# Triton-like molecules

Li Ma

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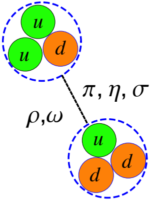
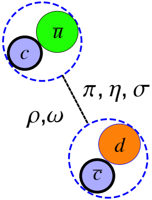
北京交通大學
BEIJING JIAOTONG UNIVERSITY

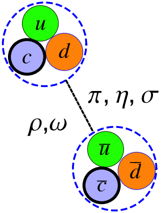
# The deuteron-like molecules




Deuteron



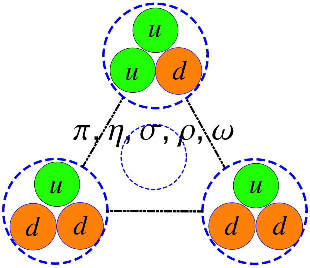

Di-meson molecule



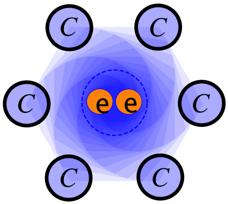

Di-baryon molecule

- $X(3872) \rightarrow D\bar{D}^*$, $D_{s1}(2460) \rightarrow D^*K$, $Z_b(10610) \rightarrow B\bar{B}^*$
- $Y(4260) \rightarrow D_1\bar{D}$, $Z_c(3900) \rightarrow D\bar{D}^*$, $Z_c(4020) \rightarrow D^*\bar{D}^*$
- $P_c(4380) \rightarrow \Sigma_c\bar{D}^*$, $P_c(4450) \rightarrow \Sigma_c^*\bar{D}^*$

# The triton-like molecules




Triton




Benzene ring

- $NDK$, $\bar{K}DN$, $ND\bar{D}$, $N\bar{K}K$, $DKK$, $DK\bar{K}$, $BBB^*$, $DDK$, $DD^*K$, $\Omega NN$ and $\Omega\Omega N$
- Faddeev equation, or FCA     Dimer or isobar formalism     Gaussian expansion method (GEM)

# Efimov effect

- $B_2 = \frac{\hbar^2}{ma^2}(1 + \mathcal{O}(\frac{r_0}{a}))$
- $B_3(1 + \mathcal{O}(\frac{r_0}{a})) = -\frac{\hbar^2}{ma^2} + [e^{-2\pi n} f(\xi)]^{\frac{1}{s_0}} \frac{\kappa_*^2}{m}$
- $s_0 = 1.00624...$, $\xi$ is defined by $\tan\xi = -(mB_3)^{1/2} a/\hbar$, $f(\xi)$is a universal function with $f(-\pi/2) = 1$, $K = sgn(E)\sqrt{m|E|}$.

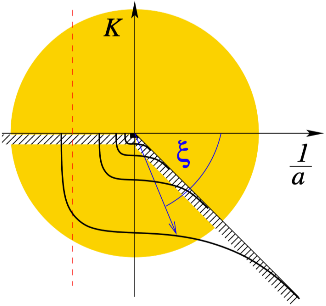



Figure: Efimov plot

# Tri-baryon systems ΛΛΛ, ΞΞΞ and ΣΣΣ

The OBE interaction indicates that there is only one virtual boson exchanged by any two constituents as shown in the following.

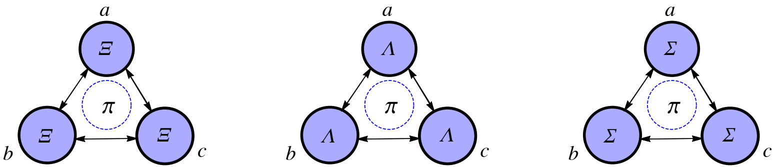
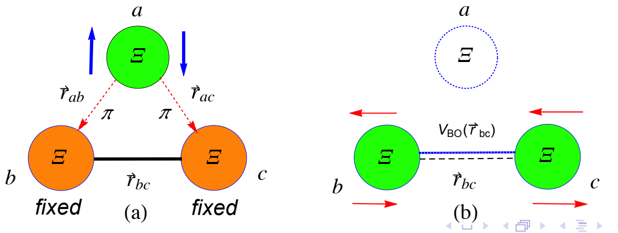



Figure: Dynamical illustration of the ΛΛΛ, ΞΞΞ and ΣΣΣ systems with a circle describing the delocalized $\pi$ *bond* inside.

