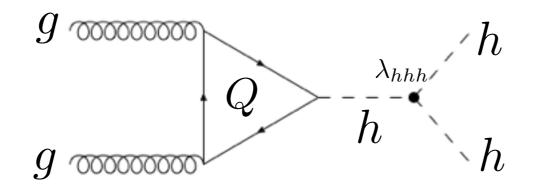
Mini-workshop on precision physics and future colliders

HIGGS BOSON PAIR PRODUCTION VIA GLUON FUSION AT N³LO IN QCD



HUA-SHENG SHAO

in collaboration with L.-B. Chen, H.T. Li and J. Wang (1909.06808)







IHEP, BEIJING 14 OCTOBER 2019

Data driven

M. Mangano's talk at Higgs Hunting 2019 in Paris

- Dark matter
- Neutrino masses
- Matter vs anti-matter asymmetry
- Dark energy
- •

Theory driven

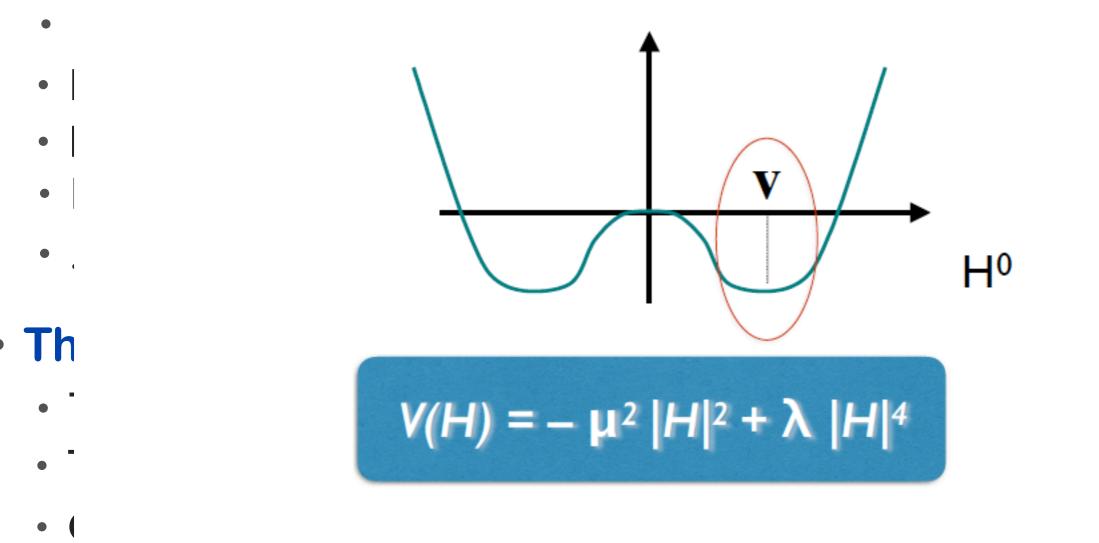
- The hierarchy problem and naturalness
- The flavour problem (origin of fermion families, mass/mixing pattern)
- Quantum gravity
- Origin of inflation

The path to answer these questions is ambiguous.

One question stemming from the LHC appears to single out a unique defined direction ...

• Data driven

M. Mangano's talk at Higgs Hunting 2019 in Paris



Who ordered that ?

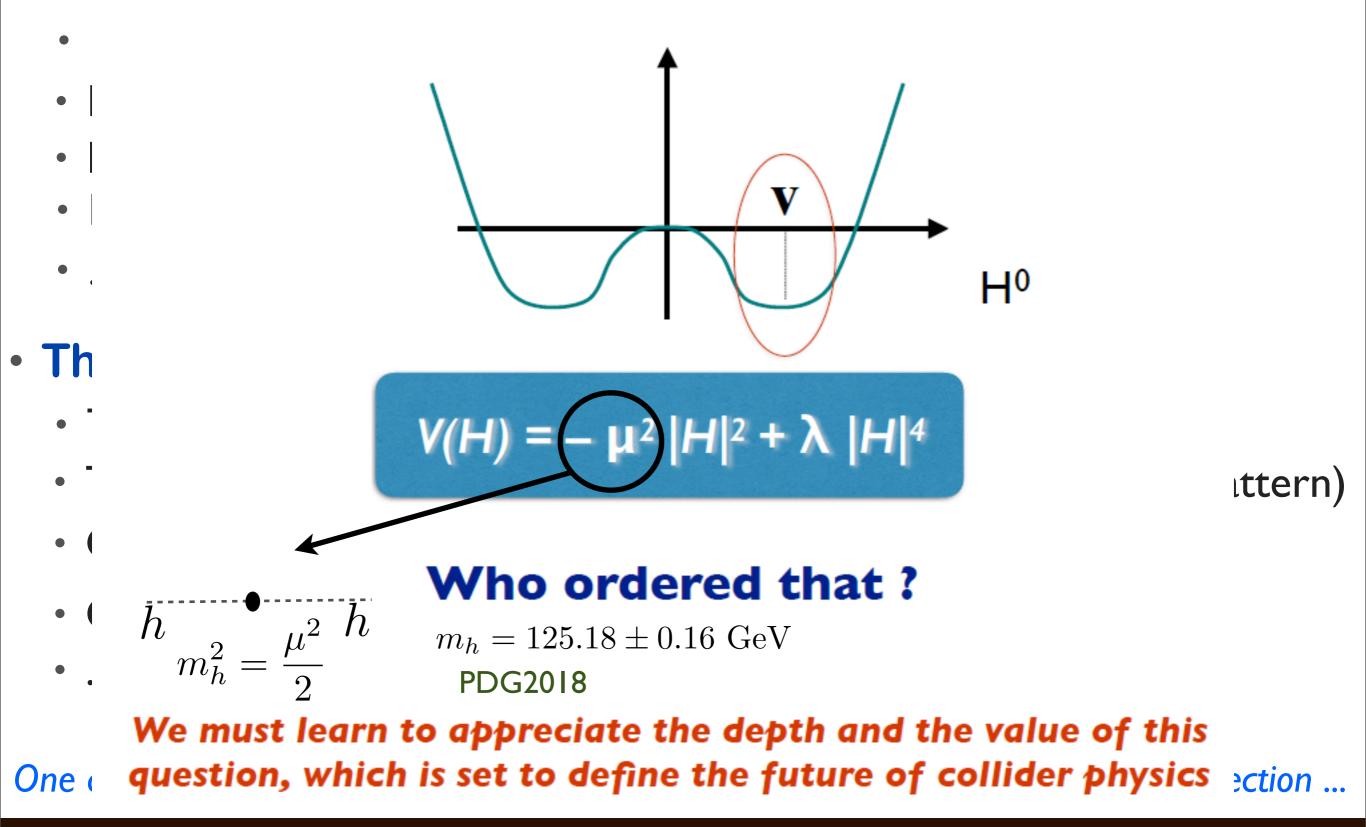
We must learn to appreciate the depth and the value of this One (question, which is set to define the future of collider physics <u>ection</u> ...

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ittern)

