粒子物理学的自旋极化
Lecture 5

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第五讲：自旋密度矩阵2

- **5.1.** 在反应截面计算中的应用
- **5.2.** 极化分析和极化观测量
- **5.3.** 自旋传递和自旋关联

5.1. 在反应截面计算中的应用

- 共振态的自旋单态和混合态
  - resonance in a given exclusive mode can be described with a pure state of spin density matrix.
  - If a given resonance has different mother particle decayed from, its spin density matrix can be described with a mixture state.
  - Mixed spin density matrix can be measured with the exclusive decay of spin analysis, but can’t measured with single tag experiment.

e.g. single tag exp. $K^* \to K\pi$, can not measure the K^* spin density matrix.

5.1. 在反应截面计算中的应用

- 角分布的计算

➢ 例：假设Λ的自旋密度矩阵为 $\rho = \frac{1}{2}\begin{pmatrix} 1+P_z & P_x - iP_y \\ P_x + iP_y & 1-P_z \end{pmatrix}$, 计算$\Lambda \to p\pi^-$衰变中的质子的角分布。

5.2. 极化分析和极化观测量

- 粒子衰变中的自旋密度矩阵元

  - 粒子的自旋密度矩阵只决定于其产生过程，与衰变无关

    例如，级联式衰变 $\mathrm{A}(\lambda_A) \to B(\lambda_B) + C(\lambda_C)$，$B \to D(\lambda_D) + E(\lambda_E)$,中，粒子$E$的振幅可表示为

$$M(\lambda_A, \lambda_C, \lambda_D \lambda_E) = \sum_{\lambda_B} Amp(\lambda_A, \lambda_B, \lambda_C, \lambda_D, \lambda_E)$$

粒子E的自旋密度矩阵元定义为：

$$\rho(\lambda_E, \lambda_E') = \frac{1}{2S_A + 1} \sum_{\lambda_A, \lambda_C, \lambda_D} M(\lambda_A, \lambda_C, \lambda_D \lambda_E)^{\dagger} M(\lambda_A, \lambda_C, \lambda_D \lambda_E)$$

5.2. 极化分析和极化观测量

- 粒子的极化分析

$$\mathcal{P}_x = \frac{\mathrm{Tr}(\rho\,\hat{S}_x)}{\mathrm{Tr}(\rho)},\ \mathcal{P}_y = \frac{\mathrm{Tr}(\rho\,\hat{S}_y)}{\mathrm{Tr}(\rho)},\ \mathcal{P}_z = \frac{\mathrm{Tr}(\rho\,\hat{S}_z)}{\mathrm{Tr}(\rho)}$$

- ➢ For $s = \frac{1}{2}$, polarization of a particle is well specified by the polarization vector, with spin operators

$$S_x = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, S_y = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, S_z = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- ➢ For $S = 1$, the polarization vector $\boldsymbol{\mathcal{P}}(\mathcal{P}_x, \mathcal{P}_y, \mathcal{P}_z)$ is calculated with the spin operators

5.2. 极化分析和极化观测量(续)

- 粒子的极化分析

$$S_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad S_y = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} .$$

Rank-2 tensor polarization

$$T_{ij} = \frac{1}{2}\sqrt{\frac{3}{2}}\left(\langle \hat{s}_i \hat{s}_j + \hat{s}_j \hat{s}_i \rangle - \frac{4}{3}\delta_{ij}\right) .$$

Overall degree of polarization: $d = [\frac{3}{4}\mathscr{P}^2 + T^2]^{1/2}.$

- For high spin state, the spin operators in Cartesian coordinate system are taken as

5.2. 极化分析和极化观测量(续)

- Spin-$\frac{3}{2}$ particle

$$J_x = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & 2 & 0 \\ 0 & 2 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix} \quad J_y = \frac{1}{2}\begin{pmatrix} 0 & -i\sqrt{3} & 0 & 0 \\ i\sqrt{3} & 0 & -2i & 0 \\ 0 & 2i & 0 & -i\sqrt{3} \\ 0 & 0 & i\sqrt{3} & 0 \end{pmatrix}$$

$$J_z = \frac{1}{2}\begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}.$$

5.2. 极化分析和极化观测量(续)

