Overlap fermion near zero mode and quark/hadron mass

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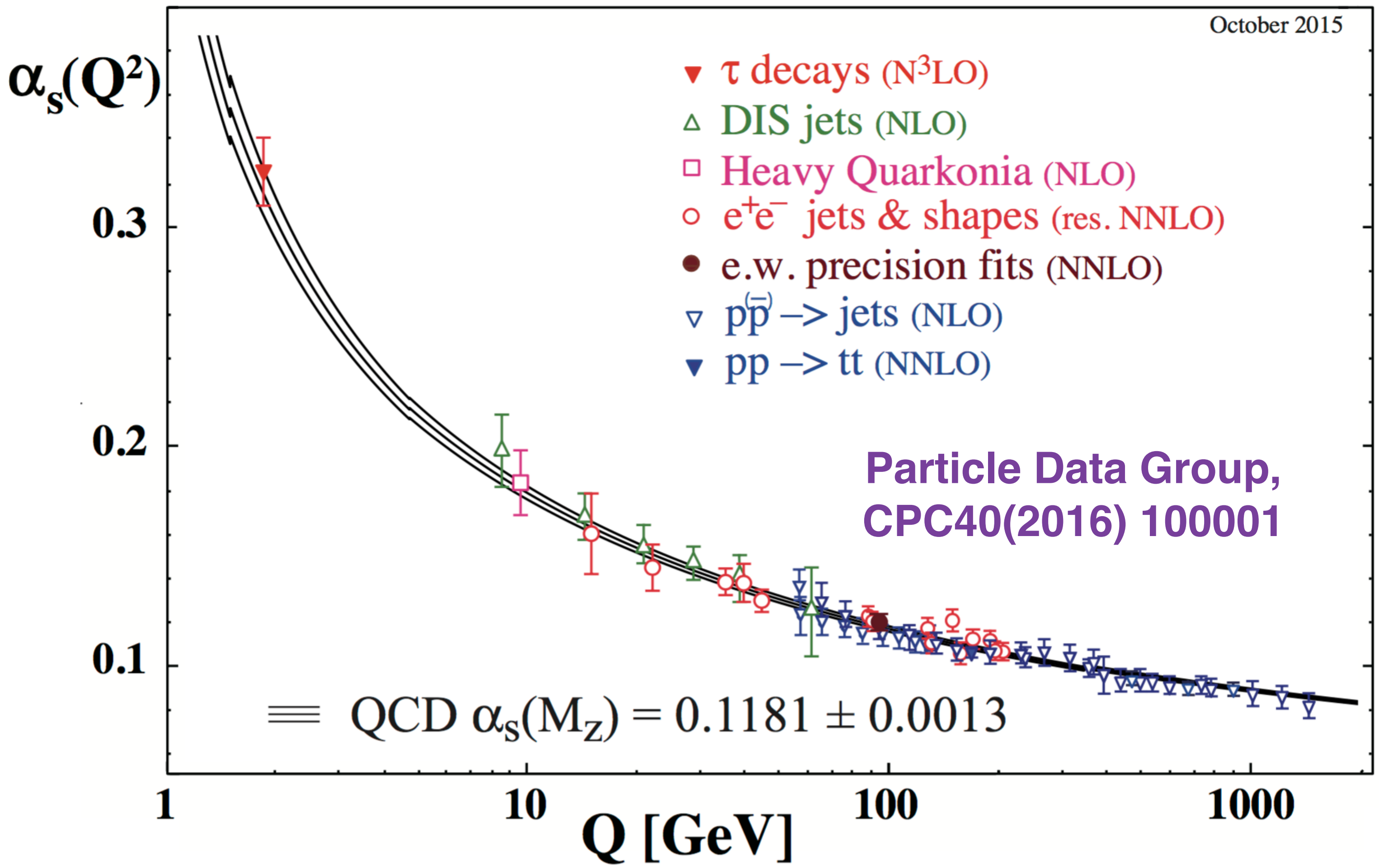
QCD

at low energy scale



- **Confinement**:
- The hadron ground state mass is higher than the vacuum by O(Λ_{QCD}), even in the chiral limit;
- Trace anomaly contributes most of the hadron mass;
- **……except pion.**

- **Spontaneous chiral symmetry breaking:**
- Obvious difference between the S/V and P/A meson/current;
- A dynamical quark mass non-vanishing in the chiral limit?

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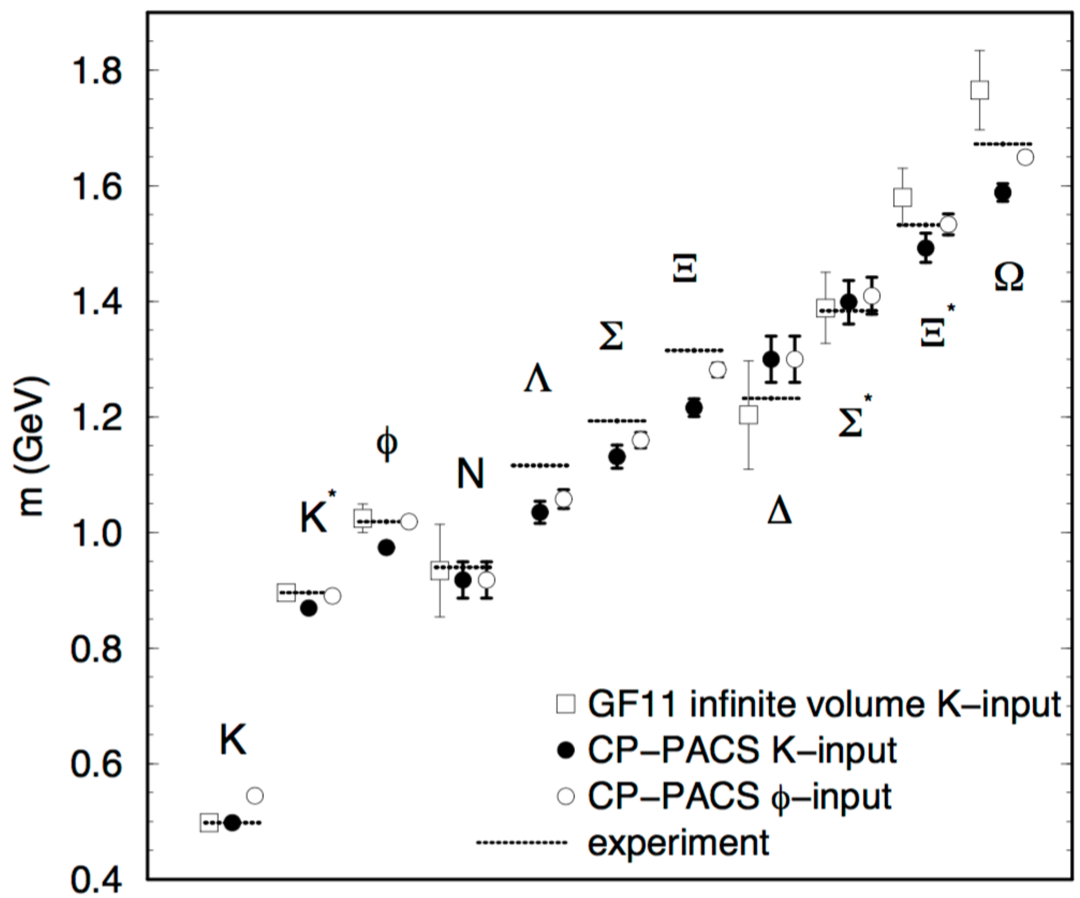
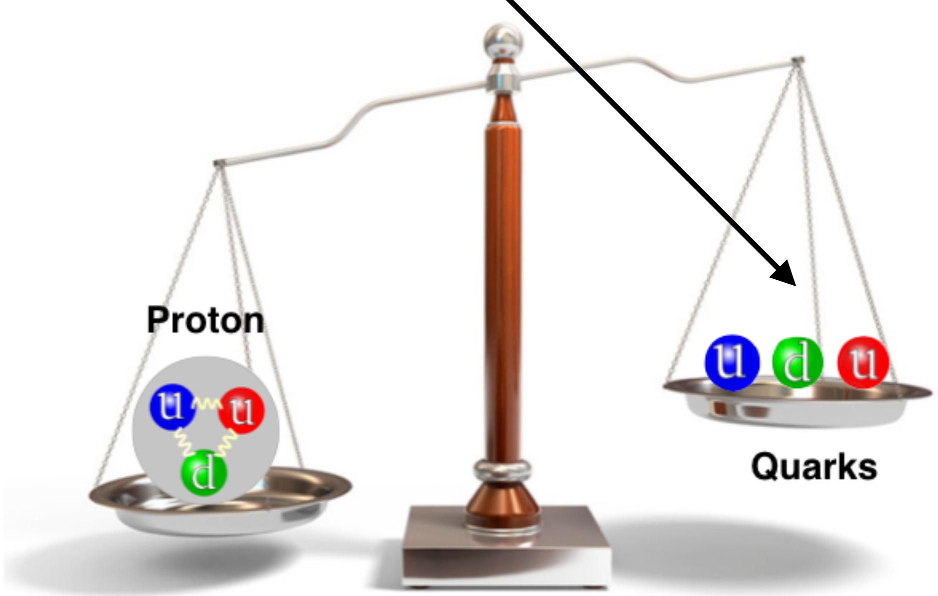


$$T^{\alpha}_{\alpha} = \underline{(1+\gamma^R_m)\left(m\bar{\psi}\psi\right)_R} + \frac{\beta_R}{2g_R}F^2_R\,.$$

