粒子物理学的自旋极化
Lecture 3

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第三讲：螺旋度振幅的运用

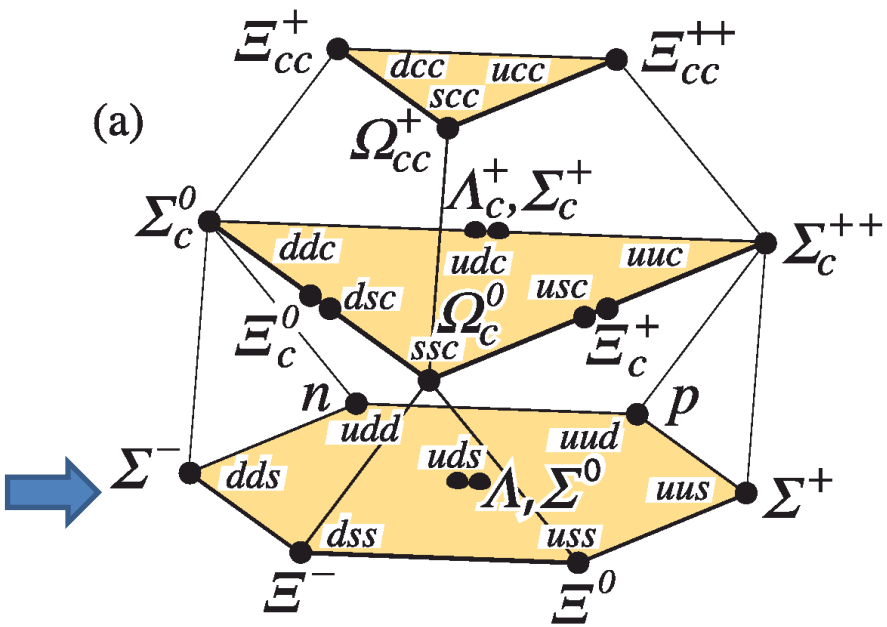
3.1 . 超子的非轻子衰变

3.2. 共振态自旋-宇称的测量

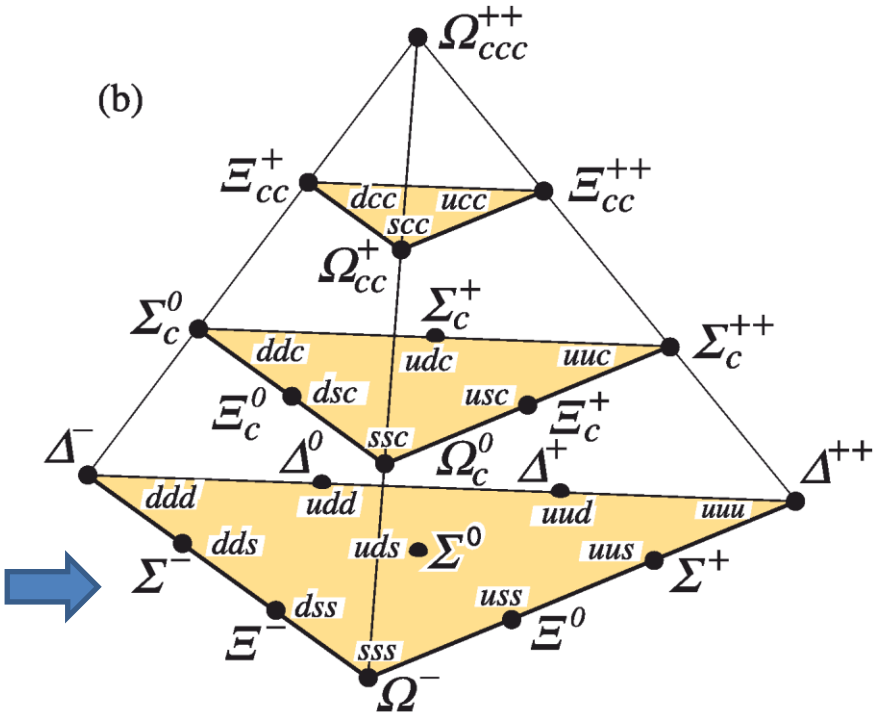
3.3. 多道级联式衰变(isobar 模型)中问题

3.1 超子的非轻子衰变

- Hyperon baryons in quark model



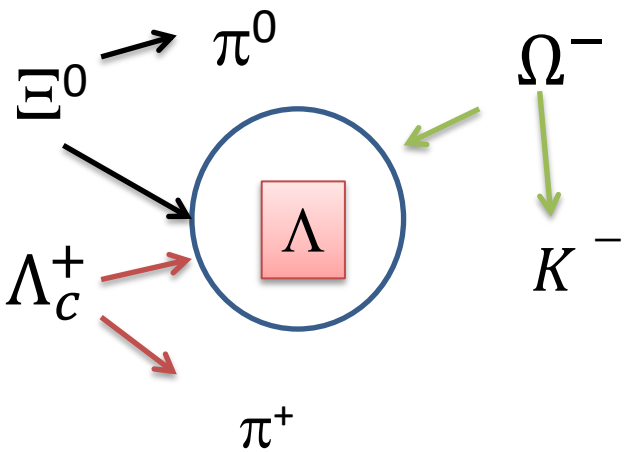
Bottom level: SU(3) octet baryons with $J = \frac{1}{2}$

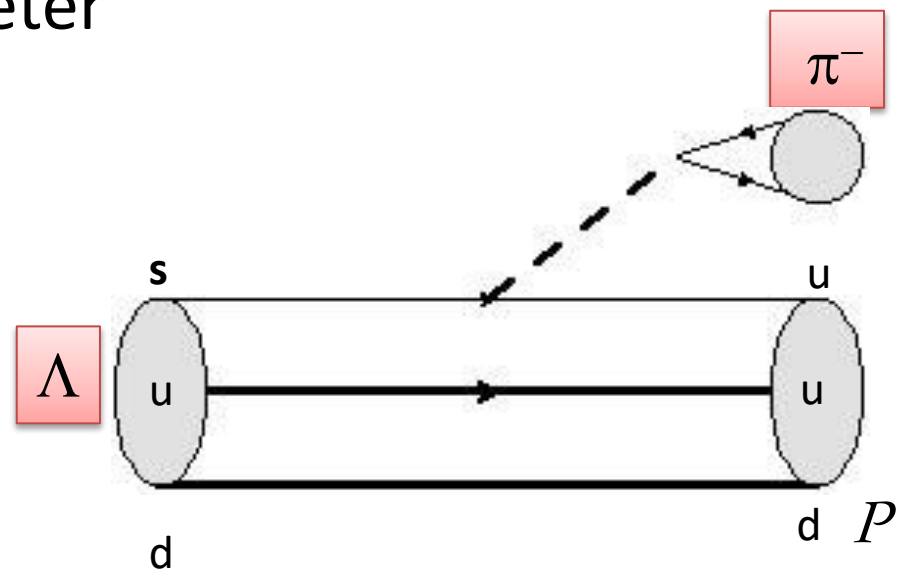


Bottom level: SU(3) decuplet baryons with $J = \frac{3}{2}$

Specific processes

1. $\Lambda \to p\pi^-$ as polarimeter





In the Λ CM system, amplitude reads

$$M \sim \psi_2^\dagger (S + P\boldsymbol{\sigma} \cdot \boldsymbol{n})\psi_1$$

Parity violation allows S- and $P-$wave, $\psi_1(\psi_2)$ is the two-component spinor for $\Lambda(p)$