- Spin-2 particle

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 2 & 0 & 0 & 0 \\ 2 & 0 & \sqrt{6} & 0 & 0 \\ 0 & \sqrt{6} & 0 & \sqrt{6} & 0 \\ 0 & 0 & \sqrt{6} & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 \end{pmatrix}, \quad S_z = \hbar\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -2 \end{pmatrix}$$

$$S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -2i & 0 & 0 & 0 \\ 2i & 0 & -\sqrt{6}i & 0 & 0 \\ 0 & \sqrt{6}i & 0 & -\sqrt{6}i & 0 \\ 0 & 0 & \sqrt{6}i & 0 & -2i \\ 0 & 0 & 0 & 2i & 0 \end{pmatrix}$$

5.2. 极化分析和极化观测量(续)

- Spin-$\frac{5}{2}$ particle

$$J_x = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{5} & 0 & 0 & 0 & 0 \\ \sqrt{5} & 0 & 2\sqrt{2} & 0 & 0 & 0 \\ 0 & 2\sqrt{2} & 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 & 2\sqrt{2} & 0 \\ 0 & 0 & 0 & 2\sqrt{2} & 0 & \sqrt{5} \\ 0 & 0 & 0 & 0 & \sqrt{5} & 0 \end{pmatrix}$$

$$J_z = \frac{1}{2}\begin{pmatrix} 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & -5 \end{pmatrix}$$

$$J_y = \frac{1}{2}\begin{pmatrix} 0 & -i\sqrt{5} & 0 & 0 & 0 & 0 \\ i\sqrt{5} & 0 & -2i\sqrt{2} & 0 & 0 & 0 \\ 0 & 2i\sqrt{2} & 0 & -3i & 0 & 0 \\ 0 & 0 & 3i & 0 & -2i\sqrt{2} & 0 \\ 0 & 0 & 0 & 2i\sqrt{2} & 0 & -i\sqrt{5} \\ 0 & 0 & 0 & 0 & i\sqrt{5} & 0 \end{pmatrix}$$

5.2. 极化分析和极化观测量(续)

- Spin observables

Consider a process e$^+$(A) + e$^-$ (B)$\xrightarrow{\gamma^*/Z^0}$ $C+D$

- If C and D are spinless particle, the only observable is angular distribution

$$\frac{d\sigma}{dcos\theta} \propto I_0(1+\alpha\ \cos\theta)\ ,$$

where α is angular distribution parameter

- If electron is polarized, the observable of transverse polarization:

$$\frac{d\sigma}{d\Omega} = I_0(1+A_n\boldsymbol{S_T}\cdot\boldsymbol{n}) = I_0(1+A_n\boldsymbol{S_T}cos\phi),$$

A_n: analysing power

5.2. 极化分析和极化观测量(续)

- Spin observables (cont.)

➢ For the case of C and D are spin-$\frac{1}{2}$ particle :

$$\frac{d\sigma}{d\Omega} = \frac{1}{n_A n_B} \sum_{a,b,c,d} | < C, D|\mathcal{M}|A, B > |^2 = \frac{1}{n_A n_B} \mathbf{Tr}\{\mathcal{M} M^\dagger\}$$

Observable: (spin average, or spin polarization)

$$C_{\lambda\mu\nu\tau} = \mathrm{Tr}\left\{\mathcal{M}\left[\sigma_\lambda(A) \otimes \sigma_\mu(B)\right] \mathcal{M}^\dagger \left[\sigma_\nu(C) \otimes \sigma_\tau(D)\right]\right\} / \mathrm{Tr}\left\{\mathcal{M}\, \mathcal{M}^\dagger\right\}$$

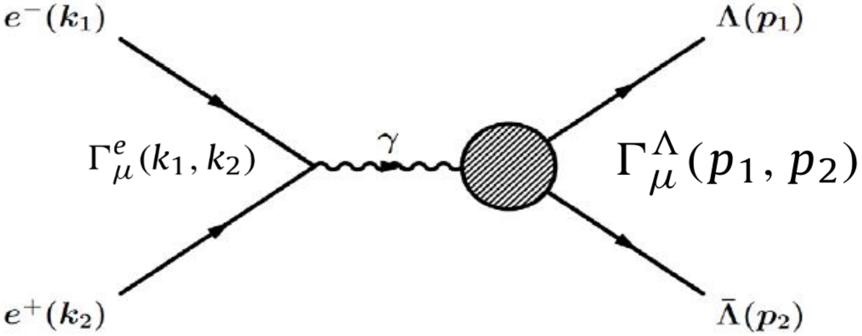
Or abbreviated by:

$$(\lambda\mu|\nu\tau) \equiv C_{\lambda\mu\nu\tau} = \langle\sigma_\lambda(A)\sigma_\mu(B)\sigma_\nu(C)\sigma_\tau(D)\rangle$$