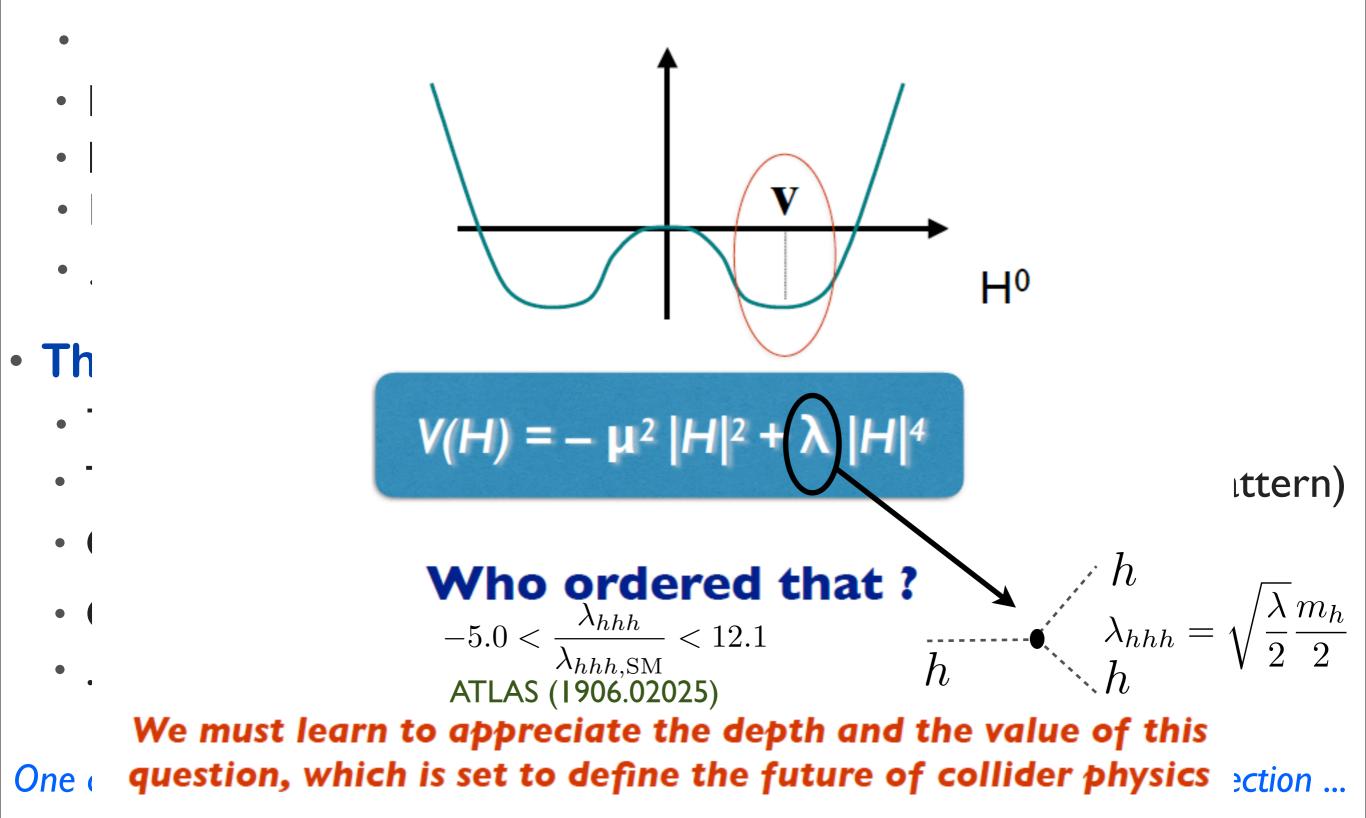
Data driven

M. Mangano's talk at Higgs Hunting 2019 in Paris



Data driven

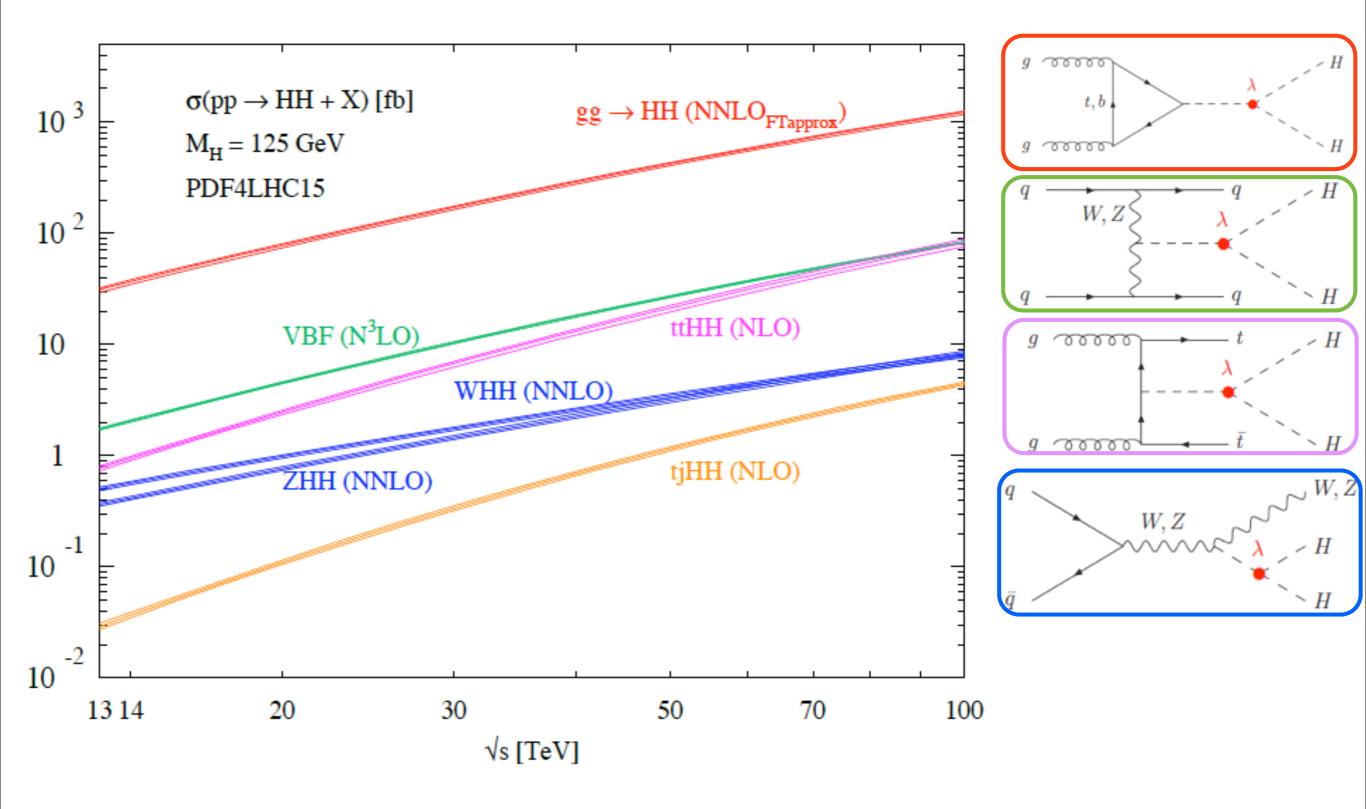
M. Mangano's talk at Higgs Hunting 2019 in Paris



HIGGS BOSON PAIR PRODUCTION



arXiv:1910.00012



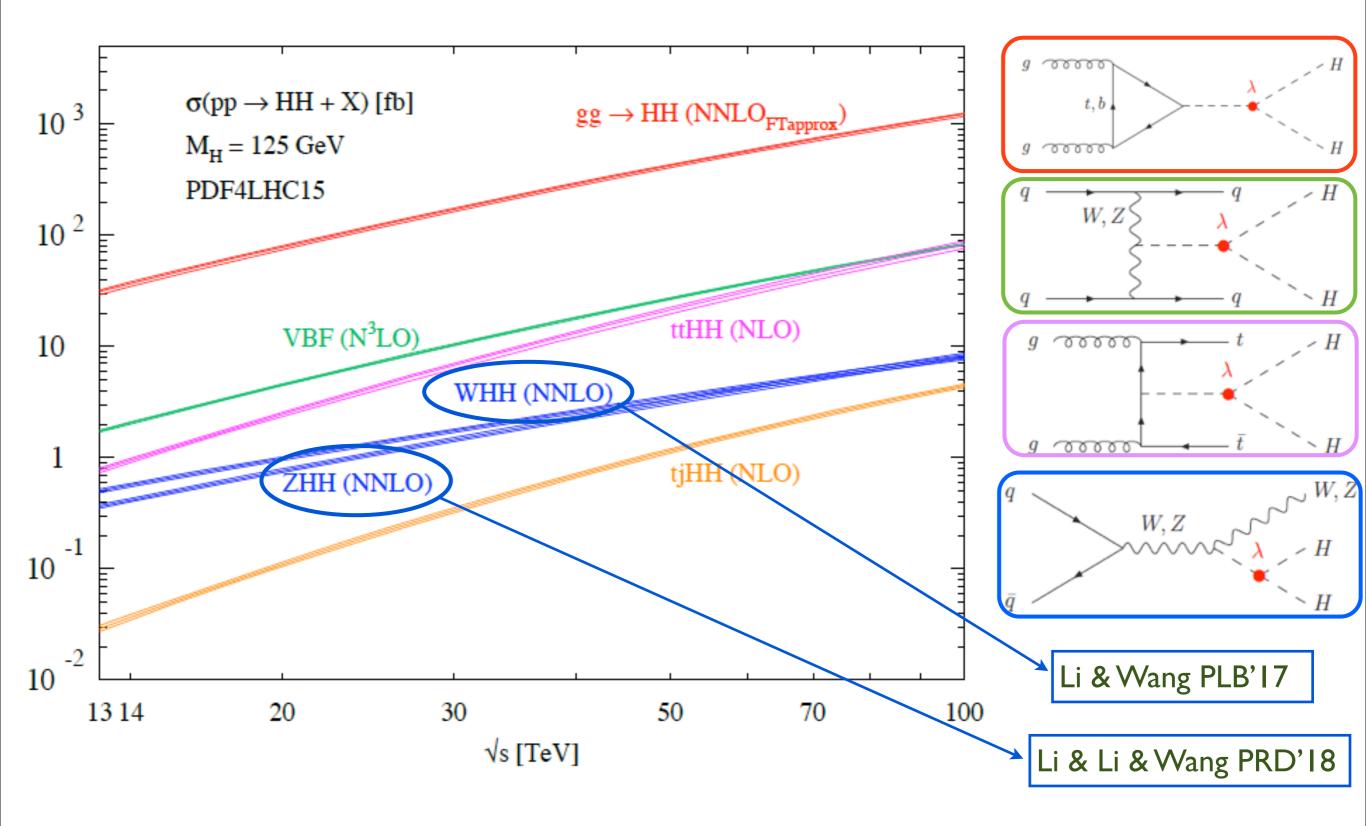
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HIGGS BOSON PAIR PRODUCTION



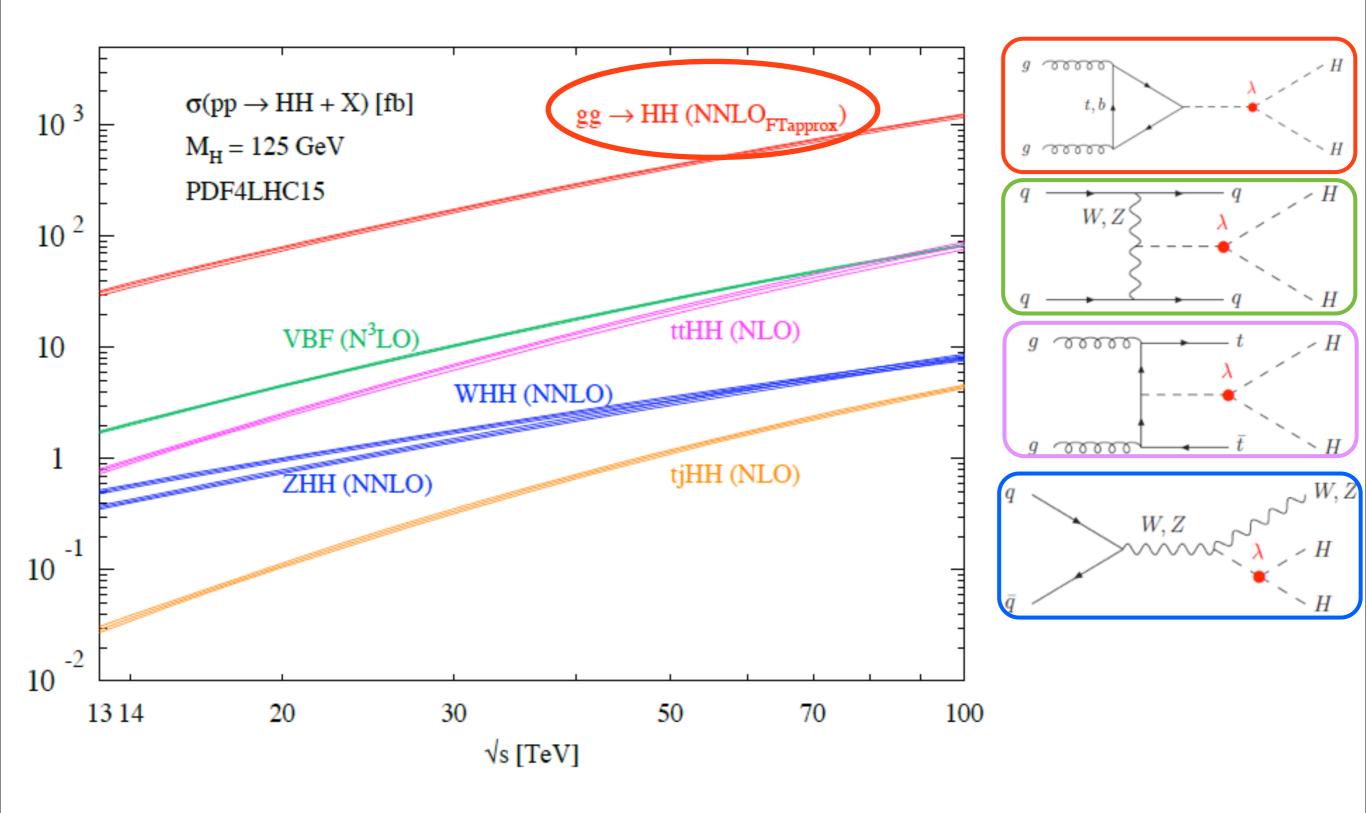
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HIGGS BOSON PAIR PRODUCTION

