

Pseudoscalar Quarkonium + γ at NLL+NLO accuracy

arXiv 1906.03275, H.S. Chung, J.H. Ee,
DK, U. Kim, J. Lee, X. P. Wang

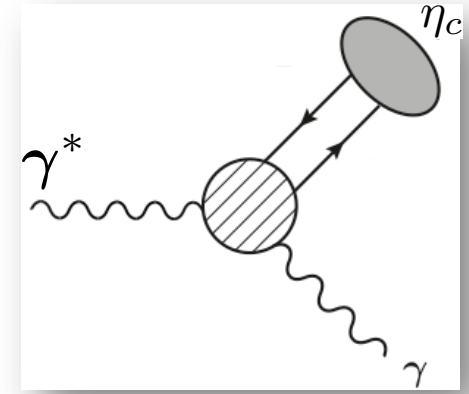
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Outline

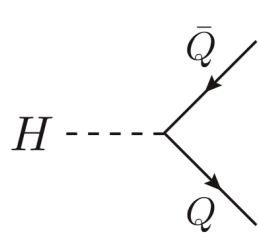
- ❑ Why exclusive quarkonium process?
- ❑ Theoretical approach
 - ❑ hierarchy in energy scales $Q, m_Q, m_Q v^2$
 - ❑ logarithms and resummation
- ❑ Numerical results
 - ❑ resummation effect
 - ❑ comparison with previous results



Why exclusive quarkonium process?

- Higgs decays into $J/\psi + \gamma$ is one of most promising channel that could measure the Yukawa couplings of the charm quarks at the LHC

Higgs-quark Yukawa coupling is given by


$$i\mathcal{M}_{HQ\bar{Q}} = -ig \frac{m_Q}{2m_W} \times \kappa_Q$$

quark mass

In the SM, $\kappa_Q = 1$

- Current CMS, ATLAS upper limits (95% CL) are $\sim 10^2$ times larger than SM values and they will be improved at high luminosity LHC.

$$\text{ATLAS: } 3.5 \times 10^{-4} \approx 117 \times \text{Br}(H \rightarrow J/\psi + \gamma)|_{\text{SM}} \quad (36.1 \text{ fb}^{-1} \text{ at } \sqrt{s} = 13 \text{ TeV})$$

[PLB 786 \(2018\) 134](#)

$$\text{CMS: } 7.6 \times 10^{-4} \approx 260 \times \text{Br}(H \rightarrow J/\psi + \gamma)|_{\text{SM}} \quad (35.9 \text{ fb}^{-1} \text{ at } \sqrt{s} = 13 \text{ TeV})$$

[EPJC 79\(2019\) 94](#)

Why exclusive quarkonium process?

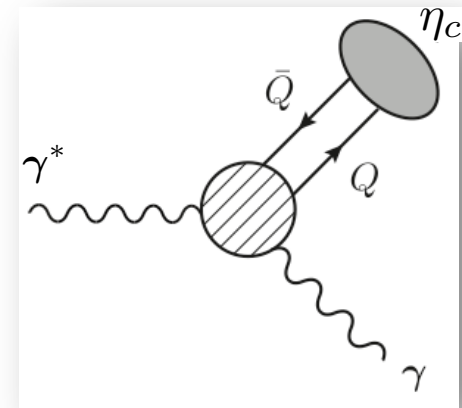
- ❑ $\eta_c + \gamma$ and $\chi_c + \gamma$ are measured by Belle and by BESIII
 - ❑ background channel in XYZ decays
 - ❑ benchmarking process w/ small samples before XYZ search
- Accurate theoretical predictions would be useful and it is a good place to test NRQCD predictions improved from previous lessons.
- from double quarkonium: **large logarithms, v^2 corrections**
- from polarization puzzle: **power corrections of m_Q/p_T**

We focus on $\eta_c + \gamma$ production in e^+e^- annihilation improved by
log resummation (NLL), α_s corr (NLO),
and v^2 and m_Q/Q power corrections

