# XYZ Particles from Lattice QCD

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# Outline

I. Introduction

II. $X(3872)$ relevant

III. $Z_c$ states from lattice QCD

IV. $Y(4260)$ relevant studies

V Summary

# I. Introduction

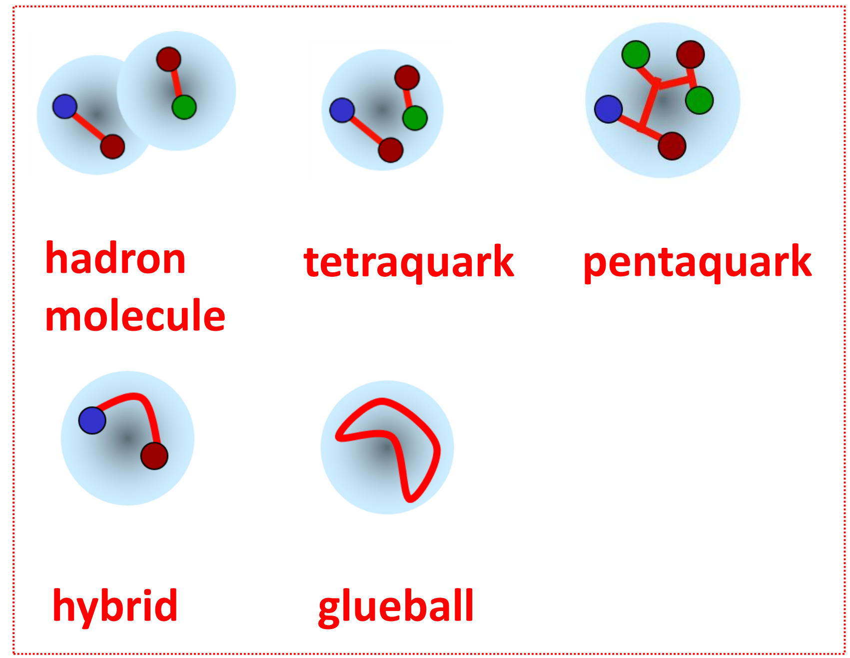
In naïve quark model, hadrons are sorted into two categories:

$\bar{q}q$ mesons and $qqq$ baryons

Extented constituent picture:
multi-quark states
$\bar{q}q\bar{q}q$ mesons
$qqqq\bar{q}$ baryons

Gluons can be also the building blocks of hadrons:

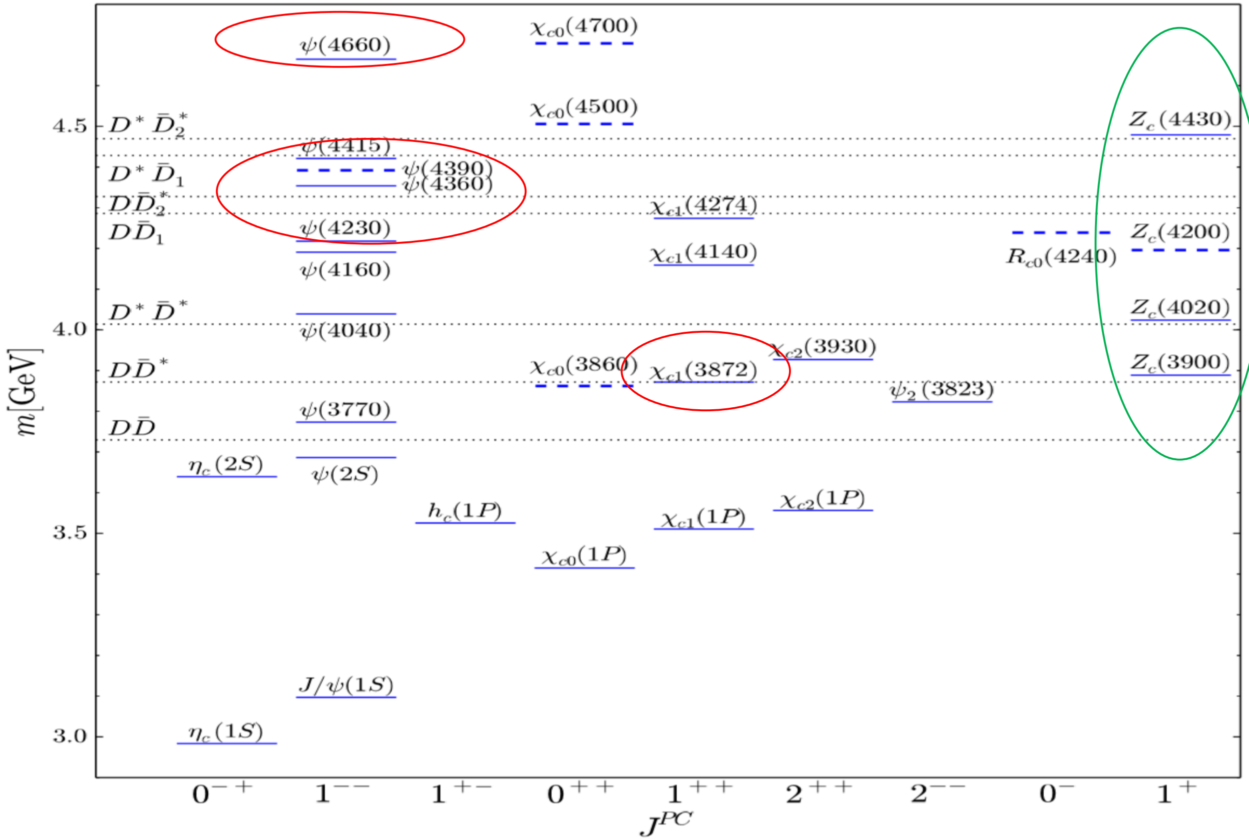
$\bar{q}qg$ mesons (hybrid)
$ggg$ mesons (glueball)


Exotic hadrons

**On the other hand, many XYZ particle have been discovered in recent years.**

## Charmoinum(-like) family




**(N. Brambilla et al, arXiv:1907.07583[hep-ph])**

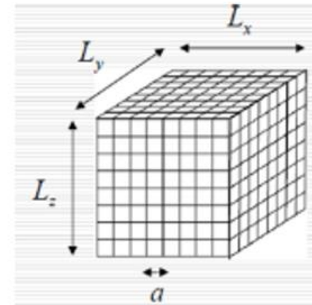
## The lattice formulation of QCD---Lattice QCD

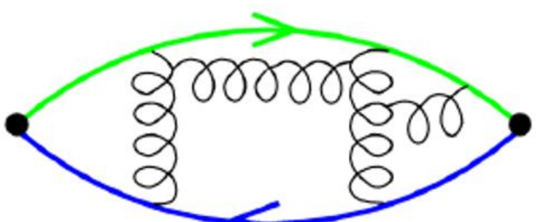
$$Z = \int \mathcal{D}A_\mu \, \mathcal{D}\psi \, \mathcal{D}\overline{\psi} \; e^{-S}$$

$$S = S_{gauge} + S_{quarks} = \int d^4x \left(\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu}\right) - \textstyle\sum_i \log(\mathrm{Det} M_i)$$

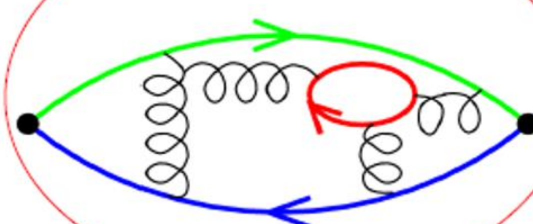
$$Z = \int \mathcal{D}A_\mu \, \det M \; e^{\int d^4x \, (-\frac{1}{4} F_{\mu\nu} F^{\mu\nu})}.$$

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A_\mu \, \mathcal{O} \, e^{-S} \; .$$



(A) Quenched QCD: quark loops neglected



(B) Full QCD

Dominated in the present era

## The methods for the hadron spectroscoapy in lattice QCD

- Interpolation field operators --- starting point for a meson (-like) system with given $J^{PC}$ and flavor quantrum numbers:

$$\mathcal{O}_i: \quad \bar{q}_1\Gamma q_2 \quad [\bar{q}_1\Gamma_1 q\,][\bar{q}\Gamma_2 q_2\,] \quad [q_1^T\Gamma_1 q][\bar{q}\Gamma_2\bar{q}_2^T\,], \ldots\ldots$$

- Two-point functions --- Observables

$$C_{ij}(t) = \left\langle 0\middle|\mathcal{O}_i(t)\mathcal{O}_j^+(0)\middle|0\right\rangle$$
$$= \sum_n \langle 0|\mathcal{O}_i|n\rangle\left\langle n\middle|\mathcal{O}_j^+\middle|0\right\rangle e^{-E_n t}$$

In principle, all the physical states with the same quantum numbers $|n\rangle$ contribute to the two point functions $C_{ij}(t)$ as the eigenstates of the QCD Hamiltonian with the energy eigenvalue $E_n$:

- " one-particle state" : $E_n = m_n$
- " two-particle state" : $E_n = \sqrt{m_1^2+\vec{p}^2} + \sqrt{m_2^2+\vec{p}^2} + \Delta E,\ \vec{p} = \frac{2\pi}{L}\vec{n}$
- ……

# Comparison of the hadron spectra

| Euclidean spacetime lattice | | Minkowski continuum spacetime |
|---|---|---|
| One particle states | ⟷ | Stable particles |
| Multiple particle states with discrete relative spatial Momentum (scattering States in a finite volume | ⟶ | Bound states of hadrons<br>Resonances |
| | | Continuum scattering states |

All the energies are Discretized.