Using:

$$\psi_1\psi_1^+ = \tfrac{1}{2}(1 + \boldsymbol{\eta}\cdot\boldsymbol{\sigma}),$$

$$\psi_2\psi_2^+ = \tfrac{1}{2}(1 + \boldsymbol{\zeta}\cdot\boldsymbol{\sigma}).$$

$$\boldsymbol{\zeta}\cdot(\boldsymbol{\eta}\times\boldsymbol{n})$$

$$\sigma_i\sigma_k = \delta_{ik} + i\varepsilon_{ikl}\sigma_l,$$

$$\mathrm{Tr}\sigma_i\sigma_k = 2\delta_{ik},$$

$$\mathrm{Tr}\,\sigma_i\sigma_k\sigma_l = 2i\varepsilon_{ikl},$$

$$\mathrm{Tr}\,\sigma_i\sigma_k\sigma_l\sigma_m = 2(\delta_{ik}\delta_{lm} + \delta_{im}\delta_{kl} - \delta_{il}\delta_{km}).$$

$$W(\boldsymbol{\eta}, \boldsymbol{\zeta}, \boldsymbol{n}) \sim |M|^2 \sim \mathrm{Tr}[(1 + \boldsymbol{\zeta}\cdot\boldsymbol{\sigma})(S + P\boldsymbol{\sigma}\cdot\boldsymbol{n})(1 + \boldsymbol{\eta}\cdot\boldsymbol{\sigma})$$
$$\times(S^* + P^*\boldsymbol{\sigma}\cdot\boldsymbol{n})]$$
$$\sim\{|S|^2(1 + \boldsymbol{\eta}\cdot\boldsymbol{\zeta}) + |P|^2(1 + 2(\boldsymbol{\eta}\cdot\boldsymbol{n})(\boldsymbol{\zeta}\cdot\boldsymbol{n}) - \boldsymbol{\eta}\cdot\boldsymbol{\zeta})$$
$$+ (SP^* + S^*P)(\boldsymbol{\eta}\cdot\boldsymbol{n} + \boldsymbol{\zeta}\cdot\boldsymbol{n}) + i(SP^* - S^*P)\boldsymbol{\zeta}[\boldsymbol{\eta}\cdot\boldsymbol{n}]\}$$

$$\sim \{1 + \alpha(\boldsymbol{\eta} \cdot \boldsymbol{n} + \boldsymbol{\zeta} \cdot \boldsymbol{n}) + \beta\ \boldsymbol{\zeta} \cdot (\boldsymbol{\eta} \times \boldsymbol{n}) + \gamma \boldsymbol{\eta} \cdot \boldsymbol{\zeta}$$
$$+ (1 - \gamma)(\boldsymbol{\eta} \cdot \boldsymbol{n})(\boldsymbol{\zeta} \cdot \boldsymbol{n})\}.$$

Lee- Yang parameters:

$$\alpha = \frac{SP^* + S^*P}{|S|^2 + |P|^2}, \qquad \beta = i\frac{SP^* - S^*P}{|S|^2 + |P|^2}, \qquad \gamma = \frac{|S|^2 - |P|^2}{|S|^2 + |P|^2}.$$

$$\alpha^2 + \beta^2 + \gamma^2 = 1$$

Comments:

- 如果核子的极化不观测, $\langle \boldsymbol{\zeta} \rangle = 0$， $W(\boldsymbol{\eta}) \sim 1 + \alpha \boldsymbol{\eta} \cdot \boldsymbol{n}$
- 如果Λ是非极化的，$\langle \boldsymbol{\eta} \rangle = 0$, $W(\boldsymbol{\zeta}) \sim 1 + \alpha \boldsymbol{\zeta} \cdot \boldsymbol{n}$
- **一般情况下，核子的极化矢量为**

$$\boldsymbol{P} = \frac{\boldsymbol{n}(\alpha + \boldsymbol{\eta} \cdot \boldsymbol{n}) + \beta \boldsymbol{\eta} \times \boldsymbol{n} + \gamma \boldsymbol{n} \times (\boldsymbol{\eta} \times \boldsymbol{n})}{\mathbf{1} + \alpha \boldsymbol{\eta} \cdot \boldsymbol{n}}$$

Angular distribution in helicity amplitude

- Lee-Yang parameters in terms of helcity amplitude

e.g. $\Lambda(\lambda) \to p(\lambda_1)\pi^-$, angle: (θ, ϕ), amplitude: H_{λ_1}

S- and P –wave in terms of H_{λ_1}

$$\because \quad |JM\ell s\rangle = \sum_{\lambda_1\lambda_2} \left(\frac{2\ell+1}{2J+1}\right)^{\frac{1}{2}} (\ell 0 s\lambda|J\lambda)(s_1\lambda_1 s_2 -\lambda_2|s\lambda)|JM\lambda_1\lambda_2\rangle$$

$\therefore \quad S = \frac{1}{\sqrt{2}}(H_+ + H_-),\ P = \frac{1}{\sqrt{2}}(H_+ - H_-)$, where $H_\pm \equiv H_{\pm\frac{1}{2}}$

$\alpha_\Lambda = \frac{|H_+|^2 - |H_-|^2}{|H_+|^2 + |H_-|^2}$, assumming $H_+ = h_+\, e^{i\delta_+},\ H_- = h_- e^{i\delta_-}$

Normalization: $|h_+|^2 + |h_-|^2 = 1$

Angular distribution in helicity amplitude

- Lee-Yang parameters in terms of helicity amplitude

$$\beta_\Lambda = \sqrt{1-\alpha_\Lambda^2}\sin\Delta\,,\ \gamma_\Lambda = \sqrt{1-\alpha_\Lambda^2}\cos\Delta, \text{ with } \Delta = \delta_+ - \delta_-$$

Satisfy: $\alpha_\Lambda^2 + \beta_\Lambda^2 + \gamma_\Lambda^2 = 1$

- Angular distribution

Given Λ spin density matrix $\rho^\Lambda = \begin{pmatrix} 1-P & 0 \\ 0 & 1+P \end{pmatrix}$,

$$|\mathcal{M}|^2 = \sum_{\lambda,\lambda',\lambda_1} \rho^\Lambda_{\lambda,\lambda'} D^{1/2*}_{\lambda,\lambda_1}(\phi,\theta,0) D^{1/2}_{\lambda',\lambda_1}(\phi,\theta,0) \left|F_{\lambda_1}\right|^2$$

Asymmetry angular distribution

$$\frac{dN}{d\Omega} = \frac{1}{4\pi}\left(1 + \alpha_\Lambda \vec{P} \cdot \hat{q}\right) = \frac{1}{4\pi}(1 + \alpha_\Lambda P_\Lambda cos\theta_p)$$

C- and P- transformation

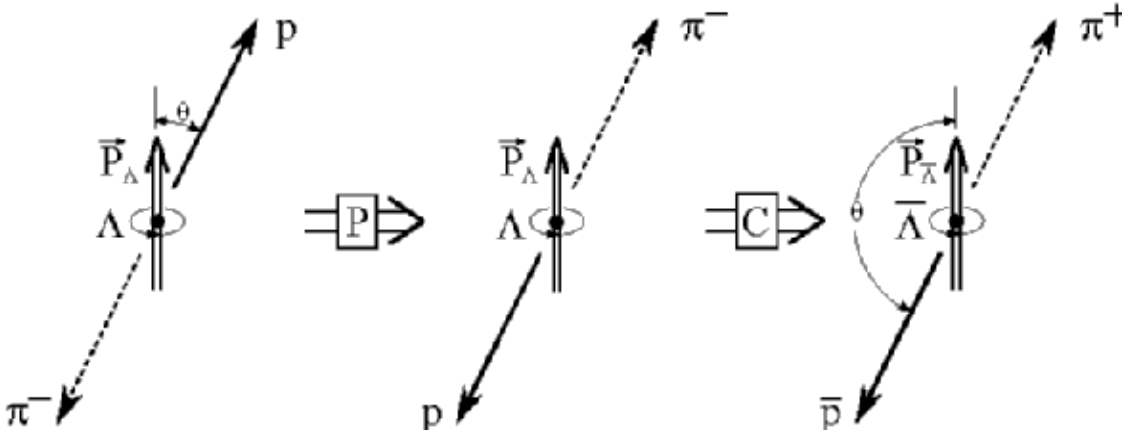
$$\alpha_\Lambda = \frac{|B_+|^2 - |B_-|^2}{|B_+|^2 + |B_-|^2}, \alpha_{\bar{\Lambda}} = \frac{|\bar{B}_+|^2 - |\bar{B}_-|^2}{|\bar{B}_+|^2 + |\bar{B}_-|^2}$$

CP **invariance:**

$$\bar{B}_{-\lambda_p} = \eta_\Lambda \eta_p \eta_\pi (-1)^{s_\Lambda - s_p - s_\pi} B_{\lambda_p} = -B_{\lambda_p}$$

