Multi-Higgs Production at Future Hadron Colliders

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Overview

Higgs mechanism

Lagrangian

$$\mathcal{L}_H = (D^\mu H)^\dagger (D_\mu H) - V(H^\dagger H)$$

Higgs potential

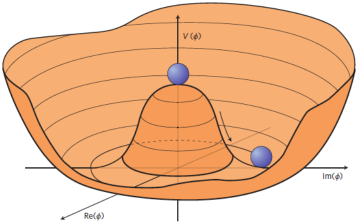
$$V(H^\dagger H) = -\mu^2 (H^\dagger H) + \lambda (H^\dagger H)^2$$

vacuum expectation value

$$v = \sqrt{\mu^2/\lambda}$$

Yukawa interaction

$$\mathcal{L}_Y = -\sum_f (y_f \bar{\psi}_{f,L} H \psi_{f,R} + h.c.)$$



[J. Ellis, 2013]

After the discovery of Higgs boson

- Are the properties of Higgs boson agreed with the SM prediction?
- What is the shape of Higgs potential?
- How New Physics contributes to the Higgs sector?

The measurements of multi-Higgs final state are important to answer these questions.

Why future collider?

We compare the cross section of mutli-Higgs process at 14 TeV LHC and a 100 TeV collider:

Process	$\sigma(14\ TeV)$ (fb)	err.[th]	err.[exp]
$gg \to h$	4.968×10^4	$^{+7.5\%}_{-9.0\%}$	$\pm 1\%$
$gg \to hh$	45.05	$^{+7.3\%}_{-8.4\%}$	$< 120fb$
$gg \to hhh$	0.0892	$^{+8.0\%}_{-6.8\%}$	$-$
	$\sigma(100\ TeV)$ (fb)	err.[th]	err. [exp]
$gg \to h$	8.02×10^5	$^{+7.5\%}_{-9.0\%}$	$\pm 0.1\%$
$gg \to hh$	1749	$^{+5.7\%}_{-6.6\%}$	$\pm 5\%$
$gg \to hhh$	4.82	$^{+4.1\%}_{-3.7\%}$	$< 30fb$

If we can reduce the background and the integrated luminosity is high enough, it is possible to observe $gg \to hhh$ at a 100 TeV machine.

The effective field theory

The effective Lagrangian

$$\mathcal{L}_{EFT} = \mathcal{L}_{\overline{SM}} + \mathcal{L}_t + \mathcal{L}_{ggh} + \mathcal{L}_{Vh} + \mathcal{L}_{VVh} + \mathcal{L}_h$$

$\mathcal{L}_{\overline{SM}}$: The SM Lagrangian after removing the Higgs interactions.
$\mathcal{L}_t$: The effective Yukawa interactions.
$\mathcal{L}_{ggh}$: The Higgs-gluon effective interactions.
$\mathcal{L}_{Vh}$: The SM-like Higgs-vector-boson interactions.
$\mathcal{L}_{VVh}$: The tensor structure interactions of Higgs to vector boson.
$\mathcal{L}_h$: The effective Higgs self-interactions.

Terms contribute to $gg \to hh$ and $gg \to hhh$

$$\mathcal{L}_t = -a_1 \frac{m_t}{v} \bar{t} t\, h - a_2 \frac{m_t}{2v^2} \bar{t} t\, h^2 - a_3 \frac{m_t}{6v^3} \bar{t} t\, h^3$$

In the SM at tree level, we have $a_1 = 1$ and $a_2 = a_3 = 0$.

$$\mathcal{L}_{ggh} = \frac{g_s^2}{48\pi^2} \left(c_1 \frac{h}{v} + c_2 \frac{h^2}{2v^2} \right) G^a_{\mu\nu} G^{a\,\mu\nu}.$$

In the SM, the couplings of Higgs to gluon is absent at tree level, and $c_1 = c_2 = 0$.

$$\mathcal{L}_h = -\lambda_3 \frac{m_h^2}{2v} h^3 - \frac{\kappa_5}{2v} h \partial^\mu h \partial_\mu h - \lambda_4 \frac{m_h^2}{8v^2} h^4 - \frac{\kappa_6}{4v^2} h^2 \partial^\mu h \partial_\mu h$$

When $\lambda_3 = \lambda_4 = 1$ and $\kappa_5 = \kappa_6 = 0$, the SM is recovered.

