

Triangle singularity in the possible reaction of
 $e^+e^- \rightarrow X(3872)\gamma$

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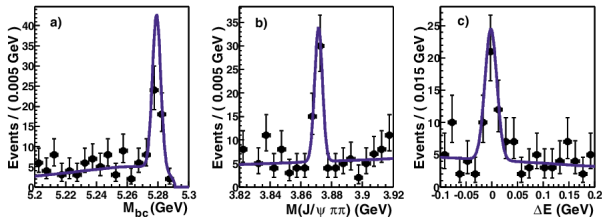
11th.Jan.2020

- The discovery of $X(3872)$ and its properties
- The possibility of triangle singularity in $e^+e^- \rightarrow X(3872)\gamma$
- The interpretation of $X(3872)$ via the molecular state
- The case for $X(3872)$ to be a hybrid state
- Conclusion

The discovery of X(3872) and its properties

The X(3872) resonance was first observed by the Belle Collaboration in 2003 in exclusive $B^\pm \rightarrow K^\pm \pi^+ \pi^- J/\psi$ decays.

S.K.Choi, et al.(Phys. Rev. Lett. 91 (2003) 262001)



The discovery of X(3872) and its properties

- Quantum numbers of X(3872) $J^{PC} = 1^{++}$
R.Aaij et al.(Phys. Rev. Lett. 110, 222001 (2013))
- The average mass of (3871.69 ± 0.17) MeV
M.Tanabashi et al. [PDG], Phys. Rev. D, 98,030001 (2018)
and a width < 1.2 MeV at 90% C.L..
S.K. Choi et al. (Phys. Rev. D 84,052004 (2011))
- The mass threshold of $M_{D^0} + M_{D^{*0}} = (3871.81 \pm 0.09)$ MeV.
Suppose X(3872) is a $D^0\bar{D}^{*0}/\bar{D}D^*$ bound state.
P.Wang, X.G.Wang, (Phys. Rev. Lett 11, 042002 (2013))
C.E.Thomas and F.E.Close, (Phys. Rev. D 78,034007 (2008))
E.Braaten, H.W.Hammer, T.Mehen, (Phys. Rev. D
82,034018 (2010))
V.Baru et al. (Phys. Lett. B 726, 537 (2013))

The discovery of $X(3872)$ and its properties

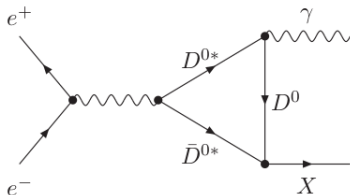
- As a virtual state of $D^0\bar{D}^{*0}/\bar{D}D^*$
C.Hanhart et al. (Phys. Rev. D 76, 034007 (2007))
X.W.Kang and J.A.Oller (Eur. Phys. J. C 77, no. 6, 399 (2017))
- As a tetraquark state
L.Maiani et al. (Phys. Rev. D 71, 014028 (2005))
T.F.Carames et al. (Phys. Rev. Lett 103, 222001 (2009))
- As a hybrid state i.e. a mixture of a charmonium $chi_{c1}(2P)$ with a $D^0\bar{D}^{*0}/\bar{D}D^*$ component
C.Meng, Y.J.Gao and K.T.Chao, (Phys. Rev. D 87, 074035 (2013))
M.Suzuki (Phys. Rev. D 72, 114013 (2005))

The possibility of the triangle singularity

There could exist a triangle loop based on:

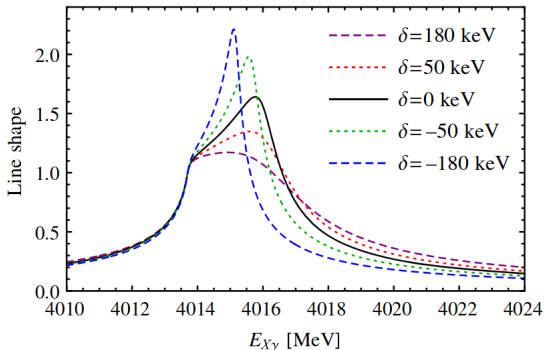
F.K.Guo (Phys. Rev. Lett 122, 202002 (2019))

- e^+e^- annihilation
- e^+e^- annihilation into a virtual can produce a pair of spin-1 charm mesons $D^{*0}\bar{D}^{*0}$
- The threshold of M_{D^0} and $M_{D^{*0}}$ is very closed to the mass of $X(3872)$.