Hierarchy in energy scales

- 3 energy scales : Q (or \sqrt{s}), $2m_Q \gg m_Q v^2$
- NRQCD allows separation between SD and LD

$$\mathcal{M}(H) \sim \sum_n \underbrace{C_n(Q, m_Q)}_{\substack{\text{short-distance coefficient} \\ \text{(perturbative } \gtrsim m_Q)}}} \underbrace{\langle H | \mathcal{O}_n | 0 \rangle}_{\substack{\text{NRQCD long-distance (decay) matrix elements} \\ \text{(non-perturbative } \lesssim m_Q v)}}}$$



✓ v = relative velocity
between quark and anti-quark

- SD contains two scales:

$$r \equiv \frac{4m_Q^2}{Q^2}$$

r spreads in wide ranges

$r \sim 1$ in BESIII (3-4 GeV)

$r \approx 0.1$ in Belle (10 GeV)

$r \approx 0.001$ in Higgs/Z decay (100 GeV)

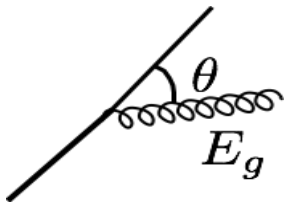
Log Log Log ...

- SD at NLO contains log of r and it is large for small r.

$$\lim_{r \rightarrow 0} \sigma^{\text{fixed}}(r; \mu) = \sigma_0 \left[1 + \frac{\alpha_s C_F}{4\pi} c^{\text{sing}} + \langle v^2 \rangle c_{v^2}^{\text{sing}} \right]$$

$$c^{\text{sing}} = -\frac{2}{3} \left[(9 - 6 \log 2) \log r + 9(3 + \log^2 2 - 3 \log 2) + \pi^2 \right]$$

- Logarithms are a generic feature in QFT at high energies and come from soft and collinear regions.



probability of splitting $\sim \frac{1}{E_g (1 - \cos \theta)}$

soft and collinear enhancements



$$\text{Log}(E_g) \text{Log}(\theta)$$

Log Log Log ...

- No problem with small log, the ordinary perturbative series works

$$\sigma = 1 + \alpha_s + \alpha_s^2 + \dots \quad \alpha_s < 1$$

- For a large log, **resummed perturbation** with log-power counting

$$\alpha_s L \sim 1$$

$$\begin{aligned} & \text{LO} \quad \text{NLO} \quad \text{NNLO} \\ \sigma^{\text{sing}} &= 1 + \alpha_s L + \alpha_s^2 L^2 + \dots \quad \text{Leading Log} \\ & \quad + \alpha_s + \alpha_s^2 L + \dots \quad \text{Next to LL} \\ & \quad \quad + \alpha_s^2 + \dots \quad \text{NNLL} \end{aligned}$$

$$\sigma = \sigma^{\text{sing}} + (v^2 \text{ corr.}) + (\text{power corr. of } O(r))$$

✓ valid in both small and large r regions

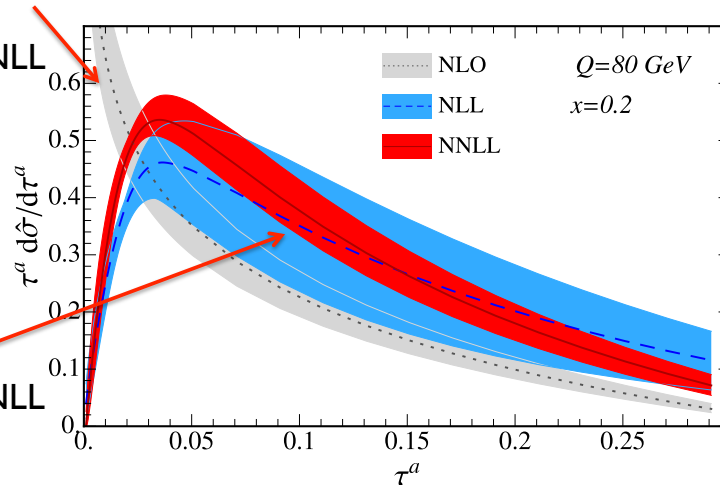
Example: An event shape distribution

- We can get an idea about resummation effect by comparing results before and after resummation in this example.
- event shape (τ^a) cross section contains $\text{Log}[\tau^a]$ c.f. $\text{Log}[r]$