Terms contribute to $pp \to hhjj$ and $pp \to hhhjj$

In addition to the Higgs self-coupling terms $\mathcal{L}_h$, the vector-boson-fusion processes also involve

$$\mathcal{L}_{Vh} = g_{W,a1} \frac{2m_W^2}{v} h W^{+,\mu} W_\mu^- + g_{W,a2} \frac{m_W^2}{v^2} h^2 W^\mu W_\mu + g_{W,a3} \frac{m_W^2}{3v^3} h^3 W^\mu W_\mu + g_{Z,a1} \frac{m_Z^2}{v} h Z^\mu Z_\mu + g_{Z,a2} \frac{m_Z^2}{2v^2} h^2 Z^\mu Z_\mu + g_{Z,a3} \frac{m_Z^2}{6v^3} h^3 Z^\mu Z_\mu + \cdots$$

The SM is recovered when $g_{W,a1} = g_{W,a2} = g_{Z,a1} = g_{Z,a2} = 1$ and $g_{W,a3} = g_{Z,a3} = 0$.

$$\mathcal{L}_{VVh} = -\left(g_{W,b1} \frac{h}{v} + g_{W,b2} \frac{h^2}{2v^2} + g_{W,b3} \frac{h^3}{6v^3} + \cdots \right) W_{\mu\nu}^+ W^{-\,\mu\nu} - \left(g_{Z,b1} \frac{h}{2v} + g_{Z,b2} \frac{h^2}{4v^2} + g_{Z,b3} \frac{h^3}{12v^2} + \cdots \right) Z_{\mu\nu} Z^{\mu\nu}$$

This part is absent at tree level in the SM, and all coefficients vanish.

Relations to New Physics scenario

- **Scenario 1:** Strong Interaction Light Higgs (SILH) [G. F. Giudice *et al.*, 2007]

$$
\begin{aligned}
\mathcal{L}_{\text{SILH}} \quad &= \frac{c_H}{2f^2}\partial^\mu\left(H^\dagger H\right)\partial_\mu\left(H^\dagger H\right) + \frac{c_T}{2f^2}\left(H^\dagger \overleftrightarrow{D^\mu} H\right)\left(H^\dagger \overleftrightarrow{D}_\mu H\right) \\
&-\frac{c_6\lambda}{f^2}\left(H^\dagger H\right)^3 + \left(\frac{c_y y_f}{f^2} H^\dagger H \bar{f}_L H f_R + \text{h.c.}\right) \\
&+\frac{c_g g_S^2}{16\pi^2 f^2}\frac{y_t^2}{g_\rho^2} H^\dagger H G^a_{\mu\nu} G^{a\mu\nu} + \frac{i c_W g}{2m_\rho^2}\left(H^\dagger \sigma^i \overleftrightarrow{D^\mu} H\right)\left(D^\nu W_{\mu\nu}\right)^i \\
&+\frac{i c_B g'}{2m_\rho^2}\left(H^\dagger \overleftrightarrow{D^\mu} H\right)\left(\partial^\nu B_{\mu\nu}\right) + \frac{i c_{HW} g}{16\pi^2 f^2}(D^\mu H)^\dagger \sigma^i (D^\nu H) W^i_{\mu\nu} \\
&+\frac{i c_{HB} g'}{16\pi^2 f^2}(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu} + \frac{c_\gamma g'^2}{16\pi^2 f^2}\frac{g^2}{g_\rho^2} H^\dagger H B_{\mu\nu} B^{\mu\nu}.
\end{aligned}
$$

Two particular cases:

$$
\begin{aligned}
\text{MCHM4:} \quad & c_H = 1, \quad c_y = 0, \quad c_6 = 1 \\
\text{MCHM5:} \quad & c_H = 1, \quad c_y = 1, \quad c_6 = 0
\end{aligned}
$$

- **Scenario 2:** Higgs inflation
  [F. L. Bezrukov and M. Shaposhnikov, 2007]

Higgs can be treated as inflaton, and couples to gravity by

$$\mathcal{L}_{inflation} = \mathcal{L}_{SM} - \frac{M^2}{2}R - \alpha H^\dagger H R,$$

The Einstein frame of the Lagrangian predicts the deviations of Higgs-gauge boson couplings and Higgs self-couplings.