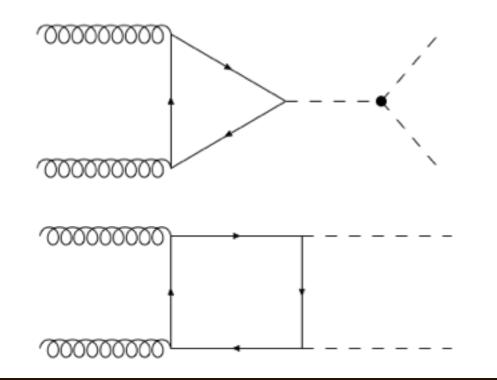


arXiv:1910.00012





- Full top-quark mass dependence
 - Leading order (LO) is a loop-induced process

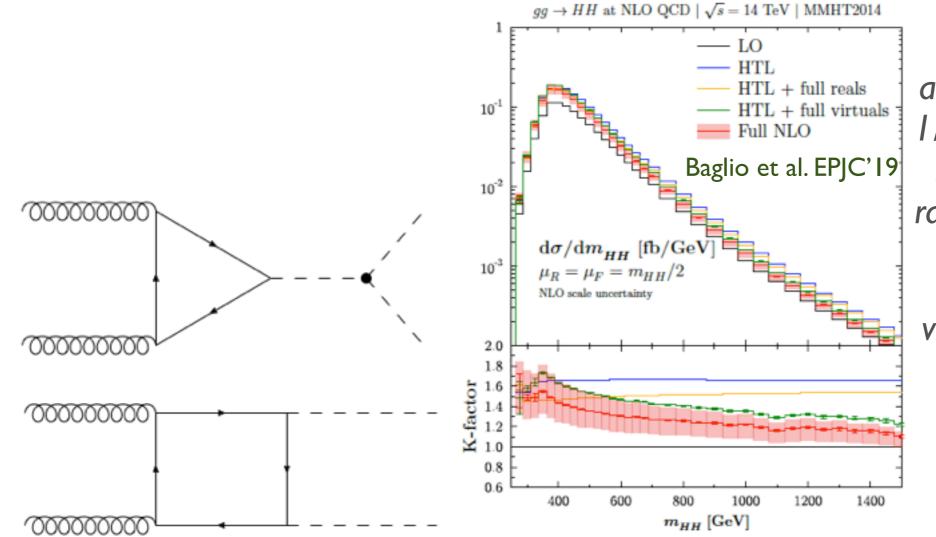




Full top-quark mass dependence

- Leading order (LO) is a loop-induced process
- Next-to-leading order (NLO) was computed numerically

Borowka et al. PRL'16, JHEP'16; Baglio et al. EPJC'19



Reasonable approximations to extend I/m_t result (rescaled exact Born, include exact real radiation) can fail the true K factor significantly.

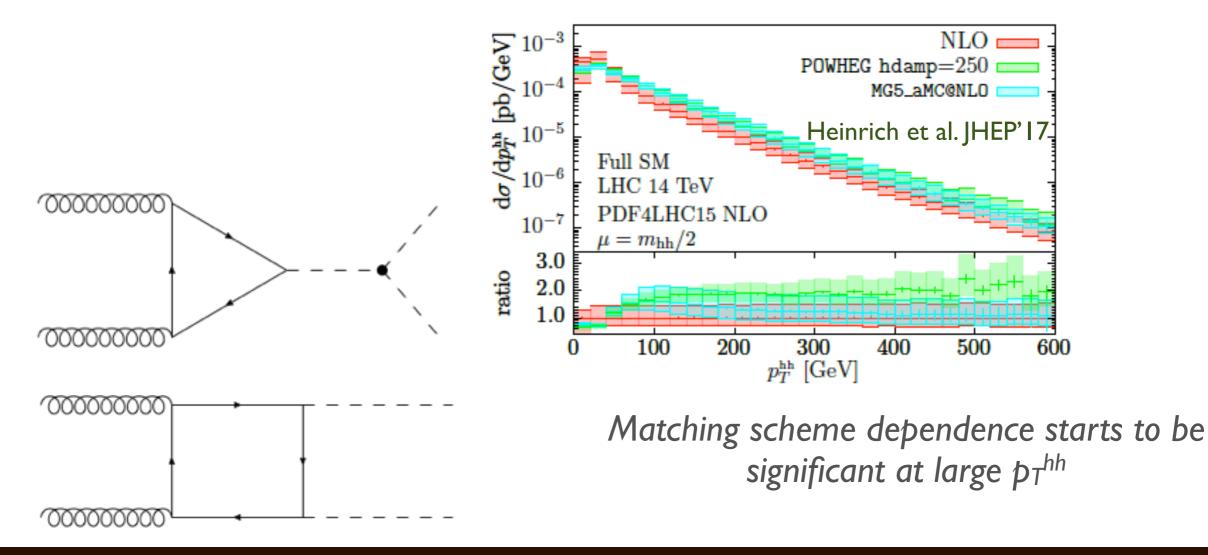
virtual is so crucial, which is remaining to be understood



Full top-quark mass dependence

- Leading order (LO) is a loop-induced process
- Next-to-leading order (NLO) was computed numerically
 - Borowka et al. PRL'16, JHEP'16; Baglio et al. EPJC'19
- ... even after matching to parton showers (i.e. NLO+PS)

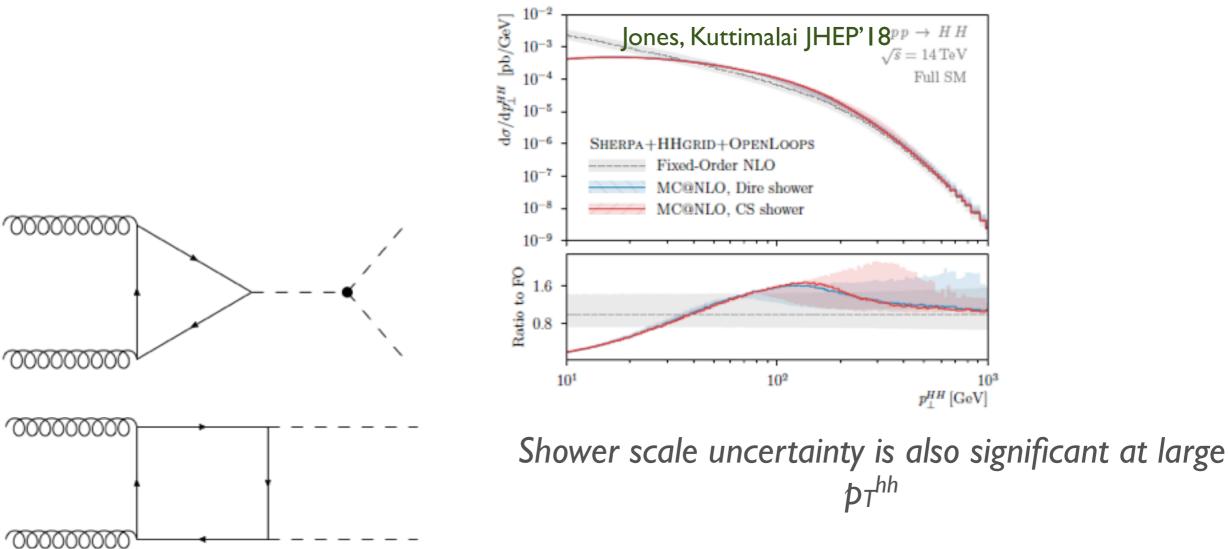
Heinrich et al. JHEP'17, JHEP'19; Jones, Kuttimalai JHEP'18





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Heinrich et al. JHEP'17, JHEP'19; Jones, Kuttimalai JHEP'18

- ... even after matching to parton showers (i.e. NLO+PS)
- Scale unc. (>10%)

 Energy
 13 TeV
 14 TeV
 27 TeV
 100 TeV

 NLO
 27.78^{+13.8%}/_{-12.8%} fb
 32.88^{+13.5%}/_{-12.5%} fb
 127.7^{+11.5%}/_{-10.4%} fb
 1147^{+10.7%}/_{-9.9%} fb