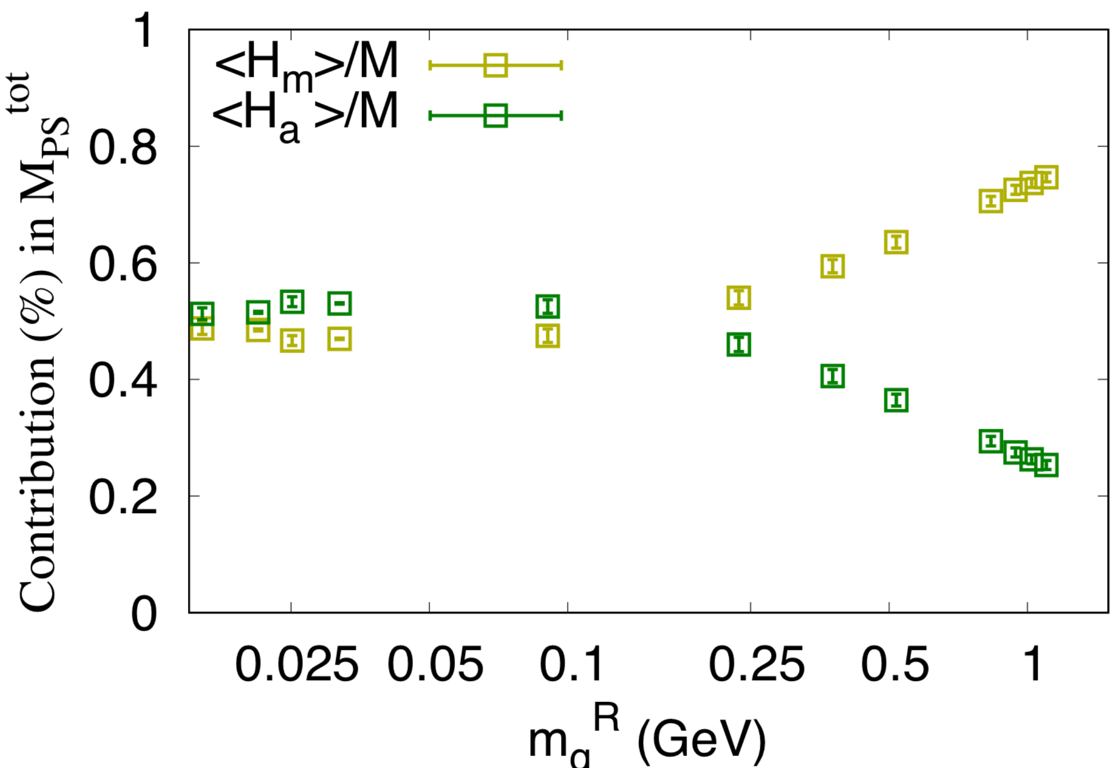


Trace anomaly in the hadron state

Thus we have the sum rule for the hadron mass

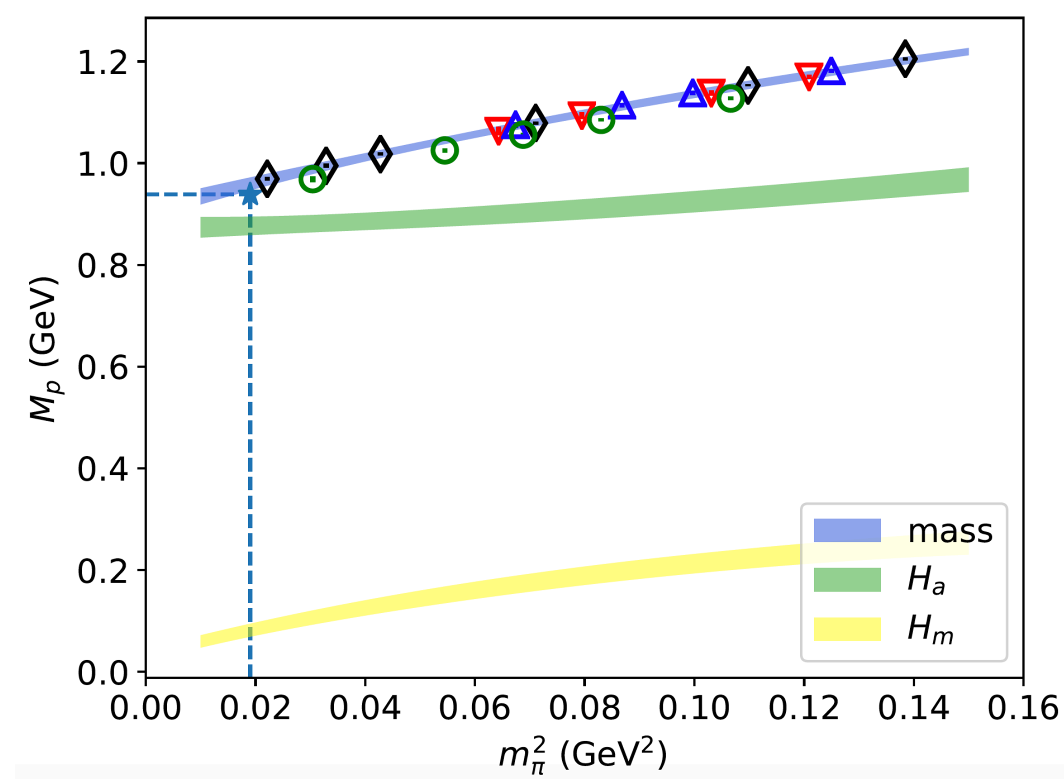
$$M = \langle M | T^{\alpha}_{\alpha} | M \rangle = \langle M | \underbrace{\sum_q m_q \bar{\psi}_q \psi_q}_{H_m} | M \rangle + \langle M | \underbrace{\gamma_m \sum_q m_q \bar{\psi}_q \psi_q + \frac{\beta}{2g} F^2}_{H_a} | M \rangle$$

which is renormalization invariant while n_f depends.



YBY, et. al., χQCD collaboration, PRD91 (2015) 074516

n_f=3: quark mass ~ 90 MeV, anomaly 850 MeV
n_f=6: quark mass ~270 MeV, anomaly 670 MeV



YBY, J. liang, et. al., χQCD collaboration, PRD94 (2016) 054503

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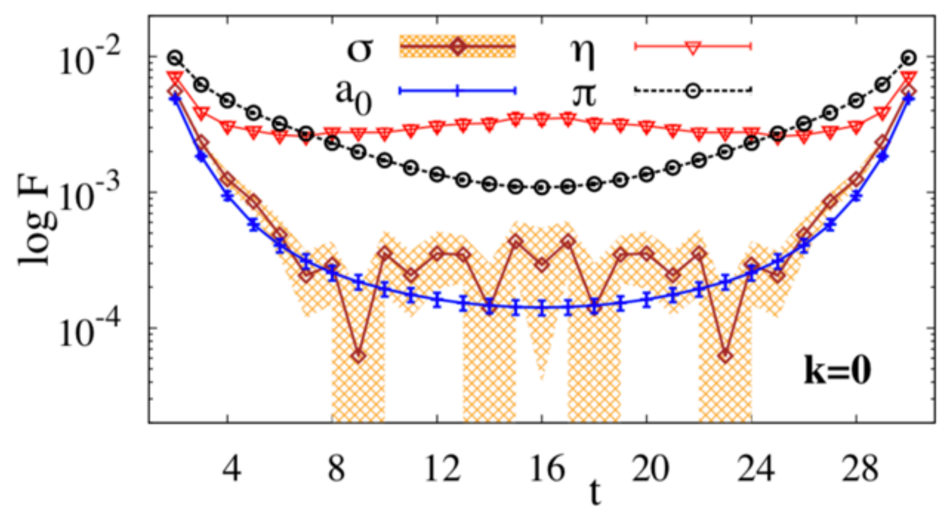
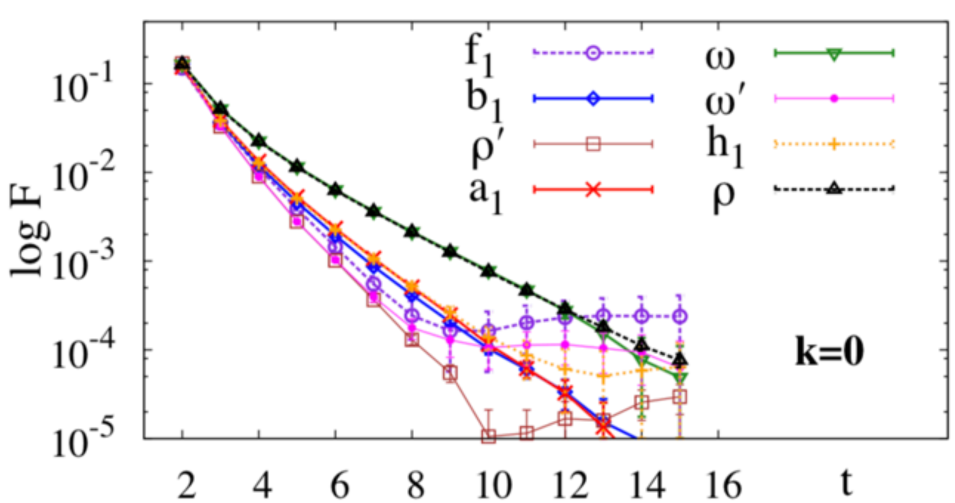
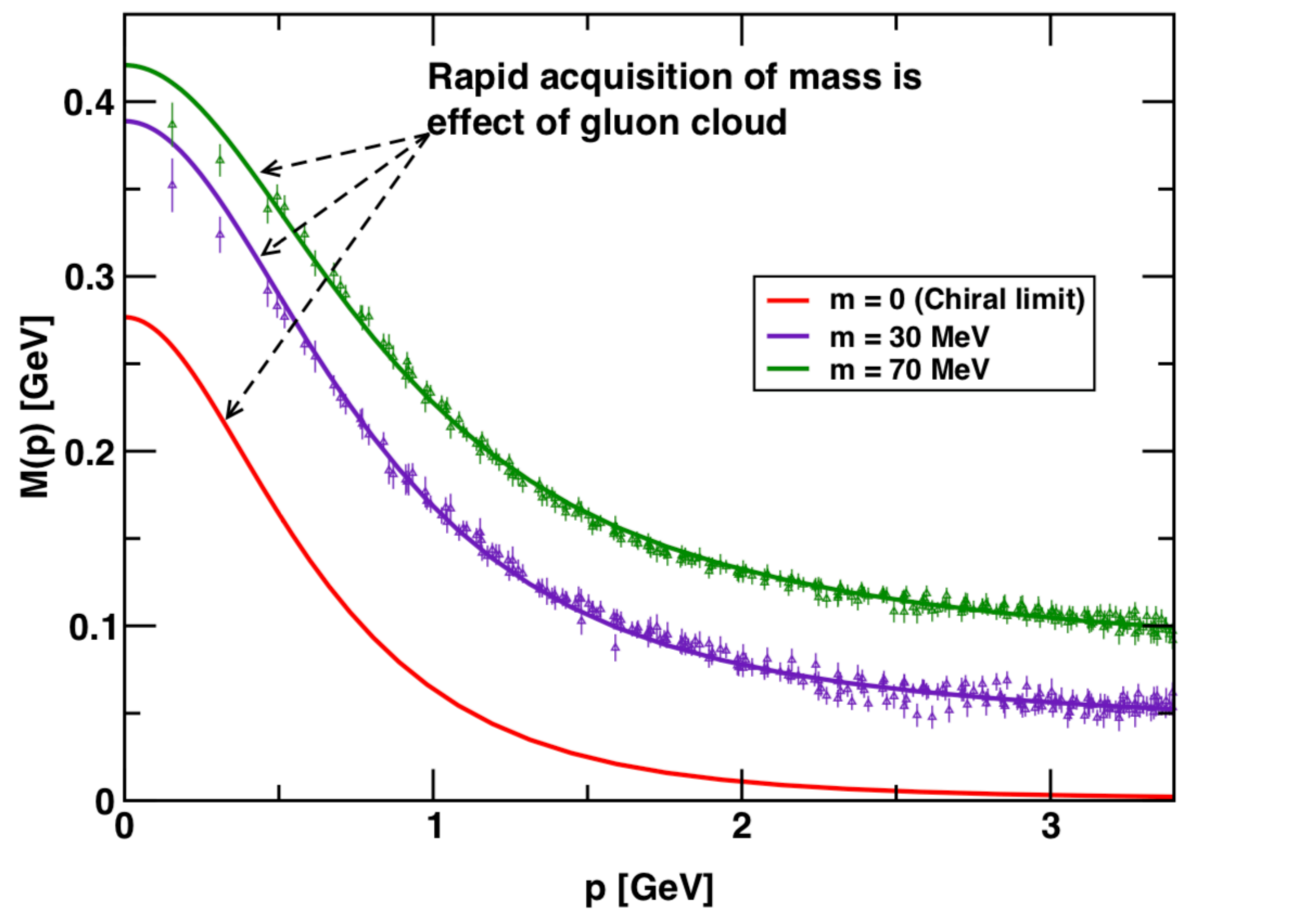