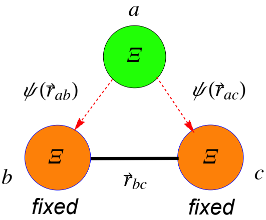
Monopole form factor $\mathcal{F}(q) = \frac{\Lambda^2 - m_\alpha^2}{\Lambda^2 - q^2}$, $(\alpha = \pi, \eta, \rho, \omega, \sigma, \phi)$ with $q$ the four-momentum of the pion and Λ the cutoff parameter.

# Born-Oppenheimer potential

- Considering that the particle *b* and *c* are static with the separation $r_{bc}$, one can separate the degree of freedom of *a* from the three-body system.
- We assume the distance $r_{bc}$ is a parameter. The mesons *b* and *c* are static, and have one-pion interactions with meson *a*, which can be viewed as two static sources.
- We explore the dynamics for the meson *a* in the limit $r_{bc} \to \infty$, and subtract the binding energy for the break-up state which is trivial for the three-body bound state.



# Interpolating wave function of meson *a*




The zero order of the final wave function for the meson *a* could be the superposition of these two components

$$\psi(\vec{r}_{ab}, \vec{r}_{ac}) \quad = \quad C[\psi(\vec{r}_{ab}) \pm \psi(\vec{r}_{ac})]|\Xi\Xi\Xi\rangle$$

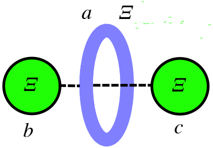
Accordingly, one can obtain the energy eigenvalue of the meson *a*

$$E_a(\Lambda, \ \vec{r}_{bc}) \quad = \quad \langle\psi(\vec{r}_{ab}, \vec{r}_{ac})|H_a|\psi(\vec{r}_{ab}, \vec{r}_{ac})\rangle$$

# BO potential and Its physical meaning (Intensity of "glue")

We define the BO potential as

$$V_{BO}(\Lambda,\ \vec{r}_{bc}) = E_a(\Lambda,\ \vec{r}_{bc}) - E_2(\Lambda).$$




The BO potential can describe the contribution for the one meson on the dynamics of the two remaining mesons. The meson *a* here works like a mass of "glue".

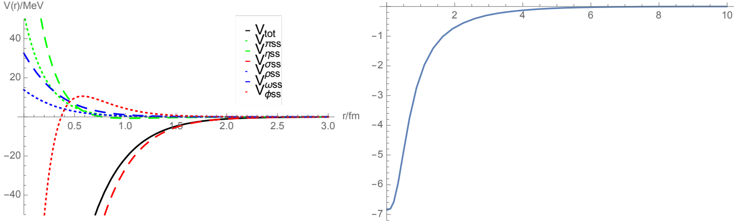



Figure: Here we chose the parameter $\Lambda = 900$ MeV. $E^{\Xi\Xi}_{I=0} = -3.09$ MeV.

$$\Psi_T = \alpha\Phi(\vec{r}_{bc})\psi(\vec{r}_{ab},\ \vec{r}_{ac}).$$

# The configurations of the three-body systems

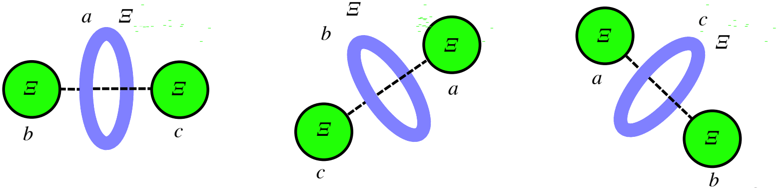



Figure: Every meson can be considered to be a lighter one and separated from the three-body system. Each of them can generate the "glue" for the remaining mesons.

