Three-photon decay of J/psi from Lattice QCD

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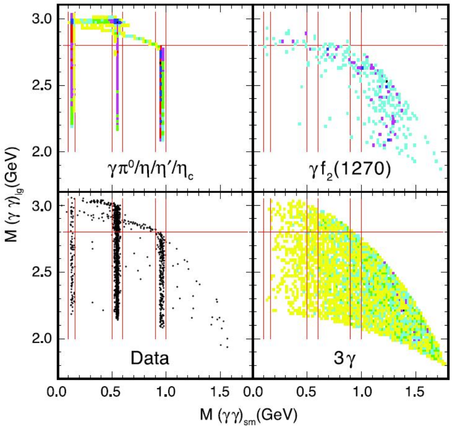
Outline

- Experiments on $J/\psi \to 3\gamma$
- How to calculate $\langle \gamma\gamma\gamma | J/\psi \rangle$ on lattice?
- Lattice simulation
- Discussion

Experiments on $J/\psi \to 3\gamma$

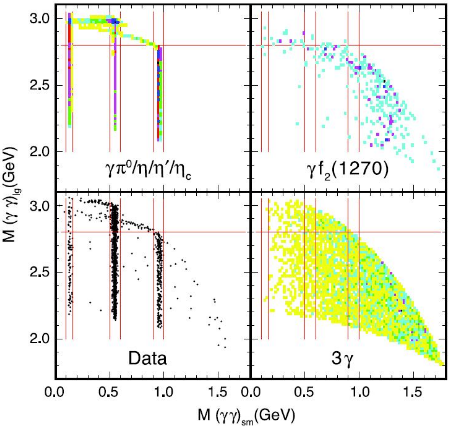
CLEO



Phys.Rev.Lett.101,101801(2008)

$\mathcal{B}(J/\psi \to 3\gamma)$
$=(1.2 \pm 0.3 \pm 0.2) \times 10^{-5}$

CLEO

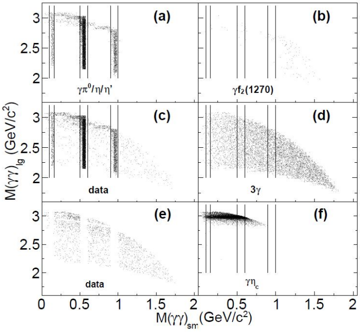


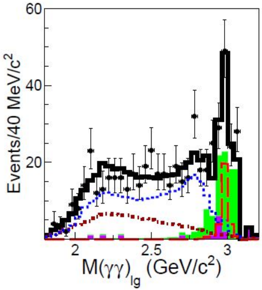
$N_{J/\psi\to3\gamma} = 37$

Phys.Rev.Lett.101,101801(2008)

$\mathcal{B}(J/\psi \to 3\gamma)$
$=(1.2 \pm 0.3 \pm 0.2) \times 10^{-5}$

BESIII

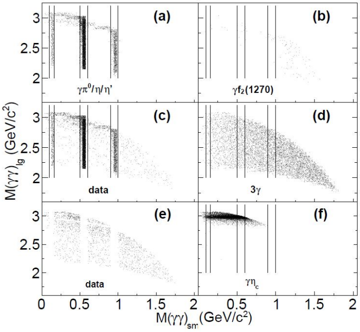


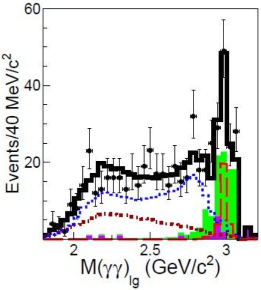


Phys.Rev.D.87,032003(2013)

$$\mathcal{B}(J/\psi \to 3\gamma) = (1.13 \pm 0.18 \pm 0.2) \times 10^{-5}$$

BESIII





Phys.Rev.D.87,032003(2013)

$$\mathcal{B}(J/\psi \to 3\gamma) = (1.13 \pm 0.18 \pm 0.2) \times 10^{-5}$$

$$N_{J/\psi \to 3\gamma} = 113$$

How to calculate $\langle\gamma\gamma\gamma|J/\psi\rangle$ on lattice ?