M. Denissenya, et.al., PRD91(2015)034505, 1410.8751



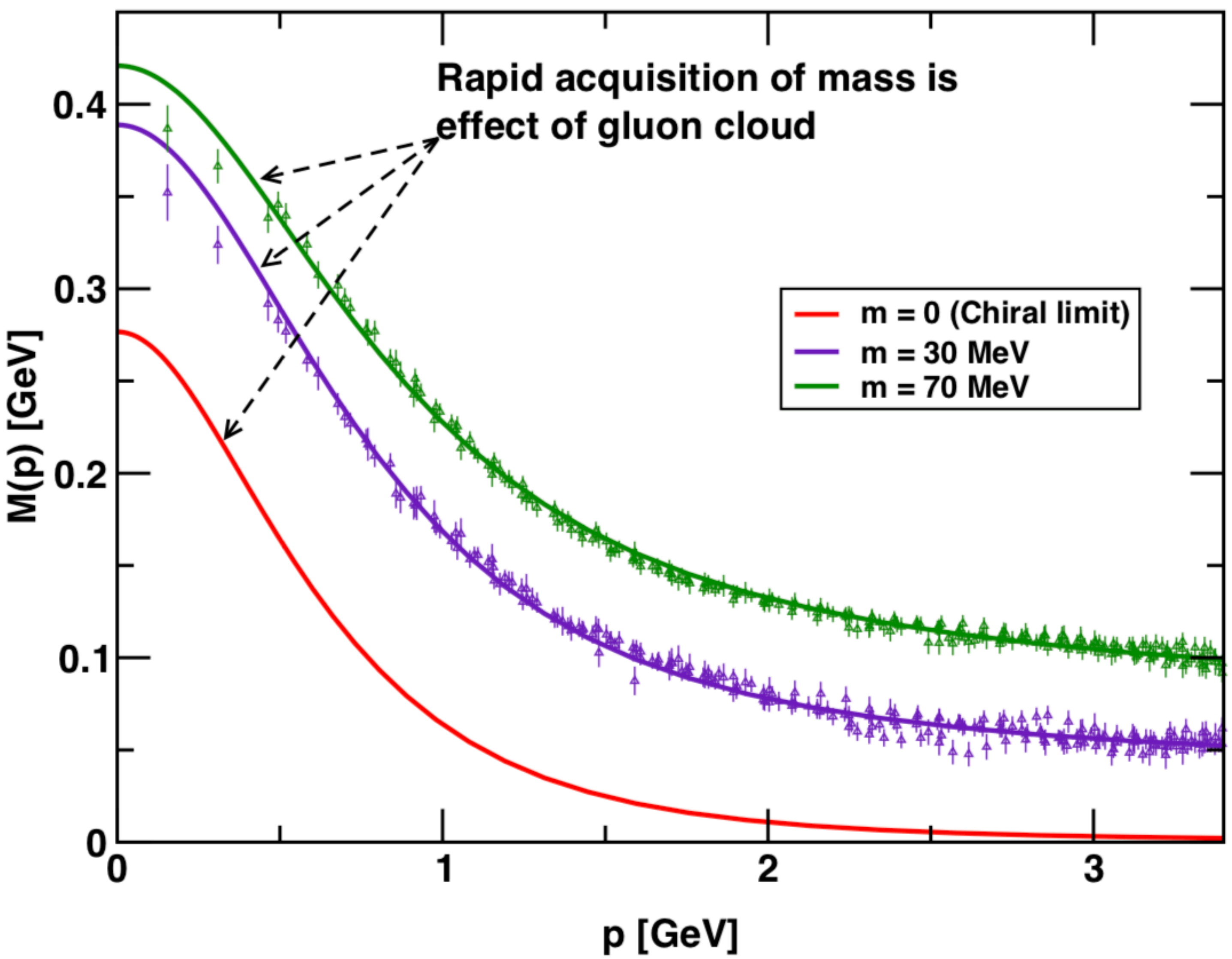
M.S.Bhagwat, I.C.Cloet and C.D.Robers,0710.2059

Quark mass at non-perturbative scale

$$S = \frac{Z_q}{\not{p} + Z_m m}, \; m^{RI} = Tr[\langle S \rangle^{-1}]$$

M.S.Bhagwat, I.C.Cloet and C.D.Robers,0710.2059

- ***Solid curves*** **– DSE results**
- **“data” – Lattice QCD simulations of the quark propagator in the Landau gauge, p is the scale of the quark external momentum.**
- The so-call gluon cloud contribution can not be obtained from the multiplicative renormalization of the quark mass.
- Related to the quark condensate through the OPE of the propagator.



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$$\mathrm{Non-zero}\ \langle F^2 \rangle$$

- **Spontaneous chiral symmetry breaking:**
- Obvious difference between the S/V and P/A meson/current;
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$$\mathrm{Non-zero}\ \langle \bar{q}q \rangle$$

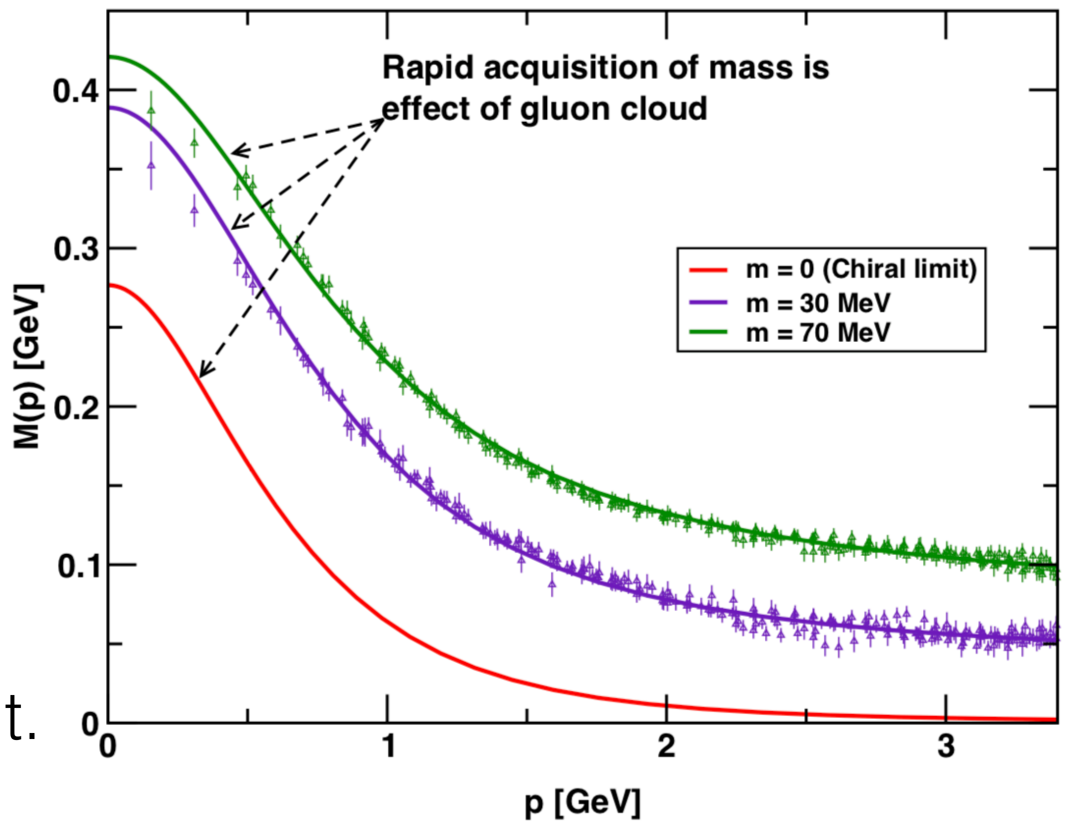
Are they related to each other in all the condition?

Trace anomaly

and quark mass ?

M.S.Bhagwat, I.C.Cloet and C.D.Robers,0710.2059

- In a private discussion, C.D.Roberts mentioned that the mass-independent mass at low energy scale would come from the trace anomaly;



- In sense of the perturbative calculation,

$$\langle q|F^2|q\rangle \propto \gamma_m m\langle q|q\rangle$$

and then should vanish in the chiral limit.

- Similar ME in the gluon state introduce the gluon trace anomaly, which is not vanishing in the chiral limit. $\langle g|F^2|g\rangle \propto \beta\langle g|F^2_{tree}|g\rangle$

- But we also know that some miracle happen in the quark mass at low scale which should also proportional to the bare quark mass;

- A similar miracle in $\langle q|F^2|q\rangle$?

Overlap fermion

the perfect action for this topic

The overlap fermion action satisfies the Ginsburg-Wilson

$$\gamma_5 D_{ov} + D_{ov}\gamma_5 = \frac{a}{\rho} D_{ov}\gamma_5 D_{ov}$$

It can be rewritten into

$$D_{ov}^{-1}\gamma_5 + \gamma_5 D_{ov}^{-1} = \frac{a}{\rho}\gamma_5, \quad (D_{ov}^{-1} - \frac{1}{2\rho})\gamma_5 + \gamma_5(D_{ov}^{-1} - \frac{1}{2\rho}) = 0$$

Thus the chiral fermion can be defined through the overlap fermion action:

$$D_c + m = \frac{D_{ov}}{1 - \frac{1}{2\rho}D_{ov}} + m, \; D_{ov} = \rho(1 + \gamma_5 \epsilon_{ov}(\rho)) \, .$$

Since such an action is highly non-local, we just consider its propagator in most the cases.