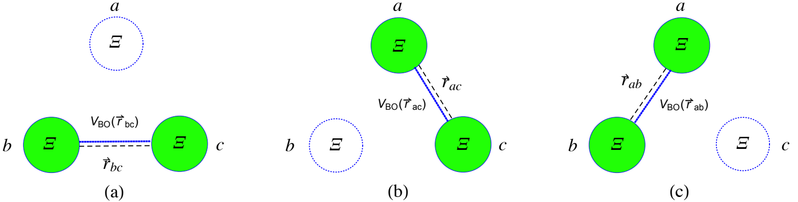



Figure: (a), (b) and (c) correspond to the wave functions $\psi_{\not{a}} = \Phi(\vec{r}_{bc})\psi(\vec{r}_{ab},\ \vec{r}_{ac})$, $\psi_{\not{b}} = \Phi(\vec{r}_{ac})\psi(\vec{r}_{ab},\ \vec{r}_{bc})$ and $\psi_{\not{c}} = \Phi(\vec{r}_{ab})\psi(\vec{r}_{bc},\ \vec{r}_{ac})$, respectively.

# Interpolating wave functions

The basis constitute a configuration space $\{\psi_{\not a},\ \psi_{\not b},\ \psi_{\not c}\}$.

$$\begin{aligned} \Psi_T &= \alpha\Phi(\vec{r}_{bc})\psi(\vec{r}_{ab},\ \vec{r}_{ac}) + \beta\Phi(\vec{r}_{ac})\psi(\vec{r}_{ab},\ \vec{r}_{bc}) + \gamma\Phi(\vec{r}_{ab})\psi(\vec{r}_{bc},\ \vec{r}_{ac}) \\ &= \alpha\psi_{\not a} + \beta\psi_{\not b} + \gamma\psi_{\not c} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}, \end{aligned}$$

Expand $\Phi(\vec{r}_{bc})$, $\Phi(\vec{r}_{ac})$ and $\Phi(\vec{r}_{ab})$ as a set of Laguerre polynomials

$$\chi_{nl}(r) = \sqrt{\frac{(2\lambda)^{2l+3} n!}{\Gamma(2l+3+n)}} r^l e^{-\lambda r} L_n^{2l+2}(2\lambda r), \qquad n = 1, 2, 3...$$

$$\psi_{\not a} = \sum_i \phi_i(\vec{r}_{bc})\psi(\vec{r}_{ab},\ \vec{r}_{ac}), \psi_{\not b} = \sum_i \phi_i(\vec{r}_{ac})\psi(\vec{r}_{ab},\ \vec{r}_{bc}), \psi_{\not c} = \sum_i \phi_i(\vec{r}_{ab})\psi(\vec{r}_{bc},\ \vec{r}_{ac}).$$

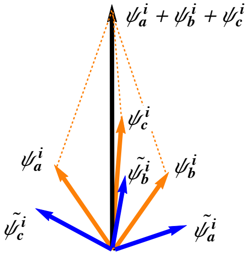
# Orthonormalization

We orthonormalize the $\{\psi_{\not{a}},\ \psi_{\not{b}},\ \psi_{\not{c}}\}$ into a new basis $\{\tilde{\psi}_{\not{a}},\ \tilde{\psi}_{\not{b}},\ \tilde{\psi}_{\not{c}}\}$.

$$\tilde{\psi}^i_{\not{a}} = \frac{1}{N_i}\Big[(\psi^i_{\not{a}} + \psi^i_{\not{b}} + \psi^i_{\not{c}}) - \sum_i x_{ij}\psi^j_{\not{a}}\Big],$$

$$\tilde{\psi}^i_{\not{b}} = \frac{1}{N_i}\Big[(\psi^i_{\not{a}} + \psi^i_{\not{b}} + \psi^i_{\not{c}}) - \sum_i x_{ij}\psi^j_{\not{b}}\Big],$$

$$\tilde{\psi}^i_{\not{c}} = \frac{1}{N_i}\Big[(\psi^i_{\not{a}} + \psi^i_{\not{b}} + \psi^i_{\not{c}}) - \sum_i x_{ij}\psi^j_{\not{c}}\Big],$$

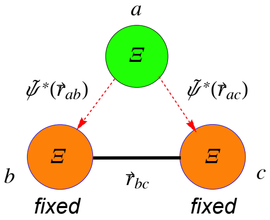



where the $x_{ij}$ is a parameter matrix which will be determined later. The $N_i$ are normalization coefficients.

