + 32x^2 - 14x^3 + 4x^4)}{(2x-1)(2x-3)(1-x^2)},$$

对于SU(2)的n维表示，有 $T(n) = \frac{n(n^2-1)}{12}$

特别地， $T(2) = \frac{1}{2}$， $T(3) = 2$， $T(5) = 10$

$F_2(A)$在$A=1$处会对数发散： $F_2(A) \overset{A\to 1}{\simeq} \frac{1}{4}\ln(1-A)+\cdots$

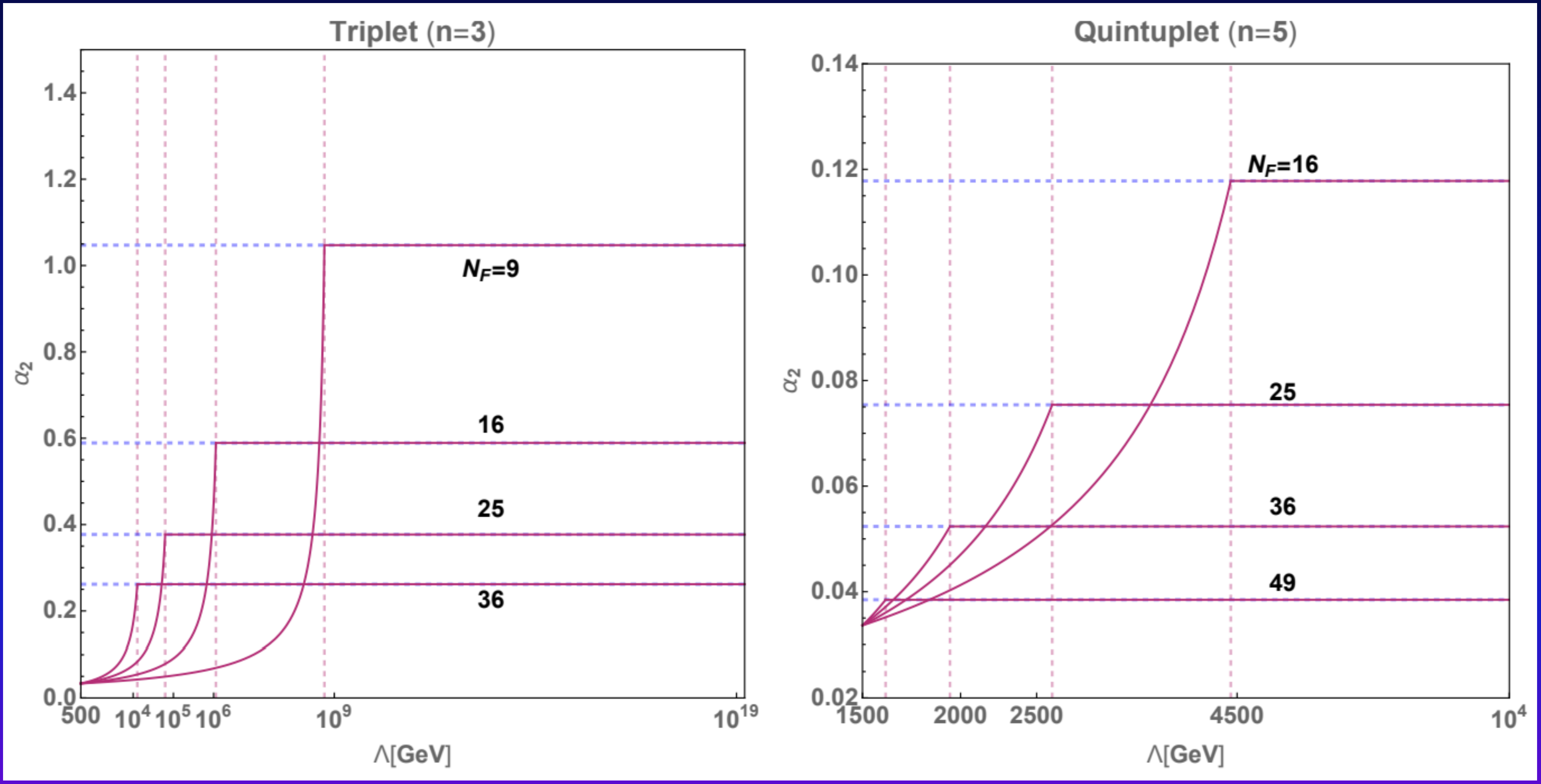
也即是说，当$A\to 1$时，$F_2(A)\to -\infty$.