## Luescher's Relation:

$$E_n = (m_1^2 + p^2)^{1/2} + (m_2^2 + p^2)^{1/2}$$

$$\tan\delta(p) = \frac{\sqrt{\pi}\ p\ L}{2\ \mathcal{Z}_{00}\left(1; (\frac{pL}{2\pi})^2\right)}$$

**Resonances**

$$T(p) = \frac{-\sqrt{s}\,\Gamma(p)}{s - m_R^2 + i\sqrt{s}\,\Gamma(p)} = \frac{1}{\cot\delta(p) - i}$$

$$\Gamma(p) = g^2\frac{p^{2l+1}}{s}, \quad \frac{p^{2l+1}}{\sqrt{s}}\cot\delta(p) = \frac{1}{g^2}(m_R^2 - s)$$

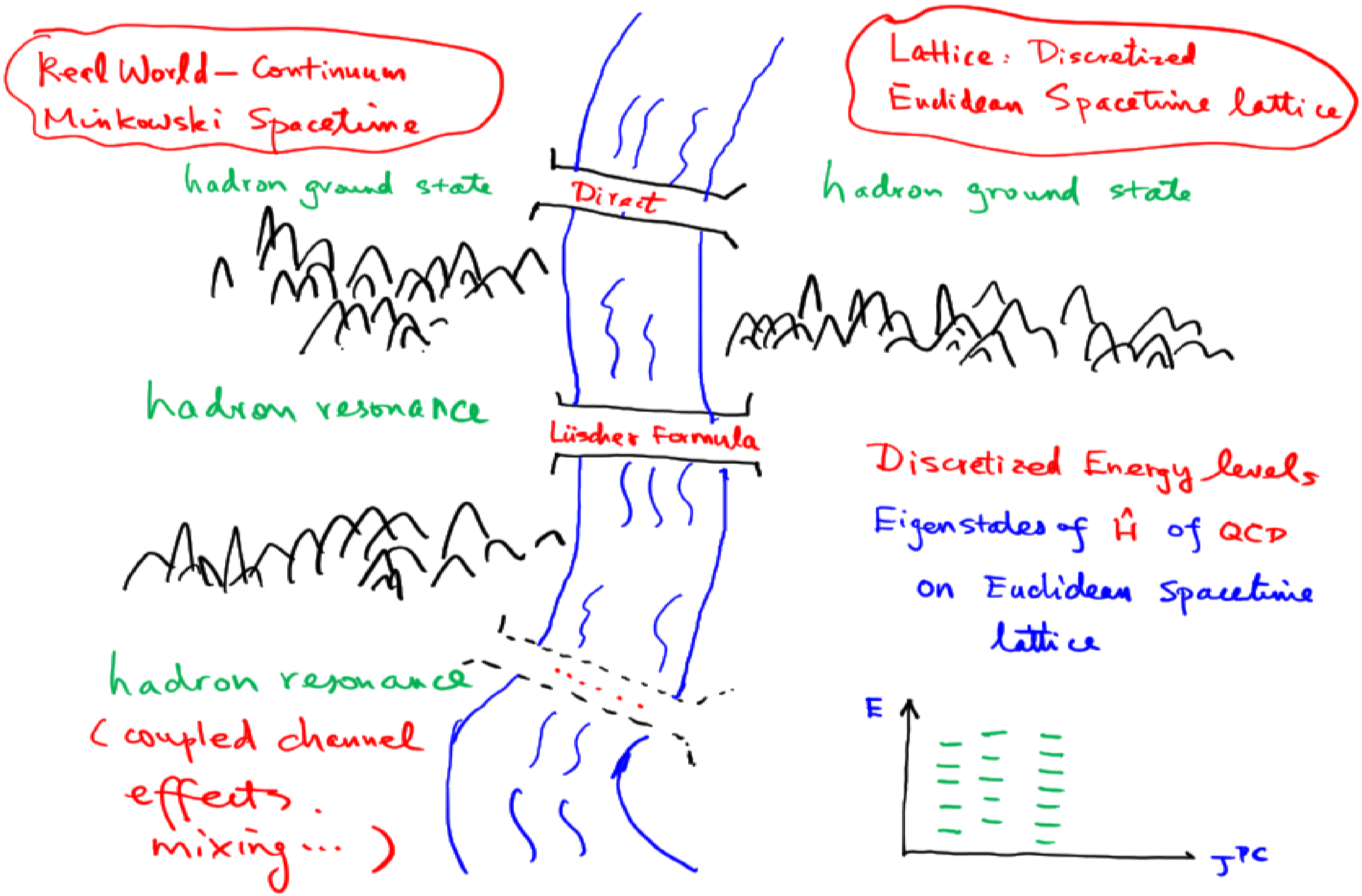
**Bound states**

$$p\cot(\delta_0(p)) = \frac{1}{a_0} + \frac{1}{2}r_0 p^2, \quad -|p_B| = \frac{1}{a_0} - \frac{1}{2}r_0|p_B|^2$$

$$T = \frac{\,}{\cot(\delta_l(p_B)) - i} = \infty$$

$$m_B = E_{H_1}(p_B) + E_{H_2}(p_B), \ p_B = i|p_B|$$

## Present status of lattice QCD study on hadron spectroscopy



# II. $X(3872)$ relevant lattice studies

## 1. $D\bar{D}^*$ scattering and $X(3872)$

S. Prelovsek & L. Leskovec, PRL111(2013)192001

**Operators** ($O_i = D\bar{D}^*,\quad J/\psi\,\omega,\quad c\bar{c}$)

$$O^{\bar{c}c}_{1-8} = \bar{c}\hat{M}_i c(0) \qquad \text{(only } I=0\text{)} \qquad (2)$$

$$O_1^{DD^*} = [\bar{c}\gamma_5 u(0)\ \bar{u}\gamma_i c(0) - \bar{c}\gamma_i u(0)\ \bar{u}\gamma_5 c(0)] + f_I\{u \to d\}$$

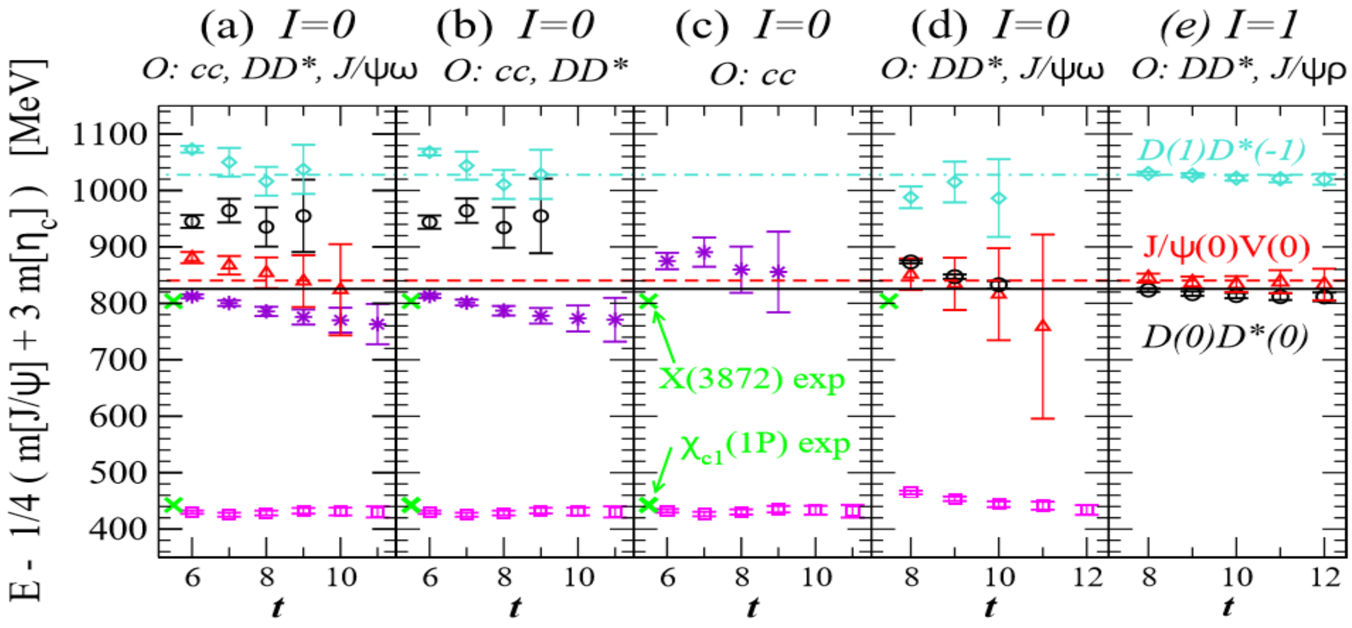
$$O_2^{DD^*} = [\bar{c}\gamma_5\gamma_t u(0)\ \bar{u}\gamma_i\gamma_t c(0) - \bar{c}\gamma_i\gamma_t u(0)\ \bar{u}\gamma_5\gamma_t c(0)] + f_I\ \{u \to d\}$$

$$O_3^{DD^*} = \sum_{e_k=\pm e_{x,y,z}} [\bar{c}\gamma_5 u(e_k)\ \bar{u}\gamma_i c(-e_k) - \bar{c}\gamma_i u(e_k)\ \bar{u}\gamma_5 c(-e_k)] + f_I\ \{u \to d\}$$

$$O_1^{J/\psi V} = \epsilon_{ijk}\ \bar{c}\gamma_j c(0)\ [\bar{u}\gamma_k u(0) + f_I\ \bar{d}\gamma_k d(0)]$$

$$O_2^{J/\psi V} = \epsilon_{ijk}\ \bar{c}\gamma_j\gamma_t c(0)\ [\bar{u}\gamma_k\gamma_t u(0) + f_I\ \bar{d}\gamma_k\gamma_t d(0)]\ ,$$

$$C_{ij}(t) = \langle O_i(t) O_j^{+}(0) \rangle = \sum_n Z_i^{(n)} Z_j^{(n)*}\, e^{-m_n t}$$




**Only $c\bar{c}$-type operator :** The 2nd state higher than $D\bar{D}^*$ threshold

**$D\bar{D}^* + c\bar{c}$-type operator :** The 2nd state lower than $D\bar{D}^*$ threshold

$$E(p) = \sqrt{m_D^2 + p^2} + \sqrt{m_{D^*}^2 + p^2} \Longrightarrow p^2 \equiv \left(\frac{2\pi}{L}\right)^2 q^2$$

**Phase shift:**

$$p\, cot\, \delta(p) = \frac{2Z_{00}(1;q^2)}{\sqrt{\pi}L}$$

**Effective range expansion:**

$$p \cot \delta(p) = \frac{1}{a_0^{D\bar{D}^*}} + \frac{1}{2} r_0^{D\bar{D}^*} p^2$$

$$a_0^{D\bar{D}^*} = -1.7 \pm 0.4 \text{ fm}$$
$$r_0^{D\bar{D}^*} = -0.5 \pm 0.1 \text{ fm}$$

**In the $L \to \infty$ limit, the existence of a bound state implies**

$$T \propto \frac{1}{\cot\delta(p_B) - i} = \infty \Longrightarrow \begin{cases} \cot\delta(p_B) = i \\ p_B = -i|p_B| \end{cases}$$

**Effective range expansion:**

$$-|p_B| = \frac{1}{a_0^{D\bar{D}^*}} - \frac{1}{2} r_0^{D\bar{D}^*} |p_B|^2$$

$$E(p_B, L = \infty) = \sqrt{m_D^2 + p_B^2} + \sqrt{m_{D^*}^2 + p_B^2} \equiv m_X$$

**Binding Energy:** $\Delta E_B = m_X - (m_D + m_{\bar{D}^*})$

| $X(3872)$ | $m_X - \frac{1}{4}(m_{\eta_c} + 3m_{J/\psi})$ | $m_X - (m_{D^0} + m_{D^{0*}})$ |
|---|---|---|
| $\mathrm{lat}^{L\to\infty}$ | $815 \pm 7$ MeV | $-11 \pm 7$ MeV |
| exp | $804 \pm 1$ MeV | $-0.14 \pm 0.22$ MeV |

# III. $Z_c$ State relevant lattice studies

$Z_c(3900)$ : first observed as a structure in $J/\psi\pi^+$ invariant mass spectrum, its "mass" is close to the $D\bar{D}^*$ threshold
$Z_c(4025)$ : first observed as a structure in $h_c\pi^+$ invariant mass spectrum, its "mass" is close to the $D^*\bar{D}^*$ threshold.
$Z_c(4430)$ : first observed as a structure in $\psi'\pi^+$ invariant mass spectrum, its "mass" is close to the $D^*\bar{D}_1$ threshold.