Overlap fermion

the perfect action for this topic

The low-lying Eigen mode of the overlap fermion to get that of the chiral fermion,

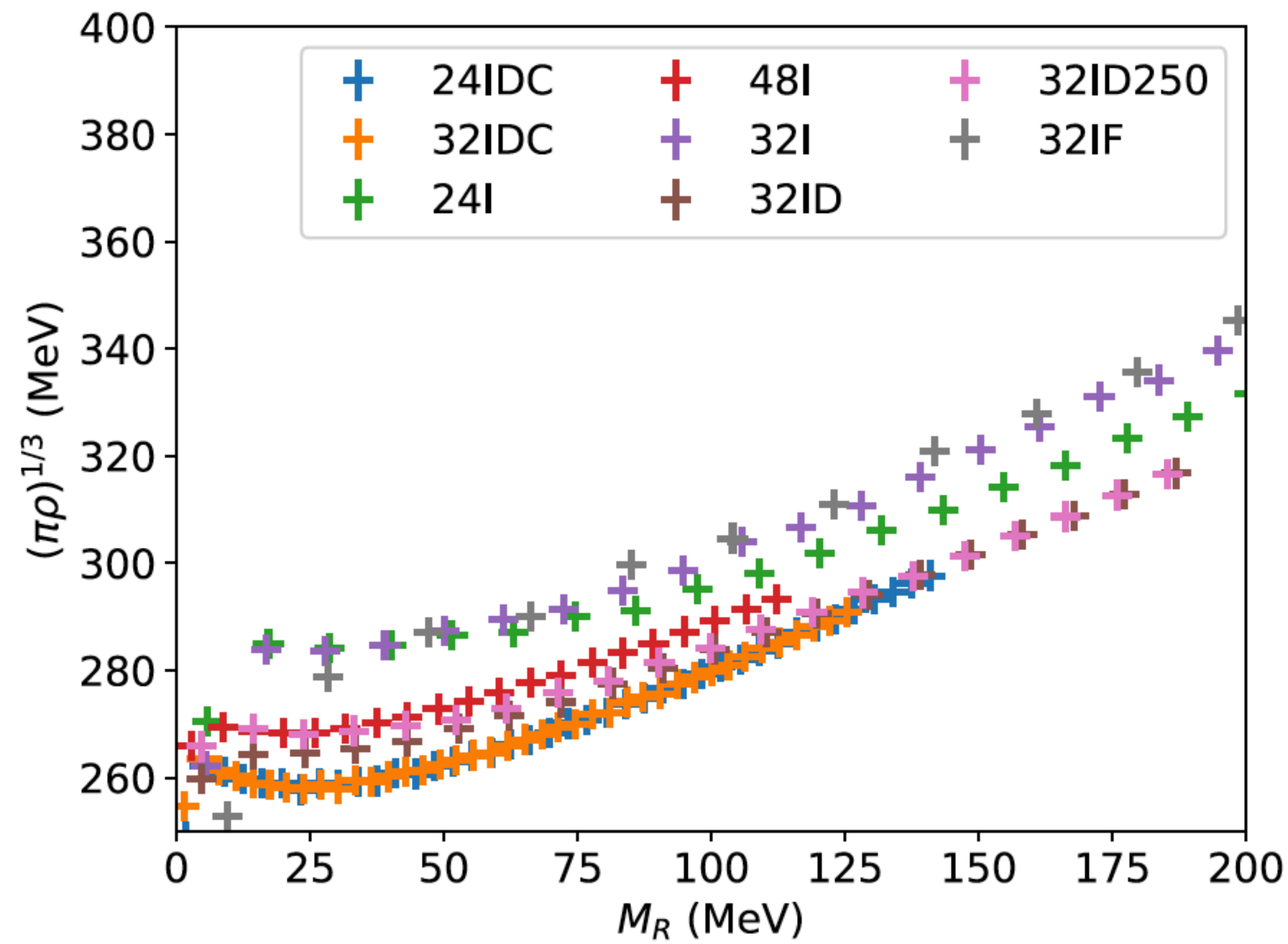
$$D_{ov} v_{ov} = \lambda_{ov} v_{ov}, \quad D_c v_{ov} = \frac{\lambda_{ov}}{1 - \frac{1}{2\rho}\lambda_{ov}} v_{ov} \,.$$

And the density of the near-zero modes can be related to the chiral condensate,

$$\lim_{M_R \to 0} \lim_{V \to \infty} \frac{\sum_{\lambda < M_R} 1}{M_R V} = \frac{\Sigma}{\pi}$$

$N_f = 2:$	$\Sigma^{1/3} = 266(10)\,\mathrm{MeV}$
$N_f = 2+1:$	$\Sigma^{1/3} = 272(5)\,\mathrm{MeV}$
$N_f = 2+1+1:$	$\Sigma^{1/3} = 286(23)\,\mathrm{MeV}$

Summary in FLAG2019



J. Liang, in progress

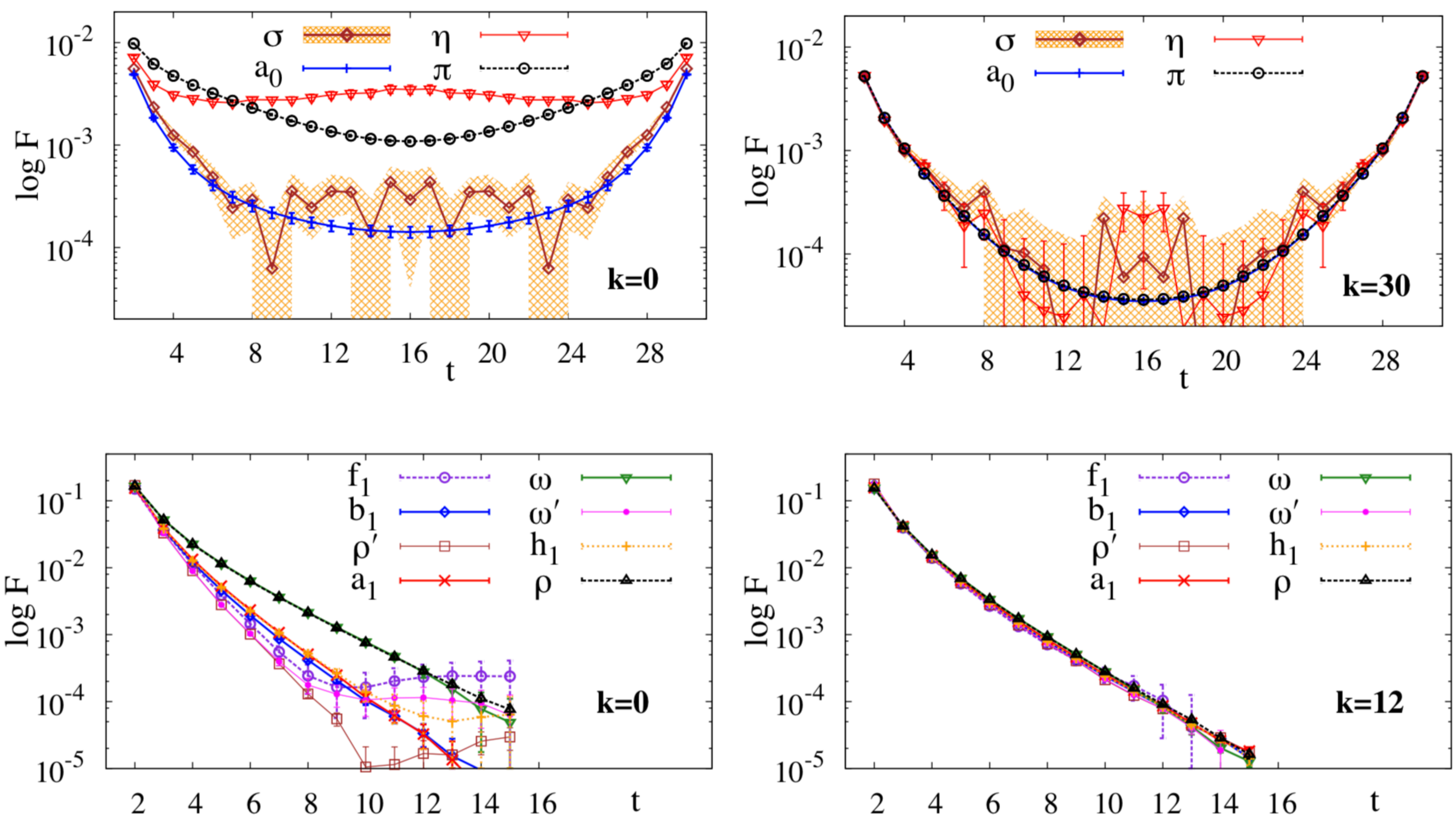
Low mode

and the chiral symmetry breaking

When the low modes are removed from the meson 2pt,

$$C_2(t,\Gamma) = \sum_{\vec{x}} \mathrm{Tr}[\langle \Gamma . S_h(\vec{0},0,\vec{x},t) . \Gamma . S_h(\vec{x},t,\vec{0},0)\rangle]$$

M. Denissenya, et.al., PRD91(2015)034505, 1410.8751



Chiral multiplets become degenerate, after the O(20) low mode are removed.

Simulation setup

24I ensemble

- 24^3x64, a=0.1105 fm, mpi=330 MeV, 40 cfgs.
- Four quark masses: m.a=0.008, 0.016 (unitary point), 0.024, 0.032
- 10 momenta: p^2a^2=0, 0.07(2pi/L_s), 0.21(2√3 pi/L_s)…6.69, corresponds to the energy scale 0-5 GeV.

Low mode

and the chiral symmetry breaking

The momentum-subtracted (RI/MOM) vertex renormalization constants are defined by:

$$\Gamma_S \equiv \frac{Z_q}{Z_S} = Tr[\langle S\rangle^{-1}.\langle S.S\rangle.\langle S\rangle^{-1}],\ \ \Gamma_P \equiv \frac{Z_q}{Z_P} = Tr[\gamma_5\langle S\rangle^{-1}.\langle S.\gamma_5.S\rangle.\langle S\rangle^{-1}],$$

We can define the above quantities without low modes,

$$\Gamma_{S,h} = Tr[\langle S_h\rangle^{-1}.\langle S_h.S_h\rangle.\langle S_h\rangle^{-1}],\ \ \Gamma_{P,h} = Tr[\langle S_h\rangle^{-1}.\langle S_h.\gamma_5.S_h\rangle.\langle S_h\rangle^{-1}],$$

Similarly, we can also consider its high mode part counterpart of the regularization independent (RI) mass under the landau gauge,

$$m^{RI} = Tr[\langle S\rangle^{-1}],\ \ m_h^{RI} = Tr[\langle S_h\rangle^{-1}]$$

How will they look like when the low mode part is removed?