Log singularity in
NLO

cured in NLL, NNLL

Good
convergence
from NLL to NNLL



- **large logs are under control after resummation**
- NLO perturbative uncertainty is not reliable without resummation

Theoretical approach

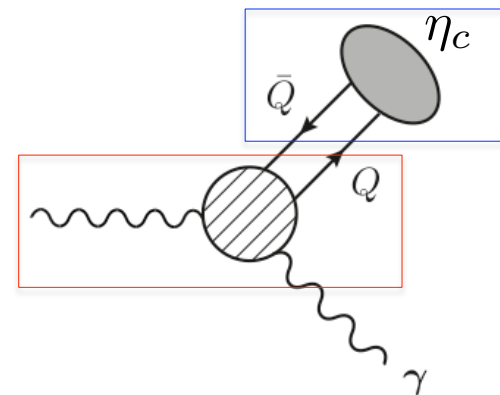
- Ordinary NRQCD factorization

$$\mathcal{M}(H) \sim \sum C_n(Q, m_Q) \langle H | \mathcal{O}_n | 0 \rangle$$

- Light-cone approach and refactorization

$$\mathcal{M} \sim T_H(x, Q) \otimes \phi_H(x, m_Q)$$

$$\sum_n \phi_n(x, m_Q) \langle H | \mathcal{O}_n | 0 \rangle$$



- **Hard-scattering kernel** produces a pair of collinear quark and anti-quark with fractional momentum $0 < x < 1$ at scale Q
- **LCDA** is the probability amplitude for the quark pair scattering at scale m_Q and later bounding into quarkonium at scale mv^2 .
- factorization valid up to the leading power in m_Q/Q

$$\mathcal{M} \sim T_H(x, Q) \otimes \phi_H(x, m_Q)$$

Lepage and Brodsky, PRD, 1980
Efremov and Radyushkin, PLB, 1980

- ❑ The light-cone approach was known since 1980 for light mesons
- ❑ The heavy-quarkonium **LCDA** is refactorized

Wang and Yang, JHEP, 2014

$$\sum_n \phi_n(x, m_Q) \langle H | \mathcal{O}_n | 0 \rangle$$

- ❑ **LDMEs** are only non-perturbative part and the rest are calculable in pert. theory

$$\phi_P(x, \mu) = \phi^{(0)}(x) + \frac{\alpha_s(\mu)}{4\pi} \phi^{(1)} + \langle v^2 \rangle \phi^{(v^2)} + O(\alpha_s^2, \alpha_s v^2, v^4)$$

$$\phi^{(0)} = \delta(x - \frac{1}{2}) \quad \phi^{(v^2)} = \frac{\delta^{(2)}(x - \frac{1}{2})}{24},$$