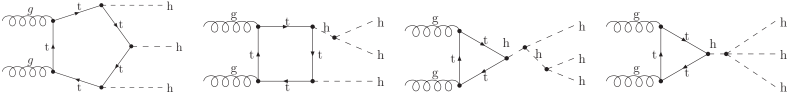
$$\begin{aligned} S_E \;=\; & \int d^4x\sqrt{-\hat{g}_W}\Big\{ -\frac{M_P^2}{2}\hat{R} + \frac{D_\mu H D^\mu H}{\Omega^2} + \frac{12\alpha^2}{M_p^2}\frac{H^2\partial_\mu H \partial^\mu H}{\Omega^4} \\ & -\frac{1}{\Omega^2}\lambda\left(H^2 - \frac{v^2}{2}\right)^2 + \frac{2H^2}{\Omega^2}(\frac{M_W^2}{v^2}W^\mu W_\mu + \frac{M_Z^2}{v^2}Z^\mu Z_\mu) + \cdots \Big\}. \end{aligned}$$

Multi-Higgs production via gluon-gluon fusion mode

Based on W. Kilian, S. Sun, Q. S. Yan, X. Zhao and ZZ,JHEP **1706**, 145 (2017) [arXiv:1702.03554 [hep-ph]]

Multi-Higgs production via gluon-gluon fusion mode

- At a hadron collider, the dominant SM process of multi-Higgs production is gluon-gluon fusion (ggF) via a heavy top quark loop.



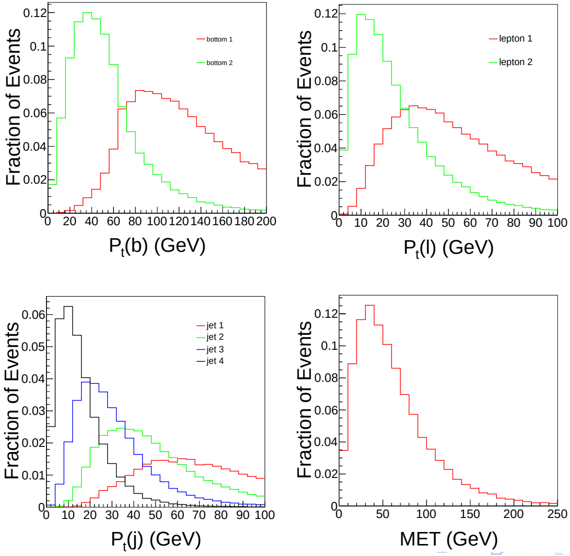
- This production mode involves many anomalous couplings.

	$gg \to h$	$gg \to hh$	$gg \to hhh$
Parameters involved	a_1, c_1	a_1, c_1	a_1, c_1
	-	a_2, c_2, λ_3, κ_5	a_2, c_2, λ_3, κ_5
	-	-	a_3, λ_4, κ_6

- The decay channel $hhh \to b\bar{b}b\bar{b}\gamma\gamma$ [C. Y. Chen *et al.*, 2015] and $hhh \to b\bar{b}b\bar{b}\tau\tau$ [B. Fuks *et al.*, 2017] have been discussed. Here we discuss the $hhh \to 2b2l^{\pm}4j + \not{E}$ channel.

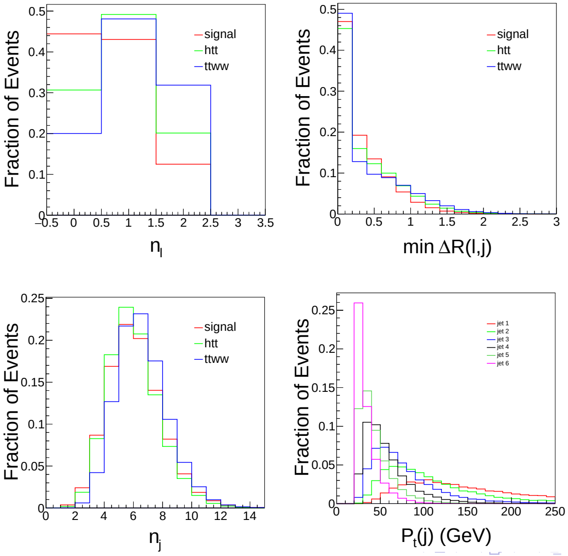
Analysis of $pp \to hhh \to 2b2l^{\pm}4j + \not{E}$ ($E_{cm} = 100$ TeV)