LSZ formula

The decay amplitude

$$\langle J/\psi(p,\lambda_0)|\gamma(q_1,\lambda_1)\gamma(q_2,\lambda_2)\gamma(q_3,\lambda_3)\rangle = \lim_{t_f-t\to\infty} e^3 \frac{\epsilon_\mu(q_1,\lambda_1)\epsilon_\nu(q_2,\lambda_2)\epsilon_\rho(q_3,\lambda_3)\epsilon_\alpha}{\frac{Z_m}{2E_p} e^{-E_p t_f}}$$
$$\times \left(\int dt^{'}\, dt_i e^{-\omega_2|t^{'}-t|} e^{-\omega_1|t_i-t^{'}|}\Gamma_{\mu\nu\rho\alpha}(t_i,t^{'},t)\right)$$

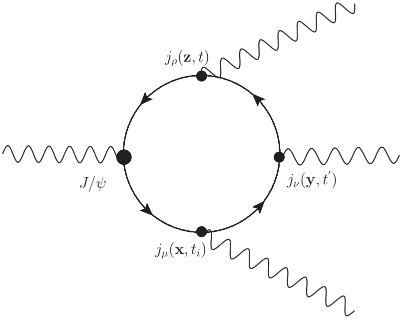
- where

$$\Gamma_{\mu\nu\rho\alpha}(t_i,t^{'},t)$$
$$=\langle 0|T\{\hat{\mathcal{O}}_\alpha(\mathbf{0},t_f)\int d^3\boldsymbol{z}e^{i\boldsymbol{q}_3\cdot\boldsymbol{z}}j^\rho(\boldsymbol{z},t)\int d^3\boldsymbol{y}e^{i\boldsymbol{q}_2\cdot\boldsymbol{y}}j^\nu(\boldsymbol{y},t^{'})\int d^3\boldsymbol{x}e^{i\boldsymbol{q}_1\cdot\boldsymbol{x}}j^\mu(\boldsymbol{x},t_i)\}|0\rangle$$

Denote the amplitude

$$M = \epsilon_\mu(q_1,\lambda_1)\epsilon_\nu(q_2,\lambda_2)\epsilon_\rho(q_3,\lambda_3)\epsilon_\alpha(p,\lambda_0)\mathcal{M}_{\mu\nu\rho\alpha}$$

Connected diagram



- The 'sequential' sources are placed at $\boldsymbol{x}, \boldsymbol{z}$.
- Local current $j_\mu = \bar{c}\gamma_\mu c$ are used.
- Renormalization constant $Z_V = 0.6095(03)$. Nucl.Phys. B887 (2014) 19-68

Decay width

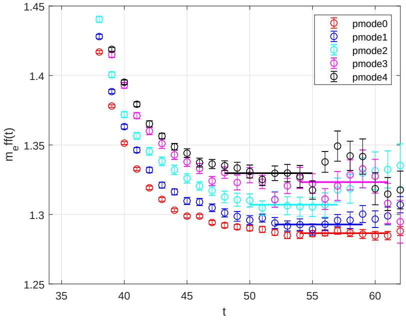
$$\Gamma(J/\psi \to 3\gamma) = \frac{1}{3!}\frac{1}{2M_V}\int \frac{d^3 q_1}{(2\pi)^3 2\omega_1}\frac{d^3 q_2}{(2\pi)^3 2\omega_2}\frac{d^3 q_3}{(2\pi)^3 2\omega_3}(2\pi^4)\delta(p-q_1-q_2-q_3)\overline{|\mathcal{M}|^2}$$
$$= \frac{m}{1536\pi^3}\int_0^1 dx_1 \int_{1-x_1}^1 dx_2\, T(x_1, x_2)$$