Resummation by RG evolution

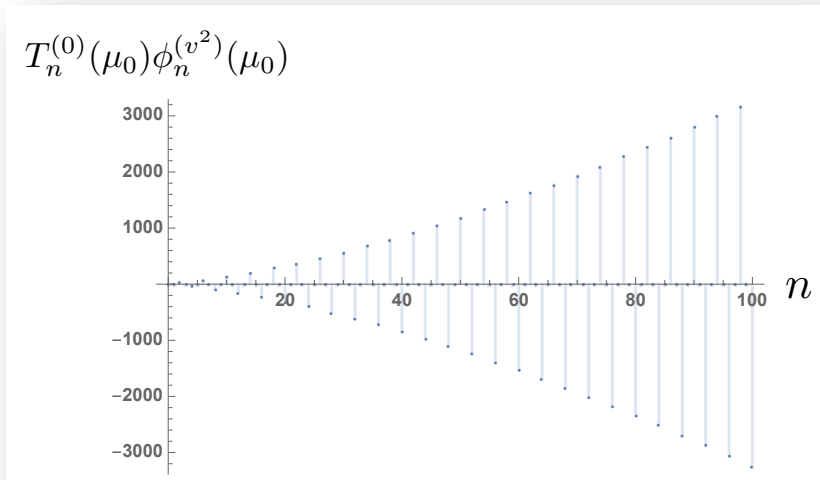
- RG eq of LCDA (called ERBL eq) was also known since 1980's
LCDA is expanded using eigenfunc. and each coeff. is evolved from m_Q to Q .

$$\phi_n(\mu) = \sum_{k=0}^n U_{nk}(\mu, \mu_0) \phi_k(\mu_0)$$

$$\phi_n(\mu_0) = N_n \int_0^1 dx \phi_P(x, \mu_0) C_n^{(3/2)}(2x-1)$$

$$\phi_n(\mu) \Big|^{LL} = \phi_n(\mu_0) \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right]^{\frac{\gamma_n^{(0)} - \gamma_0^{(0)}}{2\beta_0}}$$

- alternating between large +/- values makes the sum **NON-convergent!**



What's the problem?

for light meson, a smooth model function was taken and the sum is convergent.

for heavy quarkonium, singular LCDAs due to refactorization.

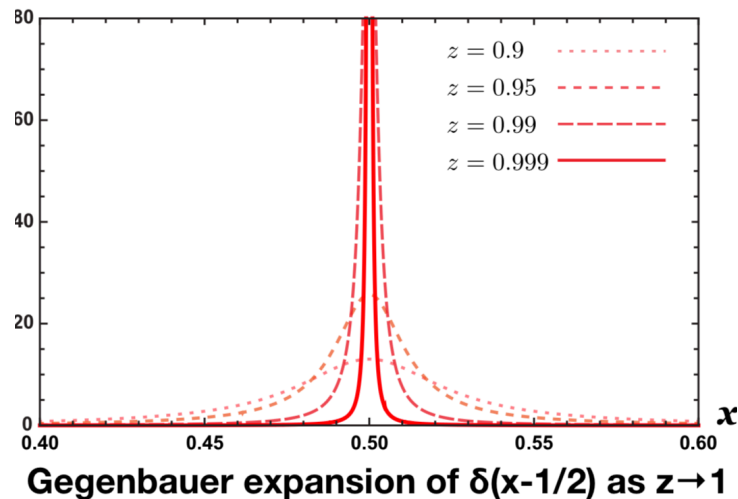
$$\phi^{(0)} = \delta\left(x - \frac{1}{2}\right) \quad \phi^{(v^2)} = \frac{\delta^{(2)}\left(x - \frac{1}{2}\right)}{24}$$

Abel-Pade method

- Smearing the LCDA with new variable $z < 1$

- Abel summation

$$\phi_n(\mu) = \sum_{k=0}^n U_{nk}(\mu, \mu_0) \phi_k(\mu_0) z^n$$



$z = 0.80$	$z = 0.90$	$z = 0.95$	$z = 0.99$	Pade
4.49305	4.24087	4.11884	4.02350	4.00000
1.28864	1.31426	1.33389	1.33386	1.33333

- Use the Pade approximation to boost the convergence.

$$f(z) = \frac{P_m(z)}{Q_n(z)} + \mathcal{O}(z^{m+n+1}) \quad P_m(z) = a_0 + a_1 z + a_2 z^2 + \dots + a_m z^m$$

γ_5 -scheme dependence

□ **Pseudoscalar LCDA:** $\langle P(p) | \bar{Q}(z) \gamma^+ \gamma_5 [z, 0] Q(0) | 0 \rangle = p^+ f_P \int_0^1 dx e^{ip \cdot zx} \phi_P(x, \mu)$