Then the eigenvector for the three-body system $B^{(*)}_a B^{(*)}_b B^{(*)}_c$ can be written as a vector in the configuration space $\{\tilde{\psi}_{\not{a}},\ \tilde{\psi}_{\not{b}},\ \tilde{\psi}_{\not{c}}\}$. Therefore, we have

$$\Psi_T = \sum_i \tilde{\alpha}_i \tilde{\psi}^i_{\not{a}} + \sum_j \tilde{\beta}_j \tilde{\psi}^j_{\not{b}} + \sum_k \tilde{\gamma}_k \tilde{\psi}^k_{\not{c}},$$

# First order correction




The first order of the final wave function for the meson *a* could be the superposition of these two components

$$\tilde{\psi}^*(\vec{r}_{ab}, \vec{r}_{ac}) \quad = \quad C^*[\tilde{\psi}^*(\vec{r}_{ab}) \pm \tilde{\psi}^*(\vec{r}_{ac})]|\Xi\Xi\Xi\rangle$$

Accordingly, one can obtain the energy eigenvalue of the meson *a*

$$E_a^*(\Lambda,\ \vec{r}_{bc}) \quad = \quad \langle\tilde{\psi}(\vec{r}_{ab}, \vec{r}_{ac})|H_a|\tilde{\psi}(\vec{r}_{ab}, \vec{r}_{ac})\rangle$$

We define the BO potential as

$$V_{BO}^*(\Lambda,\ \vec{r}_{bc}) = E_a^*(\Lambda,\ \vec{r}_{bc}) - E_2(\Lambda).$$

# Application to the *NNN* system (Triton or Helium-3 nucleus)

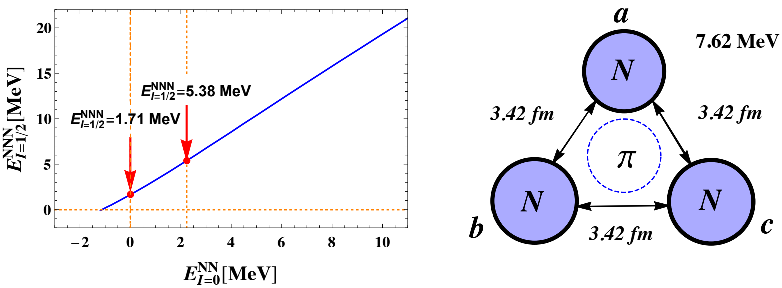


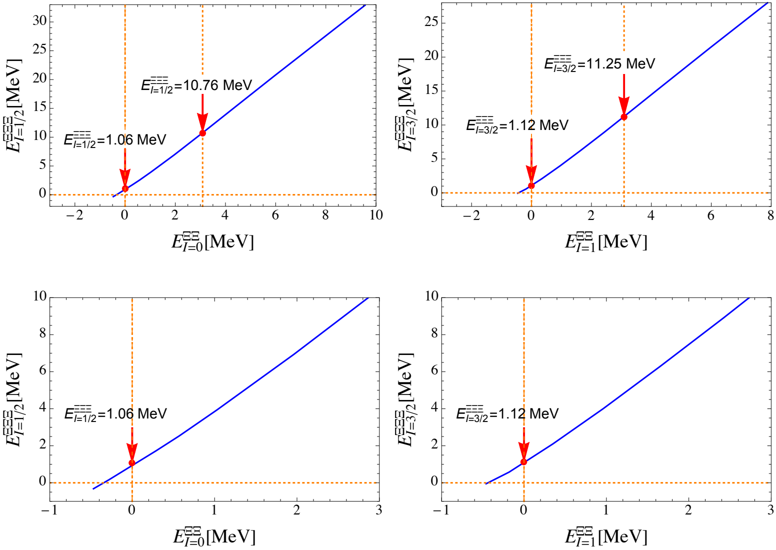

Figure: Dependence of the reduced three-body binding energy on the binding energy of its two-body subsystem (the deuteron). The result is comparable with the empirical binding energies of the triton (8.48 MeV) and helium-3 (7.80 MeV) nuclei.