这意味着在接近于1的某个点A_*处，$F_2(A_*)$的贡献会与$\beta(\alpha_2)$的第一项（$\sim N_F\alpha_2^2$的正数）抵消，这导致一个紫外固定点：

$$A_* = \Delta b_2 \frac{\alpha_2^*}{4\pi} \simeq 1 \quad \longrightarrow \quad \alpha_2^* \simeq \frac{4\pi}{\Delta b_2}$$

对于三重态$(n=3)$，根据1709.02354知，当$N_F>7$时就可用这些公式.

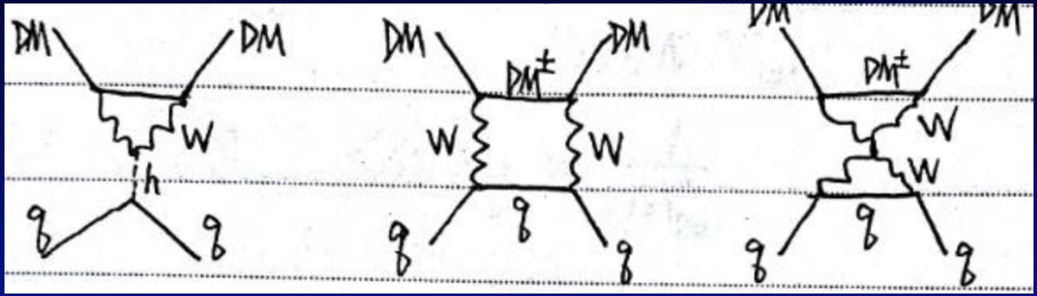
Running of the gauge coupling alpha_2

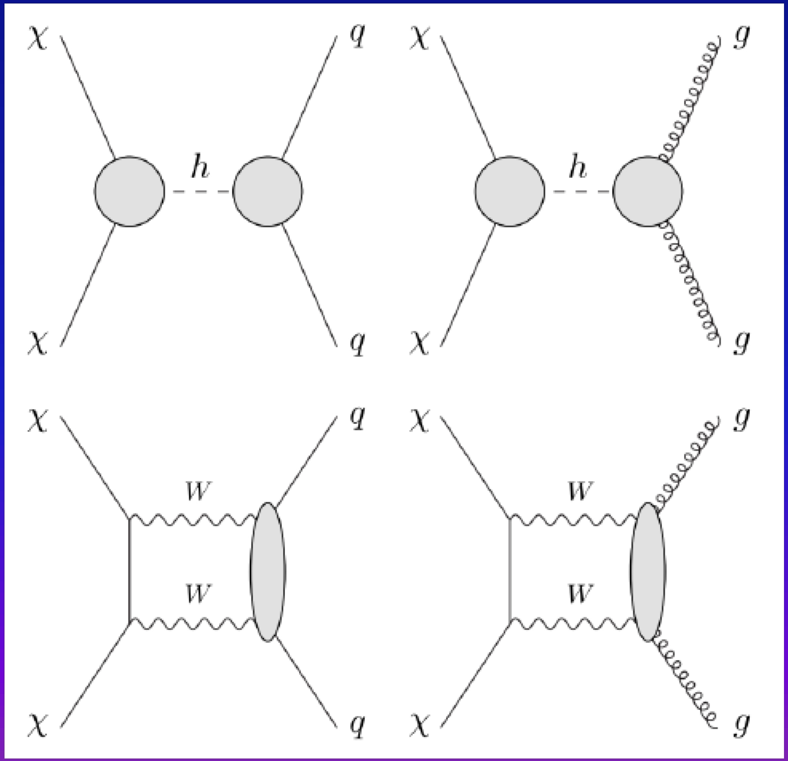


Direct detection constraints

the same as MDM

Electroweak contributions





Including QCD effects, the upper bounds are [Hisano, Ishiwata, Nagata, 1504.00915]:

- 10^(-47) cm^2 for 3plet
- 10^(-46) cm^2 for 5plet

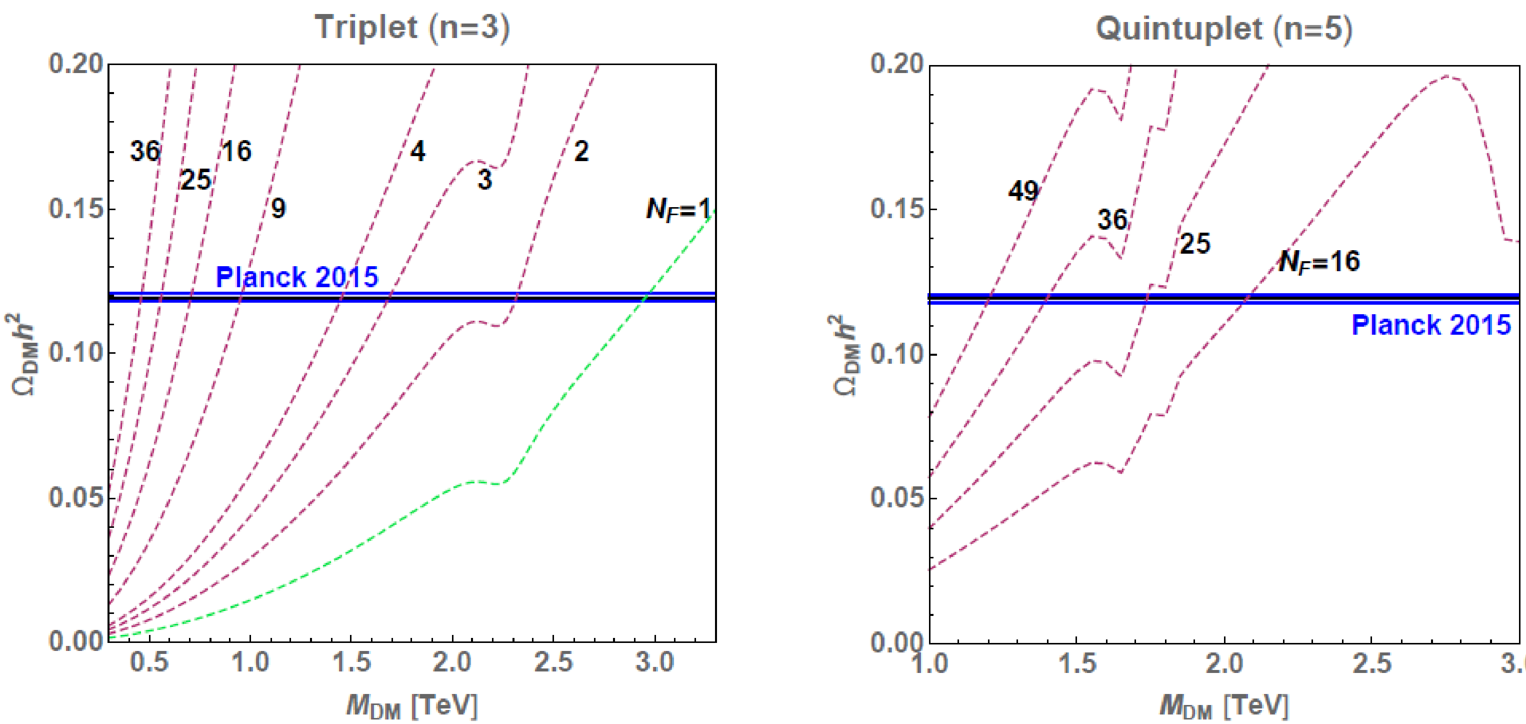
n-plets (n>5) are excluded

Dark matter relic density

The total relic abundance is N_F times of MDM.

$$\Omega_{\rm DM}h^2 \approx N_F \times \frac{1.07\times 10^9 \mathrm{GeV}^{-1}}{\sqrt{g_*}M_{\rm Pl}\int_{x_{\rm F}}^{\infty}\frac{\langle\sigma v\rangle_{eff}}{x^2}dx} \propto N_F M_{DM}^2.$$

When $\Omega_{DM}h^2$ is fixed by data, M_{DM} is suppressed by $1/\sqrt{N_F}$.