**Lattice studies from three aspects:**

1. $D\bar{D}$ scattering
   $Z_c(3900)$: Y. Chen et al. (CLQCD), PRD89(2014)094506,
   $Z_c(4020)$: Y. Chen et al. (CLQCD), PRD92(2015)054507.
   $Z_c(4430)$: T. Chen et al. (CLQCD), PRD93(2016)114501,
   G. Meng et al. (CLQCD), PRD 70(2009) 034503
2. Spectroscopy study (S. Prelovsek et al., PRD91(2015)014504)
3. Potential matrix and scattering amplitudes
   Y. Ikeda et al (HAL Collab.), PRL117(2016) 242001
4. $D\bar{D}^*$ and $J/\psi\ \pi$ coupled-channel effects relevant to $Z_c(3900)$
   T. Chen et al. (CLQCD), Chin. Phys. C43 (2019) no.10, 103103

## 1. Scattering:

### Calculation procedure:

i) The masses of $D$ mesons

ii) The energies of $D\bar{D}^*$ system

iii) Define the scattering momenta of $D$ mesons in the $D\bar{D}^*$ system

$$E_{1,2}(\vec{k}) = \sqrt{m_1^2 + \vec{k}^2} + \sqrt{m_2^2 + \vec{k}^2}, \qquad q^2 = \vec{k}^2 \left(\frac{L}{2\pi}\right)^2$$

iv) Use the Lüscher formular to get the scattering phase shift

$$q \cot \delta_0(q) = \frac{1}{\pi^{3/2}} \mathcal{Z}_{00}(1; q^2)$$

v) For near threshold scattering, one can use the effective range expansion to parameterize the phase shift versus q.

$$k^{2l+1} \cot \delta_l(k) = a_l^{-1} + \frac{1}{2} r_l k^2 + \cdots$$

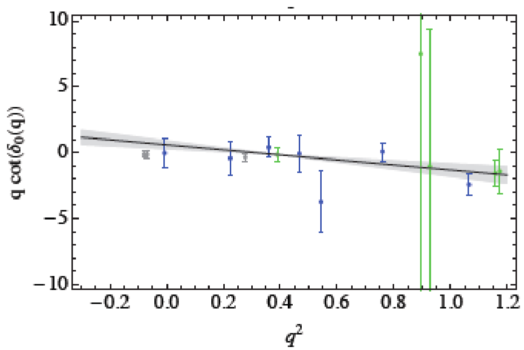
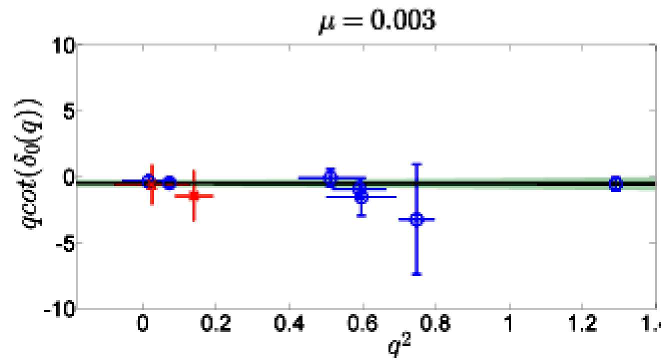



TABLE VI. The values for $a_0$ and $r_0$ in physical units obtained from the numbers for the correlated fit in Table IV.

| | $\mu = 0.003$ | $\mu = 0.006$ | $\mu = 0.008$ |
|---|---|---|---|
| $a_0$[fm] | −0.67(1) | −2.1(1) | −0.51(7) |
| $r_0$[fm] | −0.78(3) | −0.27(7) | 0.82(27) |




| | $\mu = 0.003$ | $\mu = 0.006$ | $\mu = 0.008$ |
|---|---|---|---|
| $a_0$[fm] | $-0.76^{+0.14}_{-0.21}$ | $-0.86^{+0.22}_{-0.22}$ | $-0.59^{+0.19}_{-0.25}$ |
| $r_0$[fm] | $-0.0022^{+0.18}_{-0.19}$ | $-0.14^{+0.15}_{-0.18}$ | $0.64^{+0.50}_{-0.51}$ |

**$Z_c(3900)$ and $Z_c(4020)$:** In the $J^P = 1^+$ channel, the scattering lengths are negative, indicating a weak repulsive interaction between D(D*) and D*bar. These results does not support a bound state in this channel. However, since the pion mass is still much higher than the physical pion mass, we cannot rule out the possible appearance of a bound state. A more systematic lattice study is demanding.

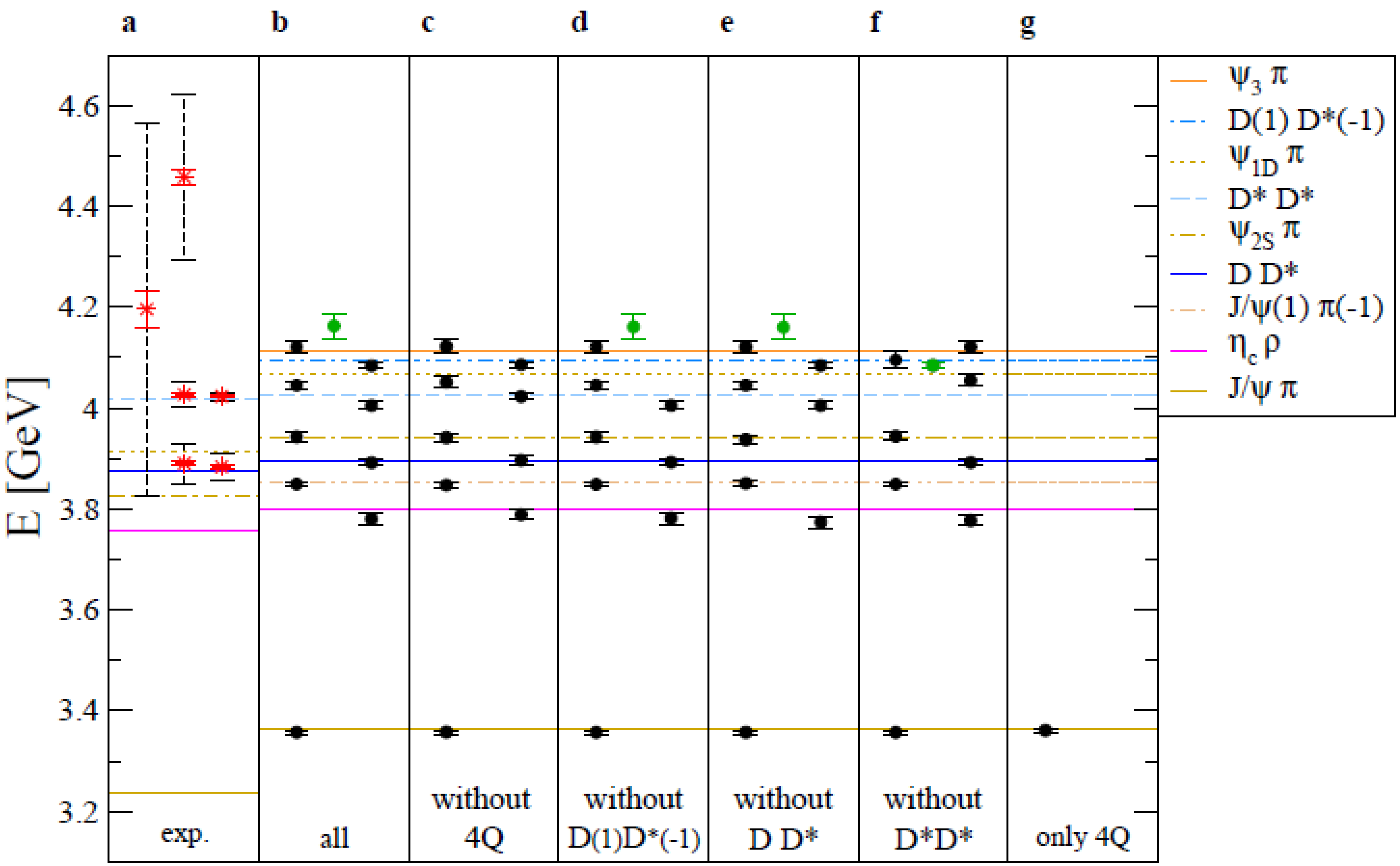
**$Z_c(4430)$** : In $J^P = 1^+, 0^-$ channels, the interaction between the two charmed mesons is attractive near the threshold in both channels.