Parton level:

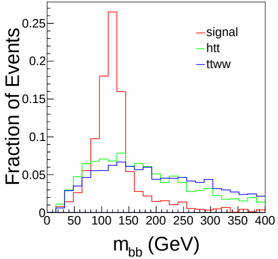


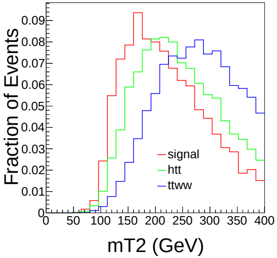
Analysis of $pp \to hhh \to 2b2l^{\pm}4j + \not{E}$ ($E_{cm} = 100$ TeV)

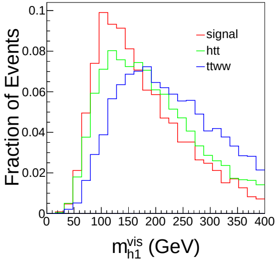
Hadron level:

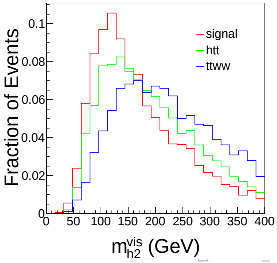


Detector level:



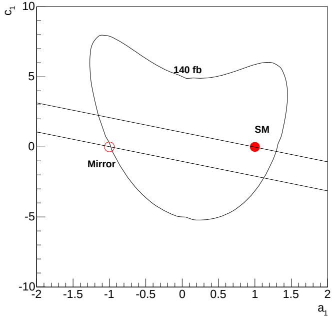




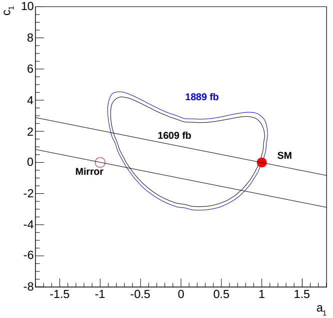


The bounds on a_1 and c_1

The processes $gg \to h$ and $gg \to hh$ give a strong constraint on a_1 and c_1 (assuming 30 ab^{-1} luminosity for 100 TeV)

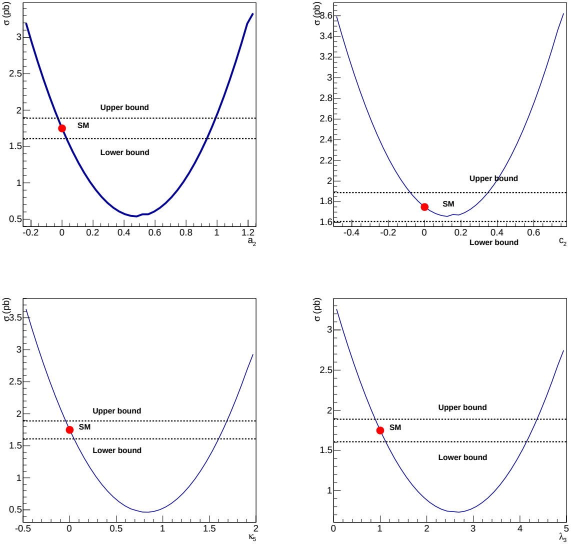


(a) 14 TeV

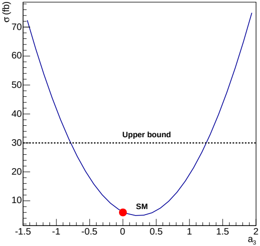


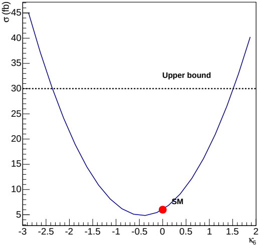
(b) 100 TeV

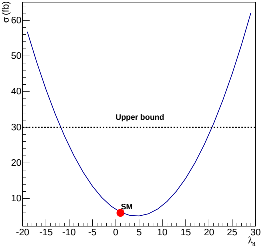
The bounds from $gg \to hh$



The bounds from $gg \to hhh$







Summary of the constraints

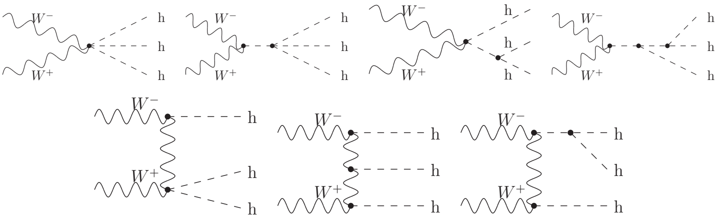
Parameters	Constraints
a_1	1%
a_2	10%
a_3	40%
λ_3	10%
λ_4	$[-13, 20]$
κ_5	10%
κ_6	$[-2.3, 1.5]$
c_1	$\leq 1.0\%$
c_2	$[-0.1, 0.4]$