Low mode

and the chiral symmetry breaking

$$\Gamma_S \equiv \frac{Z_q}{Z_S} = Tr[\langle S\rangle^{-1} . \langle S . S\rangle . \langle S\rangle^{-1}], \;\; \Gamma_P \equiv \frac{Z_q}{Z_P} = Tr[\gamma_5 \langle S\rangle^{-1} . \langle S . \gamma_5 . S\rangle . \langle S\rangle^{-1}],$$

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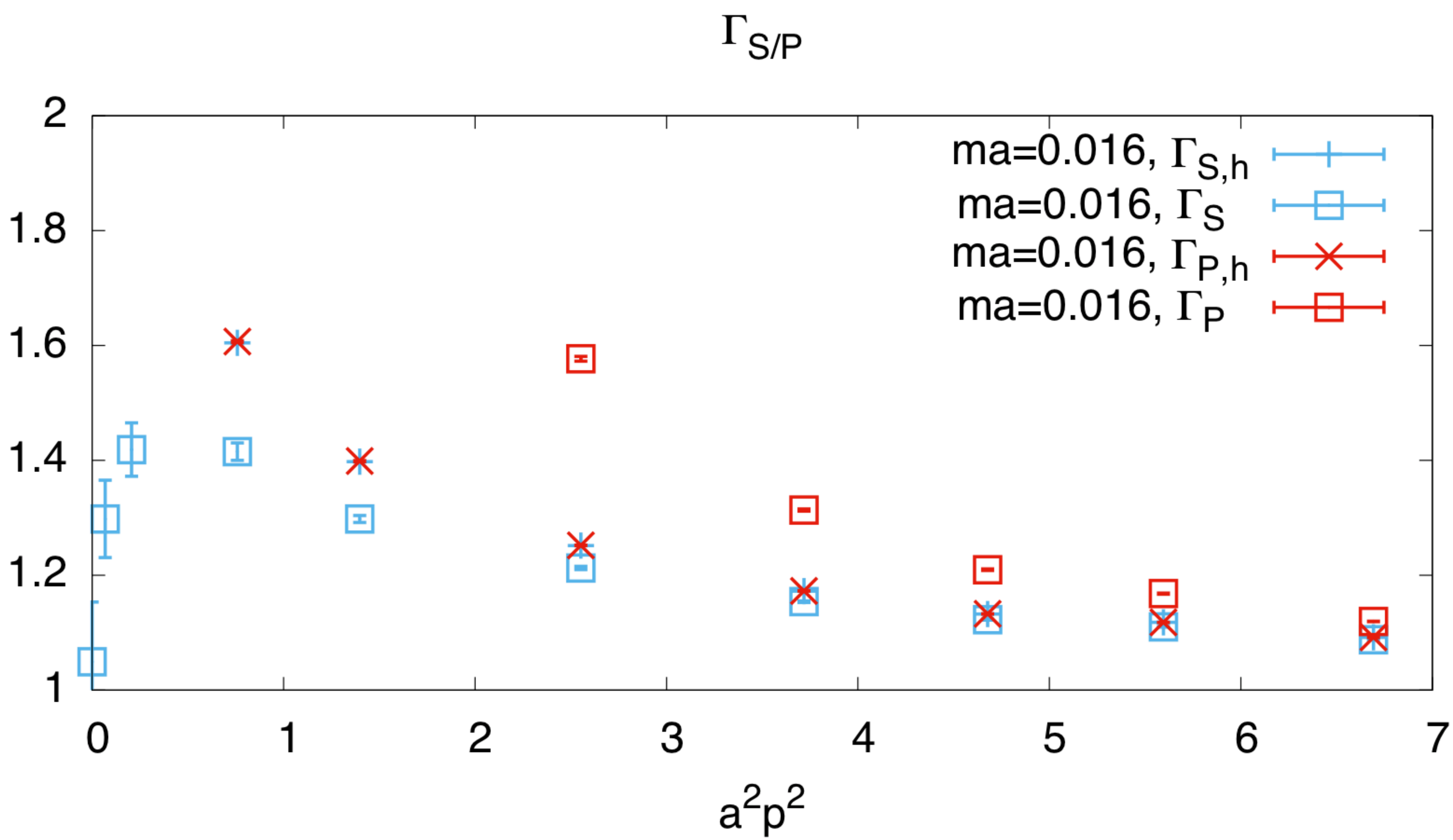


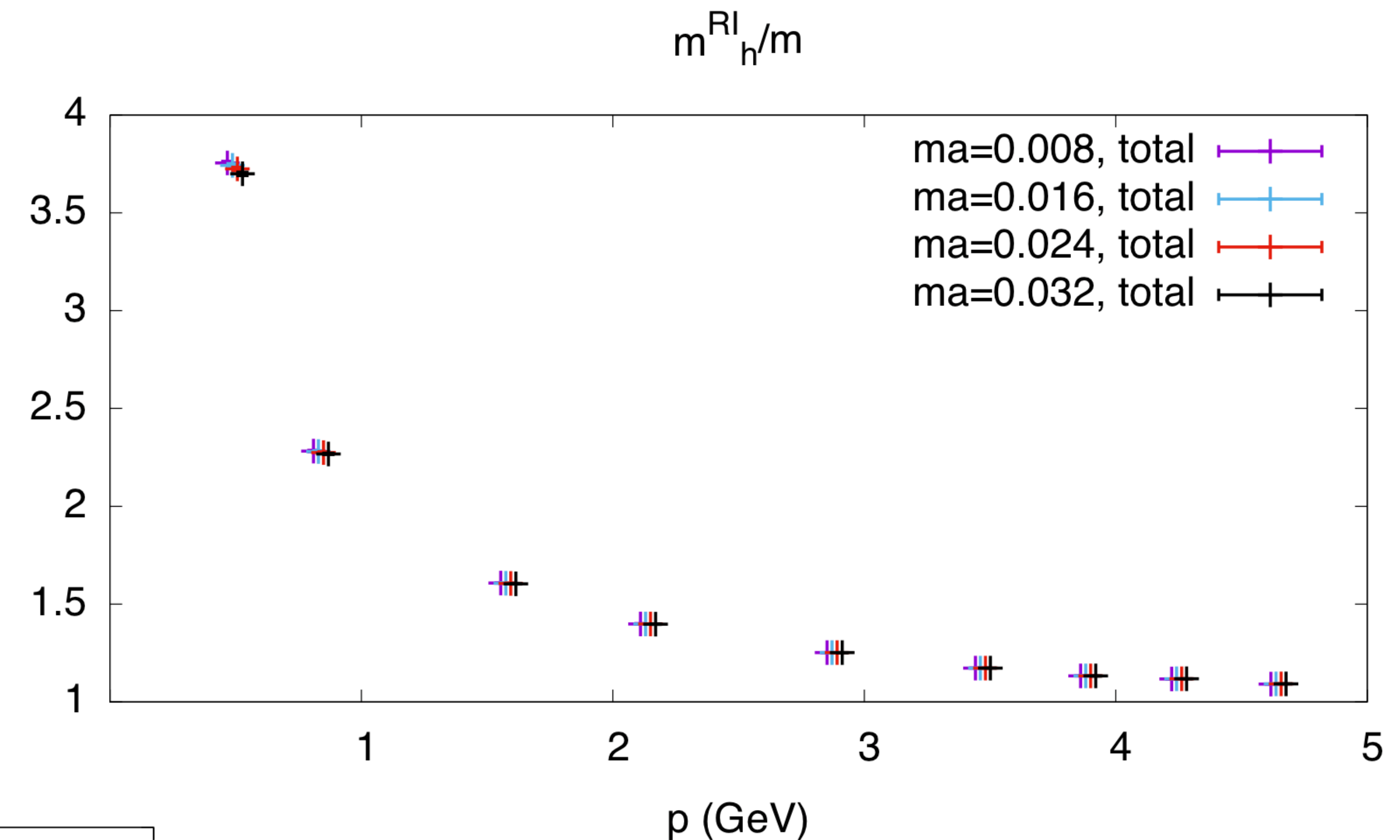
Chiral symmetry breaking also vanishes when the low modes absent, like the meson spectrum case.

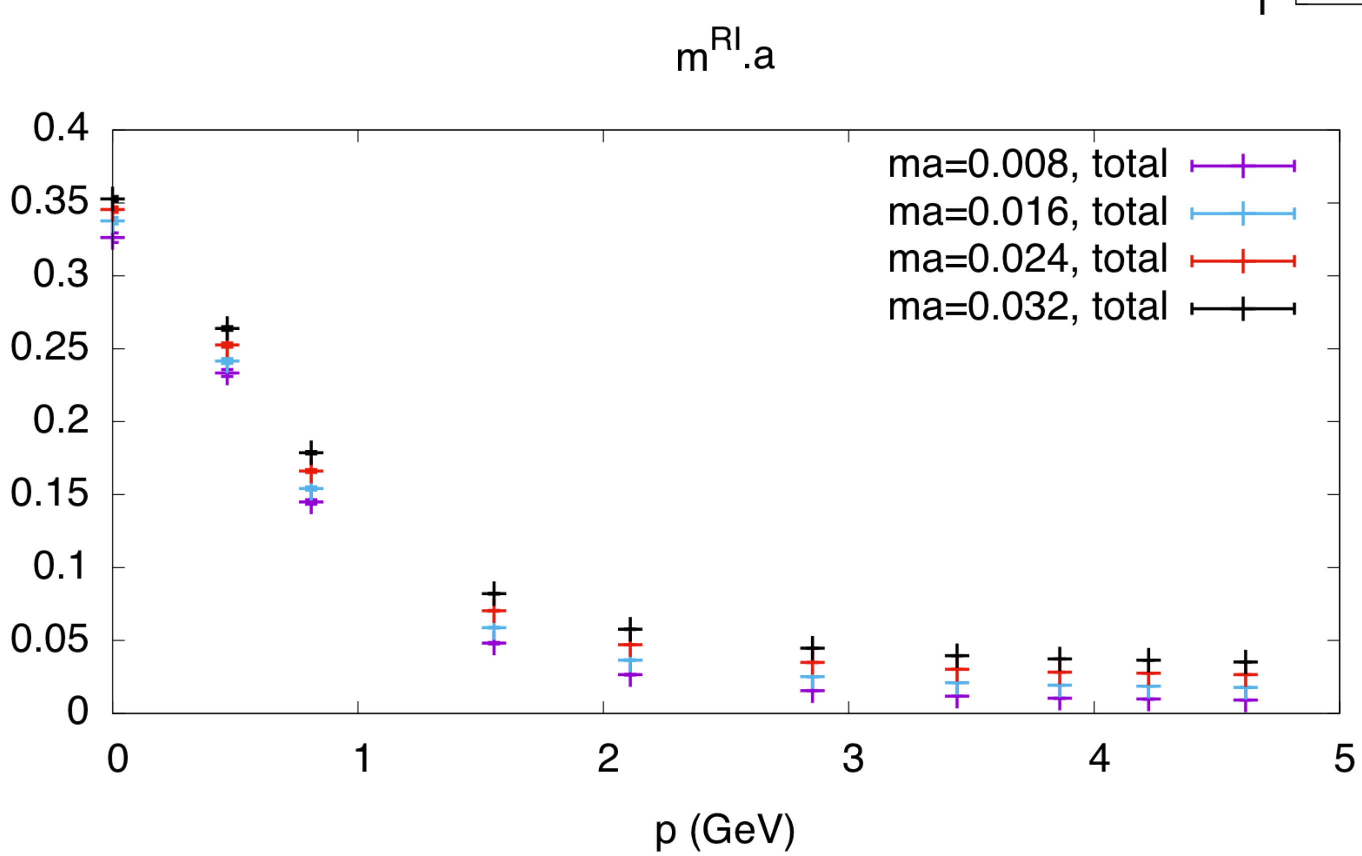
Quark mass

under the Landau gauge

$$m^{RI} = Tr[\langle S \rangle^{-1}], \quad m_h^{RI} = Tr[\langle S_h \rangle^{-1}]$$