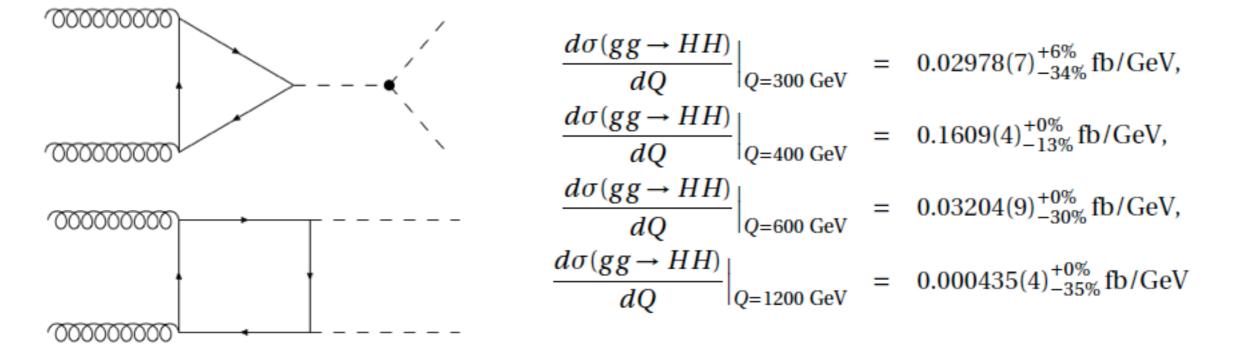


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- Scale unc. (>10%)
- ... and large top-quark mass scheme dependence



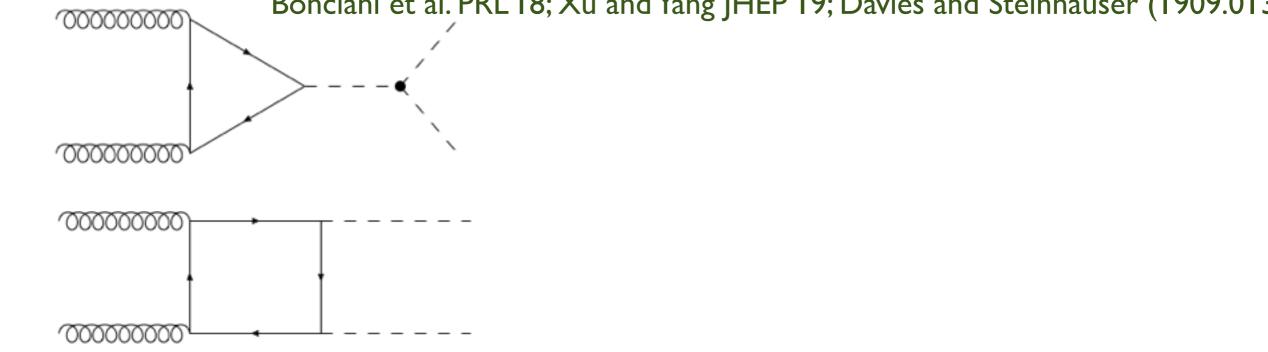


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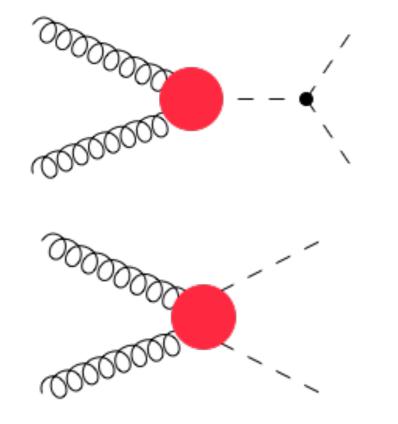
- Scale unc. (>10%)
- ... and large top-quark mass scheme dependence
- A lot of analytical approximations (well-motivated to deepen understanding) Grigo et al. NPB'13, NPB'15; Degrassi EPJC'16;, Davies et al. JHEP'18, JHEP'19; Bonciani et al. PRL'18; Xu and Yang JHEP'19; Davies and Steinhauser (1909.01361)





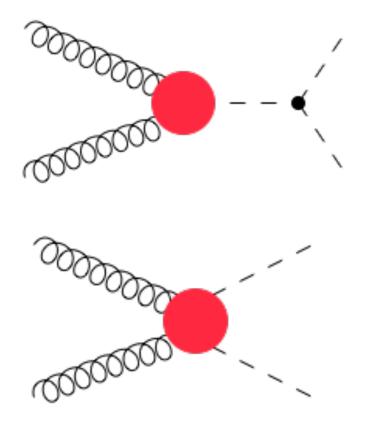
Infinite top-quark mass limit

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} G^a_{\mu\nu} G^{a\ \mu\nu} \left(C_h \frac{h}{v} - C_{hh} \frac{h^2}{2v^2} \right)$$





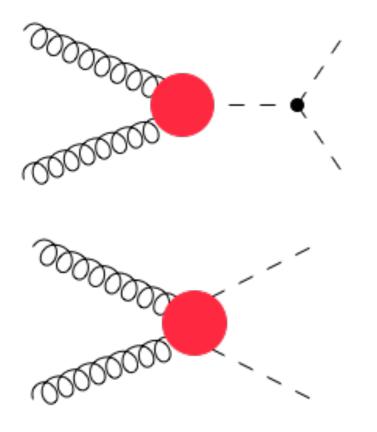
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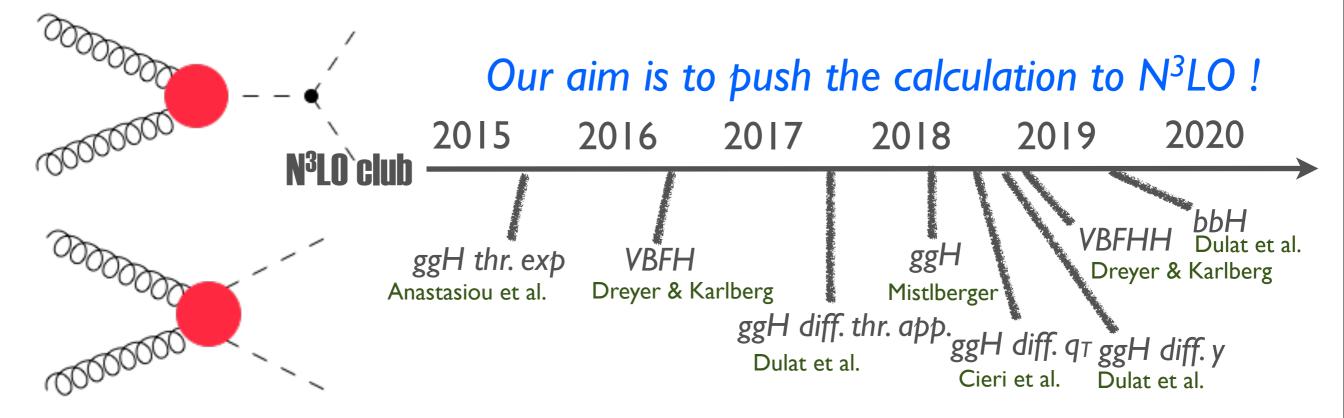
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- Technically, it is much easier to achieve high precision
 - NLO was 20 years old Dawson PRD'98
 - NNLO was known as well Florian and Mazzitelli PLB'13,PRL'13; Grigo et al. NPB'14; Florian et al. JHEP'16
 - Threshold resummation Shao et al. JHEP'13; Florian and Mazzitelli JHEP'15, JHEP'18
 - NLO_{FTapprox}: NLO plus full top quark mass in Born and real Frederix et al. PLB'14; Maltoni et al. JHEP'14
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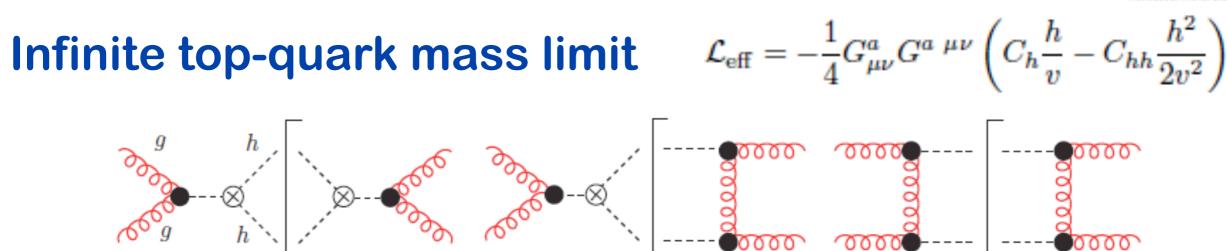
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(a)



(c)

	LO	NLO	NNLO	N ³ LO
total	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
a	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
b	0	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
с	0	0	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$

Chen, Li, HSS, Wang (1909.06808)

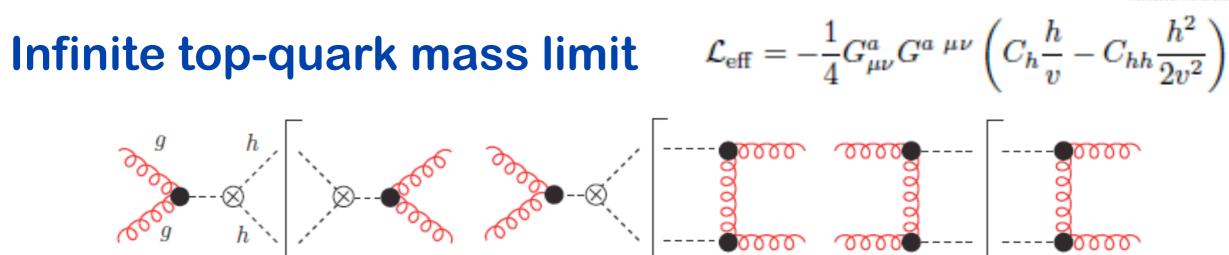
MINI-WORKSHOP, IHEP

(b)





a



(c)

	LO	NLO	NNLO	N ³ LO
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Chen, Li, HSS, Wang (1909.06808)

class-a: same topology as ggH

$$\begin{aligned} \frac{d\sigma_{hh}^a}{dm_{hh}} &= f_{h \to hh} \left(\frac{C_{hh}}{C_h} - \frac{6\lambda v^2}{m_{hh}^2 - m_h^2} \right)^2 \times \sigma_h(m_h \to m_{hh}) \\ f_{h \to hh} &= \frac{\sqrt{m_{hh}^2 - 4m_h^2}}{16\pi^2 v^2} \end{aligned}$$