If CP invariance:

$$\alpha_\Lambda = -\alpha_{\bar{\Lambda}}$$



Proton polarization from $\Lambda \to p\pi^-$

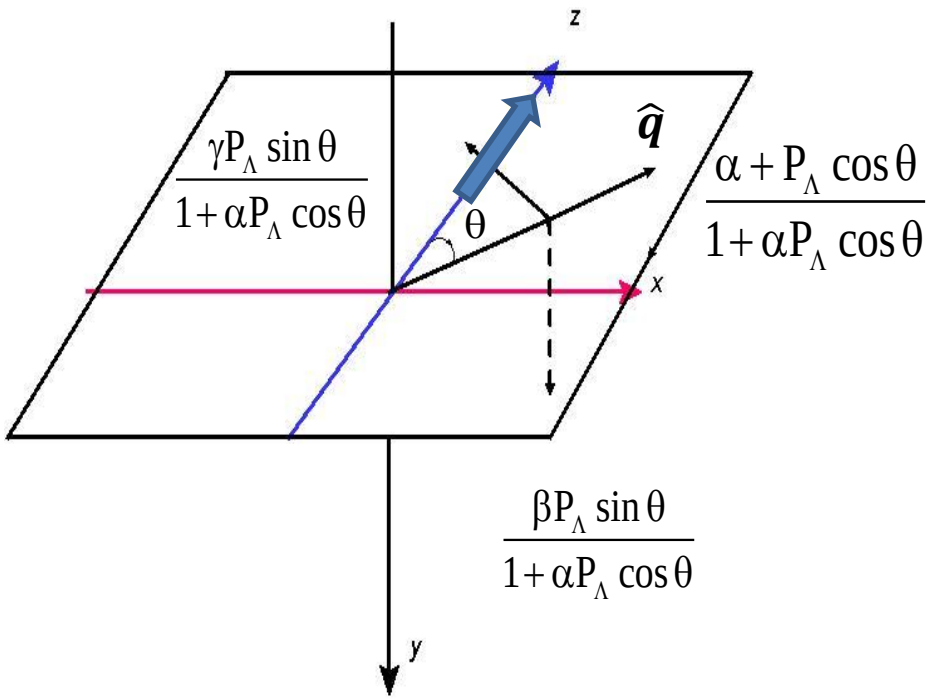
$$\vec{P}_p = \frac{(\alpha + \vec{P}_\Lambda \cdot \hat{q})\hat{q} + \beta\ (\vec{P}_\Lambda \times \hat{q}) + \gamma\ \hat{q} \times (\vec{P}_\Lambda \times \hat{q})}{(1 + \alpha \vec{P}_\Lambda \cdot \hat{q})}$$

- If $P_\Lambda = 0$ then $P_p = \alpha P_\Lambda \cdot q$
- $T-$odd transverse polarization

  $\beta \neq 0$
- If CP is conserved:

  $\alpha = -\bar{\alpha},\ \beta = -\bar{\beta},$

  $\gamma = \bar{\gamma}$ and $\Gamma = \bar{\Gamma}$



Some CP-odd observables

$$\Delta = \frac{\Gamma - \overline{\Gamma}}{\Gamma + \overline{\Gamma}} \qquad A = \frac{\alpha + \overline{\alpha}}{\alpha - \overline{\alpha}}$$

$$B = \frac{\beta + \overline{\beta}}{\beta - \overline{\beta}} \qquad B' = \frac{\beta + \overline{\beta}}{\alpha - \overline{\alpha}}$$

- **A_Λ at 10^{-5} level by CKM matrix, PDG: 0.006 ± 0.021**
- **Asymmetries B, B' require knowledge of both parent and daughter polarization**

Previous Measurement

α_- FOR $\Lambda \to p\pi^-$

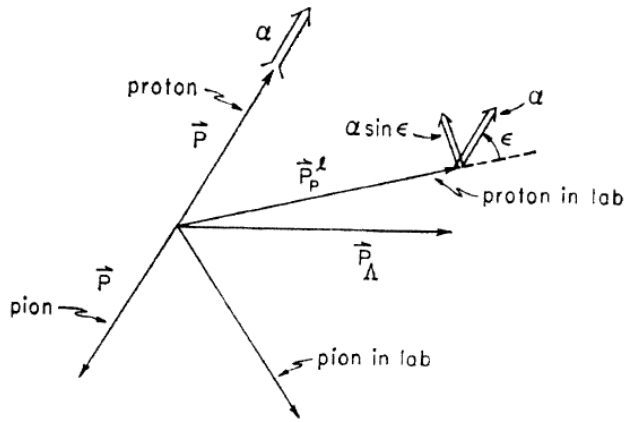
VALUE	EVTS	DOCUMENT ID		TECN	COMMENT
0.642 ± 0.013	**OUR AVERAGE**				
0.584 ± 0.046	8500	ASTBURY	1975	SPEC	
0.649 ± 0.023	10325	CLELAND	1972	OSPK	
0.67 ± 0.06	3520	DAUBER	1969	HBC	From Ξ decay
0.645 ± 0.017	10130	OVERSETH	1967	OSPK	Λ from $\pi^- p$
0.62 ± 0.07	1156	CRONIN	1963	CNTR	Λ from $\pi^- p$

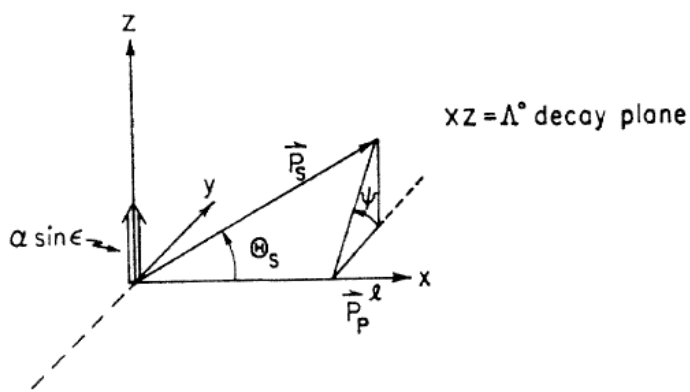
α_+ FOR $\overline{\Lambda} \to \overline{p}\pi^+$

VALUE	EVTS	DOCUMENT ID		TECN	COMMENT
-0.71 ± 0.08	**OUR AVERAGE**				
$-0.755 \pm 0.083 \pm 0.063$	$\approx$ 8.7k	ABLIKIM	2010	BES	$J/\psi \to \Lambda\overline{\Lambda}$
-0.63 ± 0.13	770	TIXIER	1988	DM2	$J/\psi \to \Lambda\overline{\Lambda}$

Most earlier measurement on α_-

- CNTR 实验， $\pi^- + p \rightarrow \Lambda + K^0$
- 非极化的Λ衰变产生的p的极化为α, 末态p的极化可以通过火花室测量





$W(\psi) = 1 + \alpha S \sin\epsilon \cos\psi$

Phys.Rev. 129 (1963) 1795-1807

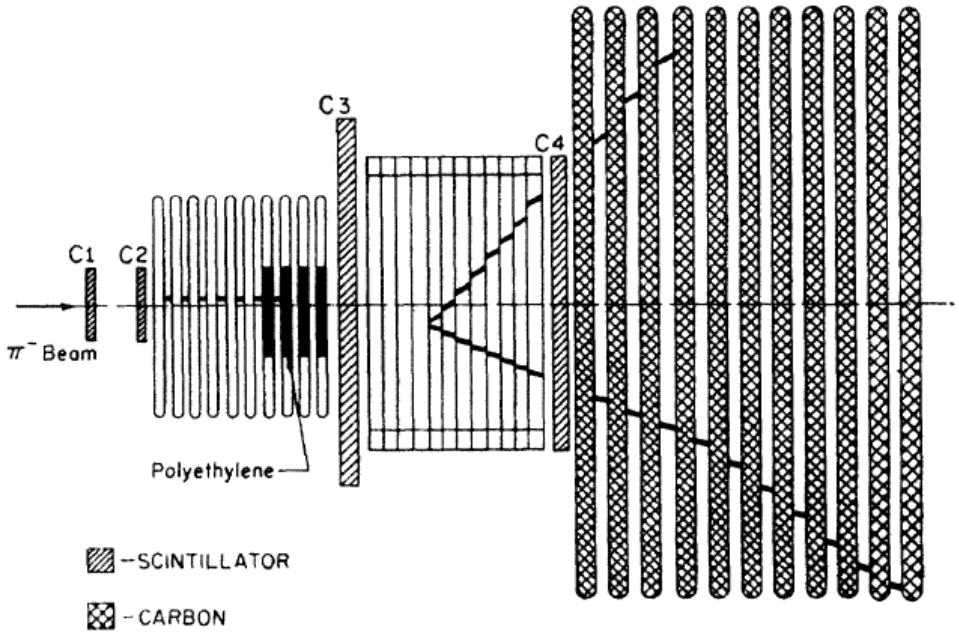


FIG. 1. Schematic diagram showing arrangement of apparatus. An example of an event has been sketched in.