- The RI renormalized quark mass is perfectly proportional to the bare quark mass, when the low modes are removed;





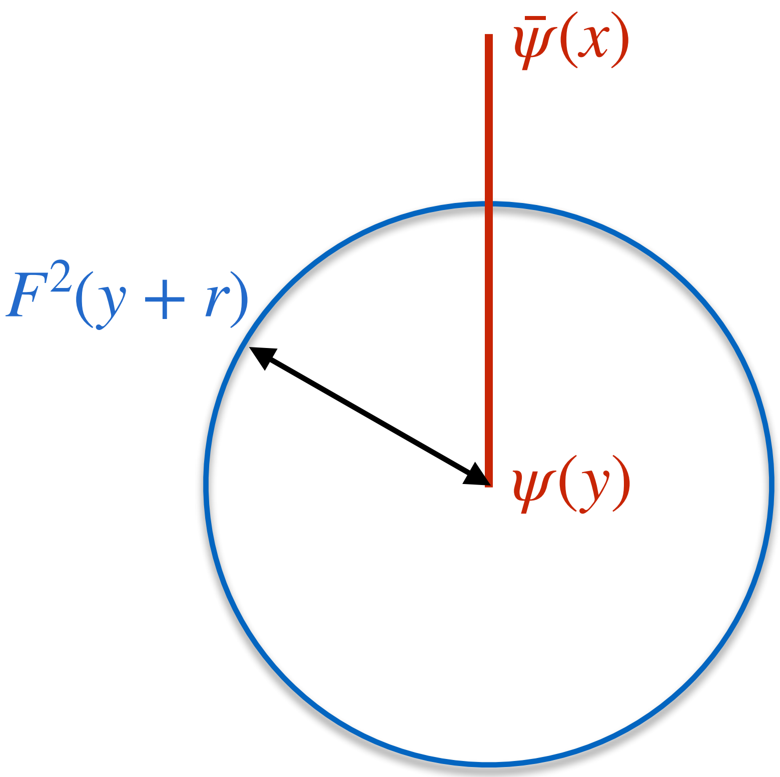
- The quark mass independent part only appears when the low modes are included in the propagator.
- Such a contribution is symbiotic with the non-zero quark condensate

The correlation <SF²> with CDER cutoff

K. Liu, J. Liang, **YBY**, PRD96, 114504(2017), 1805.00531

The gluon matrix element in a quark state can be obtained through the correlation of the corresponding operator with the quark propagator,

$$\Gamma_F = \langle q | F^2 | q \rangle = \langle S \rangle^{-1} . \langle S \mathrm{Tr}[F^2] \rangle . \langle S \rangle^{-1}, \quad S(p) = \frac{1}{V} \int d^4x d^4y e^{ip(x-y)} \bar{\psi}(x)\psi(y)$$



r: Cut on the F^2 - Ψ correlation

- The correlation decays exponentially with the distance between F^2 and the end points of the quark propagator;
- But its uncertainty is independent to the distance.
- Thus a cutoff on the distance can keep **most of** the signal but **few of** the uncertainty, given the systematic uncertainty of the cutoff dependence.

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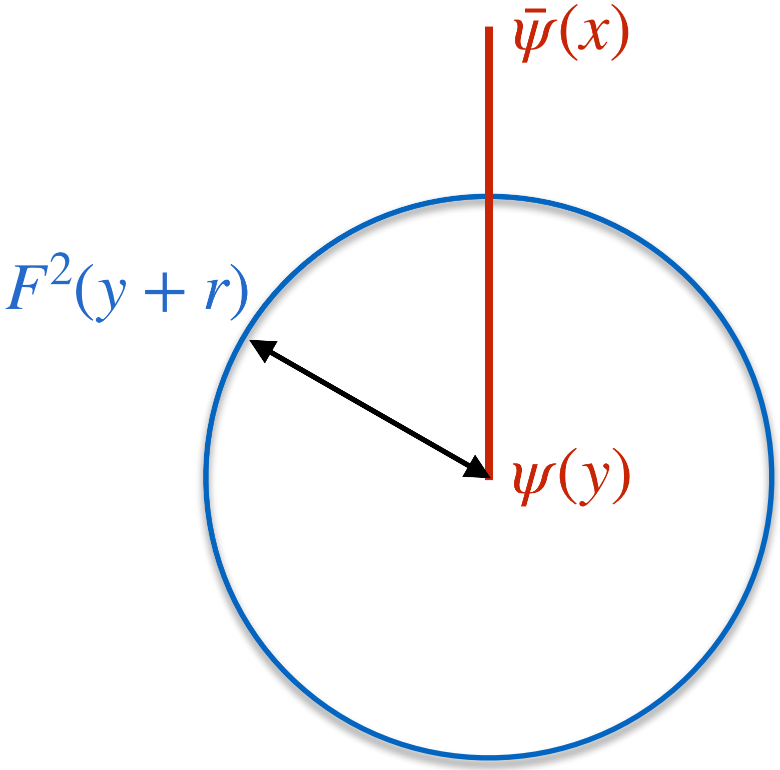