## 2. Spectroscopy study on Zc states

S. Prelovsek et al., Phys. Rev. D 91, 014504 (2015) arXiv:1405.7623(hep-lat)

- **Spectroscopic study**
- **Quite a lot of two-particle operators and tetraquark operators are involved**

$$
\begin{aligned}
\mathcal{O}_1^{\psi(0)\pi(0)} &= \bar{c}\gamma_i c(0)\ \bar{d}\gamma_5 u(0)\,, \qquad (4)\\
\mathcal{O}^{\psi(1)\pi(-1)} &= \sum_{e_k=\pm e_{x,y,z}} \bar{c}\gamma_i c(e_k)\ \bar{d}\gamma_5 u(-e_k)\,,\\
O^{\psi(2)\pi(-2)} &= \sum_{|u_k|^2=2} \bar{c}\gamma_i c(u_k)\ \bar{d}\gamma_5 u(-u_k)\,,\\
\mathcal{O}^{\eta_c(0)\rho(0)} &= \bar{c}\gamma_5 c(0)\ \bar{d}\gamma_i u(0)\,,\\
\mathcal{O}_1^{D(0)D^*(0)} &= \bar{c}\gamma_5 u(0)\ \bar{d}\gamma_i c(0) + \{\gamma_5 \leftrightarrow \gamma_i\}\,,\\
\mathcal{O}^{D^*(0)D^*(0)} &= \epsilon_{ijk}\ \bar{c}\gamma_j u(0)\ \bar{d}\gamma_k c(0)\,,\\
\mathcal{O}_1^{4q} &\propto \epsilon_{abc}\epsilon_{ab'c'}(\bar{c}_b C\gamma_5 \bar{d}_c\ c_{b'}\gamma_i C u_{c'} - \bar{c}_b C\gamma_i \bar{d}_c\ c_{b'}\gamma_5 C u_{c'})\,,\\
\mathcal{O}_2^{4q} &\propto \epsilon_{abc}\epsilon_{ab'c'}(\bar{c}_b C \bar{d}_c\ c_{b'}\gamma_i\gamma_5 C u_{c'} - \bar{c}_b C\gamma_i\gamma_5 \bar{d}_c\ c_{b'} C u_{c'})\ ,
\end{aligned}
$$




**All the scatering states below 4.2 GeV are obtained.**

**They concluded that No convincing exotic Zc state is observed.**

## 3. On the structure of Zc(3900) from lattice QCD

(Y. Ikeda et al (HAL Collab.), PRL117(2016) 242001)

a) **BS wave functions of two-meson systems are extracted from lattice QCD.**

$$C^{\alpha\beta}(\vec{r},t) \equiv \sum_{\vec{x}} \langle 0|\, \phi_1^\alpha(\vec{x}+\vec{r},t)\phi_2^\alpha(\vec{x},t)\overline{\mathcal{J}}^\beta\, |0\rangle \,/\sqrt{Z_1^\alpha Z_2^\alpha}$$

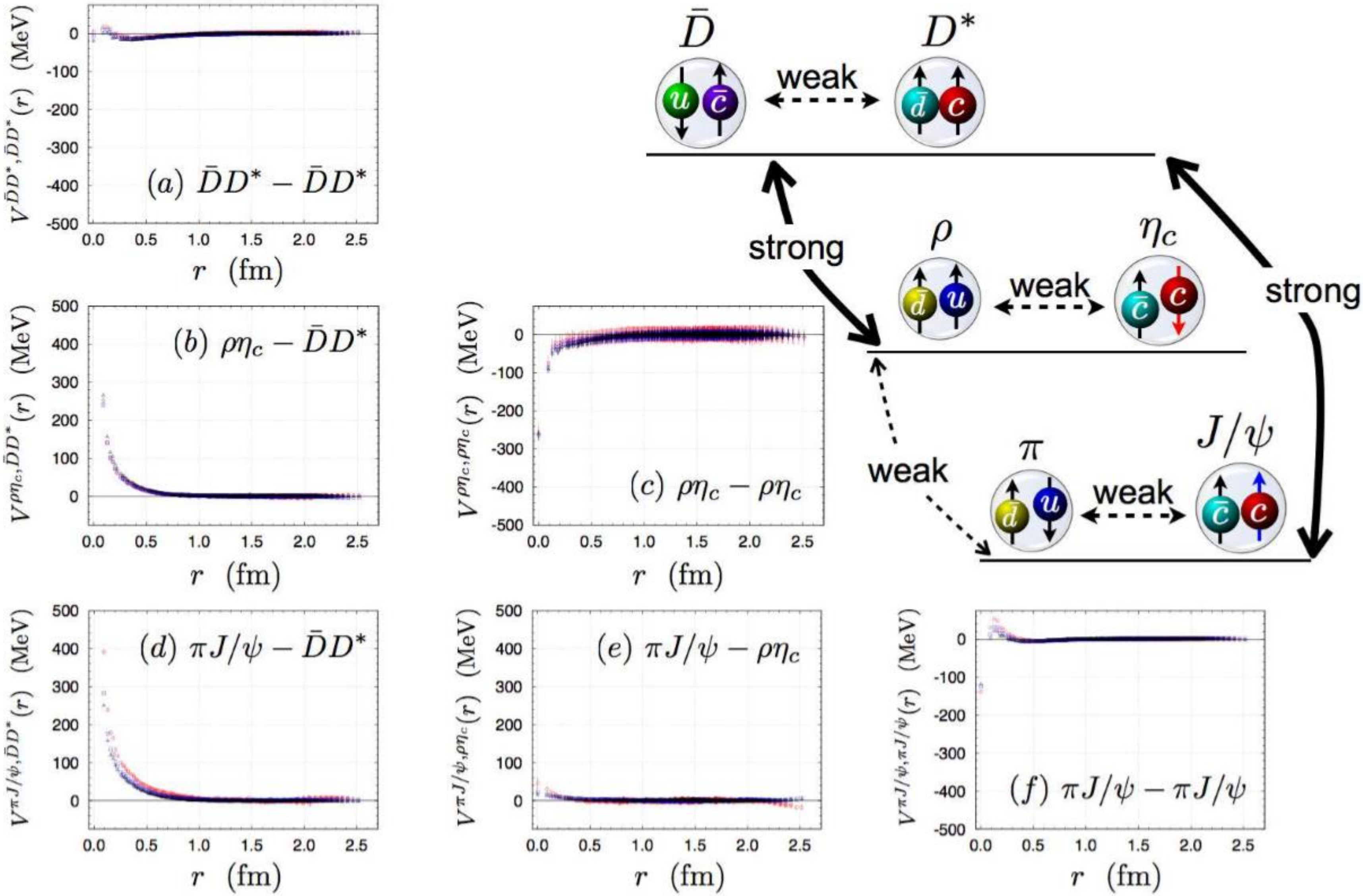
b) **Subsequently, the interaction potentials are obtained.**

$$\left(-\frac{\partial}{\partial t} - H_0^\alpha\right) R^{\alpha\beta}(\vec{r},t) = \sum_\gamma \Delta^{\alpha\gamma} \int d\vec{r'} U^{\alpha\gamma}(\vec{r},\vec{r'}) R^{\gamma\beta}(\vec{r'},t)$$

$$R^{\alpha\beta}(\vec{r},t) \equiv C^{\alpha\beta}(\vec{r},t) e^{(m_1^\alpha + m_2^\alpha)t} \qquad H_0^\alpha = -\nabla^2/2\mu^\alpha$$

c) **Couple channel effects of eta_c rho, J/psi pi and DD*bar is considered.**

d) **They conclude the near DD*bar threshold enhancement is due to the J/psi pi and DD*bar coupling.**



$\bar{D}$
$D^*$
weak
strong
$\rho$
$\eta_c$
weak
strong
$\pi$
$J/\psi$
weak
weak
(a) $\bar{D}D^* - \bar{D}D^*$
(b) $\rho\eta_c - \bar{D}D^*$
(c) $\rho\eta_c - \rho\eta_c$
(d) $\pi J/\psi - \bar{D}D^*$
(e) $\pi J/\psi - \rho\eta_c$
(f) $\pi J/\psi - \pi J/\psi$
$r$ (fm)
(MeV)

## Two-body $T$ matrix on the basis of the potential matrix $V^{\alpha\beta}$

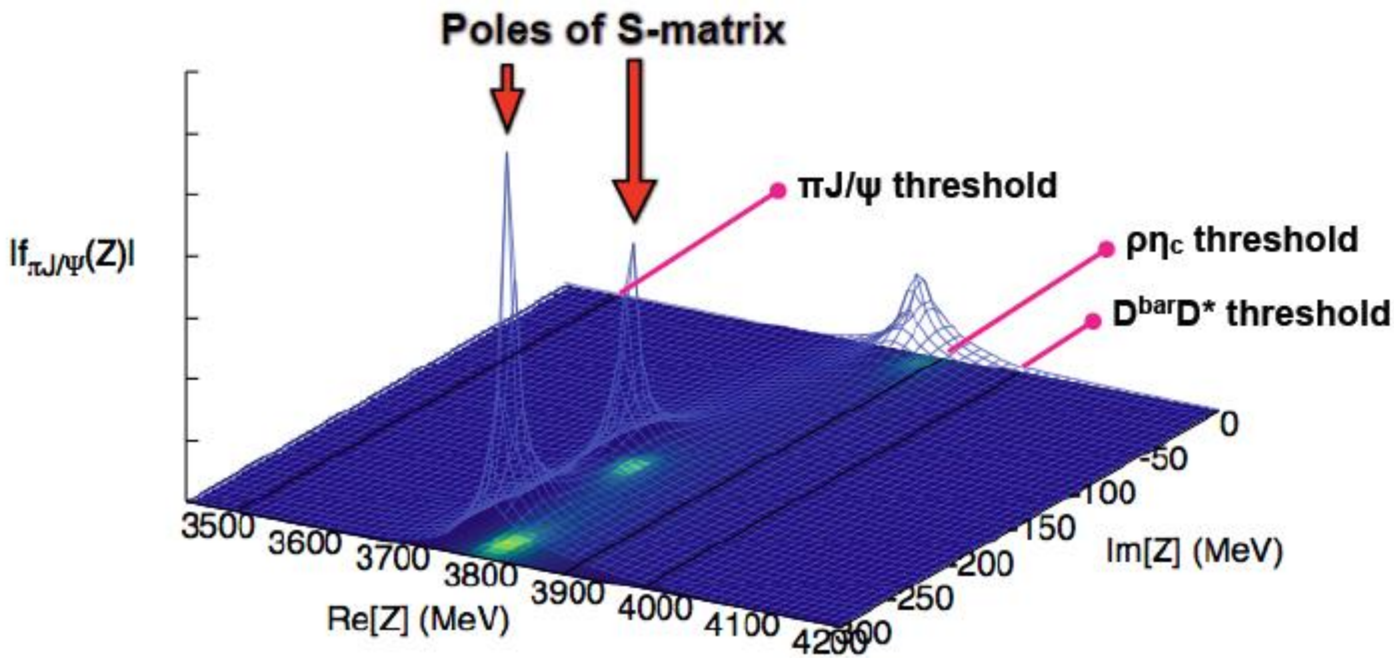
$$t^{\alpha\beta}\left(\vec{p}_\alpha,\vec{p}_\beta;W_{cm}\right) = V^{\alpha\beta}(\vec{p}_\alpha,\vec{p}_\beta)$$

$$+\sum_\gamma \int d\vec{q}_\gamma \frac{V^{\alpha\gamma}\left(\vec{p}_\alpha,\vec{q}_\gamma\right)t^{\gamma\beta}(\vec{q}_\gamma,\vec{p}_\beta;W_{cm})}{W_{cm}-E_\gamma\left(\vec{q}_\gamma\right)+i\epsilon}$$

$$\left|f^{\alpha\beta}(W_{cm})\right|^2 = \frac{d\sigma^{\alpha\beta}}{d\Omega}$$