□ γ_5 in Dim reg is scheme dependent

$$\{\gamma_5, \gamma^\mu\} = 0 \quad \text{in 4 dim (NDR) or in } d \text{ dim (HV)}$$

$$\phi^{(1)} = C_F \theta(1-2x) \left\{ \left[4x \frac{\frac{1}{2} + \bar{x}}{\frac{1}{2} - x} \left(\log \frac{\mu_0^2}{4m_Q^2} - 2 \log(\frac{1}{2} - x) - 1 \right) \right]_+ + \left[\frac{4x\bar{x}}{(\frac{1}{2} - x)^2} \right]_{++} + \Delta [16x]_+ \right\}$$

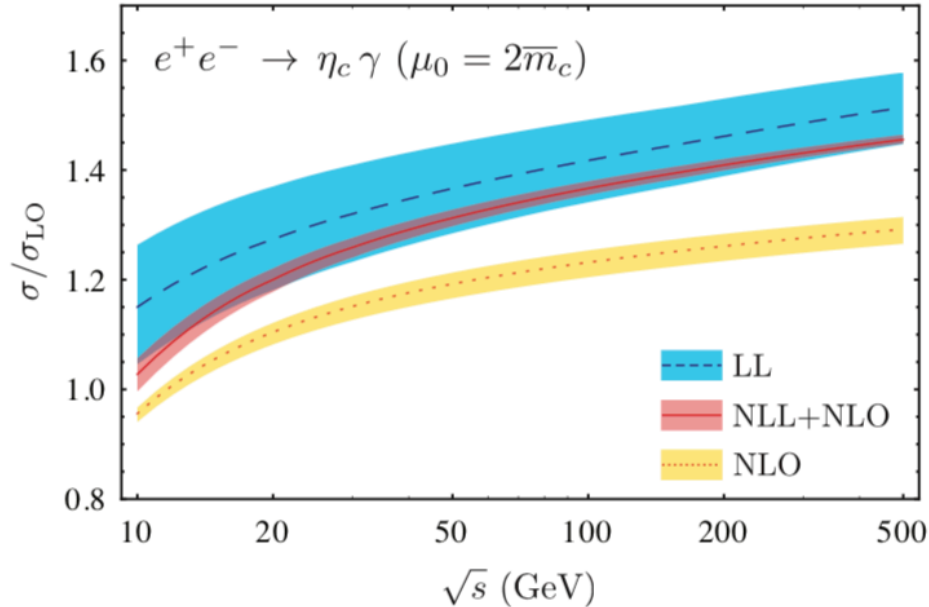
$\Delta=0$ for NDR, $\Delta=1$ for HV

□ Checking cancellation of Δ dependence is a strong constraint and nontrivial way validating our resummation.

□ Δ term in LCDA cancels against one-loop T_H and two-loop anomalous dim. in U_{nk} .

□ cancellations valid up to NLL accuracy

Log resummation effect



- ❑ Uncertainty estimated by typical scale variations
- ❑ LL and NLL are consistent and look perturbatively convergent
- ❑ fixed-order NLO w/o resummation is largely deviated from LL,NLL
 - ✓ underestimates its uncertainty

Numerical results

□ At B-, Z-boson, Higgs factories

\sqrt{s}	Cross section		Branching fraction	
	η_c	η_b	η_c	η_b
10.58 GeV	32.7 ± 2.8 fb	-		
m_Z	0.449 ± 0.037 fb	1.66 ± 0.31 fb	$(7.42 \pm 0.61) \times 10^{-9}$	$(2.80 \pm 0.53) \times 10^{-8}$
240 GeV	0.189 ± 0.016 ab	0.0934 ± 0.0176 ab		

input parameters:

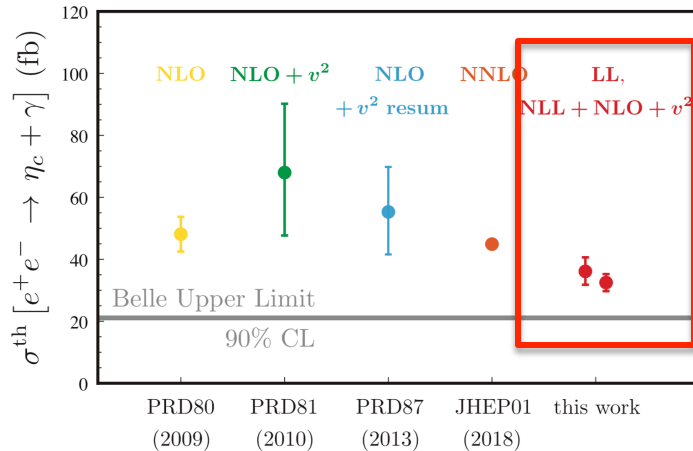
$$\overline{m}_c = 1.275^{+0.025}_{-0.035} \text{ GeV}$$

$$\Gamma[\eta_c \rightarrow \gamma\gamma] = 5.0 \pm 0.4 \text{ keV}$$

$$\langle \mathcal{O}_1 \rangle_{\eta_c} = 0.302^{+0.052}_{-0.049} \text{ GeV}^3$$

$$\langle v^2 \rangle_{\eta_c} = 0.222^{+0.070}_{-0.070}$$

□ Various prediction and Belle's upper bound

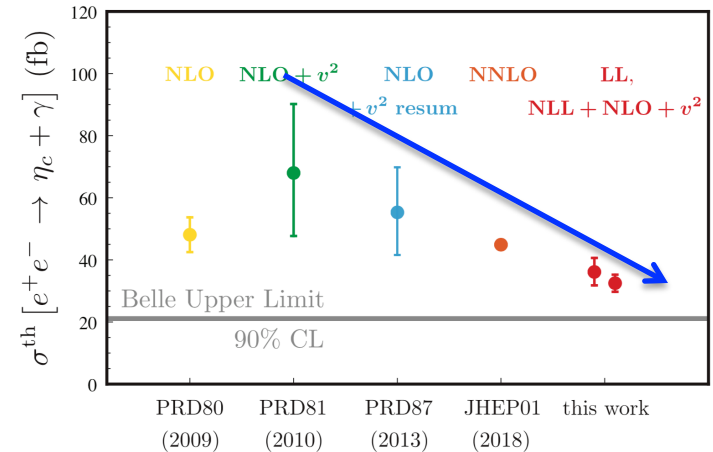


- our results are consistent with other predictions since resum. effect is not large at Belle energy
- lower central value due to smaller decay rate
- **our prediction is 4-sigma away from Belle's upper limit (20.1 fb)**

PRD98 (2018) 0922015

Summary

- ❑ Exclusive quarkonium is motivated by experiments, BESIII, Belle, LHC at wide ranges of energies 3-100 GeV
- ❑ Predictions for $\eta_c + \gamma$ valid at these energies by NRQCD and LC factorization
 - ❑ log resummation at NLL
 - ❑ v^2 corrections
 - ❑ power corrections of r at NLO
- ❑ At Belle energy, consistent with previous predictions but deviated from Belle's upper limit by 4 sigma level.



It looks like that it's moving towards a right direction with time.

Thanks!

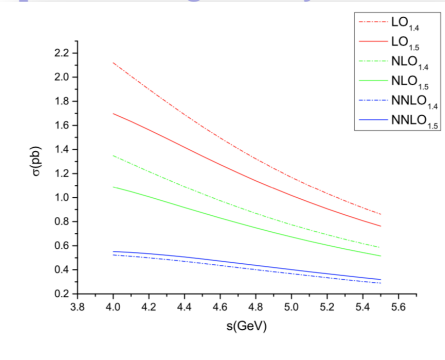
Results at BESIII energy (4 GeV)