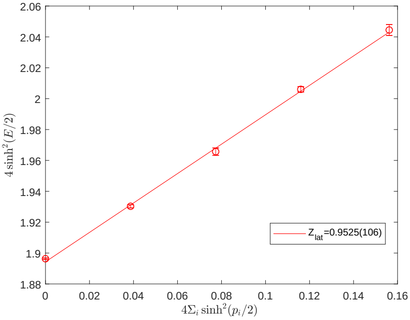
- $$T \equiv \overline{|M|^2} = \sum_{\lambda_1,\lambda_2,\lambda_3}\frac{1}{3}\sum_{\lambda_0}|M|^2 = \frac{1}{3}\mathcal{M}_{\mu\nu\rho\alpha}\mathcal{M}^*_{\mu\nu\rho\alpha} = \frac{1}{3}|\mathcal{M}|^2$$
- $$x = 1 - \frac{s_{23}}{m^2}, y = 1 - \frac{s_{31}}{m^2}, s_{jk} = 2q_j \cdot q_k$$

$$T(x, y, Q_1^2, Q_2^2, Q_3^2) = T(x, y) + b \times (\sum_{i=1}^{3} Q_i^2)$$

Lattice setup

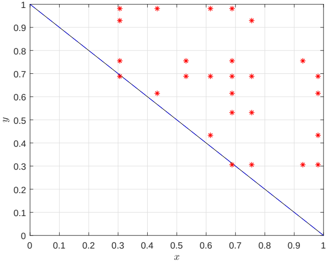
$Ensemble$	β	$a(fm)$	V/a^4	$a\mu_l$	$m_\pi[MeV]$	N_{conf}
I	1.95	0.0823	$32^3 \times 64$	0.0055	372	60

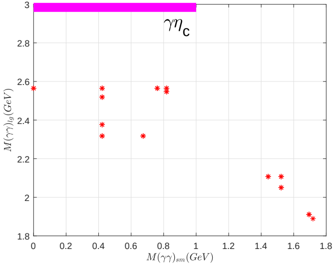




Parameters$(|Q_i^2| \ll 1)$

Q_1^2	Q_3^2	n_1	n_3	n_2	ω_1	ω_3	ω_2	x	y	$Q_2^2(GeV^2)$
0	0	002	101	-10-3	0.3952	0.2786	0.6122	0.6147	0.9521	0.004
0	0	211	-2-1-2	001	0.4865	0.5976	0.2028	0.7552	0.3154	-0.0154
0	0	201	-30-1	100	0.4426	0.6309	0.2125	0.6883	0.3305	-0.0385
0	0	201	-201	00-2	0.4426	0.4426	0.4008	0.6883	0.6234	-0.0483
0	0	012	-1-2-1	11-1	0.4426	0.4856	0.3578	0.6883	0.5565	-0.0730





Four-point function

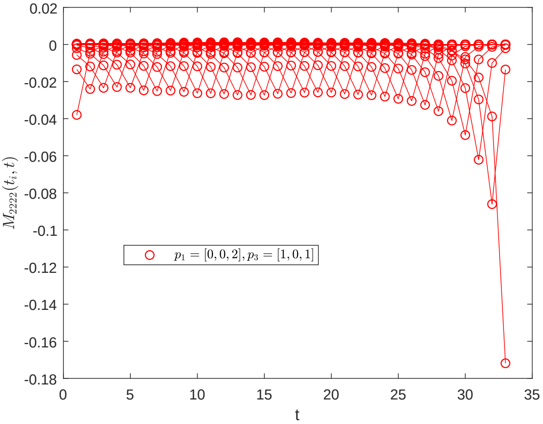
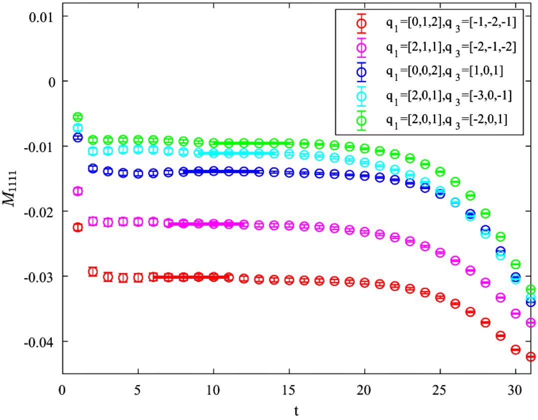
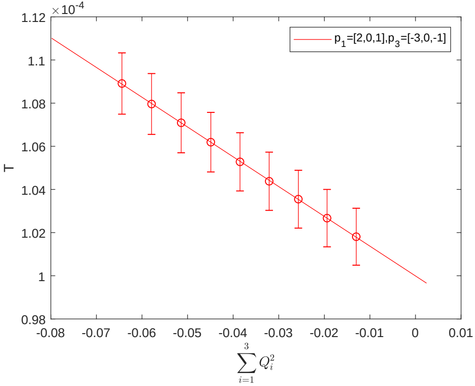