# Pole search ($\pi J/\psi$ :2nd, $\rho\eta_c$ :2nd, $D^{bar}D^*$:2nd)

✤ input : LQCD potential matrix @ $m_\pi$=410MeV




✓ "Virtual (shadow)" poles on the most adjacent complex energy plane for $Z_c$(3900) energy region are found

✓ These poles contribute to threshold enhancement of amplitude

## 4. $D\bar{D}^*$ and $J/\psi\ \pi$ coupled-channel effects relevant to $Z_c(3900)$

T. Chen et al. (CLQCD), Chin. Phys. C43 (2019) no.10, 103103

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix} = \begin{bmatrix} \eta e^{2i\delta_1} & i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{bmatrix}$$

**The two-channel Lüscher formula now takes the form**

$$\begin{vmatrix} \frac{\mathcal{M}(k_1^2)+i}{\mathcal{M}(k_1^2)-i} - S_{11} & \sqrt{\frac{k_2 m_2}{k_1 m_1}} S_{12} \\ \sqrt{\frac{k_2 m_2}{k_1 m_1}} S_{12} & \frac{\mathcal{M}(k_2^2)+i}{\mathcal{M}(k_2^2)-i} - S_{22} \end{vmatrix} = 0$$

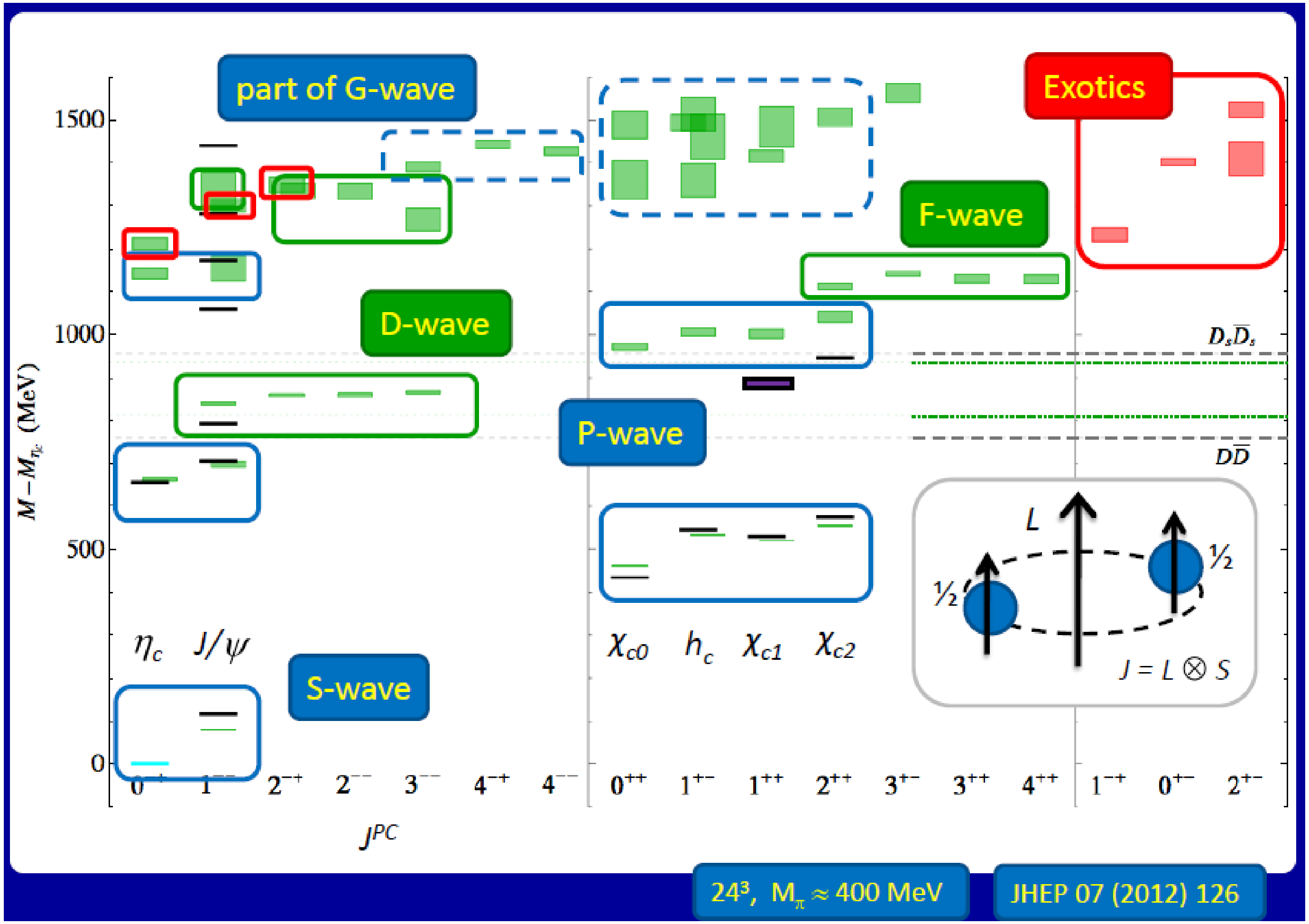
$$M_{ij}(E) = M_{ij}(E_0) + \frac{1}{2} R_i \delta_{ij} \left[ k_i^2(E) - k_i^2(E_0) \right]$$

**The best fit parameters do not correspond to the peak in the elastic scattering cross-section near the threshold**

**In the zero-range Ross-Shaw theory,the scenario of a narrow resonance close to the threshold is disfavored beyond the 3$\sigma$ level**

# IV. $Y(4260)$ relevant lattice studies

Latest charmonium spectrum from lattice QCD



## 1. 1– hybrid-like interpolation field operator

**Y. Chen, W. Chiu et al., Chin. Phys. C40, 081002 (2016)**

$$\bar{\psi}^a \gamma_5 \psi^b B_i{}^{ab}$$

**Nonrelativistic decomposition**

$$q = e^{\frac{\gamma\cdot D}{2m}} \begin{pmatrix} \psi \\ \chi \end{pmatrix} = \left[1 + \frac{\gamma\cdot D}{2m} + \frac{\gamma\cdot \vec{D}\ \gamma\cdot D}{8m^2} O(1/m^3)\right] \begin{pmatrix} \psi \\ \chi \end{pmatrix}$$

$$= \begin{pmatrix} \psi \\ \chi \end{pmatrix} + \frac{i}{2m} \begin{pmatrix} -\sigma\cdot\vec{D}\chi \\ \sigma\cdot\vec{D}\psi \end{pmatrix} + \frac{(\vec{D}^2 + \sigma\cdot B)}{8m^2} \begin{pmatrix} \psi \\ \chi \end{pmatrix} + O(1/m^3),$$

$$\bar{q} = \begin{pmatrix} \psi^\dagger & -\chi^\dagger \end{pmatrix} e^{-\frac{\gamma\cdot\overleftarrow{D}}{2m}} = \begin{pmatrix} \psi^\dagger & -\chi^\dagger \end{pmatrix} + \frac{i}{2m} \begin{pmatrix} \chi^\dagger \sigma\cdot\overleftarrow{D}^\dagger & \psi^\dagger\sigma\cdot\overleftarrow{D}^\dagger \end{pmatrix}$$

$$+ \frac{(\overleftarrow{D}^2 + \sigma\cdot B)}{8m^2} \begin{pmatrix} \psi^\dagger & -\chi^\dagger \end{pmatrix} + O(1/m^3),$$

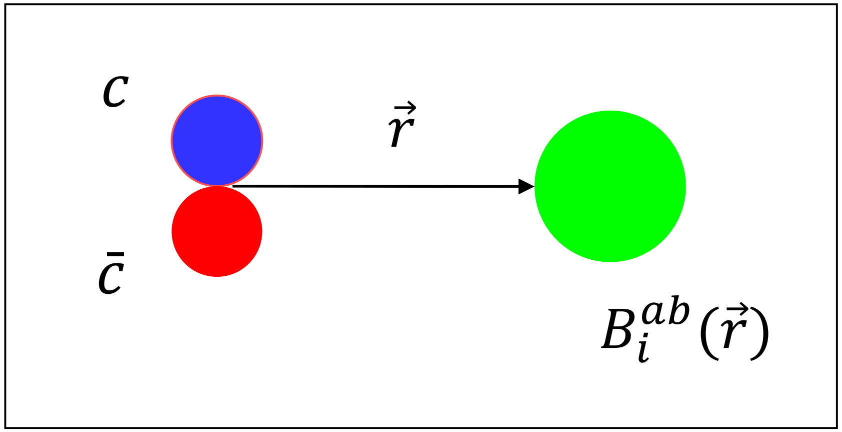
| | | |
|---|---|---|
| $0^{-+}$ | $\bar{\psi}\gamma_5\psi$ | $\chi^+\phi$ |
| $1^{--}$ | $\bar{\psi}\gamma_i\psi$ | $\chi^+\sigma_i\phi$ |
| $1^{--}{}_H$ | $\bar{\psi}^a\gamma_5\psi^b B_i{}^{ab}$ | $\chi^{a+}\phi^b B_i^{ab}$ |

$$O_i^{(H)} \equiv \bar{c}^a \gamma_5 c^b B_i^{ab} \rightarrow \chi^{a\dagger}\phi^b B_i^{ab} + O(\frac{1}{m_c}),$$ → **c-cbar spin singlet**

$$O_i^{(M)} \equiv \bar{c}^a \gamma_i c^a \rightarrow \chi^{a\dagger}\sigma_i \phi^a + O(\frac{1}{m_c}).$$ → **c-cbar spin triplet**