- ❑ BESIII's peak cross section via $Y(4260)$ resonance

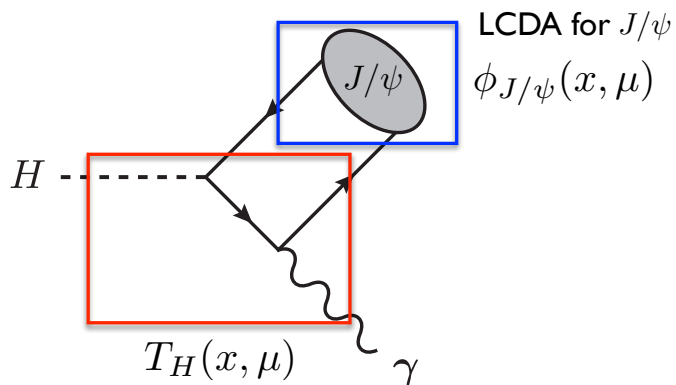
$$2.11 \pm 0.49(\text{stat.}) \pm 0.36(\text{syst.}) \text{ pb}$$

- ❑ Off-resonance cross section must be smaller than the value
- ❑ Our numerical value: 0.53 pb
- ❑ A simple extrapolation from our prediction at BELLE energy by using scaling $1/Q^2$: $7 \times (32.7 \text{ fb}) = 0.23 \text{ pb}$
- ❑ comparable with NNLO prediction $\sim 0.5 \text{ pb}$

[Chen, Liang, Qiao, JHEP 2018]



Comparison with inclusive process



exclusive production process

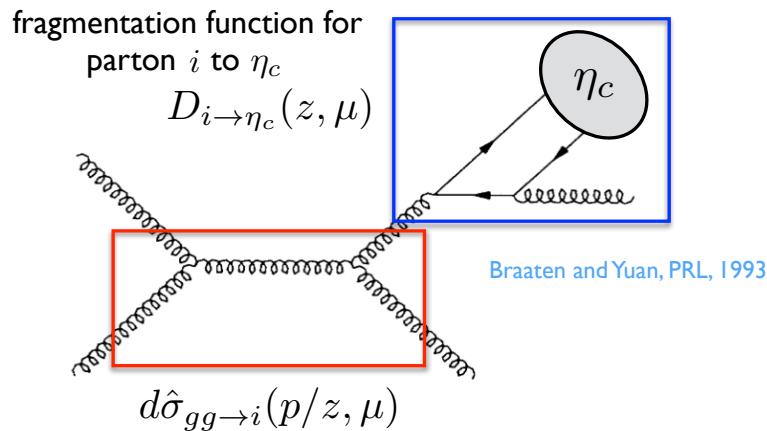
x : longitudinal momentum fraction of quark in J/ψ

$$\mathcal{M}_{H \rightarrow J/\psi + \gamma} = \int_0^1 dx T_H(x, \mu) \phi_{J/\psi}(x, \mu)$$

scale evolution: ERBL equation

$$\mu^2 \frac{\partial}{\partial \mu^2} \phi(x, \mu) = \int_0^1 dy \underbrace{V[x, y; \alpha_s(\mu)]}_{\text{Evolution kernel}} \phi(y, \mu)$$

↑
 perturbation theory



inclusive production (fragmenting) process

z : longitudinal momentum fraction of η_c to fragmenting parton i

$$d\sigma_{\eta_c}(p) = \sum_i \int_0^1 dz d\hat{\sigma}_i(p/z, \mu) D_{i \rightarrow \eta_c}(z, \mu)$$

(sum over parton i)

scale evolution: DGLAP equation

$$\mu \frac{\partial}{\partial \mu} D_{i \rightarrow \mathcal{O}}(z, \mu) = \sum_j \int_z^1 \frac{dy}{y} \underbrace{P_{i \rightarrow j}(z/y, \mu)}_{\text{DGLAP splitting function}} D_{j \rightarrow \mathcal{O}}(y, \mu)$$

↑
 perturbation theory