# Numerical results for the *NNN* system (Triton or Helium-3 nucleus)

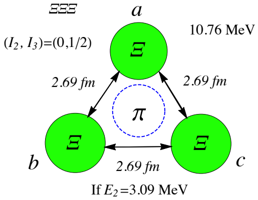
| $\Lambda$(MeV) | $E_2$(MeV) | $E_3$(MeV) | $E_T$(MeV) | $V_{BO}(0)$(MeV) | S wave(%) | D wave(%) | $r_{rms}$(fm) |
|---|---|---|---|---|---|---|---|
| 830.00 | -0.18 | -1.93 | -2.11 | -4.54 | 94.01 | 5.99 | 4.21 |
| 850.00 | -0.67 | -2.71 | -3.38 | -5.36 | 93.36 | 6.64 | 4.00 |
| 870.00 | -1.23 | -3.65 | -4.88 | -6.32 | 92.68 | 7.32 | 3.78 |
| 890.00 | -1.88 | -4.77 | -6.66 | -7.42 | 91.99 | 8.01 | 3.54 |
| 899.60 | -2.23 | -5.38 | -7.62 | -8.00 | 91.66 | 8.34 | 3.42 |
| 900.00 | -2.25 | -5.41 | -7.66 | -8.03 | 91.64 | 8.36 | 3.42 |
| 920.00 | -3.05 | -6.85 | -9.90 | -9.35 | 90.97 | 9.03 | 3.18 |
| 940.00 | -3.98 | -8.51 | -12.49 | -10.83 | 90.35 | 9.65 | 2.95 |
| 960.00 | -5.03 | -10.42 | -15.45 | -12.46 | 89.76 | 10.24 | 2.74 |
| 980.00 | -6.21 | -12.57 | -18.78 | -14.23 | 89.23 | 10.77 | 2.54 |
| 1000.00 | -7.55 | -14.97 | -22.51 | -16.14 | 88.73 | 11.27 | 2.37 |
| 1020.00 | -9.04 | -17.61 | -26.65 | -18.19 | 88.27 | 11.73 | 2.23 |
| 1040.00 | -10.69 | -20.51 | -31.20 | -20.37 | 87.84 | 12.16 | 2.10 |

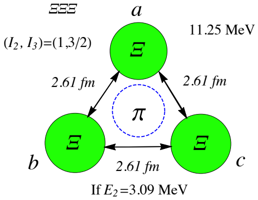
Table: Bound state solutions for the *NNN* system with isospin $I_3 = 1/2$. $E_2$ is the energy eigenvalue of its subsystem. $E_3$ is the reduced three-body energy eigenvalue relative to the break-up state of the *NNN* system. $E_T$ is the total three-body energy eigenvalue relative to the *NNN* threshold.

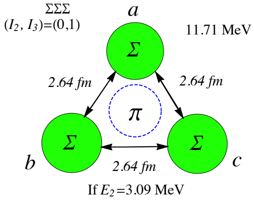
# Numerical results for the ΛΛΛ, ΞΞΞ and ΣΣΣ

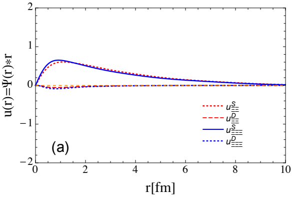


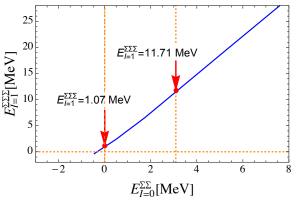
# Numerical results for the ΛΛΛ, ΞΞΞ and ΣΣΣ