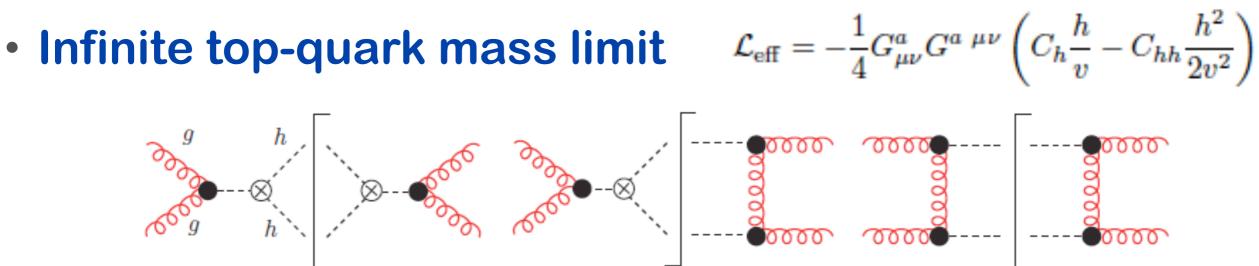
MINI-WORKSHOP, IHEP

(Ъ





(a)



(c)

N³LO NNLO LO NLO $\mathcal{O}(\alpha_s^5)$ (α_s^4) total α_s^3 $\mathcal{O}(\alpha_s^2)$ О О α_s^5 α_s^4 α_s^3 О \mathcal{O} (α_s^2) O \mathbf{a} α_s^4 α_s^5 α_s^3 О b 0 (α_s^4) Ó 0 0 \mathbf{c}

Chen, Li, HSS, Wang (1909.06808)

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From iHixs2 Dulat et al. CPC'18

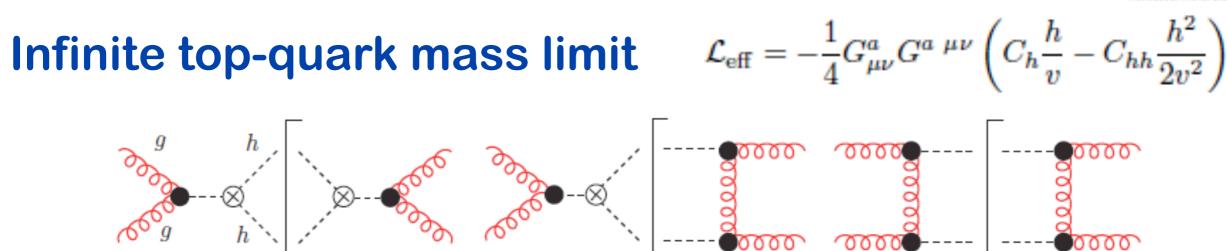
MINI-WORKSHOP, IHEP

(b)





(a)



(c)

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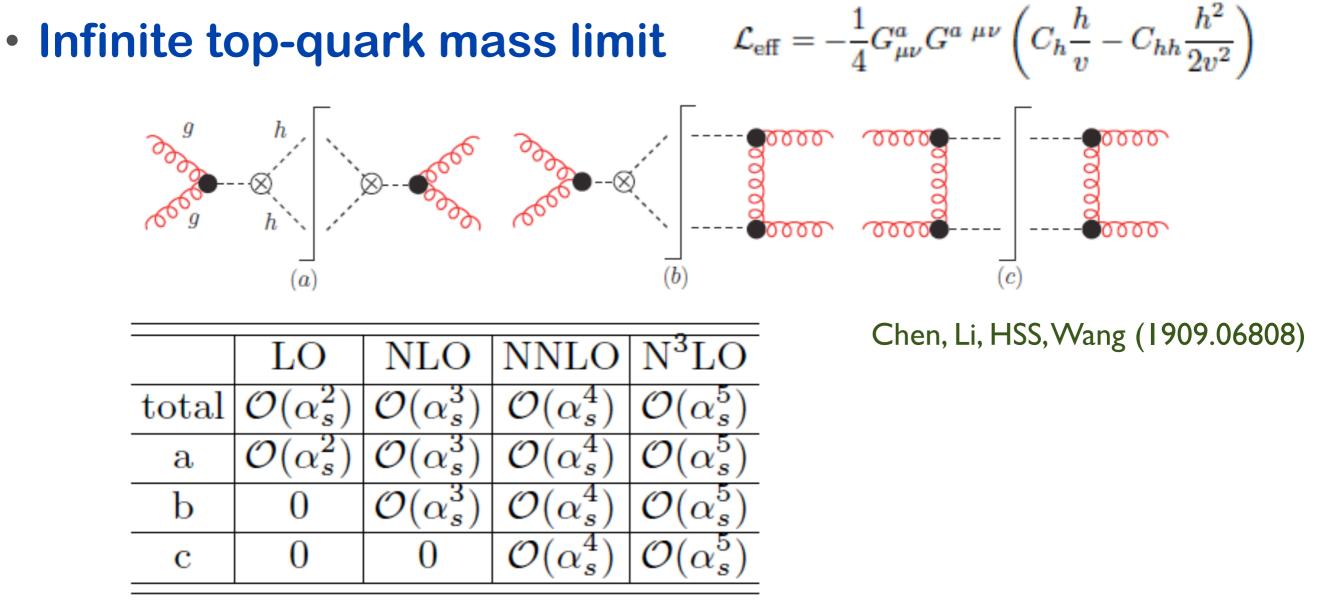
Chen, Li, HSS, Wang (1909.06808)

MINI-WORKSHOP, IHEP

(b)





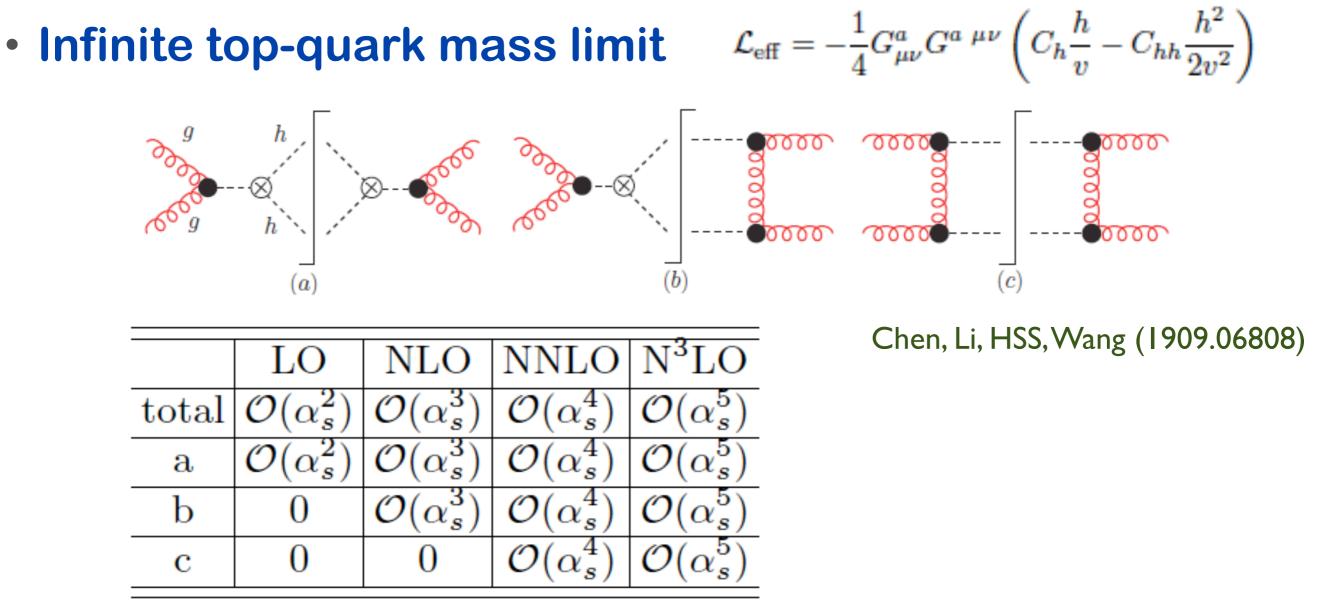


• class-b: need NNLO as its as² is zero (q_T subtraction, Catani & Grazzini PRL'07)

$$d\sigma^b_{hh} = d\sigma^b_{hh} \Big|_{p_T^{hh} < p_T^{\text{veto}}} + d\sigma^b_{hh} \Big|_{p_T^{hh} > p_T^{\text{veto}}}$$





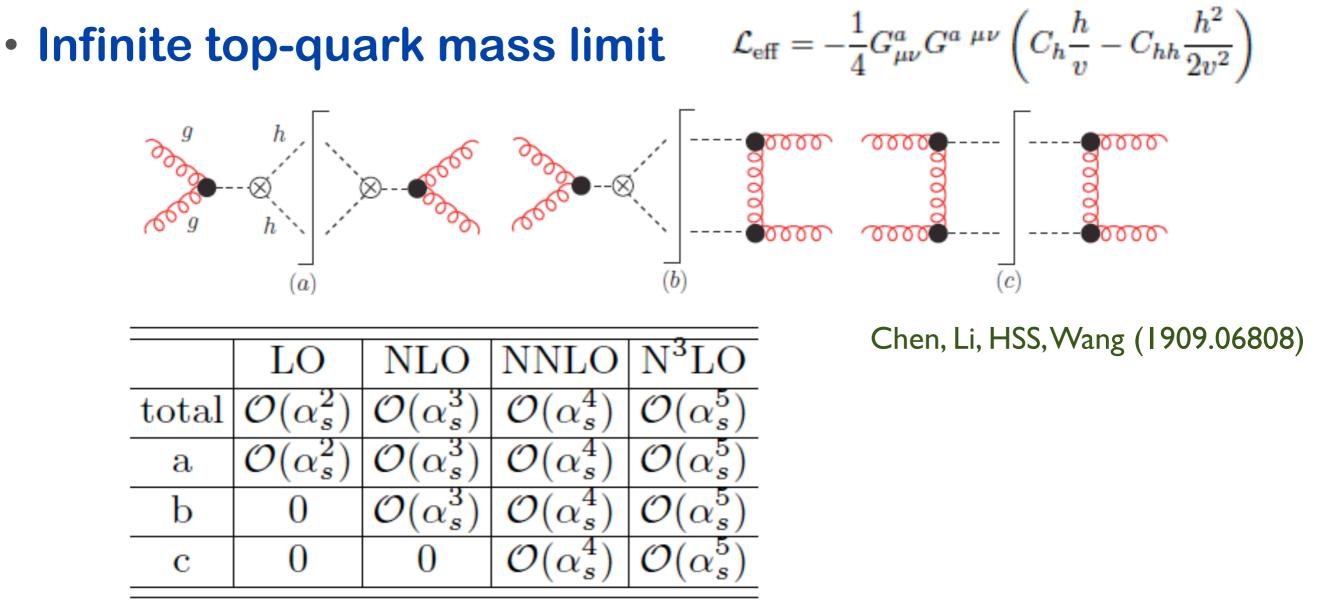


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SCET:
$$d\sigma_{hh}^{b} \right|_{p_{T}^{hh} < p_{T}^{\text{veto}}} = \mathcal{H} \otimes \mathcal{B}_{1} \otimes \mathcal{B}_{2} + \mathcal{O}\left(\left(\frac{p_{T}^{\text{veto}}}{Q} \right)^{2} \right)$$