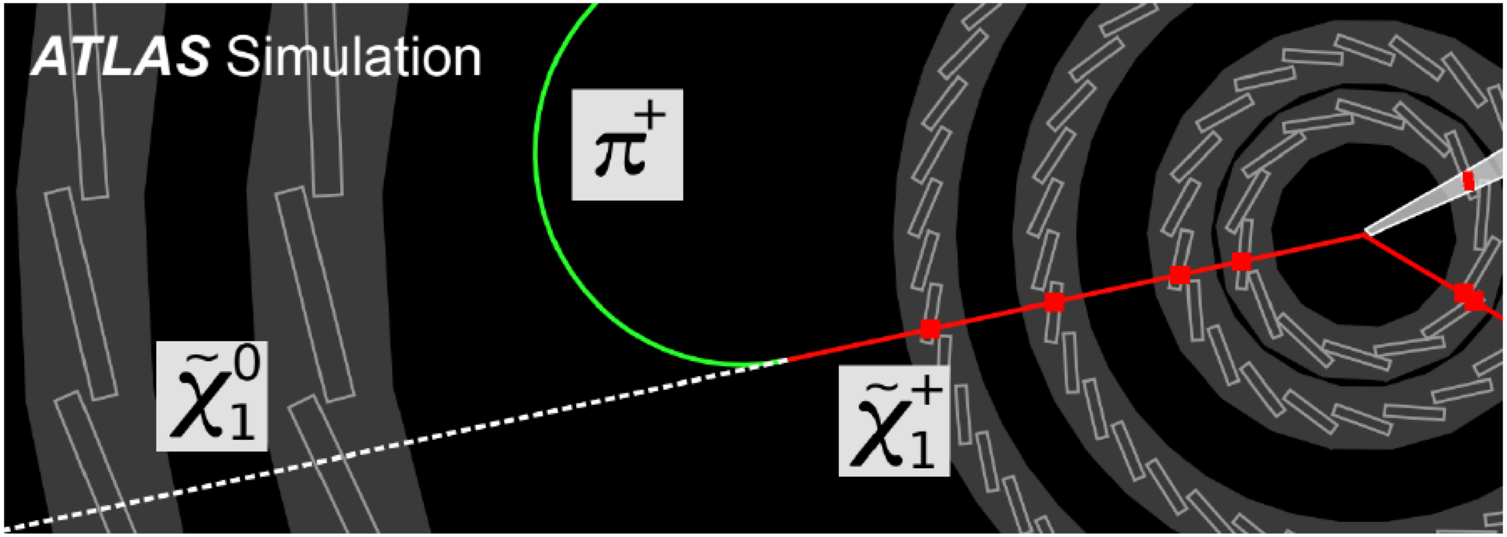


Collider searches for disappearing track of charged particles

Nearly degenerate multiplet with mass splitting

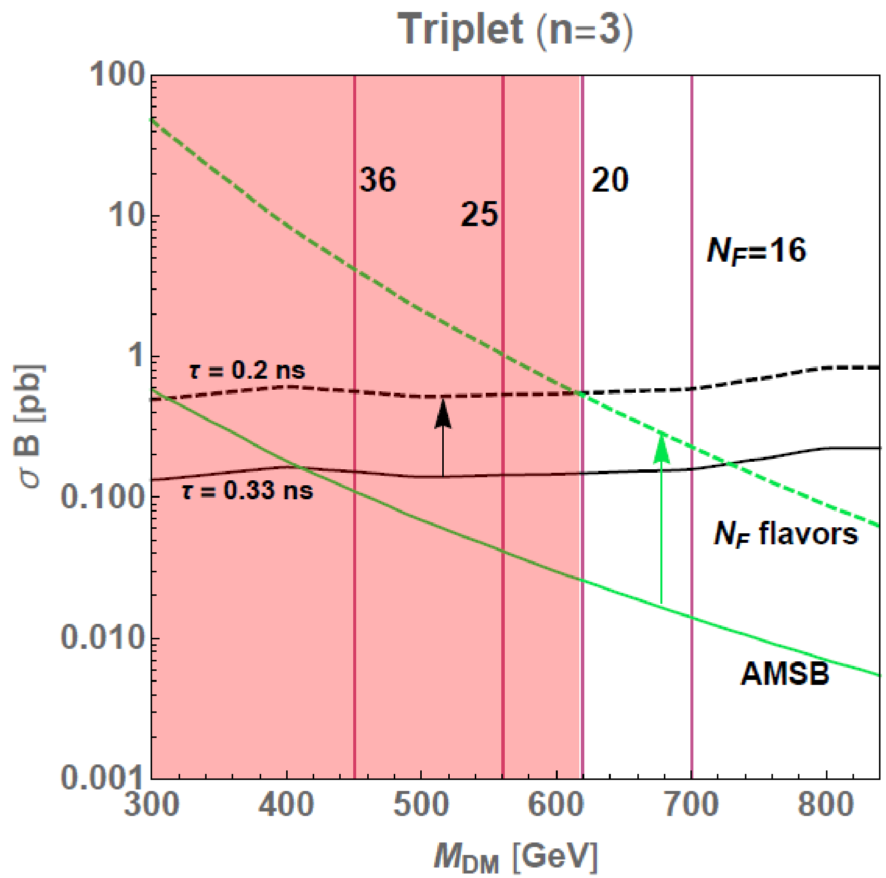
$$\Delta m_Q = M_Q - M_0 = Q^2 \delta m, \qquad \delta m \approx 164 \text{ MeV}$$