Multi-Higgs production via vector boson fusion mode

Based on W. Kilian, S. Sun, Q. S. Yan, X. Zhao and ZZ, arXiv:1808.05534 [hep-ph]

Multi-Higgs production via vector boson fusion mode

- The subdominant process of multi-Higgs production in hadron collisions is so-called vector-boson-fusion (VBF).



- This production mode involves different set of anomalous couplings.

	$VV \to h$	$VV \to hh$	$VV \to hhh$
Parameters involved	$g_{V,a1}$, $g_{V,b1}$	$g_{V,a1}$, $g_{V,b1}$	$g_{V,a1}$, $g_{V,b1}$
	-	$g_{V,a2}$, $g_{V,b2}$, λ_3, κ_5	$g_{V,a2}$, $g_{V,b2}$, λ_3, κ_5
	-	-	$g_{V,a3}$, $g_{V,b3}$, λ_4, κ_6

Constraints on parameters from the unitarity of S matrix

The scattering of observable particles is described by a $\mathcal{S}$ operator, which satisfies $\mathcal{S}^\dagger\mathcal{S} = 1$. Its nontrivial part is defined by $\mathcal{S} = 1 + i\mathcal{T}$, where the $\mathcal{T}$ satisfies the universal relation

$$-i(\mathcal{T} - \mathcal{T}^\dagger) = \mathcal{T}^\dagger\mathcal{T}$$

The matrix elements of the scattering amplitude operator $\mathcal{M}$ between the initial state $|\alpha, \Phi_a\rangle$ and the final state $|\beta, \Phi_b\rangle$ can be written as

$$\langle\beta, \Phi_b|\mathcal{T}|\alpha, \Phi_a\rangle = (2\pi)^4\delta^{(4)}(p_a - p_b)\,\langle\beta, \Phi_b|\mathcal{M}|\alpha, \Phi_a\rangle.$$

We can insert a complete set of multi-particle states $|\gamma, \Phi_c\rangle$ to the right-handed side, and obtain

$$\begin{aligned} &-i\,[\langle\beta, \Phi_b|\mathcal{M}|\alpha, \Phi_a\rangle - \langle\alpha, \Phi_a|\mathcal{M}|\beta, \Phi_b\rangle^*] \\ &\quad = \sum_\gamma \int \mathrm{d}\Phi_c\,\langle\gamma, \Phi_c|\mathcal{M}|\beta, \Phi_b\rangle^*\langle\gamma, \Phi_c|\mathcal{M}|\alpha, \Phi_a\rangle \end{aligned}$$

Constraints on parameters from the unitarity of S matrix

For convenience, we introduce a bijective mapping between the unit hypercube in $d_a = 3n_a - 4$ dimensions, $\{x_a \in \mathbb{R}^{d_a};\ 0 < (x_a)_i < 1\}$ and the manifold $\{\Phi_a\}$, for each fixed n_a. This mapping introduces a Jacobian $J_a(x_a) = \mathrm{d}\Phi_a/\mathrm{d}x_a$ and we have

$$M_{\beta\alpha}(x_b, x_a) = J_b^{1/2}(x_b)\,\langle\beta, \Phi_b(x_b)|\mathcal{M}|\alpha, \Phi_a(x_a)\rangle\, J_a^{1/2}(x_a)$$

Now we have a matrix elements relation

$$-i\left[M^{\beta\alpha*}(x_b, x_a) - M^{\alpha\beta}(x_a, x_b)\right] = \sum_\gamma \int \mathrm{d}x_c\, M^{\gamma\beta*}(x_c, x_b)\, M^{\gamma\alpha}(x_c, x_a)$$