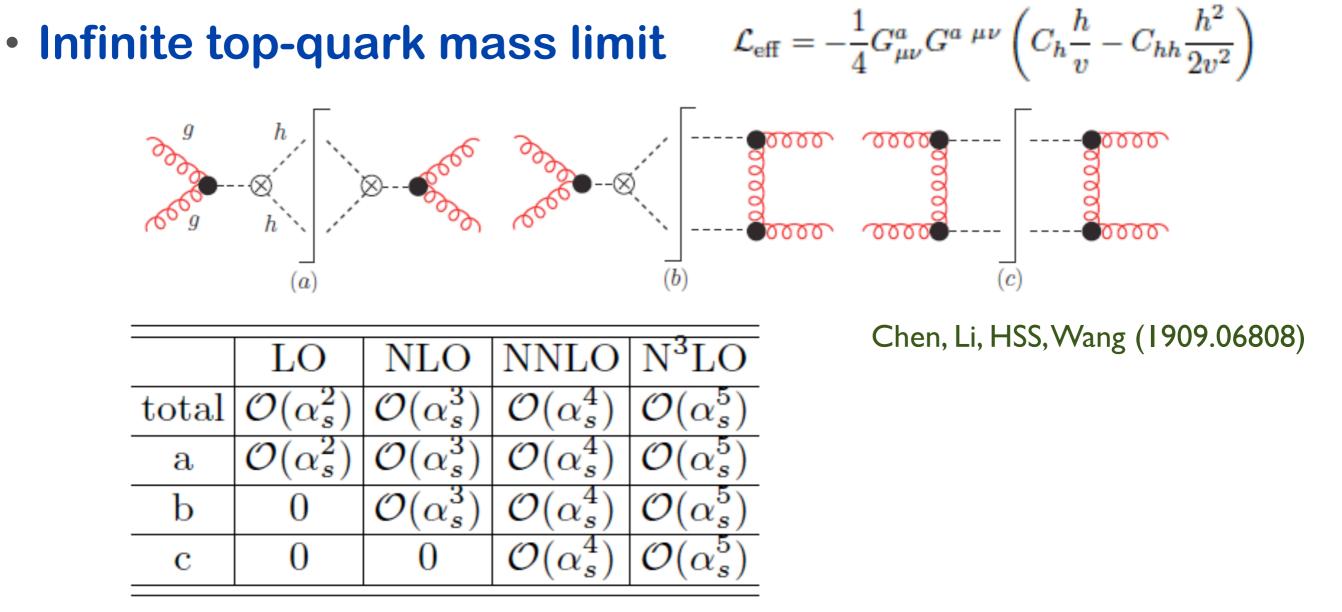
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H hard function
two-loop amplitude
Banerjee et al., JHEP'18
new one-loop amplitude



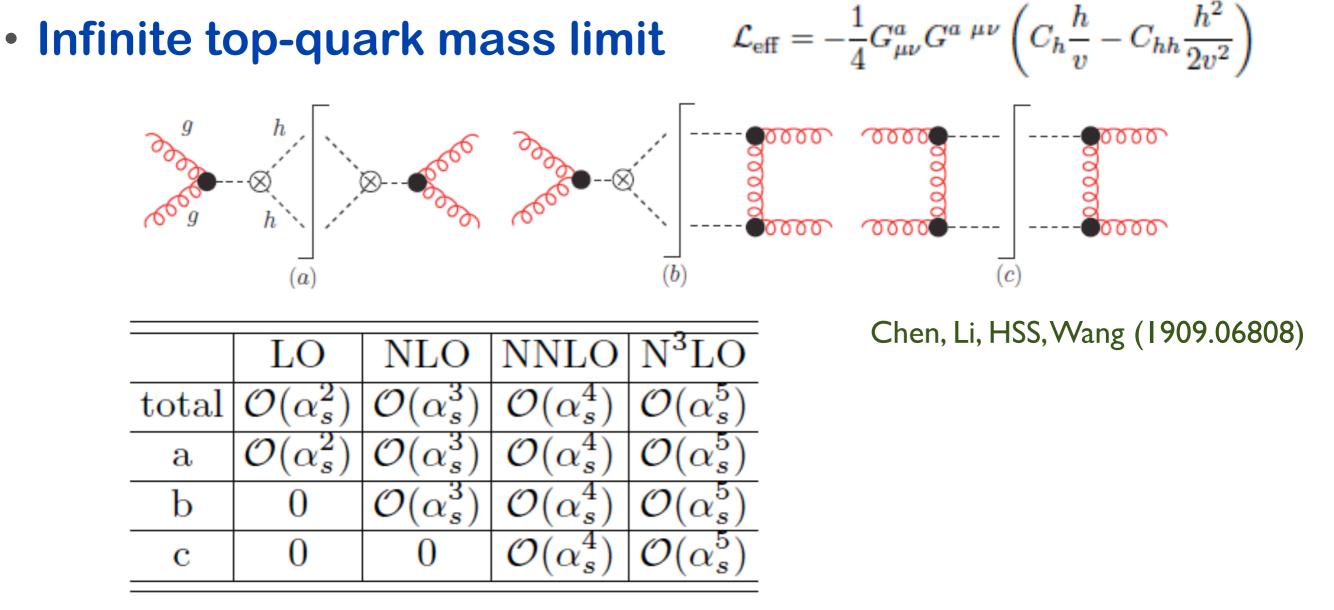




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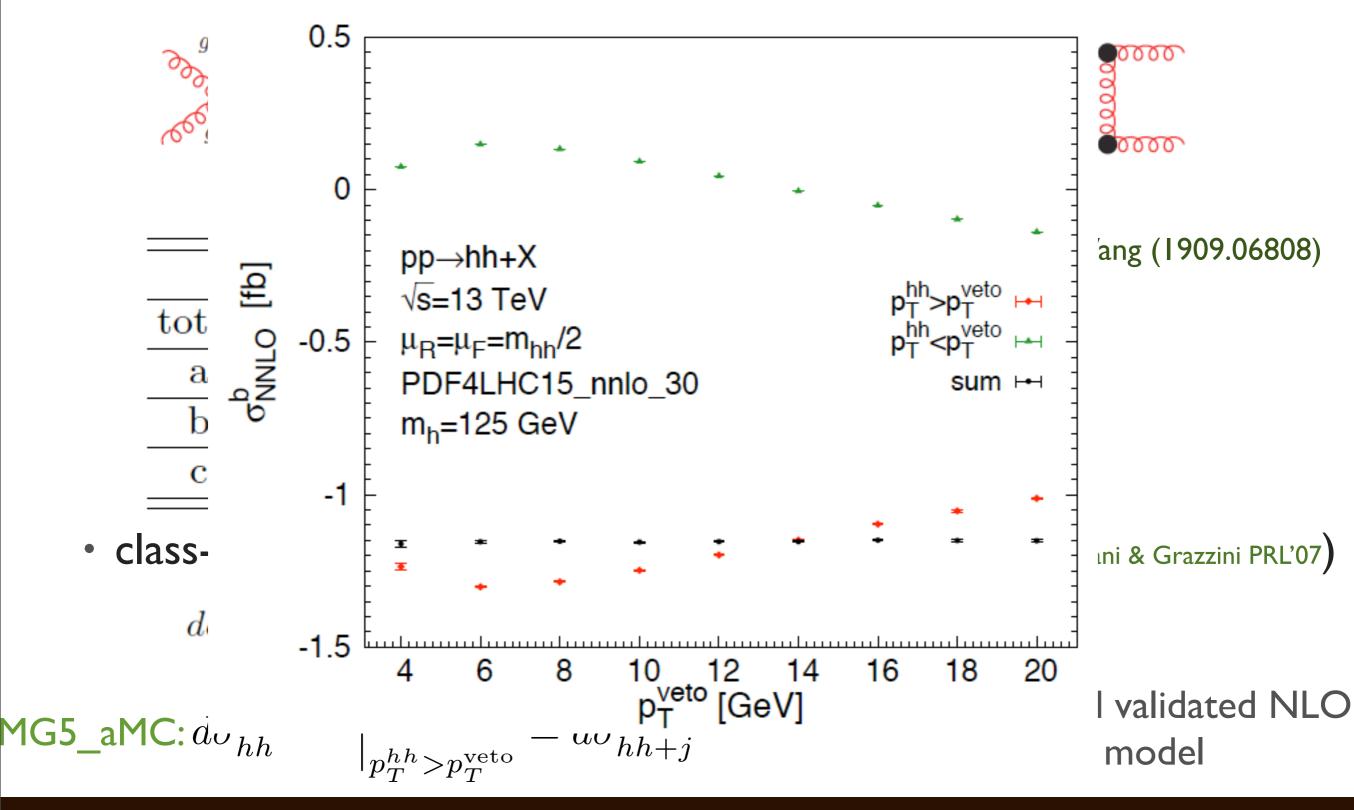
 $\mathsf{MG5_aMC:} d\sigma_{hh}^{b,\mathrm{NNLO}} \Big|_{p_T^{hh} > p_T^{\mathrm{veto}}} = d\sigma_{hh+j}^{b,\mathrm{NLO}}$