Figure: $M_{\mu\nu\rho\alpha}(t_i, t) = M_{\mu\nu\rho\alpha}(t_i, \sum t', t)$



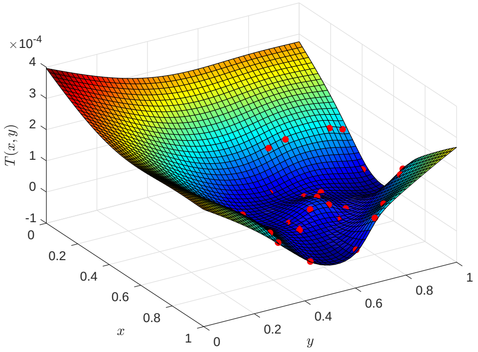
q_1=[0,1,2],q_3=[-1,-2,-1]
q_1=[2,1,1],q_3=[-2,-1,-2]
q_1=[0,0,2],q_3=[1,0,1]
q_1=[2,0,1],q_3=[-3,0,-1]
q_1=[2,0,1],q_3=[-2,0,1]
0.01
0
-0.01
-0.02
-0.03
-0.04
M_{1111}
0
5
10
15
20
25
30
t

On-shell fitting

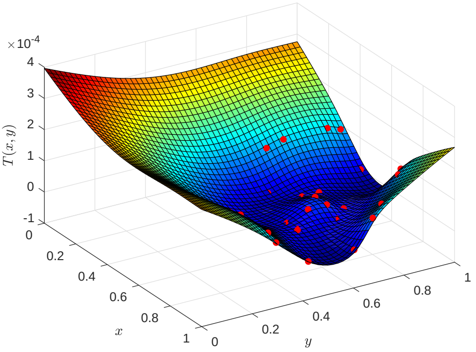


$$T = 1.000(12) \times 10^{-4}$$

T-amplitude



T-amplitude



$$\mathcal{B}(J/\psi \to 3\gamma) = 3.75(0.12)(1.50) \times 10^{-5}$$

Error sources

$$\mathcal{B}(J/\psi \to 3\gamma) = 3.75(0.12)(1.50) \times 10^{-5}$$

(a) 0.12— statistical error, Z_V error, sum error, on-shell fitting error.

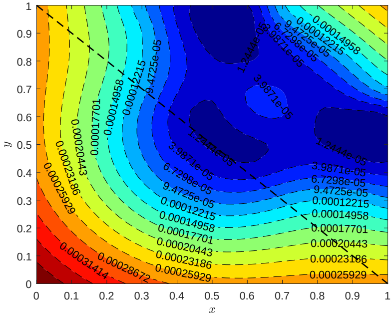
(b) 1.50— Twisted-mass chiral error.