Constraints on parameters from the unitarity of S matrix

For simplicity, we adopt the following canonical scalar product and a corresponding orthonormal basis $\{H_A^\alpha(x_a)\}$ on each α phase space,

$$\int \mathrm{d}x_a\, H_A^{\alpha *}(x_a)\, H_B^\alpha(x_a) = \delta_{AB},$$

The amplitudes can be expanded as

$$M^{\beta\alpha}(x_b, x_a) \quad = 2 \textstyle\sum_{AB} a_{AB}^{\alpha\beta} H_A^\alpha(x_a)\, H_B^{\beta *}(x_b).$$

and the coefficients are

$$a_{AB}^{\alpha\beta} = \frac{1}{2} \int \mathrm{d}x_a\, \mathrm{d}x_b\, H_A^{\alpha *}(x_a)\, H_B^\beta(x_b)\, M^{\beta\alpha}(x_b, x_a).$$

These coefficients satisfy

$$- i(a_{AB}^{\alpha\beta} - a_{BA}^{\beta\alpha *}) = 2 \sum_\gamma \sum_C a_{AC}^{\alpha\gamma} a_{BC}^{\beta\gamma *},$$

Constraints on parameters from the unitarity of S matrix

We may derive less comprehensive but phenomenologically more useful relations by focusing on diagonal matrix elements, i.e., $\alpha = \beta$ and $A = B$,

$$\begin{aligned} -i(a_{AA}^{\alpha\alpha} - a_{AA}^{\alpha\alpha *}) &= 2\sum_{\gamma}\sum_{C}|a_{AC}^{\alpha\gamma}|^2 \\ &= 2|a_{AA}^{\alpha\alpha}|^2 + 2\sum_{C\neq A}|a_{AC}^{\alpha\alpha}|^2 + 2\sum_{\gamma\neq\alpha}\sum_{C}|a_{AC}^{\alpha\gamma}|^2 \end{aligned}$$

we rewrite the diagonal amplitude in terms of its real and imaginary parts:

$$|\mathrm{Re}\, a_{AA}^{\alpha\alpha}|^2 + |\mathrm{Im}\, a_{AA}^{\alpha\alpha} - \frac{1}{2}|^2 + \sum_{C\neq A}|a_{AC}^{\alpha\alpha}|^2 + \sum_{\gamma\neq\alpha}\sum_{C}|a_{AC}^{\alpha\gamma}|^2 = \tfrac{1}{4}$$

This equation gives the bounds

$$|\mathrm{Re}\, a_{AA}^{\alpha\alpha}|^2 \leq \frac{1}{4},\ \ |\mathrm{Im}\, a_{AA}^{\alpha\alpha} - \frac{1}{2}|^2 \leq \frac{1}{4}$$

$$\sum_{C\neq A}|a_{AC}^{\alpha\alpha}|^2 \leq \frac{1}{4},\ \ \sum_{\gamma\neq\alpha}\sum_{C}|a_{AC}^{\alpha\gamma}|^2 \leq \frac{1}{4}$$

Constraints on parameters from the unitarity of S matrix

The last inequality gives the unitarity constraints on inelastic scattering, and we note that it is independent of the basis H^{γ}. To see this, we define the coefficients $b_A^{\alpha\gamma}$:

$$b_A^{\alpha\gamma} \equiv \frac{1}{4} \int \mathrm{d}x_a \mathrm{d}x_b \mathrm{d}x_c H_A^{\alpha *}(x_a) H_A^{\alpha}(x_b) M^{\gamma\alpha *}(x_c, x_b) M^{\gamma\alpha}(x_c, x_a),$$

which is clearly independent of H^{γ}. And we find that

$$b_A^{\alpha\gamma} = \sum_C |a_{AC}^{\alpha\gamma}|^2 \geq 0$$

Finally, the unitarity constraint for inelastic scattering can be written as

$$\sum_{\gamma \neq \alpha} b_A^{\alpha\gamma} \leq \frac{1}{4}$$

Unitarity Constraints from $VV \to hh$

In the high energy limit $s \gg m_W^2, m_h^2$, we can expand the amplitudes by the rescaled energy $\sqrt{s/v^2}$ as a dimensionless expansion parameter

$$M(W^+W^- \to hh) = \sum_{i=0}^{+\infty} m_i (\frac{\sqrt{s}}{v})^{2-i},$$

where m_i are the coefficients in the expansion. Its leading term m_0 are

helicity configuration	$++$	$+-$	00	$+0$
s-channel	$\frac{1}{2}\kappa_5 g_{W,b1}$	0	$\frac{1}{2}\kappa_5 g_{W,a1}$	0
t, u-channel	$2g_{W,b1}^2$	$\mathcal{O}(g_{W,b1}^2)$	$-g_{W,a1}^2$	0
contact interaction	$g_{W,b2}$	0	$g_{W,a2}$	0