$$\tau \approx 0.2 \text{ ns}, \qquad c\tau \approx 6\text{cm}$$

Charged state is EW produced. Traveling a distinguishable distance in the detector before it decays through $\chi^{\pm} \to \chi^{0} + \pi^{\pm}$.



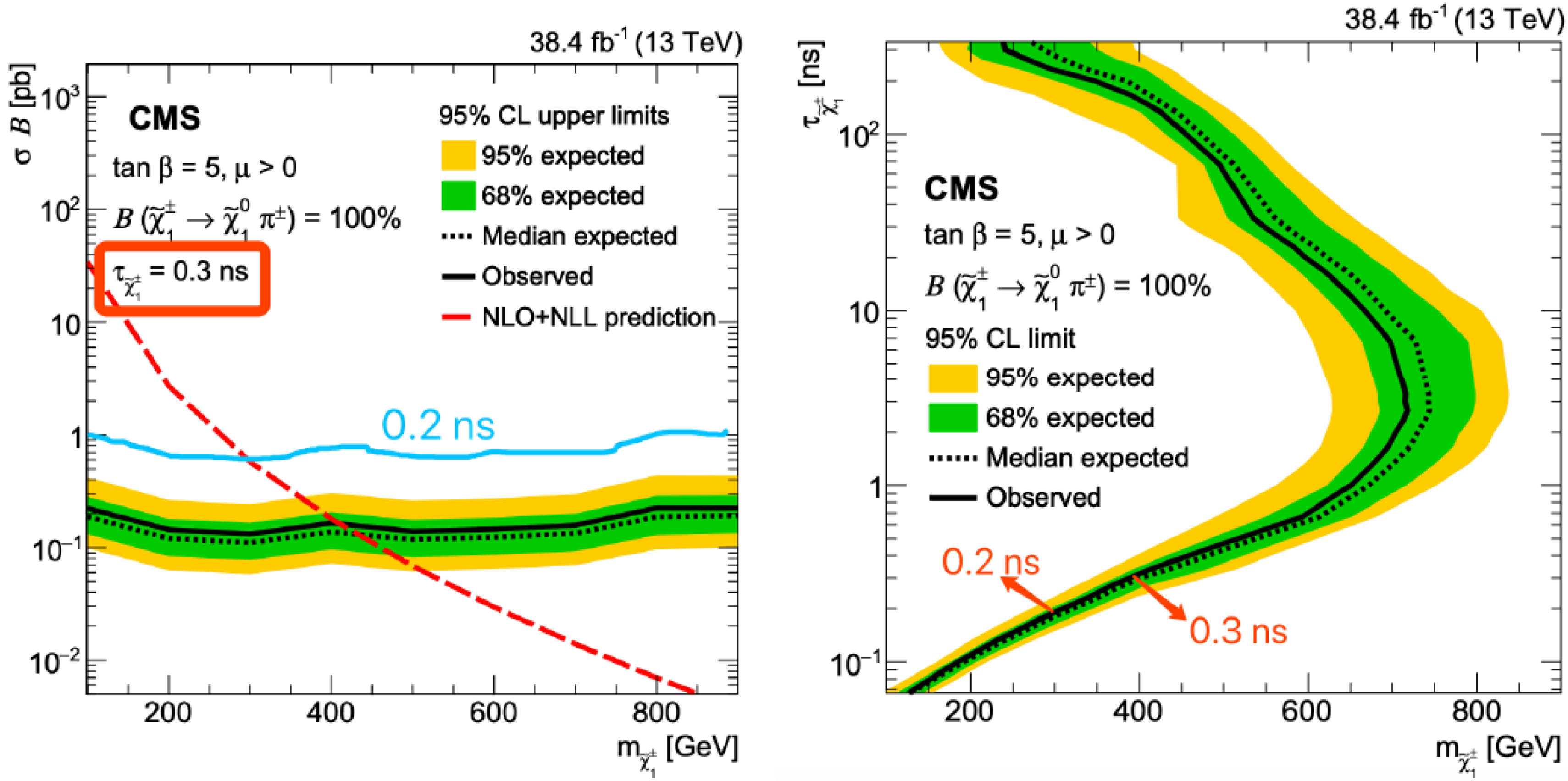
Constraints from disappearing track of charged particles