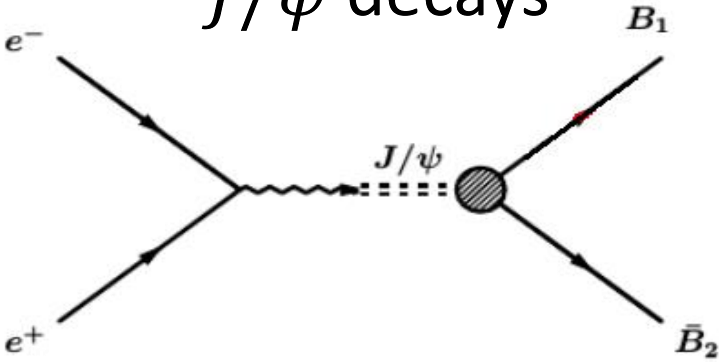
5.2. 极化分析和极化观测量(续)

- Transverse polarization of baryons in e^+e^- collisions

FFs in continuum



J/ψ decays



Time likespin ½ baryon FFs:

Dubnickova, Dubnicka, Rekalo
Nuovo Cim. A109 (1996) 241
W. Lu, et.al., Phys.Lett., B368, 261 (1996)
Gakh, Tomasi-Gustafsson Nucl.Phys. A771 (2006) 169
Czyz, Grzelinska, Kuhn PRD75 (2007) 074026
FäldtEPJ A51 (2015) 74; EPJ A52 (2016)141
Fäldt, G. & Kupsc, A, Phys. Lett. B 772 (2017) 16

$$\Gamma^e_\mu(k_1,k_2) = -ie_\psi\gamma_\mu$$

$$\Gamma^\Lambda_\mu(p_1,p_2) =$$

$$-ie_g\left[G^\psi_M\gamma_\mu - \frac{2M}{Q^2}(G^\psi_M - G^\psi_E)Q_\mu\right]$$

5.2. 极化分析和极化观测量(续)

- Transverse polarization of baryons in e^+e^- collisions

Example: With the unpolarized beam, estimate the poalrization of Λ in $e^+e^- \to J/\psi \to \Lambda(\lambda_1)\overline{\Lambda}(\lambda_2)$, assuming that we don't observe $\overline{\Lambda}$ spin configuration.

$$\rho_{\lambda_1,\lambda_1'} = \sum_{\substack{M=\pm 1/2 \\ \lambda_2=\pm 1/2}} D^{1*}_{M,\lambda_1-\lambda_2}(\phi,\theta,0) D^{1}_{M,\lambda_1'-\lambda_2}(\phi,\theta,0)\, A_{\lambda_1,\lambda_2} A^*_{\lambda_1',\lambda_2}$$

$$= \frac{1}{2}\begin{pmatrix} 1+\alpha\cos^2\theta & -i\frac{1}{2}\sqrt{1-\alpha^2}\sin\Delta\sin 2\theta \\ i\frac{1}{2}\sqrt{1-\alpha^2}\sin\Delta\sin 2\theta & 1+\alpha\cos^2\theta \end{pmatrix}$$

α: antular distribution parameter for Λ,

Δ: phase angle difference in $A_{\frac{1}{2},-\frac{1}{2}}$ and $A_{\frac{1}{2},-\frac{1}{2}}$

5.2. 极化分析和极化观测量(续)

Polarization vector components are calculated as

$$\mathcal{P}_x = \frac{\mathrm{Tr}(\rho\sigma_x)}{\mathrm{Tr}(\rho)} = 0, \mathcal{P}_z = \frac{\mathrm{Tr}(\rho\sigma_z)}{\mathrm{Tr}(\rho)} = 0$$