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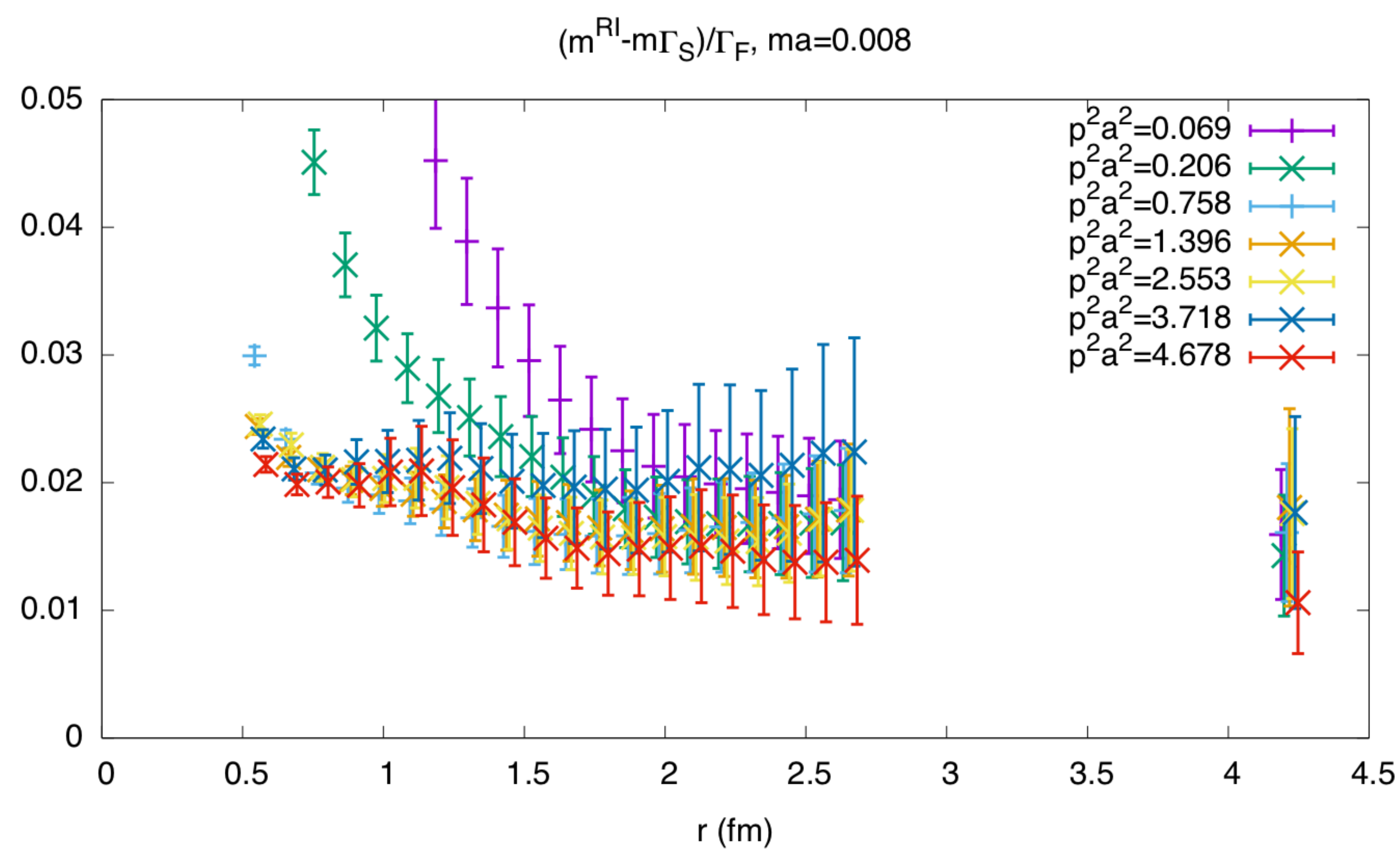
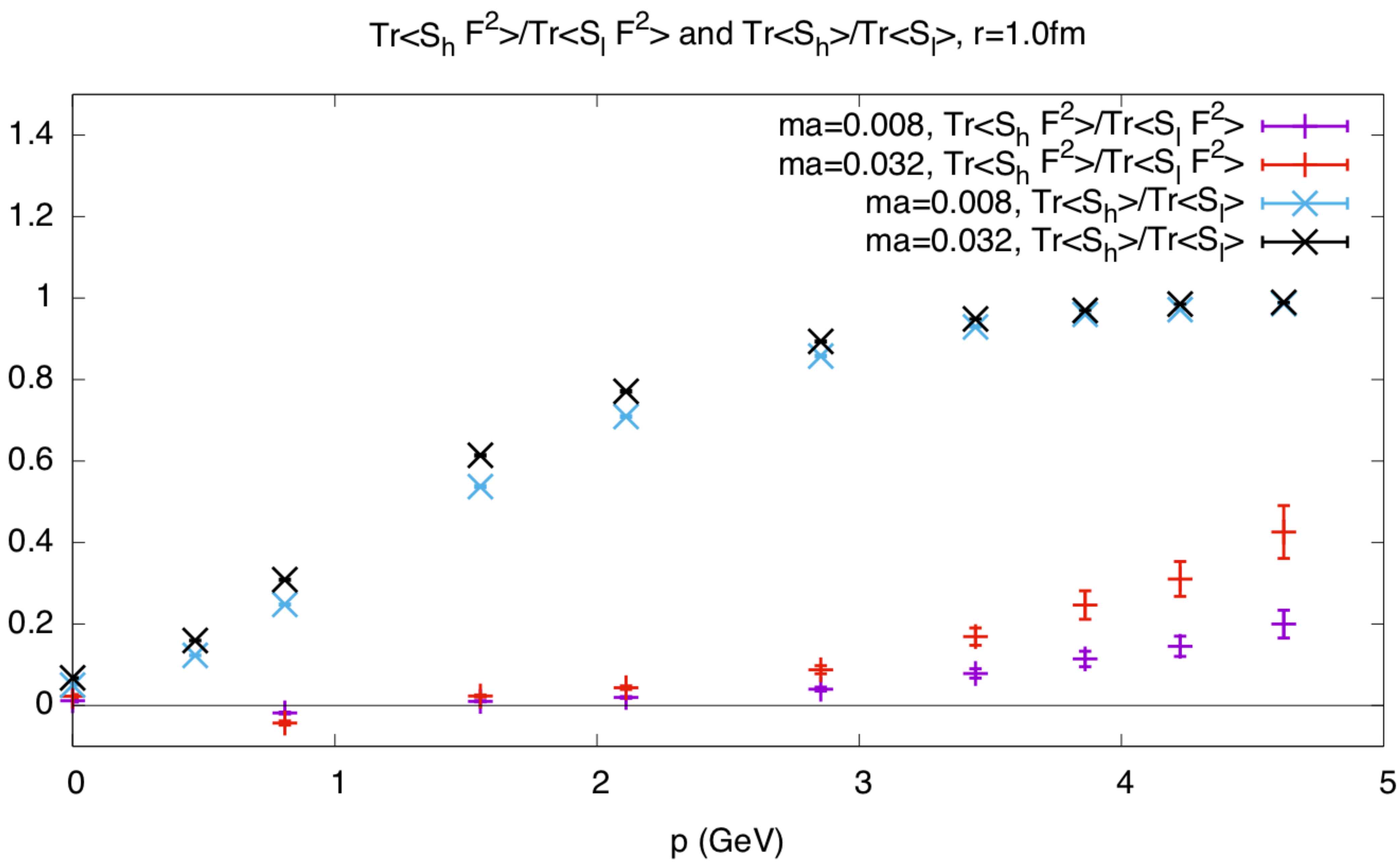
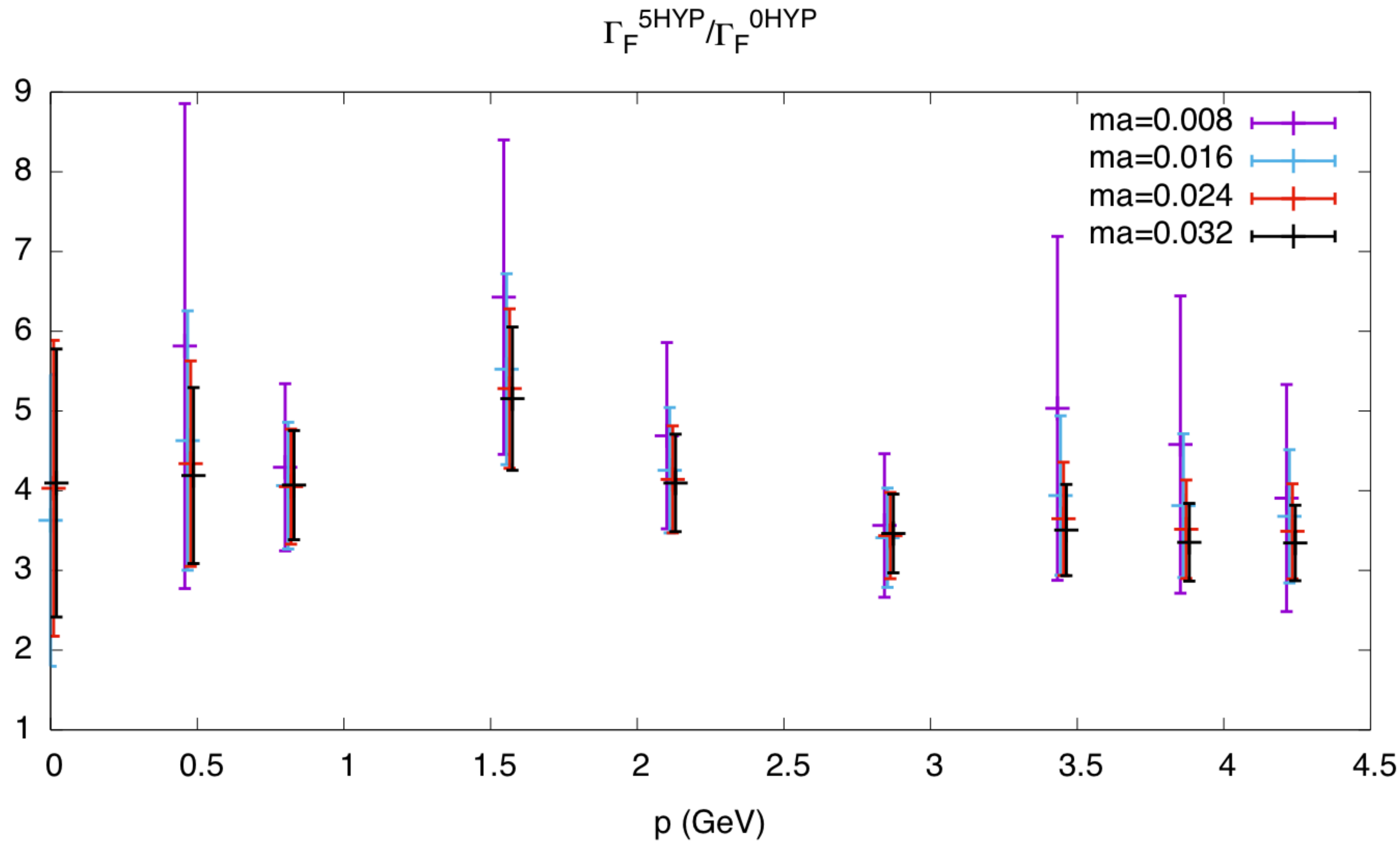


The correlation <SF^2> at different scale



- The 5HYP gluon operator couples to the low mode propagator majorly, comparing to the high mode part;
- The coupling to the high mode will be even smaller with lighter quark mass.

HYP smearing steps dependence at different scale

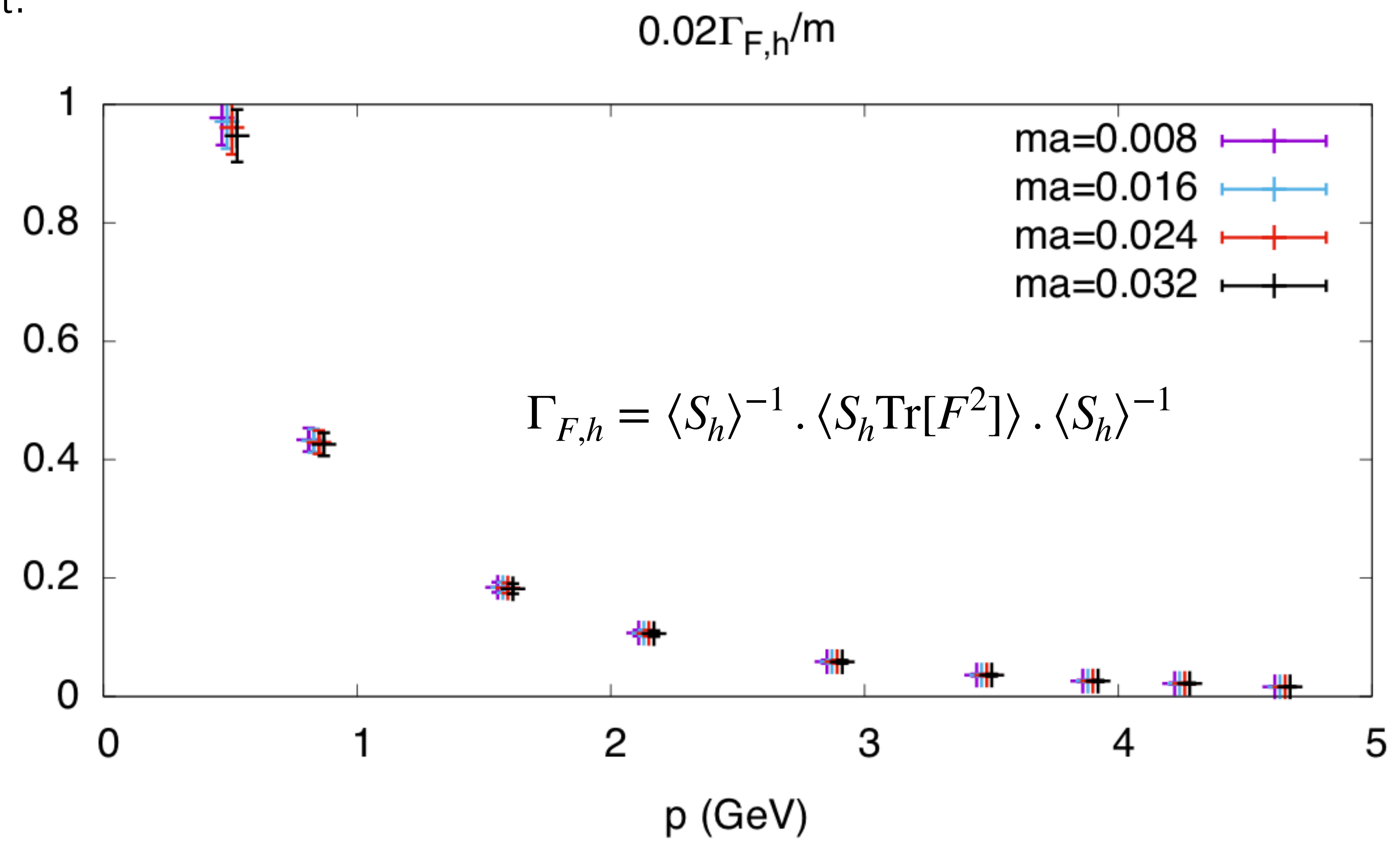


- The ratio is independent to the quark mass and also the scale;
- Roughly the same as what we can expect from the NPR of the gluon traceless EMT