$$\alpha = \frac{2}{\pi} \frac{1}{\langle S \rangle \langle \sin\epsilon \rangle} \frac{N_+ - N_-}{N_+ + N_-},$$

1156 events

$\langle S \rangle = 0.565$

$\langle \sin\epsilon \rangle = 0.84,$

$\alpha = 0.62.$

• Folding law for sequential decays

Consider decay
$\Xi^- \to \Lambda\pi^-, \Lambda \to p\pi^-$

particle	Unit 3-momentum	polarization
Ξ^-	$\hat{q}_\Xi$	$\boldsymbol{P}_\Xi$
Λ	$\hat{q}_\Lambda$	$\boldsymbol{P}_\Lambda$
p	$\hat{q}_p$	$\boldsymbol{P}_p$

Method I: merge angular distributions of two step decays

$$W_\Xi(P_\Xi, P_\Lambda, \hat{q}_\Lambda)=1+\alpha_\Xi(P_\Xi \cdot \hat{q}_\Lambda + P_\Lambda \cdot \hat{q}_\Lambda) + \beta P_\Lambda \cdot (P_\Xi \times \hat{q}_\Lambda)$$
$$+\gamma P_\Xi \cdot \hat{q}_\Lambda+(1-\gamma)(P_\Xi \cdot \hat{q}_\Lambda)(P_\Lambda \cdot \hat{q}_\Lambda)$$

$$W\left(P_\Lambda, 0, \hat{q}_p\right) = 1 + \alpha_\Lambda\, P_\Lambda \cdot \hat{q}_p$$

$$W = \sum_{P_\Lambda} \left\langle W_\Xi(P_\Xi, P_\Lambda, \hat{q}_\Lambda) W\left(P_\Lambda, 0, \hat{q}_p\right)\right\rangle_{P_\Lambda}$$
$$=1+\alpha_\Xi P_\Xi \cdot \hat{q}_\Lambda+\alpha_\Xi\alpha_\Lambda \hat{q}_\Lambda \cdot \hat{q}_p+\alpha_\Lambda\beta\, \hat{q}_p \cdot (P_\Xi \times \hat{q}_\Lambda)+\alpha_\Lambda\gamma\, \left(P_\Xi \cdot\ \hat{q}_p\right)$$
$$+(1-\gamma)(P_\Xi \cdot \hat{q}_\Lambda)(\hat{q}_\Lambda \cdot \hat{q}_p)$$

- Method II: polarization transfer

Λ polarization :

$$P_\Lambda = \frac{\hat{q}_\Lambda(\alpha_\Xi + P_\Xi \cdot \hat{q}_\Lambda) + \beta(P_\Xi \times \hat{q}_\Lambda) + \gamma \hat{q}_\Lambda \times (P_\Xi \times \hat{q}_\Lambda)}{1 + \alpha_\Xi P_\Xi \cdot \hat{q}_\Lambda}$$

In Λ CM system, if proton polarization is not observed, then

$$W \sim 1 + \alpha_\Lambda P_\Lambda \cdot \hat{q}_p$$

$$= \ 1 + \alpha_\Lambda \frac{\hat{q}_\Lambda \cdot \hat{q}_p(\alpha_\Xi + P_\Xi \cdot \hat{q}_\Lambda) + \beta \hat{q}_p \cdot (P_\Xi \times \hat{q}_\Lambda) + \gamma \hat{q}_p \cdot [\hat{q}_\Lambda \times (P_\Xi \times \hat{q}_\Lambda)]}{1 + \alpha_\Xi P_\Xi \cdot \hat{q}_\Lambda}$$

$$\sim 1 + \alpha_\Xi P_\Xi \cdot \hat{q}_\Lambda + \alpha_\Xi \alpha_\Lambda \hat{q}_\Lambda \cdot \hat{q}_p + \alpha_\Lambda \beta \ \hat{q}_p \cdot (P_\Xi \times \hat{q}_\Lambda)$$
$$+ \alpha_\Lambda \gamma \left(P_\Xi \cdot \ \hat{q}_p\right) + (1 - \gamma)(P_\Xi \cdot \hat{q}_\Lambda)(\hat{q}_\Lambda \cdot \hat{q}_p) \ \alpha_\Lambda$$

If Ξ is unpolarized, then $P_\Xi = 0$, has $W = 1 + \alpha_\Xi \alpha_\Lambda \ cos\theta_p$

The HyperCP Experiment at Fermilab

Obtain $\Xi^+ \to \Lambda\pi^-$ and $\bar{\Xi}^+ \to \bar{\Lambda}\pi^+$ yield Λ and $\bar{\Lambda}$ samples having polarization absolutely determined by the Ξ^- and $\bar{\Xi}^+$ decay parameters:

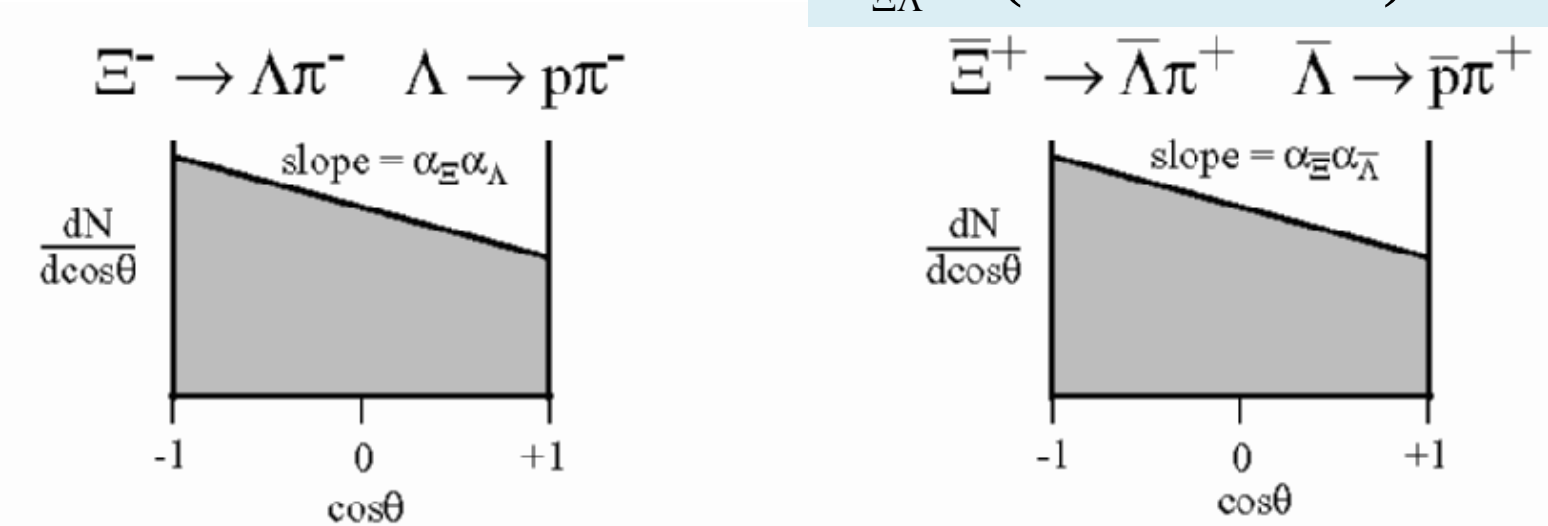
$$\bar{P}_\Lambda = \alpha_\Xi \hat{q}_\Lambda \qquad \bar{P}_{\bar{\Lambda}} = \alpha_{\bar{\Xi}} \hat{q}_{\bar{\Lambda}}$$

In the Λ and $\bar{\Lambda}$ helicity frames:

PRL93,262001 (2005)

$$\frac{dN_p}{d\cos\theta_{p\Lambda}} = 1 + \alpha_\Xi \alpha_\Lambda \cos\theta_{pL} \qquad \frac{dN_p}{d\cos\theta_{p\bar{\Lambda}}} = 1 + \alpha_{\bar{\Xi}} \alpha_{\bar{\Lambda}} \cos\theta_{p\bar{\Lambda}}$$