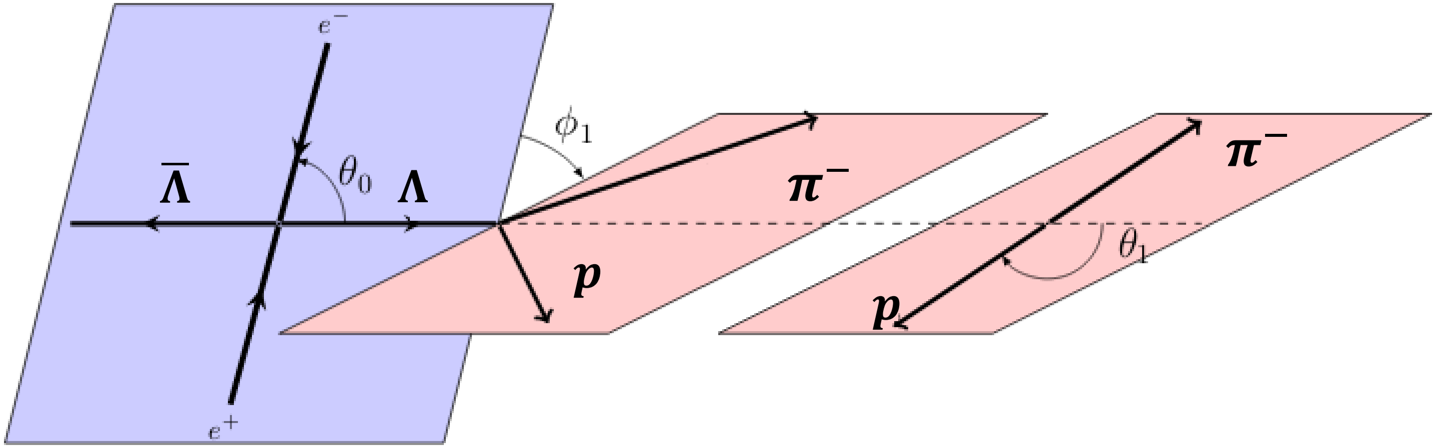
$$\mathcal{P}_y = \frac{\mathrm{Tr}\left(\rho\sigma_y\right)}{\mathrm{Tr}(\rho)} = \frac{\sqrt{1-\alpha^2}\sin\Delta\sin\theta\ \cos\theta}{1+\alpha\cos^2\theta}$$

- The decay $\Lambda \rightarrow p\pi^-$ acts as a polarimeter

Joint angular distribution for $e^+e^- \rightarrow J/\psi \rightarrow \Lambda\bar{\Lambda}, \Lambda \rightarrow p\pi^-$ reads

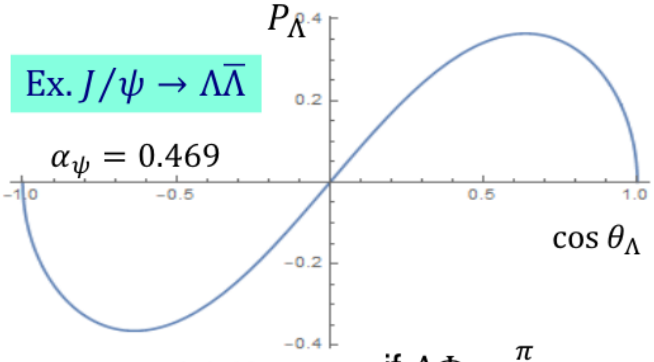
$$\mathcal{W}(\theta, \theta_1, \phi_1)$$
$$\propto 1 + \alpha\cos^2\theta + (\sqrt{1-\alpha^2}\sin\Delta\sin\theta\ \cos\theta)\ \alpha_\Lambda\sin\theta_1\sin\phi_1$$

5.2. 极化分析和极化观测量(续)



$$\langle \sin\theta_1 \sin\phi_1 \rangle = \frac{\int \mathcal{W}(\theta,\theta_1\phi_1)\sin\theta_1\sin\phi_1 d(\cos\ \theta_1\ )d\phi_1}{\int \mathcal{W}(\theta,\theta_1\phi_1)d(\cos\ \theta_1\ )d\phi_1}$$

$$= \frac{1}{3}\,\alpha_\Lambda \mathcal{P}_y$$



Ex. $J/\psi \to \Lambda\bar{\Lambda}$

$\alpha_\psi = 0.469$

Max P_Λ= 36.4%

if $\Delta\Phi = \frac{\pi}{2}$

5.2. 极化分析和极化观测量(续)

- $e^+e^- \rightarrow \psi(2S) \rightarrow \Omega^-\left(\frac{3}{2}^+\right)\overline{\Omega}^+\left(\frac{3}{2}^-\right)$