New and validated NLO model

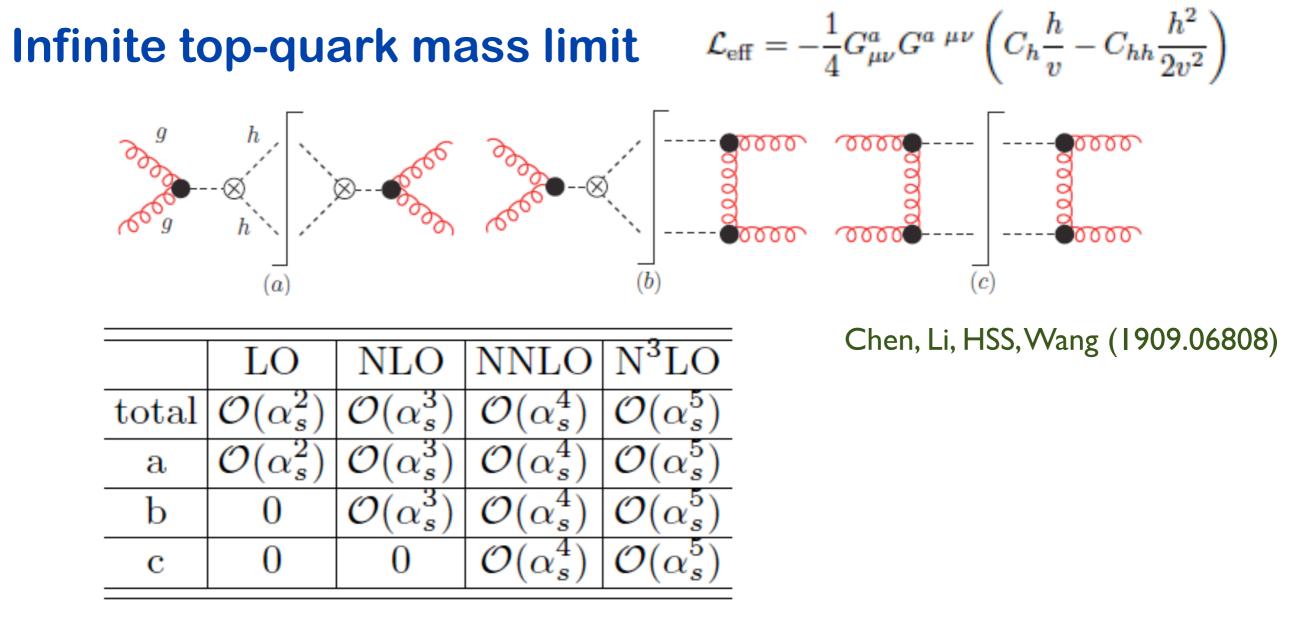












class-c: need NLO (full fledged)

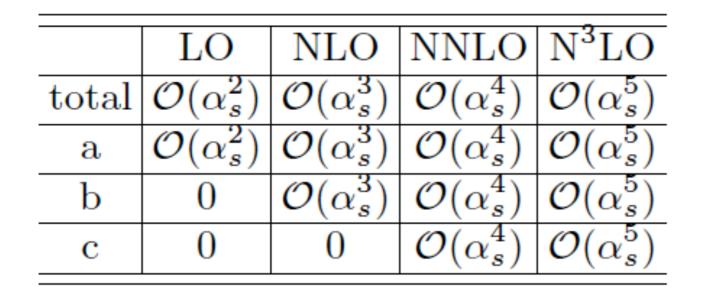
MINI-WORKSHOP, IHEP

•



- Infinite top-quark mass limit $\mathcal{L}_{eff} = -\frac{1}{4}G^a_{\mu\nu}G^{a\ \mu\nu}\left(C_h\frac{h}{v} C_{hh}\frac{h^2}{2v^2}\right)$
 - A lot of cross checks

Chen, Li, HSS, Wang (1909.06808)



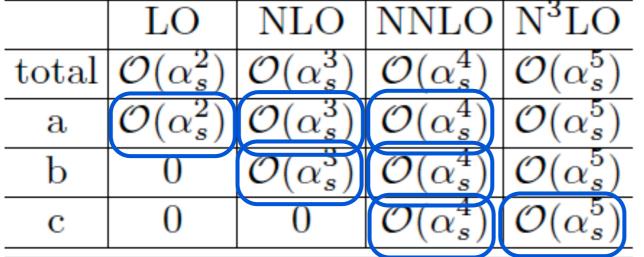
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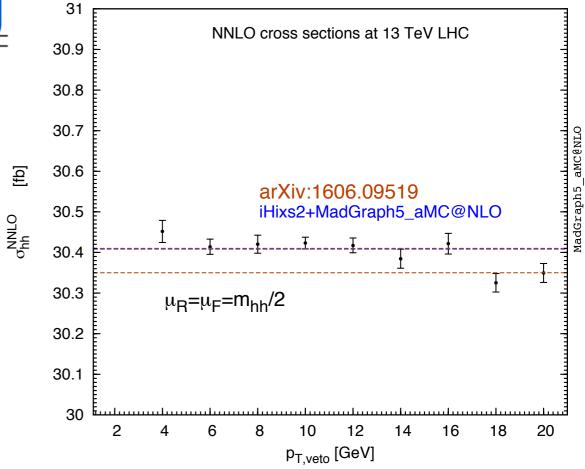
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Chen, Li, HSS, Wang (1909.06808)

At least two independent calculations

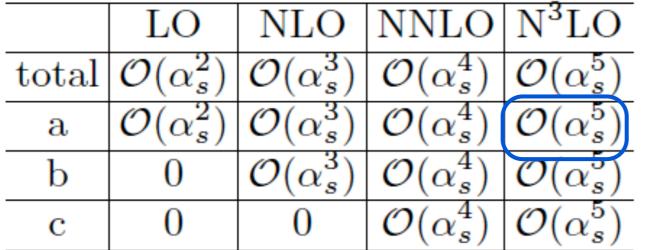




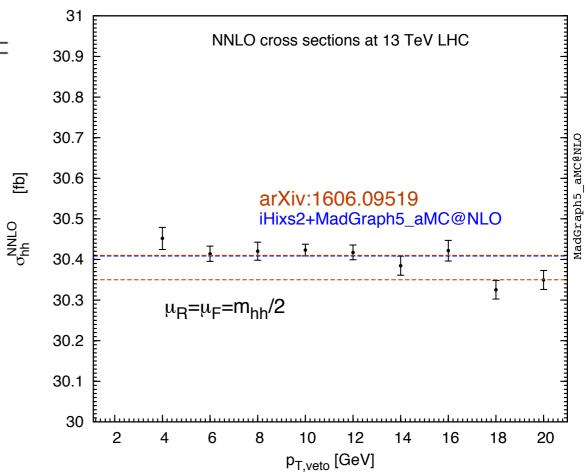
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Chen, Li, HSS, Wang (1909.06808)







- Infinite top-quark mass limit
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Chen, Li, HSS, Wang (1909.06808)

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 $\sqrt{}$ At least two independent calculations $\sqrt{}$ Orthogonal check with NNLO ggHH

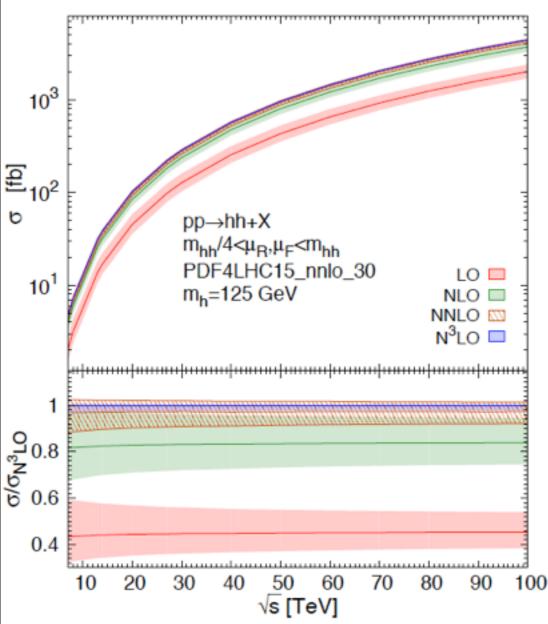
 $\sqrt{\text{Check piece-by-piece}}$





Infinite top-quark mass limit

• N³LO cross sections



in unit of fb		Chen, Li, HSS, Wang (1909.06808)		
\sqrt{s} order	$13 { m TeV}$	$14 { m TeV}$	$27 { m ~TeV}$	$100 { m TeV}$
LO	$13.80^{+31\%}_{-22\%}$	$17.06^{+31\%}_{-22\%}$	$98.22^{+26\%}_{-19\%}$	$2015^{+19\%}_{-15\%}$
NLO	$25.81^{+18\%}_{-15\%}$	$31.89^{+18\%}_{-15\%}$	$183.0^{+16\%}_{-14\%}$	$3724_{-11\%}^{+13\%}$
NNLO	$30.41^{+5.3\%}_{-7.8\%}$	$37.55^{+5.2\%}_{-7.6\%}$	$214.2_{-6.7\%}^{+4.8\%}$	$4322_{-5.3\%}^{+4.2\%}$
$N^{3}LO$	$31.31^{+0.66\%}_{-2.8\%}$	$38.65^{+0.65\%}_{-2.7\%}$	$220.2^{+0.53\%}_{-2.4\%}$	$4438^{+0.51\%}_{-1.8\%}$

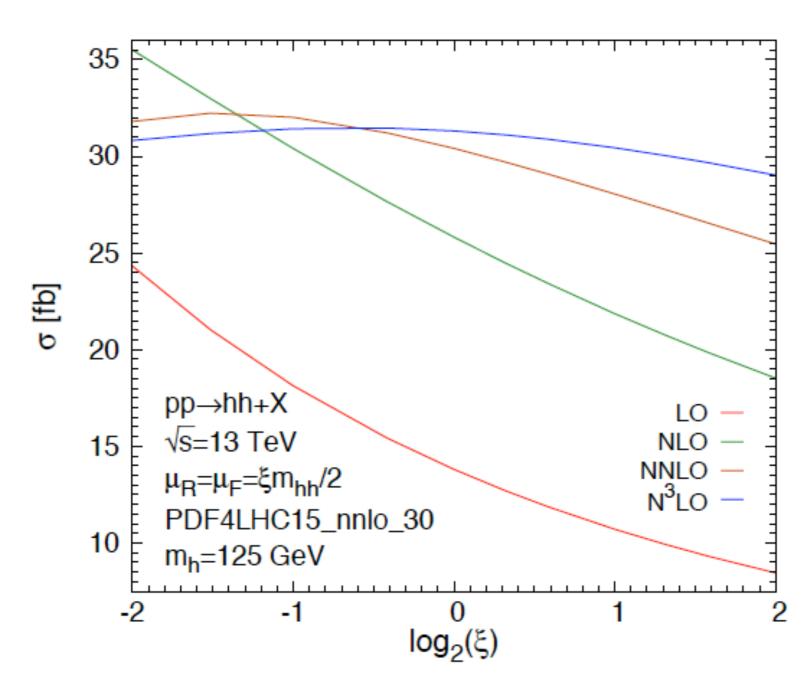
- Scale unc. is significantly reduced !
- PDF unc. > Scale unc. now !
- Very good perturbative convergence !