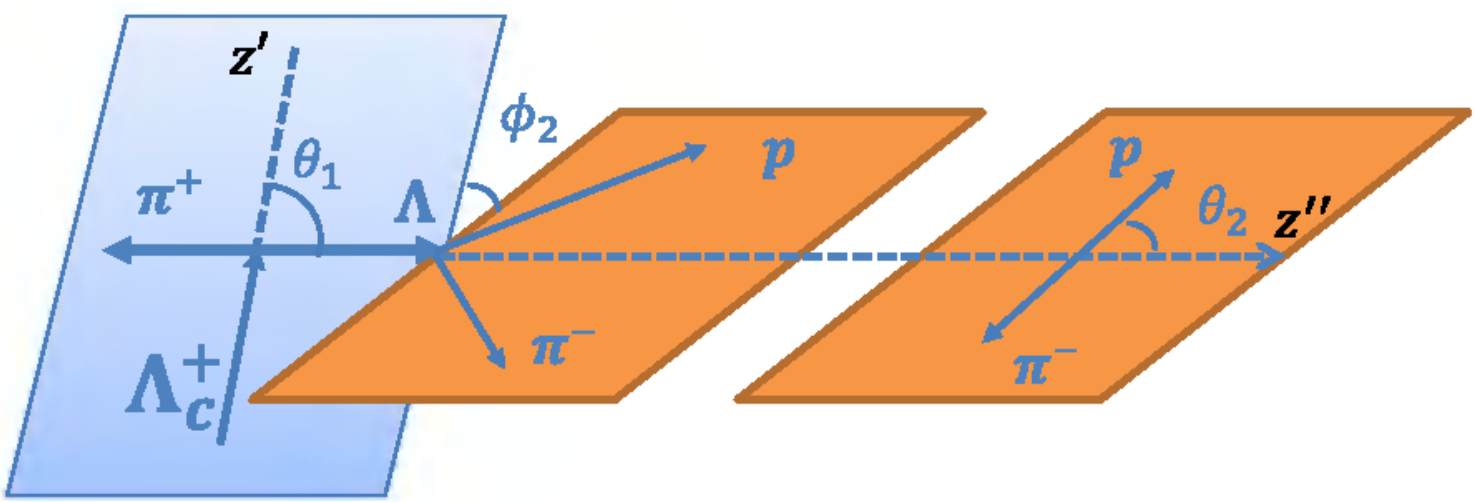
$$A_{\Xi\Lambda} = (0.0 \pm 5.1 \pm 4.4) \times 10^{-4}$$



Helicity method for sequential decays

- Example: $e^+e^- \to \gamma^* \to \Lambda_c^+\bar{\Lambda}_c^-,\ \Lambda_c^+ \to \Lambda\pi^+, \Lambda \to p\pi^-$

decay	$\gamma^* \to \Lambda_c^+\bar{\Lambda}_c^-$	$\Lambda_c^+ \to \Lambda\pi^+,$	$\Lambda \to p\pi^-$
Helicity	$M \to \lambda_1\lambda_2$	$\lambda_1 \to \lambda_2 0$	$\lambda_2 \to \lambda_3 0$
angles	θ, ϕ	θ_1, ϕ_1	θ_2, ϕ_2
amplitude	A_{λ_1,λ_2}	B_{λ_2}	C_{λ_3}



Helicity method for sequential decays

- Example: $e^+e^- \to \gamma^* \to \Lambda_c^+\overline{\Lambda}_c^-,\ \Lambda_c^+ \to \Lambda\pi^+, \Lambda \to p\pi^-$ (cont.)

$$|\mathcal{M}|^2 \propto \sum_{M=\pm 1\,,\lambda_i} D^{1*}_{M,\lambda_1-\lambda_2}(\phi,\theta,0) D^{1}_{M,\lambda_1'-\lambda_2}(\phi,\theta,0) A_{\lambda_1,\lambda_2} A^*_{\lambda_1,\lambda_2}$$
$$\times D^{1/2*}_{\lambda_1,\lambda_3}(\phi_1,\theta_1,0) D^{1/2}_{\lambda_1',\lambda_3'}(\phi_1,\theta_1,0) B_{\lambda_3} B^*_{\lambda_3'}$$
$$\times D^{1/2\,*}_{\lambda_3,\lambda_4}(\phi_2,\theta_2,0) D^{1/2}_{\lambda_3',\lambda_4}(\phi_2,\theta_2,0) \left|C_{\lambda_4}\right|^2$$

Decay conserves parity, one has

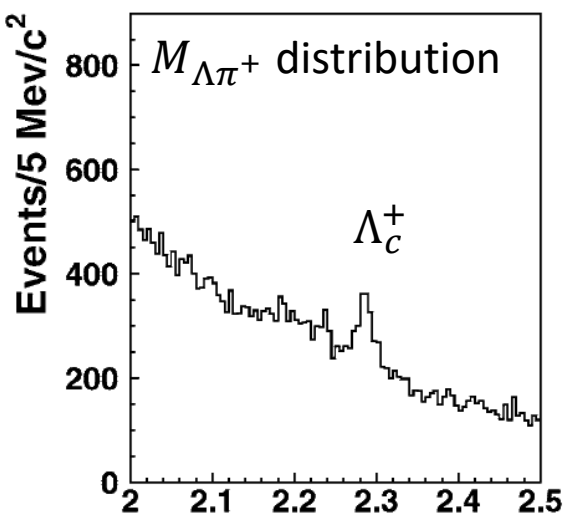
$$A_{-\frac{1}{2},-\frac{1}{2}} = A_{\frac{1}{2},\frac{1}{2}}, A_{-\frac{1}{2},\frac{1}{2}} = A_{\frac{1}{2},-\frac{1}{2}}$$
$$\alpha_{\Lambda_c} = \frac{|B_+|^2 - |B_-|^2}{|B_+|^2 + |B_-|^2}$$
$$\alpha_{\Lambda} = \frac{|C_+|^2 - |C_-|^2}{|C_+|^2 + |C_-|^2}$$

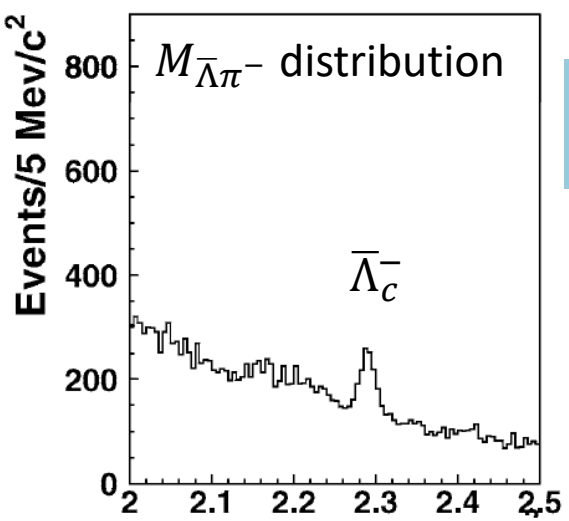
Helicity method for sequential decays

- Example: $e^+e^- \to \gamma^* \to \Lambda_c^+\overline{\Lambda}_c^-,\ \Lambda_c^+ \to \Lambda\pi^+, \Lambda \to p\pi^-$ (cont.)

$$\frac{dN}{d\cos\theta_2} \propto 1 + \alpha_{\Lambda_c}\alpha_\Lambda \cos\theta_2$$

- **FOCUS experiment: Phys.Lett.B634, 165 (2006)**
  - FOCUS, $\gamma^*\,(beam) + \mathrm{BeO}(target) \to \Lambda_c^+ + X$
  - Assume unpolarized Λ_c^+, Λ is polarized longitudinally with α_{Λ_c} degree
  - Polarimetry: $\Lambda \to p\pi^-$