## 2. Spatially extended interpolation field operator for the vector charmonium-like state

**In the Coulomb gauge,** $O(\vec{r}) = (\bar{c}^a \gamma_5 c^b)(0) B_i{}^{ab}(\vec{r})$



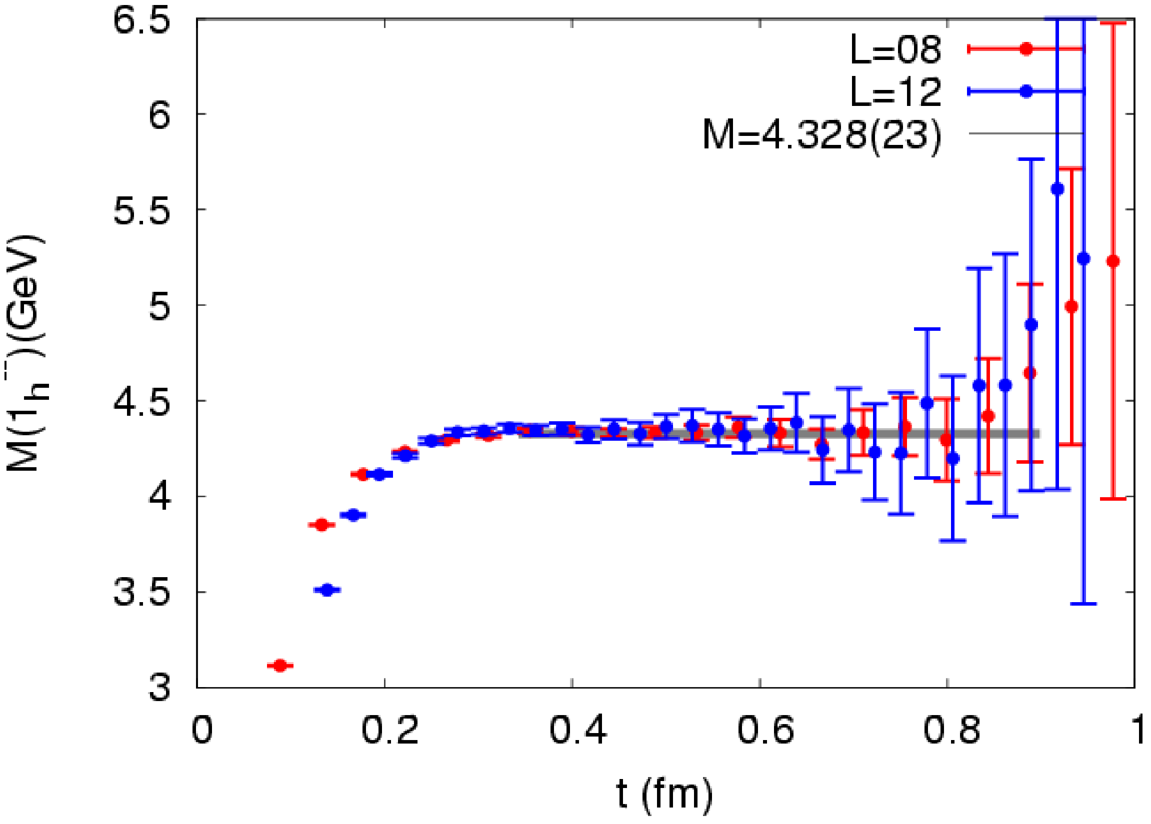

**This is equivalent to giave a c-cbar center of mass motion, which describes the recoil of the c-cbar against additional degrees of freedom.**

**Intuitively, the coupling of this kind of operators to conventional vector charmonia can be suppressed from two aspects:**

**a) spin states of the c-cbar (spin flipping is suppressed by the heavy quark mass.**

**b) center-of-mass motion ( to the leading order of NR, there is no cneter-of-mass motions for conventional charmonia.)**

**This kind of operator does couple strongly to a state with mass near 4.3 GeV, as seen in the effective mass plateau**



## 3. The leptonic decay width of the exotic vector charnomium

1. The leptonic decay width of this exotic vector chamonium is an important quantity, which can shed light on the nature of Y(4260).

$$\Gamma(Y(4260) \to e^+e^-)\Gamma(Y(4260) \to J/\psi\pi^+\pi^-)/\Gamma_{tot} = 5.8eV$$

2. The leptonic decay constant of the exotic state can be calculated directly in lattice QCD.

**The decay constant of a vector meson is defined as**

$$\langle 0|\bar{q}\gamma_\mu q|V(\vec{p},r)\rangle = m_V f_V \varepsilon_\mu(\vec{p},r)$$

**where the matrix element on the left can be derived by calculate the two point function**

$$\sum_{\vec{x}} \langle 0|\bar{q}\gamma_\mu q(\vec{x},t) O^{(w)}(0)|0\rangle = \sum_{i,r} \frac{1}{2M_i} \langle 0|\, \bar{q}\gamma_\mu q|V_i,r\rangle\langle V_i,r|O^{(w)}|0\rangle e^{-M_i t}$$

**Using the formula** $$\Gamma(V_{c\bar{c}} \to e^+e^-) = \frac{16\pi}{27}\alpha^2_{\rm QED}\frac{f_V^2}{M_V}$$

**One can predict** $$\Gamma(X \to e^+e^-) < 40\,{\rm eV}.$$

$$\Gamma(Y(4260) \to e^+e^-)\Gamma(Y(4260) \to J/\psi\pi^+\pi^-)/\Gamma_{tot} = 5.8eV$$

$\Gamma(Y(4260) \to J/\psi\pi^+\pi^-)/\Gamma_{tot} > 10\%$

**"Halo charmonium":**

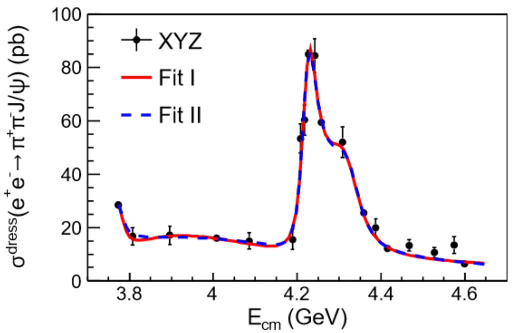
**A relatively localized kernal of color octet ccbar surrounded by a gluonic cloud.**

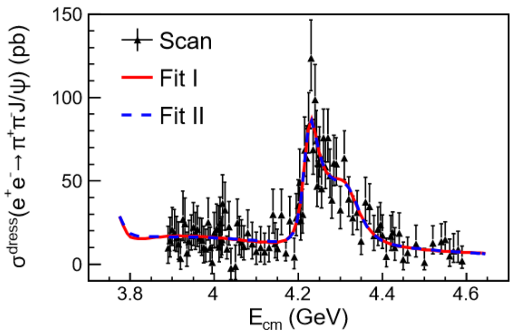
**The glounic cloud can be easily hadronized into light hadrons by emitting or absorbing a soft gluon.**

**Consequently halo charmonium has large branching ratios to decay into a conventional charmoium by emitting light hadrons.**

# $e^+e^- \to J/\psi\pi^+\pi^-$

**BESIII, PRL118(2017)092001 [arXiv: 1611.01317(hep-ex)]**

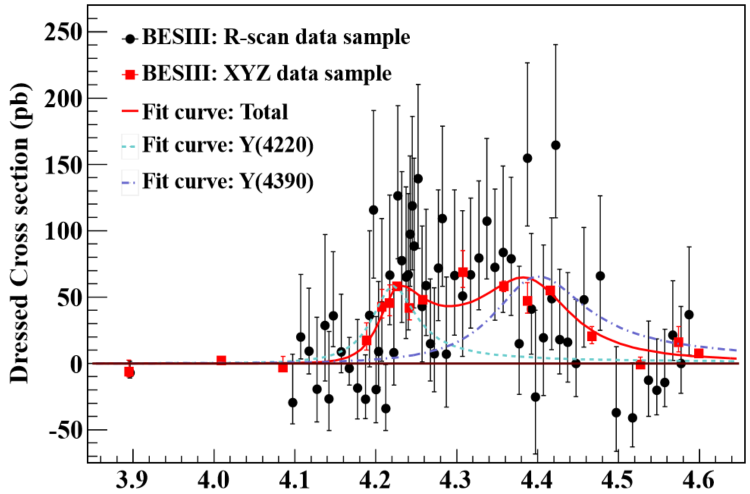



| Parameters | Fit result |
|---|---|
| $M(R_1)$ | $3812.6^{+61.9}_{-96.6}$ $(\cdots)$ |
| $\Gamma_{tot}(R_1)$ | $476.9^{+78.4}_{-64.8}$ $(\cdots)$ |
| $M(R_2)$ | $4222.0 \pm 3.1$ $(4220.9 \pm 2.9)$ |
| $\Gamma_{tot}(R_2)$ | $44.1 \pm 4.3$ $(44.1 \pm 3.8)$ |
| $M(R_3)$ | $4320.0 \pm 10.4$ $(4326.8 \pm 10.0)$ |
| $\Gamma_{tot}(R_3)$ | $101.4^{+25.3}_{-19.7}$ $(98.2^{+25.4}_{-19.6})$ |

| Parameters | Solution I | Solution II | Solution III | Solution IV |
|---|---|---|---|---|
| $\Gamma_{e^+e^-}\mathcal{B}[\psi(3770) \to \pi^+\pi^- J/\psi]$ | $0.5 \pm 0.1$ $(0.4 \pm 0.1)$ | | | |
| $\Gamma_{e^+e^-}\mathcal{B}(R_1 \to \pi^+\pi^- J/\psi)$ | $8.8^{+1.5}_{-2.2}$ $(\cdots)$ | $6.8^{+1.1}_{-1.5}$ $(\cdots)$ | $7.2^{+0.9}_{-1.5}$ $(\cdots)$ | $5.6^{+0.6}_{-1.0}$ $(\cdots)$ |
| $\Gamma_{e^+e^-}\mathcal{B}(R_2 \to \pi^+\pi^- J/\psi)$ | $13.3 \pm 1.4$ $(12.0 \pm 1.0)$ | $9.2 \pm 0.7$ $(8.9 \pm 0.6)$ | $2.3 \pm 0.6$ $(2.1 \pm 0.4)$ | $1.6 \pm 0.4$ $(1.5 \pm 0.3)$ |
| $\Gamma_{e^+e^-}\mathcal{B}(R_3 \to \pi^+\pi^- J/\psi)$ | $21.1 \pm 3.9$ $(17.9 \pm 3.3)$ | $1.7^{+0.8}_{-0.6}$ $(1.1^{+0.5}_{-0.4})$ | $13.3^{+2.3}_{-1.8}$ $(12.4^{+1.9}_{-1.7})$ | $1.1^{+0.4}_{-0.3}$ $(0.8 \pm 0.3)$ |
| $\phi_1$ | $-58 \pm 11$ $(-33 \pm 8)$ | $-116^{+9}_{-10}$ $(-81^{+7}_{-8})$ | $65^{+24}_{-20}$ $(81^{+16}_{-14})$ | $8 \pm 13$ $(33 \pm 9)$ |
| $\phi_2$ | $-156 \pm 5$ $(-132 \pm 3)$ | $68 \pm 24$ $(107 \pm 20)$ | $-115^{+11}_{-9}$ $(-95^{+6}_{-5})$ | $110 \pm 16$ $(144 \pm 14)$ |