How to deal with the triangle loop?

- F.K.Guo pointed out $M_{\chi(3872)}$ could be indirectly measured by the line shape between 4010MeV and 4020MeV precisely.

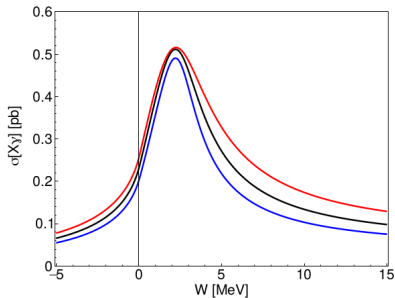


$$\delta \equiv m_{D^0} + m_{D^{*0}} - m_{\chi(3872)}$$

The line shape is sensitive to δ

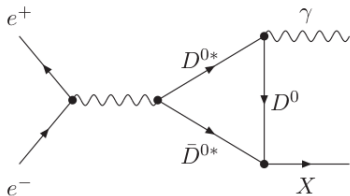
How to deal with the triangle loop?

- E.Braaten, L.P.He and K.Ingles also use the nonrelativistic normalizations to discuss on the triangle singularity.



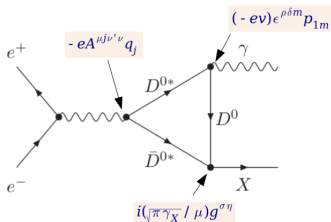
E.Braaten, L.P.He and K.Ingles (Phys. Rev. D 100,031501(R) (2019))

How to deal with the triangle loop?



- We adopt the method of QFT, with advantages:
 1. Standard techniques of QFT to calculate the loop functions.
 2. High-effective LoopTools numerical software.

The effective vertices of the triangle loop



- $\gamma \rightarrow D^{*0}\bar{D}^{*0}$: $-eA^{\mu j\nu'\nu}q_j$

$$A^{ijkl} = A_0\delta^{ij}\delta^{kl} + \frac{3}{2\sqrt{5}}A_2(\delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk} - \frac{2}{3}\delta^{ij}\delta^{kl})$$

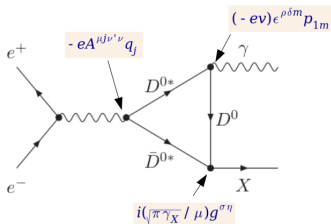
$$A_0 = 8\text{GeV}^{-1}, A_2 = 15\text{GeV}^{-1}$$

T.V.Uglov et al.(JETP Lett. 105, 1 (2017))

- We revise the vertex into four indices...

$$A^{\mu j\nu'\nu} = A_0g^{\mu j}g^{\nu'\nu} + \frac{3}{2\sqrt{5}}A_2(g^{\mu\nu'}g^{j\nu} + g^{\mu\nu}g^{j\nu'} - \frac{2}{3}g^{\mu\nu'}g^{j\nu})$$

The effective vertices of the triangle loop



- $D^{*0}D^0 \rightarrow \gamma$: $(-ev)\epsilon^{\rho\delta m} p_{1m}$
 $v = 0.95\text{GeV}^{-1}$

determined by the radiative decay width of D^{*0}

J.L.Rosner, (Phys. Rev. D 88, 034034 (2013))

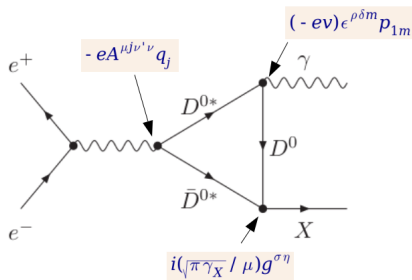
- $\bar{D}^{*0}D^0 \rightarrow X(3872)$: $i(\sqrt{\pi\gamma_X}/\mu)g^{\sigma\eta}$

determined by XEFT, related to $\gamma_X = \sqrt{2\mu|E_X|}$, where E_X is the binding energy of $X(3872)$.

S.Fleming et al. (Phys. Rev. D 76, 034006 (2007))

E.Braaten (Phys. Rev. D 91, 114007 (2015))

How to deal with the triangle loop?