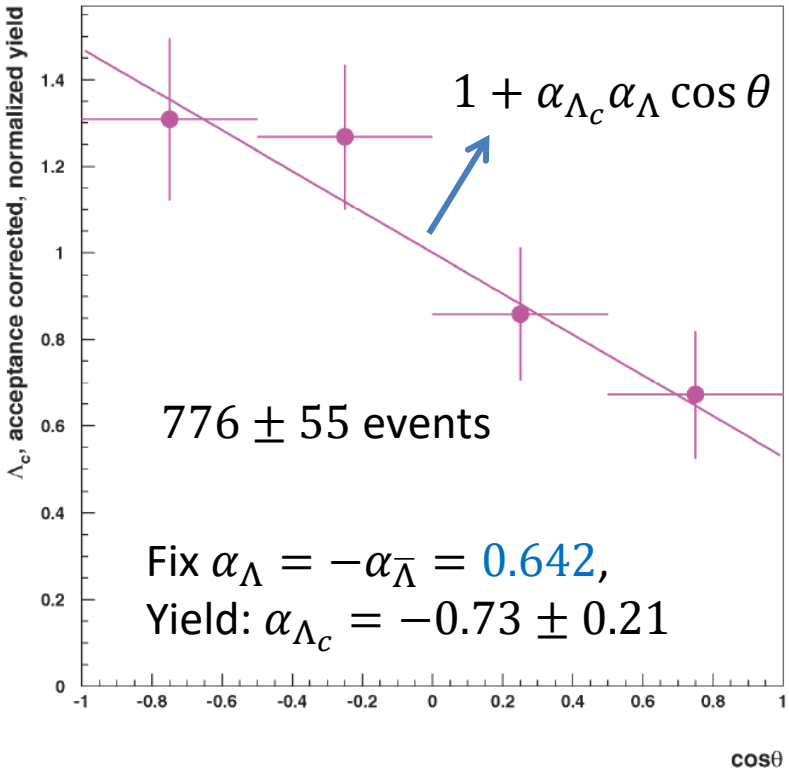


FOCUS:

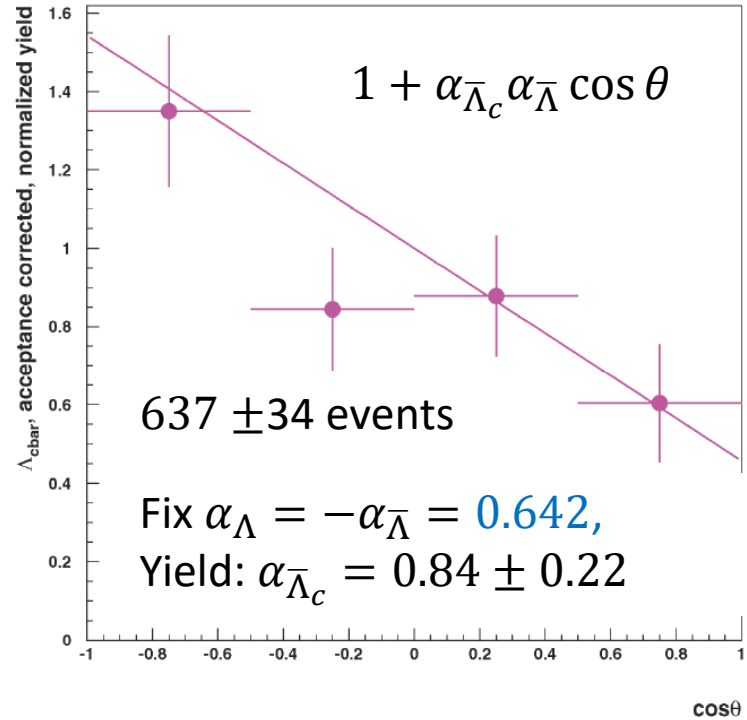
Phys.Lett.B634, 165

Previous measurements of Λ_c^+ asymmetry parameters (cont.)

α_{Λ_c} : for $\Lambda_c^+ \to \Lambda\pi^+$



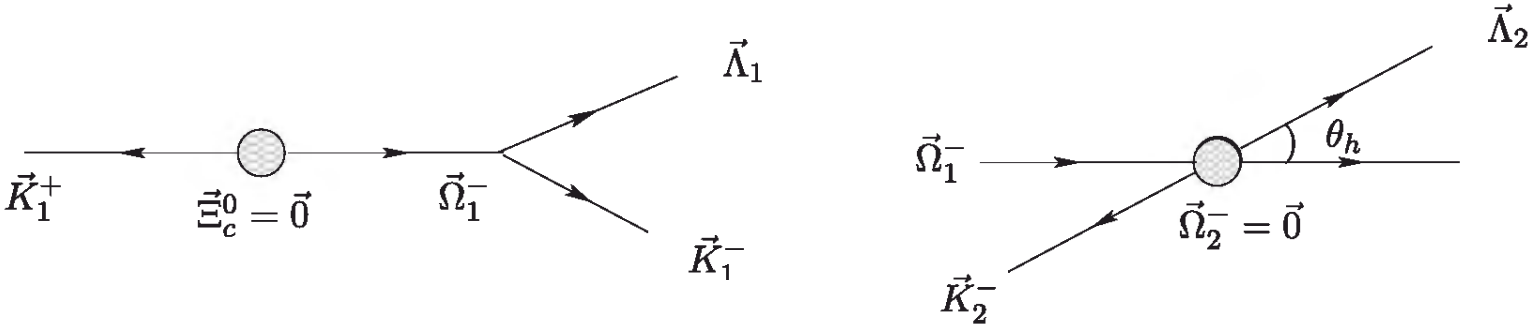
$\alpha_{\bar{\Lambda}_c}$: for $\bar{\Lambda}_c^- \to \bar{\Lambda}\pi^-$



After bias correction: $\alpha_{\Lambda_c} = -0.78 \pm 0.16 \pm 0.19$, $\mathcal{A} = \frac{\alpha_{\Lambda_c} - \alpha_{\bar{\Lambda}_c}}{\alpha_{\Lambda_c} + \alpha_{\bar{\Lambda}_c}} = -0.07 \pm 0.16 \pm 0.19$

3.2共振态自旋-宇称的测量

- Determination of Ω^- spin [PRL, 97, 112001(2006)]
  - Babar $\mathcal{L} = 116\,\mathrm{fb}^{-1}$, using $\Xi_c^0 \to \Omega^- K^+$, $\Omega^- \to \Lambda K^-$



$$J = 1/2: \quad dN/d\cos\theta_h \propto 1 + \beta\cos\theta_h,$$

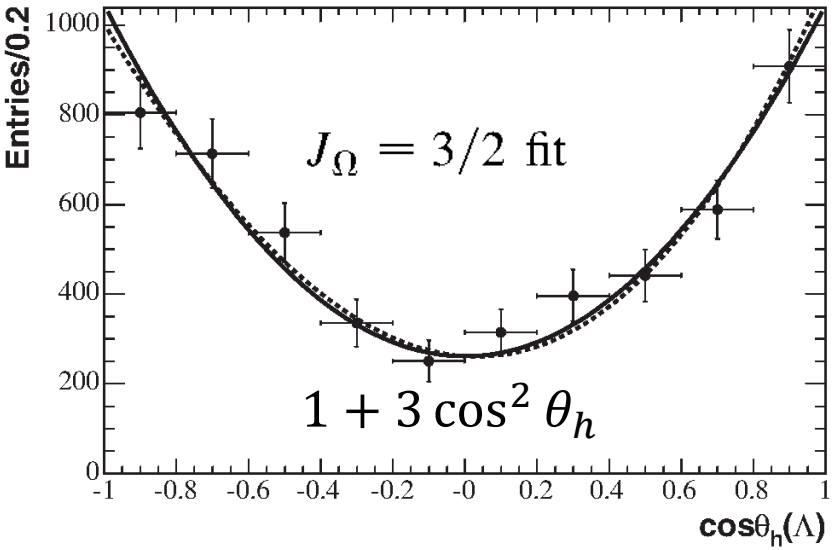
$$J = 3/2: \; dN/d\cos\theta_h \propto 1 + 3\cos^2\theta_h + \beta\cos\theta_h(5 - 9\cos^2\theta_h),$$