Infinite top-quark mass limit

• N³LO cross sections

Chen, Li, HSS, Wang (1909.06808)

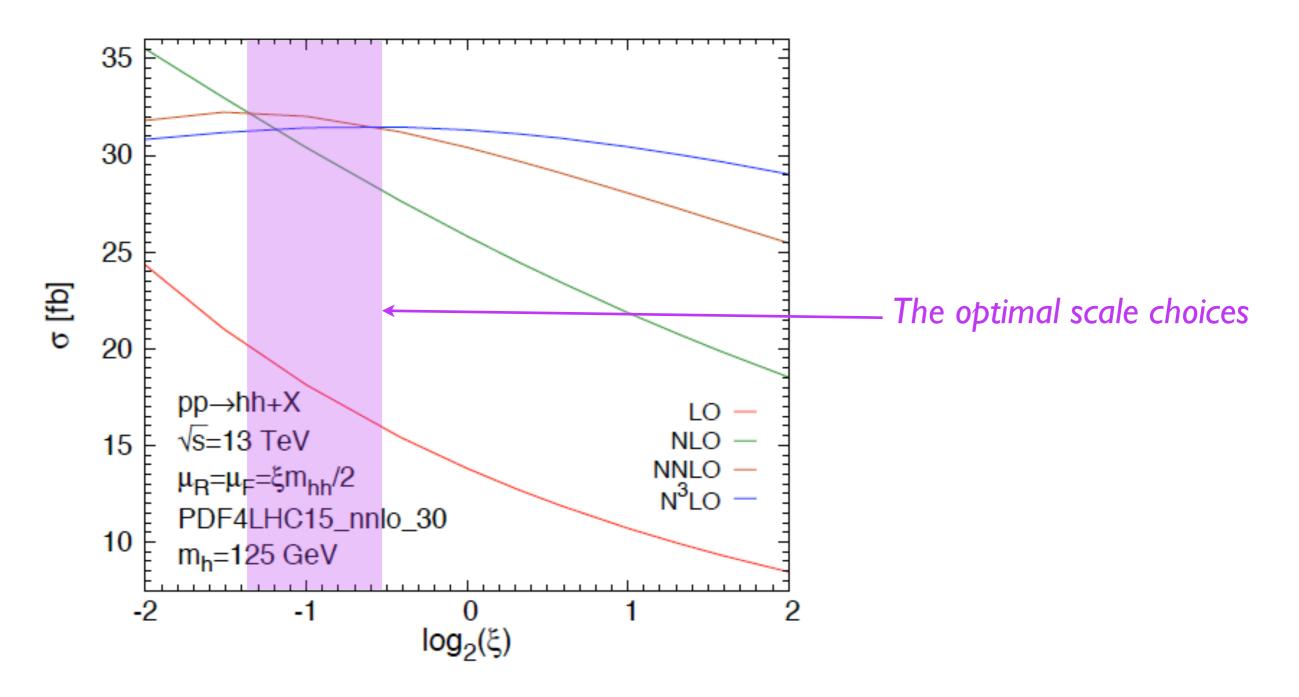




Infinite top-quark mass limit

• N³LO cross sections



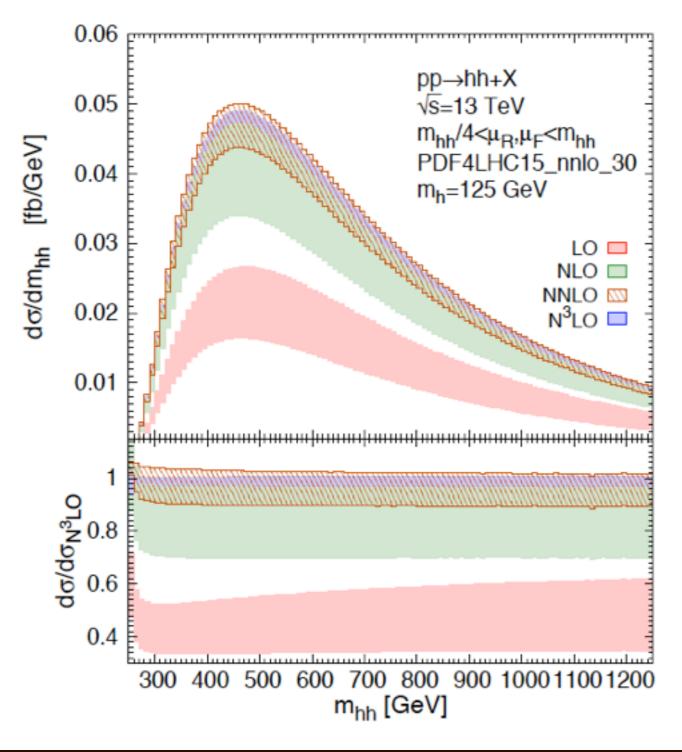




Infinite top-quark mass limit

• N³LO differential distribution

Chen, Li, HSS, Wang (1909.06808)



- Scale unc. is significantly reduced !
- Very good perturbative convergence !
- N³LO/NNLO is quite flat



- We have carried out N³LO calculations for Higgs pair production in the gluon fusion channel with the infinite top-quark mass limit.
- The scale uncertainty is significantly reduced to be below 3% (2%) at I3 (100) TeV. PDF uncertainty is bigger than scale uncertainty.
- The perturbative convergence in the process shows pretty good at N³LO.

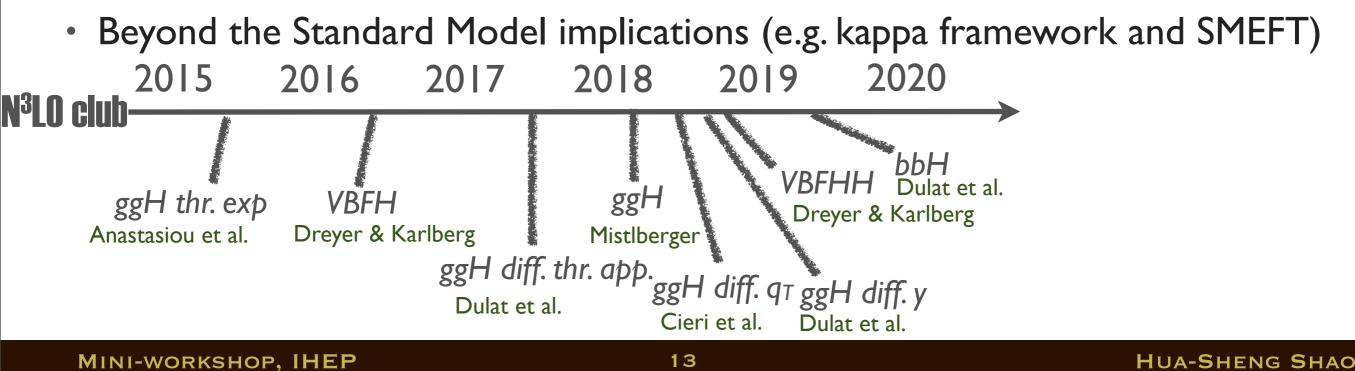


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- In the future:
 - Combine N³LO with full top-quark mass dependent NLO (NLO_{mt})
 - Combine N³LO with threshold resummation
 - Beyond the Standard Model implications (e.g. kappa framework and SMEFT)

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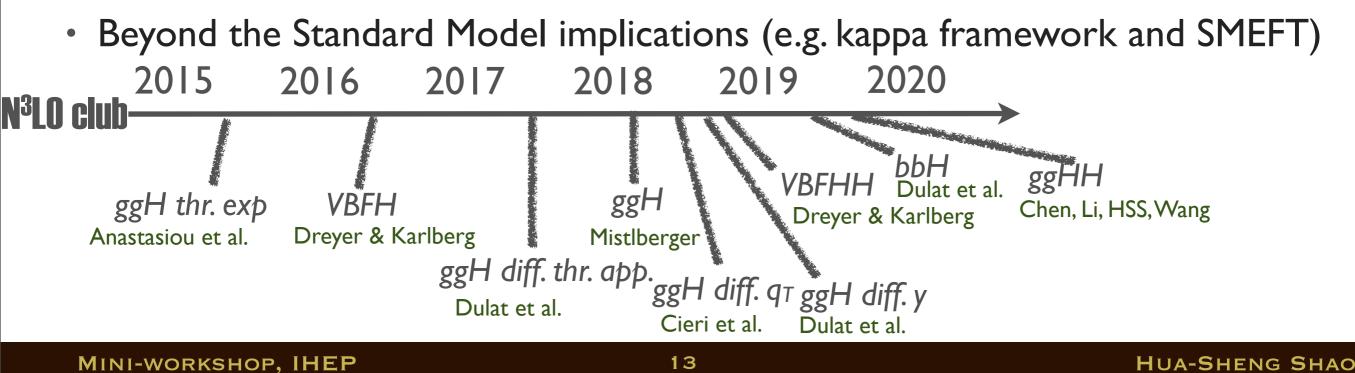


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