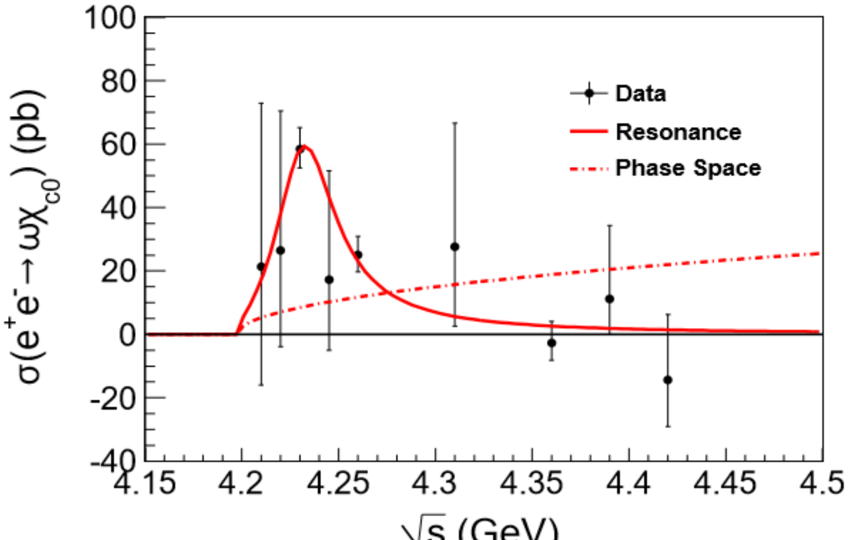
$e^+e^- \to h_c\pi^+\pi^-$




**BESIII, PRL118(2017)092002**
**arXiv: 1610.07044(hep-ex)**

$$M = (4218.4 \pm 4.0 \pm 0.9)\ \mathrm{MeV}/c^2$$
$$\Gamma = (66.0 \pm 9.0 \pm 0.4)\ \mathrm{MeV}$$
$$\Gamma^{el} = (4.6 \pm 4.1 \pm 0.8)\ \mathrm{eV}$$

$e^+e^- \to \chi_{c0}\omega$




$$M = (4230 \pm 8)\ \mathrm{MeV}/c^2$$
$$\Gamma_t = (38 \pm 12)\ \mathrm{MeV}$$
$$\Gamma_{ee}\mathcal{B}(\omega\chi_{c0}) = (2.7 \pm 0.5)\ \mathrm{eV}$$

$J/\psi\pi^+\pi^-$ **mode: relative S-wave between** $J/\psi$ **and** $\pi^+\pi^-$

$\chi_{c0}\omega$ **mode: relative S-wave between** $\chi_{c0}$ **and** $\omega$

$h_c\pi^+\pi^-$ **mode: relative P-wave between** $h_c$ **and** $\pi^+\pi^-$

**The $c\bar{c}$ in the halo charmonium is spin singlet (S=0),**

**$J/\psi\pi^+\pi^-$ mode: $J/\psi$ (S=1), spin flipping, $m_c$ suppressed,**
**no refugal barrier**

**$\chi_{c0}\omega$ mode: $\chi_{c0}$ (S=1), spin flipping, $m_c$ suppressed,**
**no centrifugal barrier**

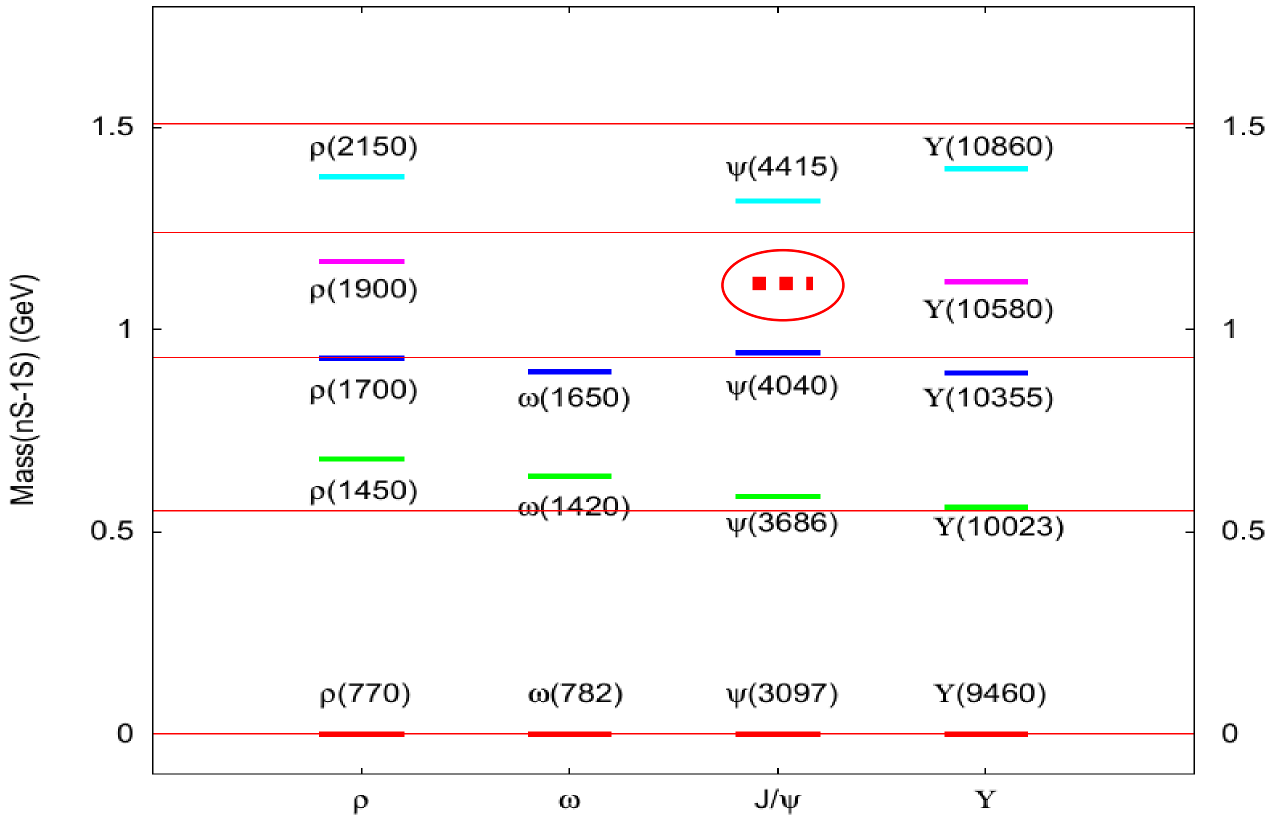
**$h_c\pi^+\pi^-$ mode: $h_c$ (S=0), no spin flipping,**
**but suppressed by the centrifugal barrier .**

**In this picture, it is understandable that the above three modes have similar cross section at $\sqrt{s} \sim 4.22\ GeV$**

A state is missing between ψ(4040) and ψ(4415) in the ψ(nS) family.

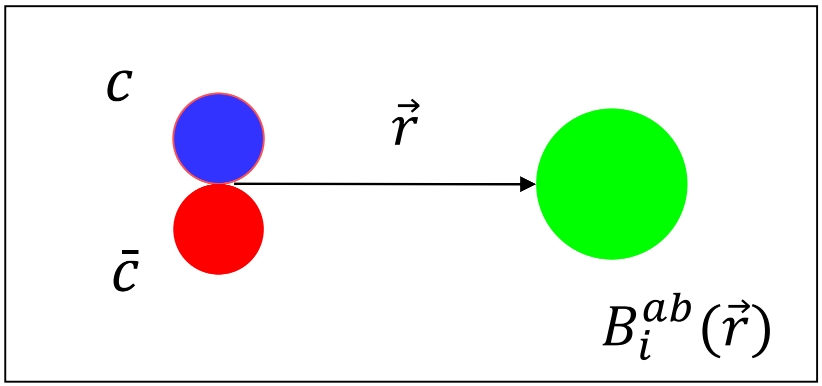
Y (4230) can be a candidate for this state, but properties are very different.

Therefore, it is intriguing that there be an additional ψ state here to be discovered.



# 3: Bethe-Salpeter Wave functions of Hybrid Charmonia

Y. Ma, W. Sun, Y. Chen M. Gong, Z. Liu, arXiv: 1910.09819 (hep-lat)




In the Coulomb gauge,

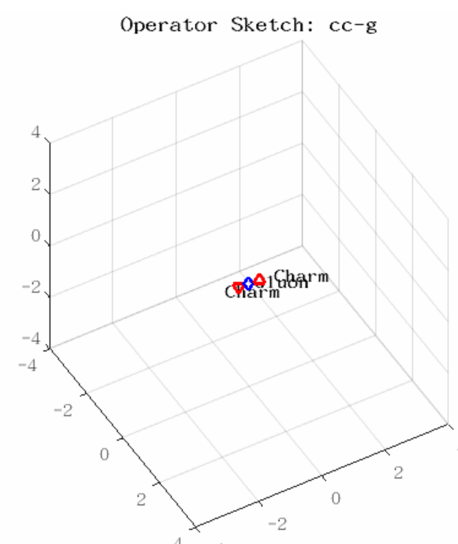
$$O(\vec{r}) = (\bar{c}^a \gamma_5 c^b)(0) B_i{}^{ab}(\vec{r})$$

This is equivalent to give a c-cbar center of mass motion, which describes the recoil of the c-cbar against additional degrees of freedom.