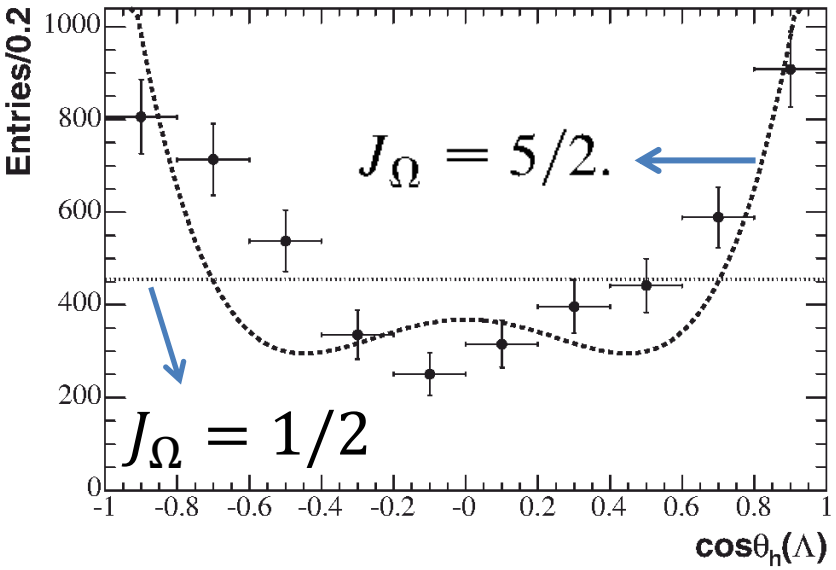
$$J = 5/2:$$

$$dN/d\cos\theta_h \propto 1 - 2\cos^2\theta_h + 5\cos^4\theta_h + \beta\cos\theta_h(5 - 26\cos^2\theta_h + 25\cos^4\theta_h),$$

3.2共振态自旋-宇称的测量(续)

- Determination of Ω^- spin [PRL, 97, 112001(2006)]



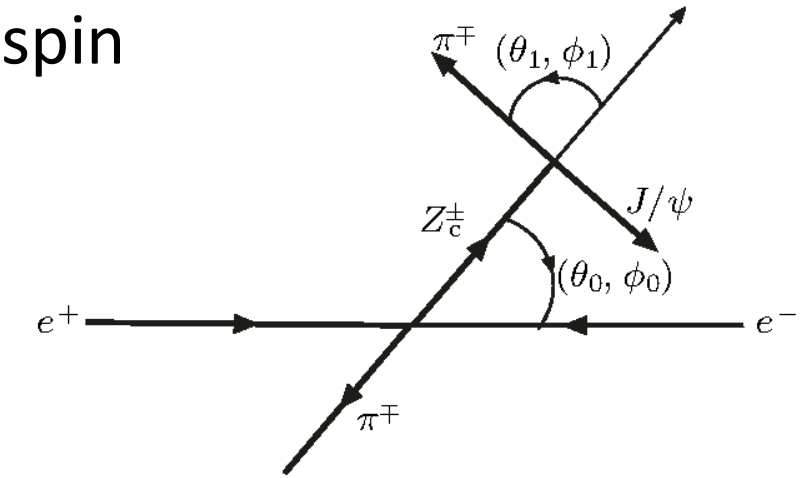


➢ data favor for $J_\Omega = 3/2$ hypothesis, and reject spin ½ and 5/2

3.2共振态自旋-宇称的测量(续)

- Determination of $Z_c(3900)^{\pm}$ spin
  - $J_Z = 1^+$

$$\frac{d|\mathcal{M}|^2}{d\cos\theta_i} \propto 1 + \alpha_i \cos^2\theta_i \quad (i = 0, 1),$$



$e^+e^- \to \gamma^* \to Z_c^{\pm}\pi^{\mp}, \qquad Z_c^{\pm} \to J/\psi\,\pi^{\pm}$

  - $J_Z = 1^-$

$$\frac{d|\mathcal{M}|^2}{d\cos\theta_i} \propto 1 + \cos^2\theta_i \quad (i = 0, 1).$$

  - $J_Z = 2^-$

$$\frac{d|\mathcal{M}|^2}{d\cos\theta_0} \propto 1 + \alpha_0 \cos^2\theta_0,$$

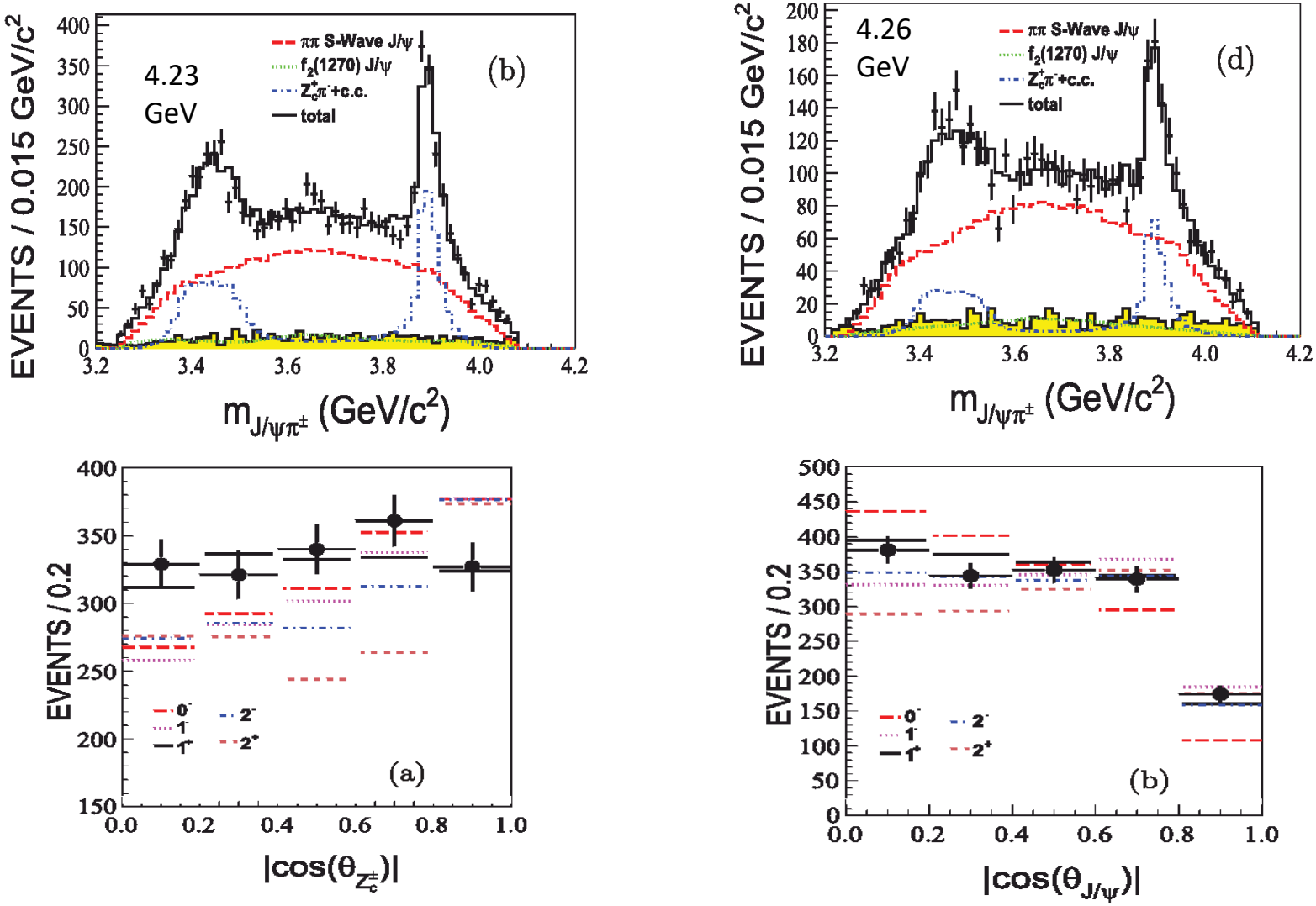
  - $J_Z = 2^+$

$$\frac{d|\mathcal{M}|^2}{d\cos\theta_0} \propto 1 + \cos^2\theta_0, \qquad \frac{d|\mathcal{M}|^2}{d\cos\theta_1} \propto 2 + \cos(2\theta_1) + \cos(4\theta_1).$$

3.2共振态自旋-宇称的测量(续)

- Determination of $Z_c(3900)^{\pm}$ spin

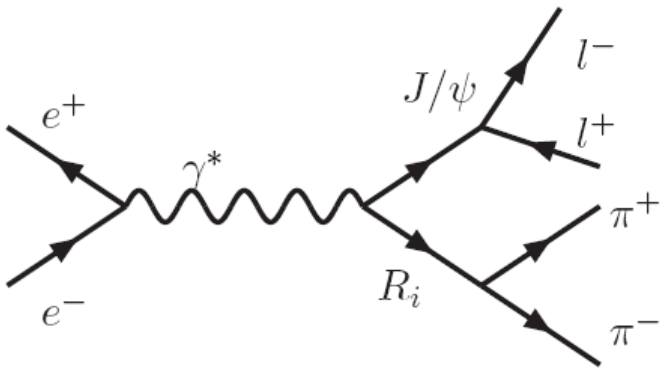
PRL, 119, 072001(2017)