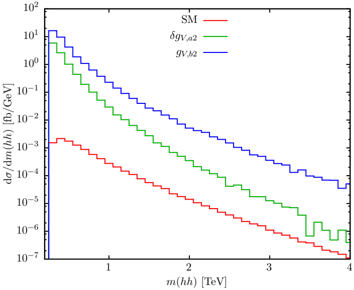
We can derive individual bounds for amplitude coefficients $b_j(h_1 h_2)$,

$$b_j(h_1 h_2) \leq \frac{1}{4}, \qquad \text{where } h_i = + - 0.$$

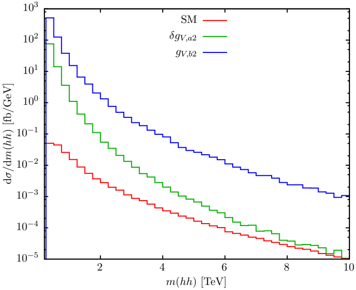
Finally, we obtain the strongest bounds on the EFT parameters:

$$\begin{aligned}
b_0(00) &= \frac{s^2}{2^9 \pi^2 v^4} \left| g_{W,a2} - g_{W,a1}^2 + \frac{1}{2}\kappa_5 g_{W,a1} \right|^2 \leq \frac{1}{4} \\
b_0(++) &= \frac{s^2}{2^9 \pi^2 v^4} \left| g_{W,b2} + 2 g_{W,b1}^2 + \frac{1}{2}\kappa_5 g_{W,b1} \right|^2 \leq \frac{1}{4} \\
b_2(+-) &= \frac{s^2}{3 \times 2^{10} \pi^2 v^4} g_{W,b1}^4 \leq \frac{1}{4}
\end{aligned}$$

Unitarity Constraints from $VV \to hh$



(a) 14 TeV



(b) 100 TeV

Unitarity Constraints from $VV \to hhh$

Analogy to $W^+W^- \to hh$, in the high energy limit, the amplitude can be expanded as a series in powers of $\sqrt{s/v^2}$

$$M(W^+W^- \to hhh) = \sum_{i=0}^{+\infty} m_i v^{-1} (\frac{\sqrt{s}}{v})^{2-i}$$

The leading term m_0 are

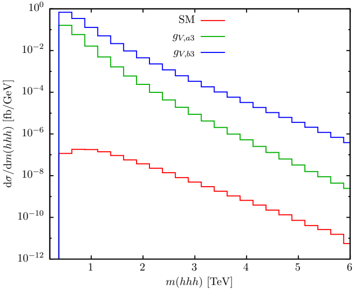
	$++$	$+-$	00	$+0$
a	$g_{W,b3}$	0	$g_{W,a3}$	0
b	$\frac{1}{2} g_{W,b1}\kappa_6$	0	$\frac{1}{2} g_{W,a1}\kappa_6$	0
c	$\frac{3}{2} g_{W,b2}\kappa_5$	0	$\frac{3}{2} g_{W,a1}\kappa_5$	0
d	$g_{W,b1}\kappa_5^2$	0	$g_{W,a1}\kappa_5^2$	0
e	$6 g_{W,b1} g_{W,b2}$	$\mathcal{O}(g_{W,b1} g_{W,b2})$	$-4 g_{W,a1} g_{W,b1}$	0
f	$\mathcal{O}(g_{W,b1}^3)$	$\mathcal{O}(g_{W,b1}^3)$	$4 g_{W,a1}^3$	0
g	$3 g_{W,b1}^2 \kappa_5$	$\mathcal{O}(g_{W,b1}^2 \kappa_5)$	$-2 g_{W,a1}^2 \kappa_5$	0