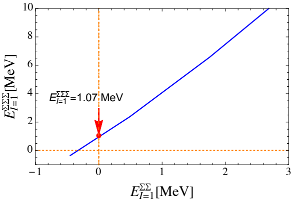


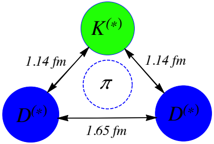



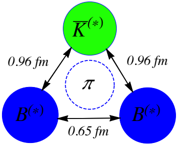



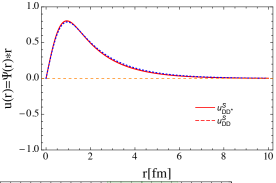



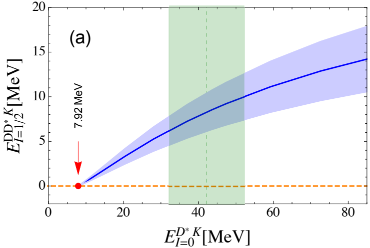



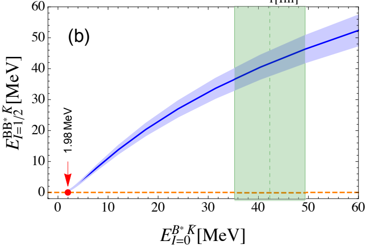
# Numerical results for the Doubly heavy tri-meson bound states




| (MeV) | $DD^*K$ | $BB^*\bar{K}$ | $D\bar{D}^*K$ | $B\bar{B}^*\bar{K}$ |
|---|---|---|---|---|
| Binding energy | $8.29^{+4.32}_{-3.66}$ | $41.76^{+8.84}_{-8.49}$ | $8.29^{+6.55}_{-6.13}$ | $41.76^{+9.02}_{-8.68}$ |
| Mass | $4317.92^{+3.66}_{-4.32}$ | $11013.65^{+8.49}_{-8.84}$ | $4317.92^{+6.13}_{-6.55}$ | $11013.65^{+8.68}_{-9.02}$ |

# Numerical results for the tri-meson bound state $BBB^*$

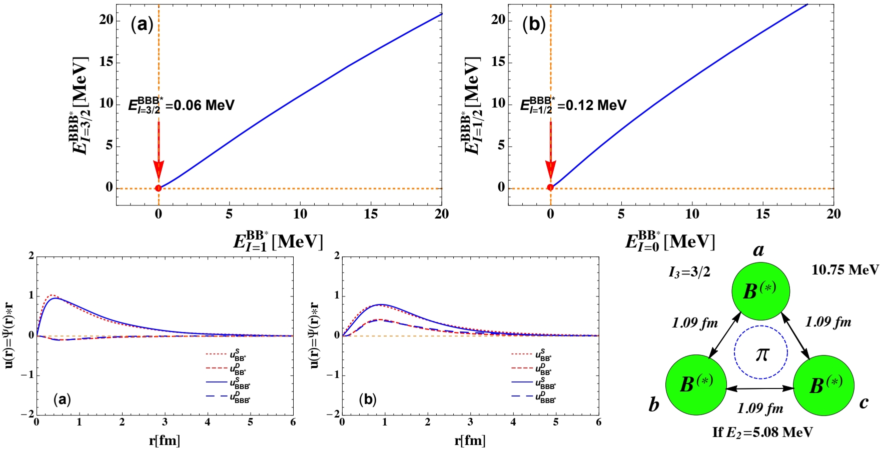



Figure: Here we chose the parameter $\Lambda = 1440$ MeV in (a) and $\Lambda = 1107.7$ MeV in (b) for a better comparison of all the cases, since they have the same two-body binding energy of 5.08 MeV.

# Summary

- We predict that the triton-like molecular states for the $\Xi\Xi\Xi$ and $\Sigma\Sigma\Sigma$ systems are probably existent as long as the molecular states of their two-body subsystems exist.
- In our calculations, we use the Born-Oppenheimer potential method to construct our interpolating wave functions, which can be regarded as a version of the variational principle which always gives an upper limit of the energy of a system.
- Other configurations $NN\Lambda$, $N\Lambda\Lambda$, $NN\Xi$, $N\Xi\Xi$, $NN\Sigma$, $N\Sigma\Sigma$ will be explored in a future work.

*Thank you very much!*