Quark mass

and trace anomaly

Then we can start to consider the following matrix element and its high mode counter part:



- 5-step HYP smearing applied on the gluon operator;
- CDER cutoff at ~1.0fm

- The character seems to be pretty similar:
- Proportional to the quark mass when the low modes are removed;

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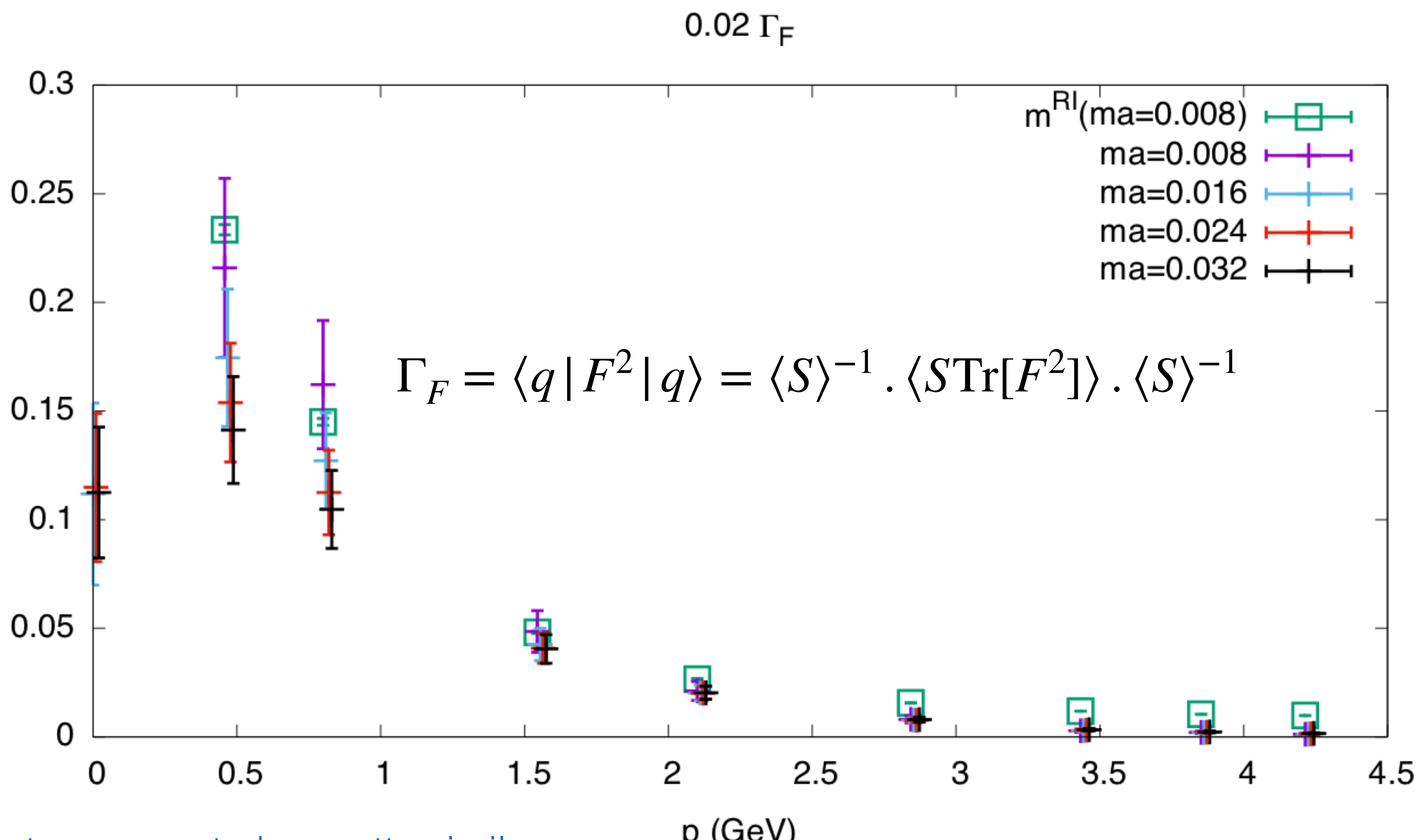
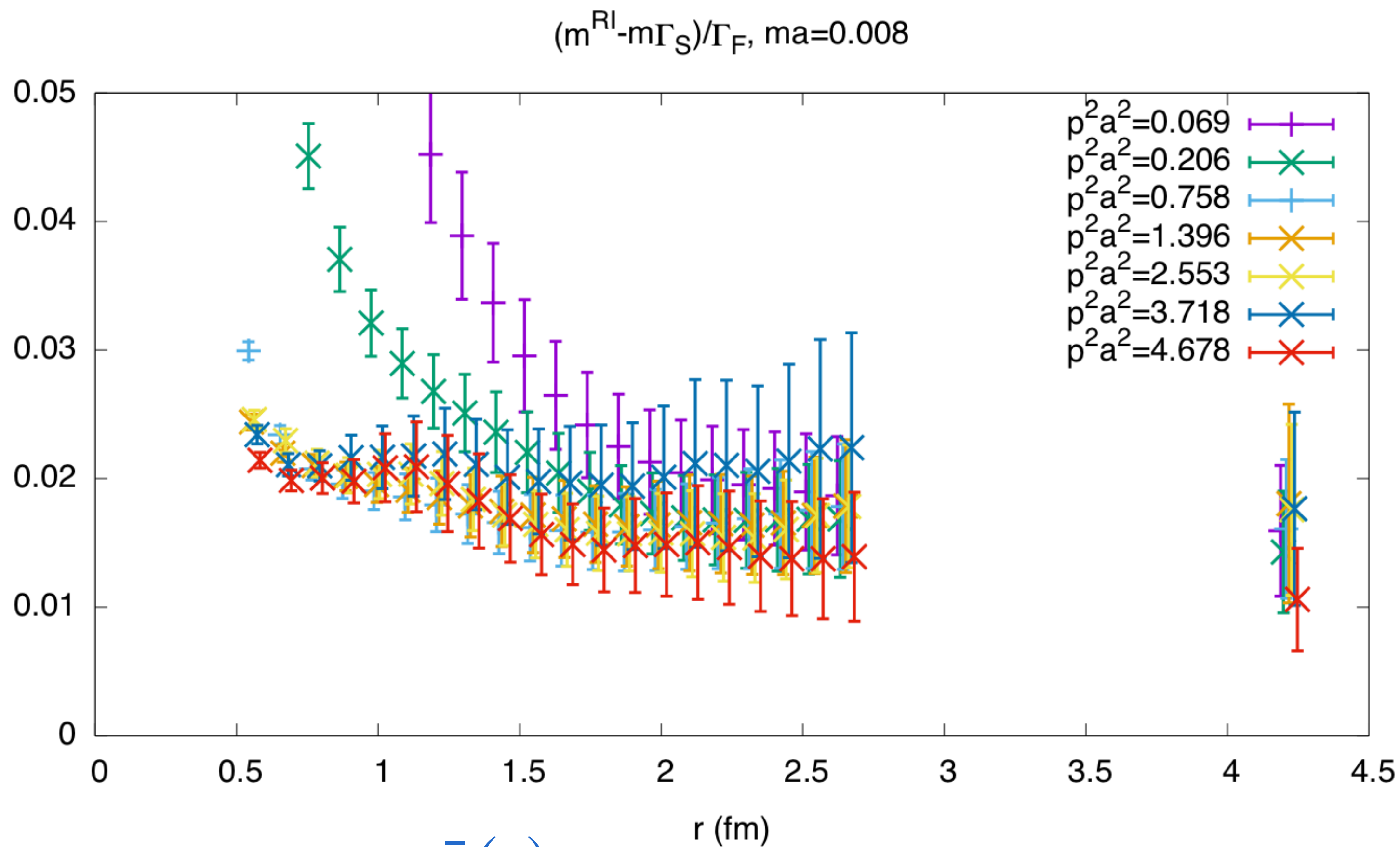


- The character seems to be pretty similar:
- Proportional to the quark mass when the low modes are removed;
- Sizable mass independent contribution at the low scale for the full propagator.

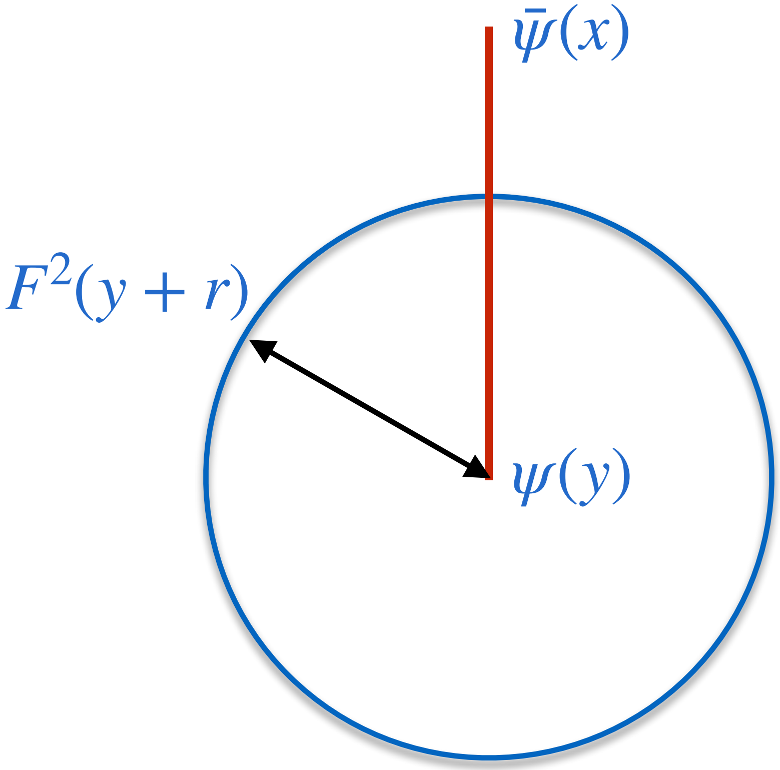
Trace anomaly



cutoff dependence

$$m^{RI} = m\Gamma_S + C_0\Gamma_F?$$

$$C_0 = (m^{RI} - m\Gamma_S)/\Gamma_F$$



r: Cut on the F^2 - Ψ correlation

- The equation seems to hold with C0~0.020
- CDER cutoff should be less aggressive with smaller p
- The EMT trace anomaly requires the coefficient to be ~0.0285

Summary

- The overlap low modes are related to the **spontaneous chiral symmetry breaking, and also the additive quark mass at low scale;**

- The **gluon trace anomaly** has very strong correlation on those low modes, and proportional to **the additive quark mass.**

- Study on the lattice with larger volume would be helpful and in progress
- Closer to the chiral limit;
- CDER would be more efficient;
- Better resolution at non-perturbative scale

- Relation to the trace anomaly in the hadron level?