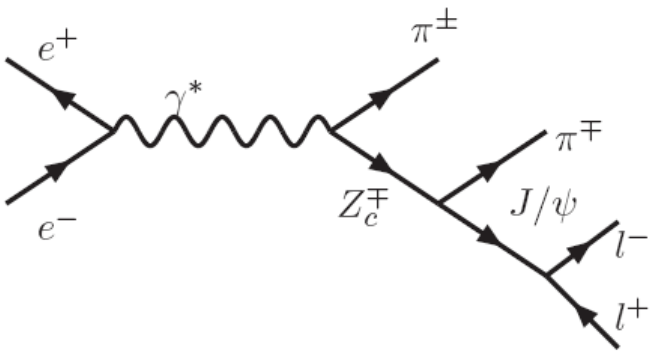


3.3多道级联式衰变(isobar 模型)中问题

- isobar model

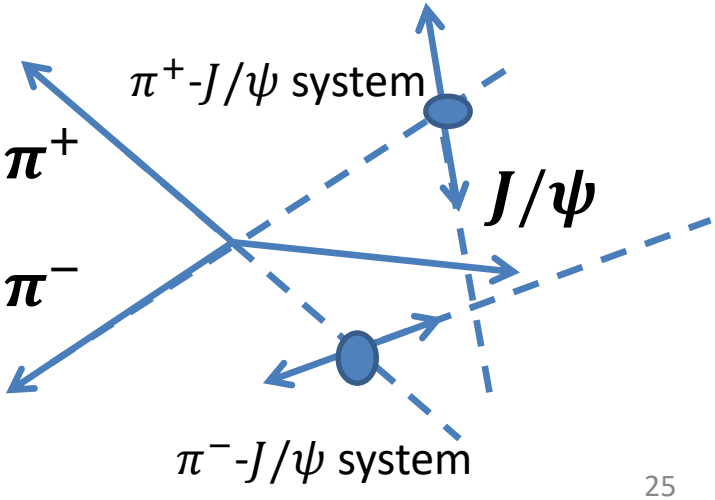
  e.g. $e^+e^- \to \gamma^* \to \pi^+\pi^- J/\psi$ via





Warning:

J/ψ quantization axis lies different direction in π^+-J/ψ and π^--J/ψ system, so that sum of these two isobar processes can not give correct interference effects.

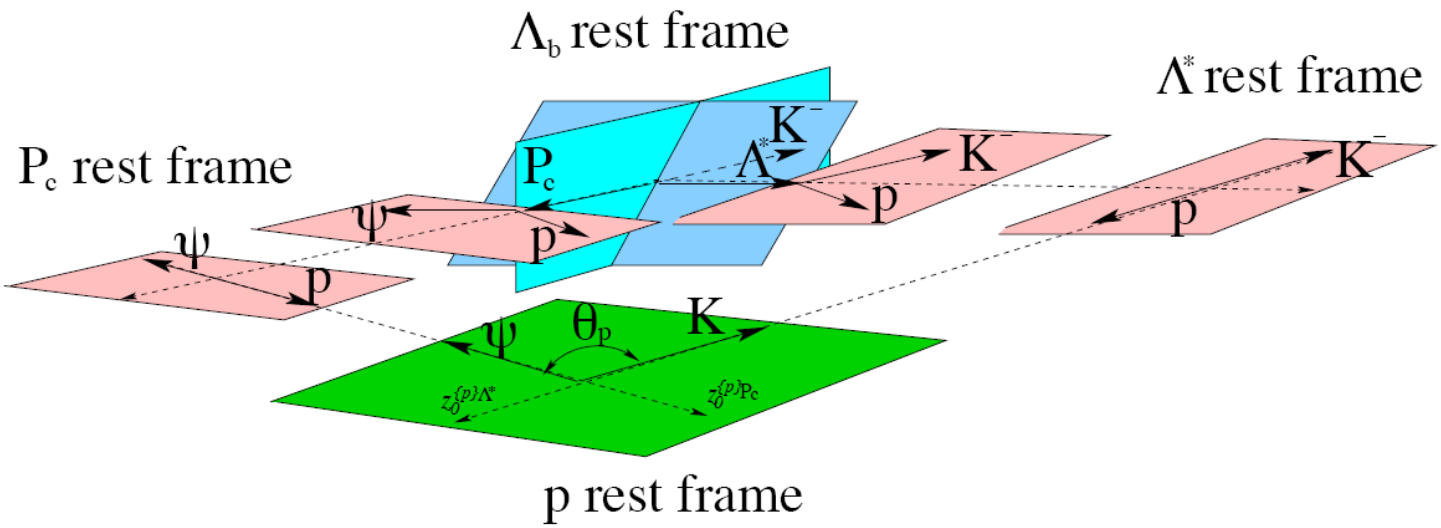


3.3多道级联式衰变(isobar 模型)中问题

- isobar model

This problem is addressed in the BELL and Babar analyses.

Belle, Phys. Rev. D 88, 074026 (2013).
LHCb, Phys. Rev. Lett. 115, 072001 (2015).



Proton in pentaquark and Λ^* system has different quantization axis

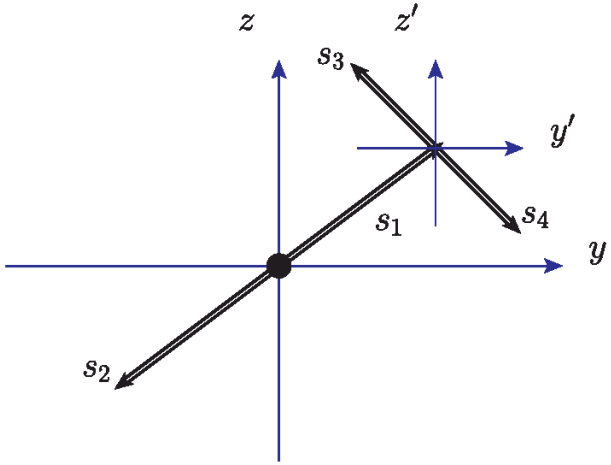
3.3多道级联式衰变(isobar 模型)中问题

- Coherent amplitude for sequential decay

For sequential decay

$$J \to s_1 + s_2, \quad s_1 \to s_3 + s_4.$$

Refer to PRD95, 076010



$$\mathcal{A}_{\mathrm{I}}(\theta_1, \theta_2, \phi_1, \phi_2; J, M, m_2, m_3, m_4)$$
$$= \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} D^{s_2}_{m_2,-\lambda_2}(\phi_1, \theta_1, 0) D^{s_3}_{m_3,\lambda_3}(\phi_2, \theta_2, 0)$$
$$\times D^{s_4}_{m_4,-\lambda_4}(\phi_2, \theta_2, 0) D^{J*}_{M,\lambda_1-\lambda_2}(\phi_1, \theta_1, 0)$$
$$\times D^{s_1*}_{\lambda_1,\lambda_3-\lambda_4}(\bar{\phi}_2, \bar{\theta}_2, 0) F^{J}_{\lambda_1,\lambda_2} F^{s_1}_{\lambda_3,\lambda_4}.$$

Questions and exercise

- 推导以下级联式衰变的角分布，只考虑纵向极化，并假设母粒子Λ_c^+， Ξ_c^0， Ω_c^0是非极化的。 [see Chin.Phys.C41, 023106(2017)]
- (1) $\Lambda_c^+ \to \Lambda\pi^+ , \Lambda \to p\pi^-$
- (2) $\Xi_c^0 \to \Xi^-\pi^+,\ \Xi^- \to \Lambda\pi^-, \Lambda \to p\pi^-$
- (3) $\Omega_c^0 \to \Omega^-\pi^+, \Omega^- \to \Lambda\pi^-, \Lambda \to p\pi^-$
- (4) $\Lambda_c^+ \to \Sigma^0\pi^+, \Sigma^0 \to \gamma\Lambda, \Lambda \to p\pi^-$ (选做)