Table: Permuting the photon momentum($\boldsymbol{k}_1 \leftrightarrow \boldsymbol{k}_3$)

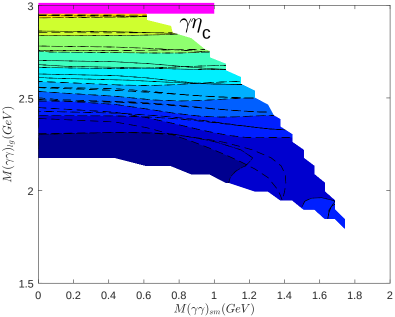
Q_1^2	Q_3^2	n_1	n_3	n_2	ω_1	ω_3	ω_2	x	y	$Q_2^2(GeV^2)$	$T(\times 10^{-4})$
0	0	002	101	-10-3	0.3952	0.2786	0.6122	0.6147	0.9521	0.004	0.1309(19)
0	0	101	002	-10-3	0.2786	0.3952	0.6122	0.4332	0.9521	0.004	0.1793(33)

What is direct observable for experiment about three-body decay ?

Dalitz plot

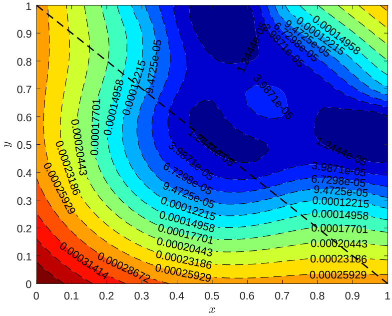


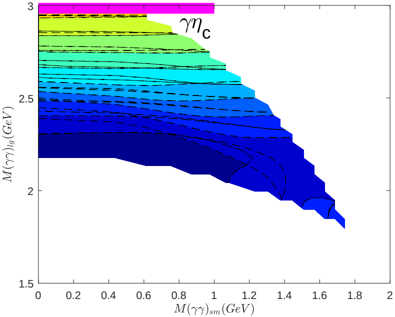
y
x
0.00031414
0.00028672
0.00025929
0.00023186
0.00020443
0.00017701
0.00014958
0.00012215
9.4725e-05
6.7298e-05
3.9871e-05
1.2444e-05



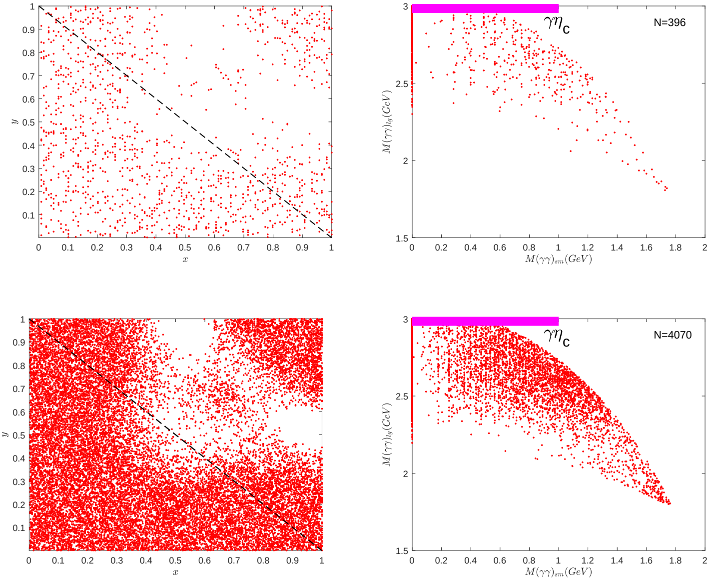
$\gamma\eta_c$
$M(\gamma\gamma)_{lg}(GeV)$
$M(\gamma\gamma)_{sm}(GeV)$

Dalitz plot



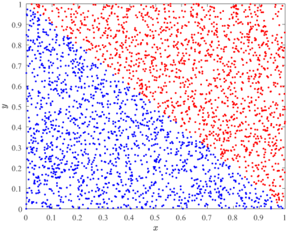


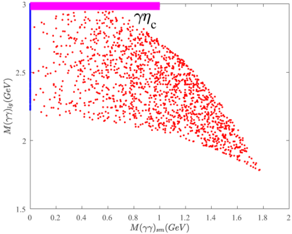
Equal-energy enhancement region is discovered.