- The vector meson D^{*0} is unstable with the width $\Gamma_{D^{*0}} = 55.3 \pm 1.4 \text{ keV} \ll m_{D^{*0}}$, then the propagators of D^{0*} and \bar{D}^{0*} have the form (Breit-Wigner mass): $\frac{i}{p^2 - m_p^2 + im_p \Gamma_{tot}}$.
- The pseudoscalar meson D^0 decays weakly, $\Gamma_{D^0} \approx 0$.

The construction of the triangle loop

The amplitude of the Feynman diagram can be expressed as:

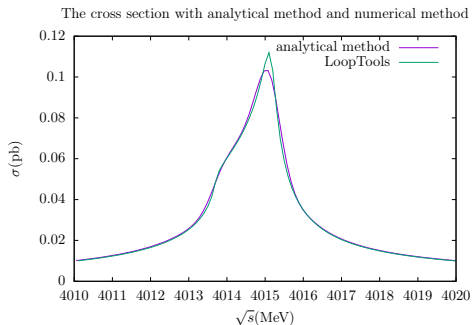
$$I^\mu = (-ev) A^{\mu j \nu'} \epsilon^{\rho \delta m} p_{1m} \epsilon_{\gamma \delta}^*(p_1, \lambda_1) i(\sqrt{\pi \gamma_X} / \mu) (-g^{\sigma \eta}) \epsilon_{X \eta}^*(p_2, \lambda_2) \\ \cdot \int \frac{d^4 q}{(2\pi)^4} \frac{-ig_{\sigma \nu} + \frac{q_\sigma q_\nu}{M_*^2}}{q^2 - M_*^2 + i\epsilon} q_j \frac{-ig_{\nu' \rho} + \frac{(q+k)_{\nu'}(q+k)_\rho}{M_*^2}}{(q+k)^2 - M_*^2 + i\epsilon} \frac{1}{(q+k-p_1)^2 - M_D^2},$$

neglect the longitudinal part, where the integral part can be expressed as:

$$\Pi_j = \int \frac{d^4 q}{(2\pi)^4} \frac{q_j}{[q^2 - M_*^2 + iM_*\Gamma_*][(q+k)^2 - M_*^2 + iM_*\Gamma_*][(q+k-p_1)^2 - M_D^2 + i\epsilon]}.$$

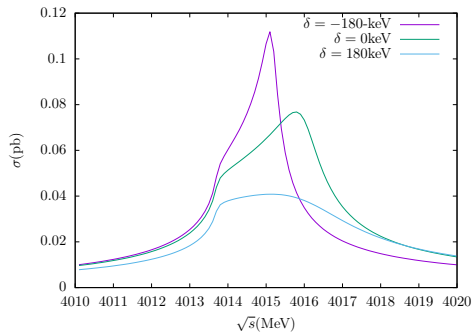
The construction of the triangle loop

We use semi-analytical method with dimensional regularization and numerical tools to calculate the triangle loop respectively.

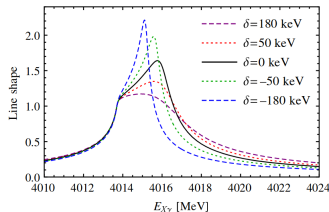
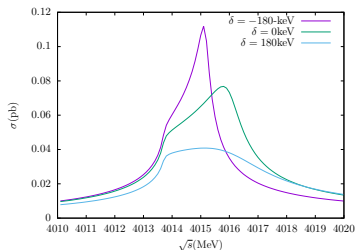


The result of the triangle singularity

$$\delta \equiv m_{D^0} + m_{D^{*0}} - m_{\chi(3872)}$$

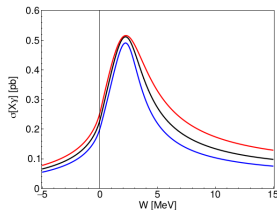
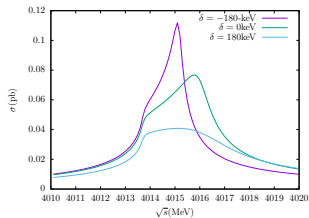


The comparison with F.K.Guo



- The line shape of $X(3872)\gamma$ is consistent with the prediction by F.K.Guo.
- There exists a sensitive effect on the line shape via tuning $M_{X(3872)}$.
- The positions of the peaks are consistent with the prediction of F.K.Guo.