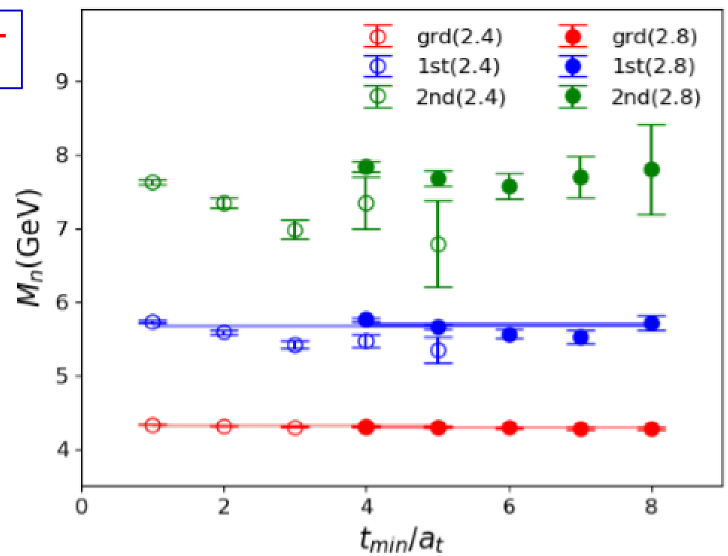
Intuitively, the coupling of this kind of operators to conventional charmonia can be suppressed from two aspects:

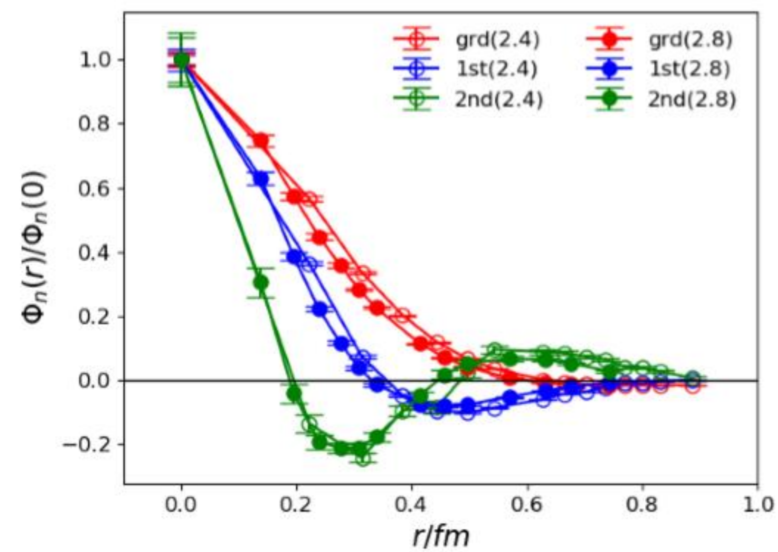
a) spin states of the c-cbar (spin flipping is suppressed by the heavy quark mass.

b) center-of-mass motion ( to the leading order of NR, there is no cneter-of-mass motions for conventional charmonia.)

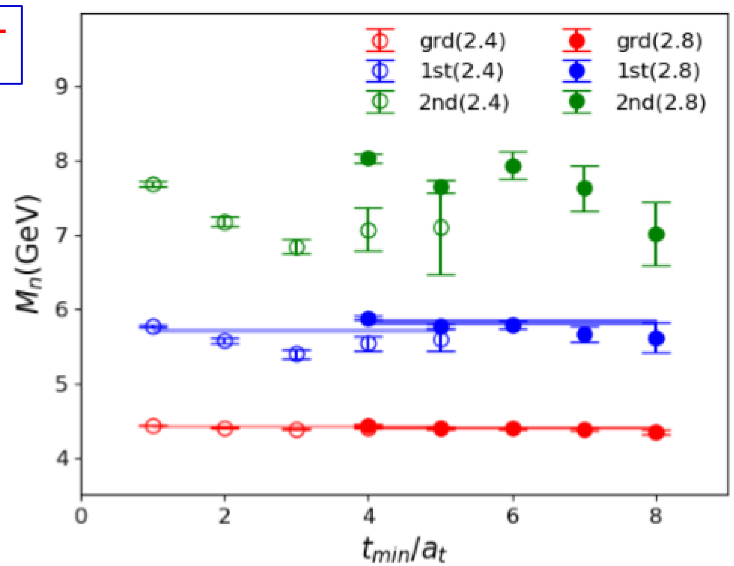


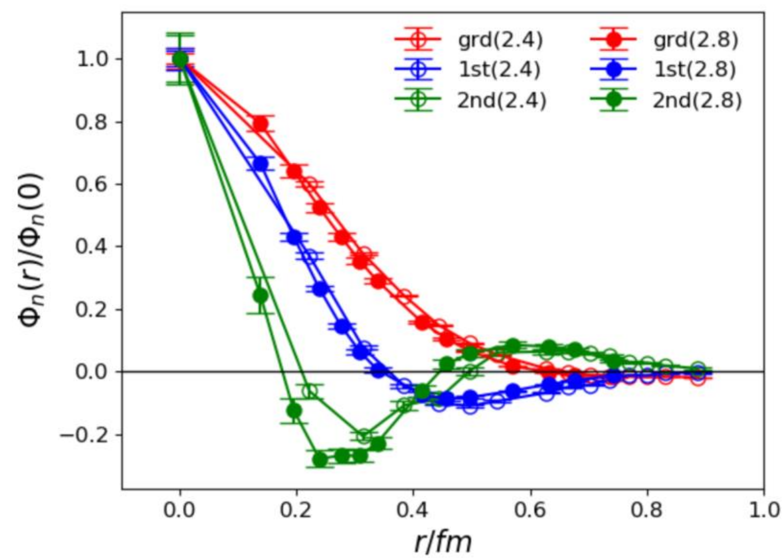
**$1^{-+}$**


grd(2.4)
1st(2.4)
2nd(2.4)
grd(2.8)
1st(2.8)
2nd(2.8)
$M_n$(GeV)
$t_{min}/a_t$

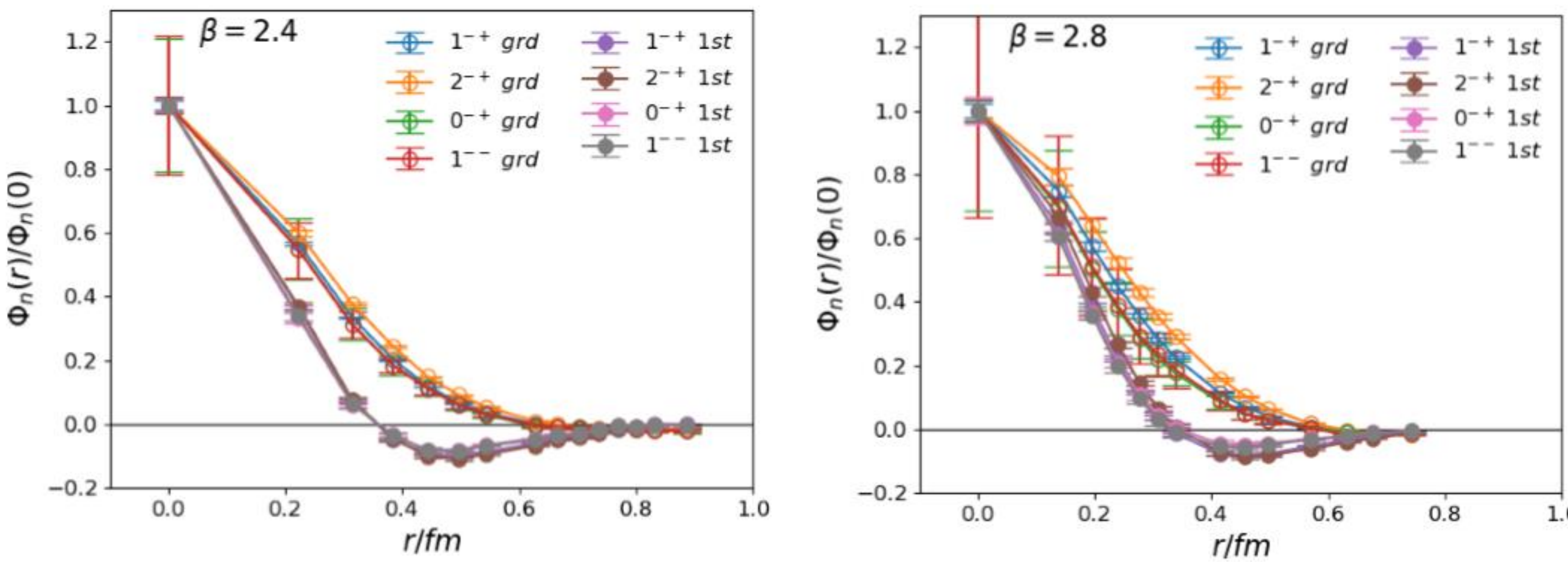

grd(2.4)
1st(2.4)
2nd(2.4)
grd(2.8)
1st(2.8)
2nd(2.8)
$\Phi_n(r)/\Phi_n(0)$
$r/fm$

**$2^{-+}$**


grd(2.4)
1st(2.4)
2nd(2.4)
grd(2.8)
1st(2.8)
2nd(2.8)
$M_n$(GeV)
$t_{min}/a_t$


grd(2.4)
1st(2.4)
2nd(2.4)
grd(2.8)
1st(2.8)
2nd(2.8)
$\Phi_n(r)/\Phi_n(0)$
$r/fm$

| #node | $m(1^{--})$ (GeV) | $m(0^{-+})$ (GeV) | $m(1^{-+})$ (GeV) | $m(2^{-+})$ (GeV) |
|---|---|---|---|---|
| 0 | 3.109(5) | 3.010(4) | - | - |
| 0 | 3.703(82) | 3.672(76) | - | - |
| 0 | 4.591(69) | 4.551(63) | 4.309(2) | 4.419(3) |
| 1 | 5.460(31) | 5.393(28) | 5.693(12) | 5.779(12) |
| 2 | 8.226(99) | 8.286(109) | 7.661(31) | 7.708(29) |




**Decay modes of $(0,1,2)^{-+}$ charmonia: $J/\psi\,\omega(\phi)$ (P-wave), $\chi_{cJ}\eta$ (S-wave)**

**BelleII and LHCb could take the mission to search them.**

# IV. Summary

- XYZ particles are hot topics in lattice QCD study, but no decisive conclusions have not be obtained.

- There are still many difficulties for lattice QCD to study exotic hadrons, from both the theoretical tools and numerical calculations.

# Thanks!