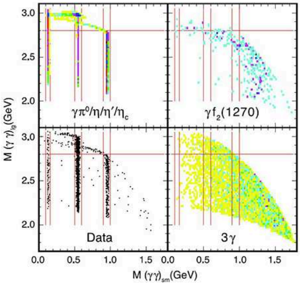


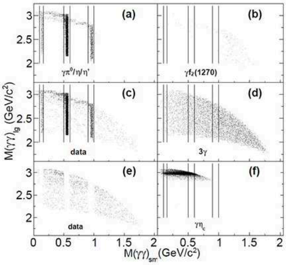
$\gamma\eta_c$
N=396
$\gamma\eta_c$
N=4070
x
y
$M(\gamma\gamma)_{sm}(GeV)$
$M(\gamma\gamma)_{lg}(GeV)$

Compare with experiment

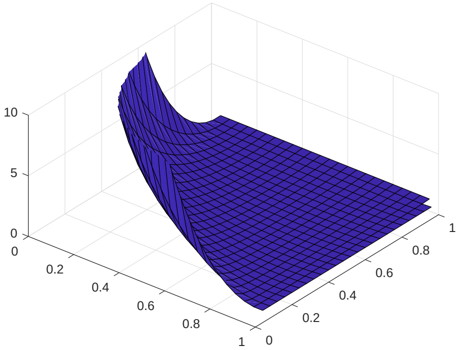








Compare with experiment



$$T = \frac{2}{9\pi} m \times \alpha^6 \times [(\frac{1-x}{y(2-x-y)})^2 + (\frac{1-y}{x(2-x-y)})^2 + (\frac{x+y-1}{(xy)})^2]$$

Conclusion and outlook

Conclusion

- First calculation of $J/\psi \to 3\gamma$ on lattice is given.

$$\mathcal{B}(J/\psi \to 3\gamma) = 3.75(0.12)(1.50) \times 10^{-5}$$

- We propose a general method to deal with three-body decay problem by combining lattice method and dalitz plot analysis.
- A special dalitz structure of $J/\psi \to 3\gamma$ is discovered which can be checked in the future by experiment.

Outlook

- A high precision analytical T-amplitude, using for the dalitz plot analysis for experiment, would be obtained.

Thank you!

System errors

- permute the photon momentum($\boldsymbol{k}_1 \leftrightarrow \boldsymbol{k}_3$)

Q_1^2	Q_3^2	n_1	n_3	n_2	ω_1	ω_3	ω_2	x	y	$Q_2^2(GeV^2)$	$T(\times 10^{-4})$
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0	0	101	002	-10-3	0.2786	0.3952	0.6122	0.4332	0.9521	0.004	0.1793(33)

Table:

$\mathcal{M}_{\mu\nu\rho\alpha} \times 10^{-3}, [x, y] = [0.6147, 0.9521]$

$\mu\nu\rho$ \ α	1	2	3
111	-13.90(23)	-2.89(23)	-4.11(17)
112	-3.70(18)	8.47(20)	-1.02(14)
121	-4.76(13)	-4.81(14)	1.13(12)
122	1.23(10)	4.43(14)	-4.66(11)
211	-4.36(17)	1.66(11)	1.92(12)
212	-4.43(14)	5.19(13)	-5.16(16)
221	7.54(21)	3.31(15)	3.49(13)
222	3.21(21)	-13.49(20)	0.33(9)

Table:

$\mathcal{M}_{\mu\nu\rho\alpha} \times 10^{-3}, [x, y] = [0.4332, 0.9521]$

$\mu\nu\rho$ \ α	1	2	3
111	-15.56(30)	-2.43(33)	-7.42(26)
112	-4.29(37)	1.04(29)	3.34(16)
121	-4.46(26)	-7.33(34)	2.40(17)
122	-1.82(15)	2.19(24)	5.70(14)
211	-3.77(14)	0.029(256)	-1.91(10)
212	-5.69(20)	5.25(19)	-5.91(15)
221	9.96(25)	4.70(26)	-4.60(22)
222	3.10(22)	-14.93(36)	1.04(17)