In ST measurement, the spin density matrix for Ω, calculated by helicity method:

$$\rho_{\overline{3/2}}(\theta) = \begin{array}{c|c} \begin{array}{cccc} -3/2 & -1/2 & 1/2 & 3/2 \end{array} & \\ \hline \begin{pmatrix} m_{11} & c_{12} & c_{13} & 0 \\ c_{12}^* & m_{22} & im_{23} & c_{13}^* \\ c_{13}^* & -im_{23} & m_{22} & -c_{12}^* \\ 0 & c_{13} & -c_{12} & m_{11} \end{pmatrix} & \begin{array}{c} -3/2 \\ -1/2 \\ 1/2 \\ 3/2 \end{array} \end{array}$$

Hermitian !

$$m_{11} = \frac{1+\cos^2\theta}{2}|h_3|^2$$

$$m_{22} = |h_1|^2\sin^2\theta + \frac{1+\cos^2\theta}{2}|h_2|^2$$

$$m_{23} = \sqrt{2}\,\Im(h_1h_2^*)\cos\theta\sin\theta$$

$$c_{12} = -\frac{h_3h_1^*\cos\theta\sin\theta}{\sqrt{2}}$$

$$c_{13} = \frac{1}{2}h_3h_2^*\sin^2\theta$$

5.2. 极化分析和极化观测量(续)

- Polarization of spin-3/2 particle

$$\rho_{3/2} = r_0\left(Q_0 + \frac{3}{4}\sum_{M=-1}^{1} \overbrace{r^1_M}^{r^1_{-1}\to P_y\quad r^1_0\to P_x\quad r^1_1\to P_z} Q^1_M + \frac{3}{4}\sum_{M=-2}^{2} r^2_M Q^2_M + \frac{3}{4}\sum_{M=-3}^{3} r^3_M Q^3_M\right)$$

M.G.Doncel, L.Michel, P.Minnaert Nucl. Phys. B38, 477(1972)

real coefficients,
scalable J=1/2,3/2,...

$$Q^L_M \to Q_\mu\,, \mu = 0, \ldots, 15$$

Degree of polarization

$$d(\rho_{3/2}) = \sqrt{\sum_{L=1}^{3}\sum_{M=-L}^{L}\left(r^L_M\right)^2}$$

5.3. 自旋传递和自旋关联

- Reaction $I_1 + I_2 + \cdots + I_n \to F_1 + F_2 + \cdots + F_m$

Initial spin state: $\chi_i = \chi_{i1} \otimes \chi_{i2} \ldots \otimes \chi_{in}$
Final spin state: $\chi_f = \chi_{f1} \otimes \chi_{f2} \ldots \otimes \chi_{fm}$

Reaction can be described by a transformation between the initial and final spin state

$$\chi_f = M\, \chi_i$$

Spin density matrix for initial state

$$\rho_i = \rho_{i1} \otimes \rho_{i2} \ldots \otimes \rho_{in}$$

ρ_f for final state : $\rho_f = |\chi_f\rangle\langle\chi_f| = M\rho_i M^\dagger$

5.3. 自旋传递和自旋关联(续)

- induced polarization for k-th particle in final states

$$\mathcal{P}_k = \frac{\mathrm{Tr}(M\rho_i M^\dagger \hat{S}_\mathrm{k})}{\mathrm{Tr}(M\rho_i M^\dagger)}$$

- Analysing power of reaction for the initial j-th polarization component

$$A_j(\Omega_j) = \frac{\mathrm{Tr}(M\rho_i(\hat{S}_\mathrm{j})M^\dagger)}{\mathrm{Tr}(M\rho_i M^\dagger)},\ \rho_i\left(S_j\right) = \rho_i(\rho_{ij} \to S_j)$$

- Spin transfer relate the initial i-th particle to the final j-th particle

$$K_j^i = \frac{\mathrm{Tr}(M\rho_i(\hat{S}_\mathrm{j})M^\dagger \hat{S}_\mathrm{j})}{\mathrm{Tr}(M\rho_i M^\dagger)}$$

5.3. 自旋传递和自旋关联(续)

Example:

- In the decay $J/\psi \to \Lambda\bar{\Lambda}$, assuming the spin density matrix for J/ψ taken as

$$\rho_J = \frac{1}{3}\begin{pmatrix} 1 + Pzz/2 & 0 & 0 \\ 0 & 1 - Pzz & 0 \\ 0 & 0 & 1 + Pzz/2 \end{pmatrix}$$

Calculate the spin transfer of J/ψ to the final state Λ particle.