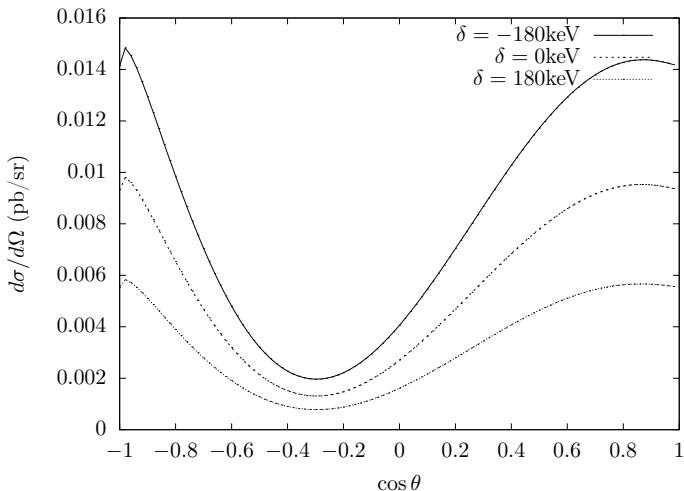
The comparison with E.Braaten, L.P.He and K.Ingles



- The value of the peak is less than their result about one order magnitude.
- The position of the peak of our prediction is less than the authors' about 1MeV.

Something wrong?

The differential cross-section at $\sqrt{s} = (4015.2 - i0.08)\text{MeV}$



The case for $X(3872)$ being a hybrid state

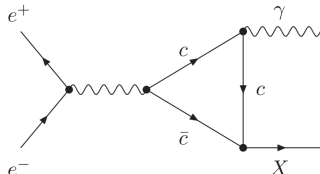
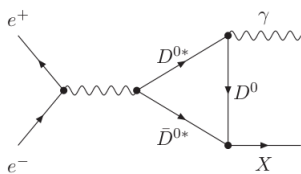
If the $X(3872)$ particle is assumed to be a hybrid state of $D^{*0}\bar{D}^0/\bar{D}^{*0}D^0$ and $c\bar{c}$ charmonium state, then an interference effect will be hold:

$$J_\nu^\xi = \sqrt{1 - \xi^2} J_\nu^m + \xi \sigma J_\nu^h,$$

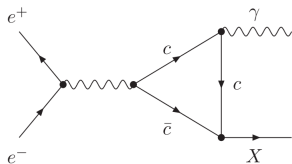
set $\sigma = 1\text{GeV}$,

where J_ν^h is the charmonium hybrid current,
 J_ν^m is the molecular current.

W.Chen et al.(Phys. Rev. D 88.045027)



The effective vertices $X(3872)$ being a charmonium state

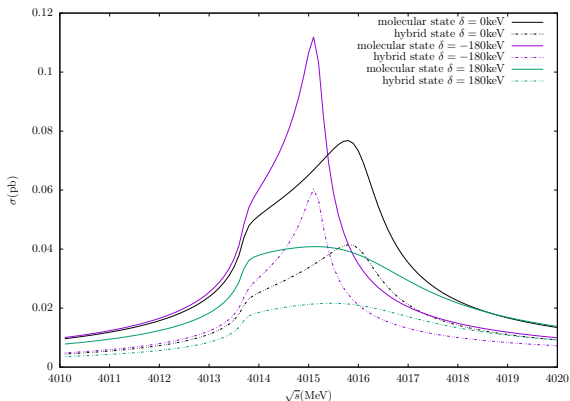


- $\gamma \rightarrow c\bar{c}$: $(-ieQ_n\gamma^\nu)$
- $cc \rightarrow \gamma$: $(-ieQ_n\gamma^\rho)$
- $cc \rightarrow X$: $[-ig_X(\frac{\lambda^a}{2})(\frac{\lambda^a}{2})\gamma^\sigma\gamma^5]$

No triangle singularity!

Interference effect of $X(3872)$ being a hybrid state

Consider $X(3872)$ as a hybrid state, and set $\xi = \frac{1}{\sqrt{2}}$:



$$\delta \equiv m_{D^0} + m_{D^{*0}} - m_{X(3872)}.$$

- The results by QFT in line shape and for the energy of the peaks is consistent with the prediction of F.K.Guo with non-relativity approximation.
However, the value of the cross-section is less than the prediction of L.P.He et al. for about one order of magnitude.
- Something wrong with the differential cross-section, perhaps due to the definition of the effective vertices.
- The situation of $X(3872)$ being a hybrid state has been considered, which has a respectively large effect on the interference.

Thank you!