Unitarity Constraints from $VV \to hhh$

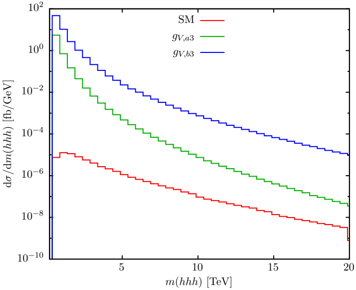
Similarly, we obtain the strongest bounds on the EFT parameters:

$$
\begin{aligned}
b_0(00) &= \frac{s^3}{3\times 2^{14}\pi^4 v^6}\Big|g_{W,a3} + \frac{1}{2}g_{W,a1}\kappa_6 + \frac{3}{2}g_{W,a2}\kappa_5 + g_{W,a1}\kappa_5^2 \\
&\quad -4g_{W,a1}g_{W,a2} + 4g_{W,a1}^3 - 2g_{W,a1}^2\kappa_5\Big|^2 \le \frac{1}{4} \\
b_0(++) &= \frac{s^3}{3\times 2^{14}\pi^4 v^6}\Big(\Big|g_{W,b3} + \frac{1}{2}g_{W,b1}\kappa_6 + \frac{3}{2}g_{W,b2}\kappa_5 + g_{W,b1}\kappa_5^2 \\
&\quad +6g_{W,b1}g_{W,b2} + f_1 g_{W,b1}^3 - 3g_{W,b1}^2\kappa_5\Big|^2 + f_2 g_{W,b1}^6\Big) \le \frac{1}{4} \\
b_2(+-) &= \frac{s^3}{3\times 2^{14}\sqrt{6}\pi^4 v^6}\left|g_{W,b1}g_{W,b2} + 2g_{W,b1}^3 + \frac{1}{2}g_{W,b1}^2\kappa_5\right|^2 \le \frac{1}{4}
\end{aligned}
$$

Unitarity Constraints from $VV \to hhh$



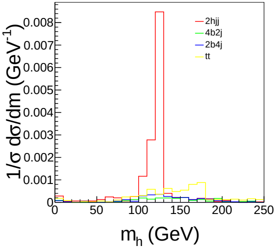
(a) 14 TeV



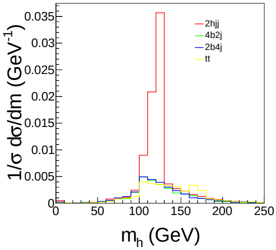
(b) 100 TeV

Highly boosted Higgs

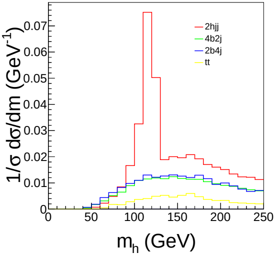
We reconstruct the Higgs bosons in $pp \to hhjj \to 4b2j$, and find



(a) 2-boosted



(b) 1-boosted



(c) 0-boosted

Conclusion

- After the discovery of Higgs boson, measurements on its properties and couplings to SM particles become essential to search for new physics.
- We use the EFT method to discuss the new physics effects in multi-Higgs production.
- The channel $pp \to hhh \to 2b2l^{\pm}4j + MET$ is studied at 100 TeV.
- We also derive the generic formalism of unitarity to vector-boson-fusion process
- The bounds on Higgs couplings to SM particles via multi-Higgs final states are obtained at 14 TeV LHC and 100 TeV hadron collider.
- Our study can be applied to constrain the new physics model such as SILH and Higgs inflation.

Thank You

Backup Slides

Triple-Higgs Decay Channel

Decay channel	Branching ratio
$HHH \to b\bar{b}b\bar{b}W^+W^-$	22.34%
$HHH \to b\bar{b}b\bar{b}b\bar{b}$	20.30%
$HHH \to b\bar{b}W^+W^-W^+W^-$	8.20%
$HHH \to b\bar{b}b\bar{b}\tau^+\tau^-$	7.16%
$HHH \to b\bar{b}b\bar{b}gg$	6.54%
$HHH \to b\bar{b}b\bar{b}ZZ$	2.69%
$HHH \to W^+W^-W^+W^-W^+W^-$	1.00%
$HHH \to W^+W^-W^+W^-\tau^+\tau^-$	0.96%
$HHH \to W^+W^-W^+W^-gg$	0.88%
$HHH \to W^+W^-W^+W^-ZZ$	0.36%
$HHH \to b\bar{b}b\bar{b}\gamma\gamma$	0.29%