For N_F triplet, life time $\tau \approx 0.2$ ns and the traveling distance of charged state is the same as wino. However, the production rate is amplified as $\sigma \to N_F \sigma$.

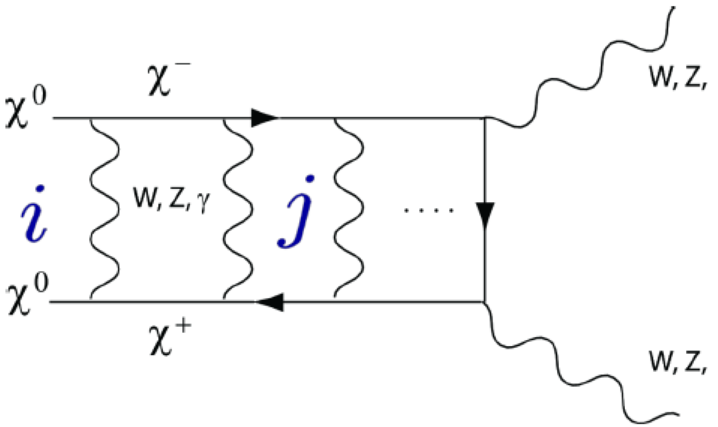


How to estimate the τ=0.2 ns data from CMS

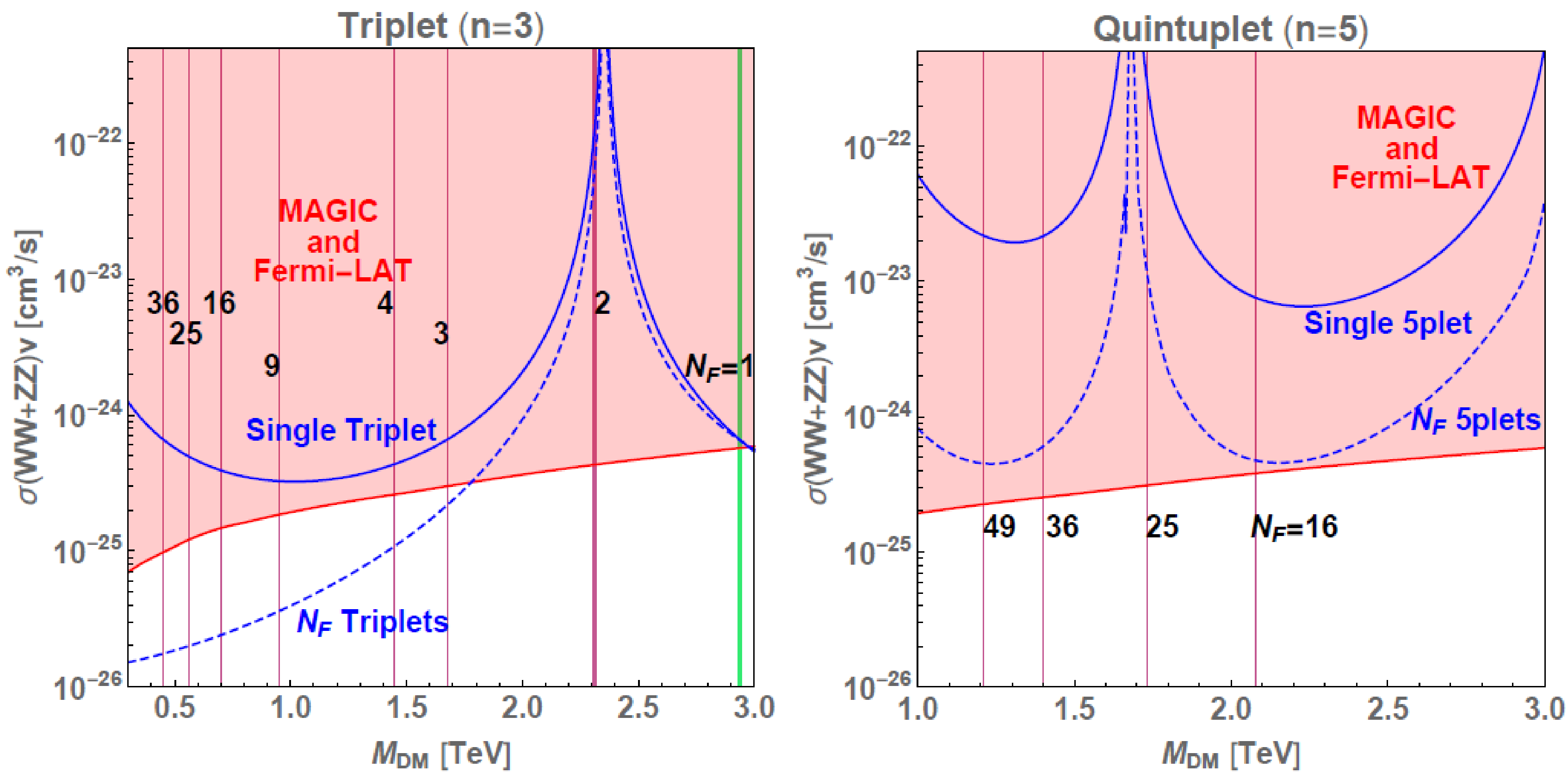
CMS bounds [JHEP 08 (2018) 016]



Continuous spectrum of γ-ray comes from DM pair annihilation to WW, ZZ.



Continuous spectrum of gamma-ray



- For triplet, models with $N_F > 2$ survive.
- For quintuplet, all cases ($N_F \geq 16$) are still disfavored.

Summary

- Extending SM with N_F fermionic n-plets is reasonable and motivative. $SU(2)_L$ gauge coupling is asymptotically safe.
- n=3 and 5 is favored by direct detection.
- Large N_F implies smaller DM mass and higher production rate in collider.
- Disappearing track searches set a bound that $N_F < 20$ for the triplet models. In the future HL-LHC experiment, this model can be further tested.
- Although triplet MDM model has been excluded by the observation of γ-ray continuous spectrum, $N_F \geq 3$ survive due to a $1/N_F$ suppression.
- Quintuplet models are still disfavored even it has $1/N_F$ suppression.

Triplet models with 3 <= N_F < 20 flavors are consistent with all current experiments