粒子物理学的自旋极化
Lecture 6

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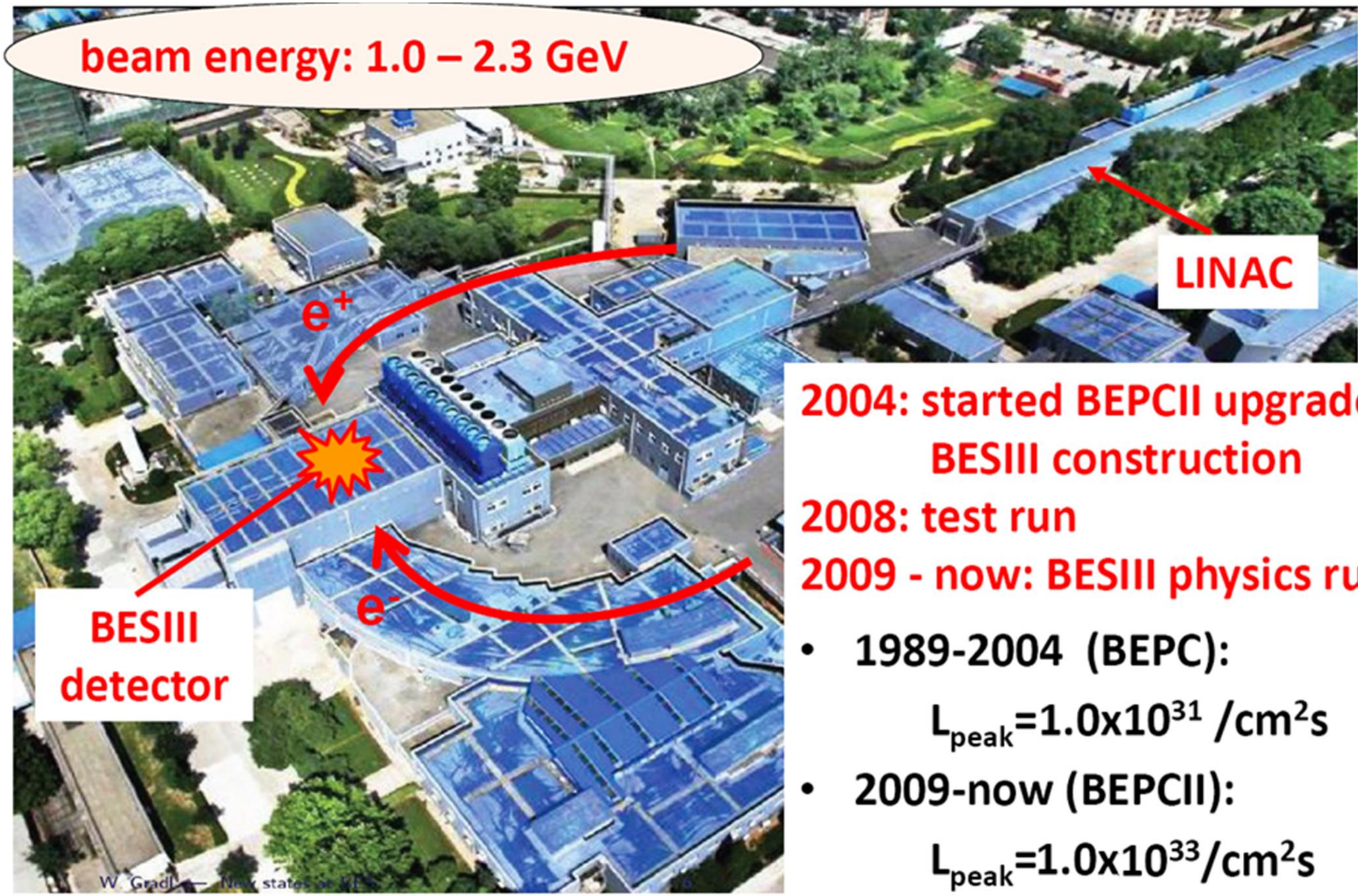
2019-10-8, 兰州大学物理学院

第六讲：粒子物理学中的自旋极化的应用特例

Test CP symmetry in hyper decays at BESIII

Beijing Electron Positron Collider (BEPC)



2004: started BEPCII upgrade,
BESIII construction

2008: test run

2009 - now: BESIII physics run

- 1989-2004 (BEPC):

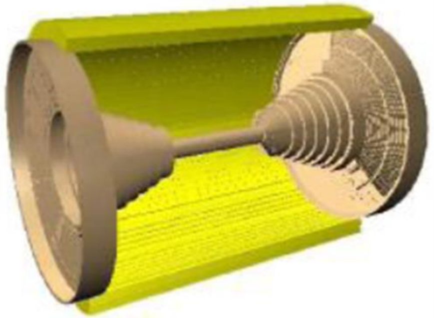
  L_{peak}=1.0x10^{31} /cm^{2}s

- 2009-now (BEPCII):

  L_{peak}=1.0x10^{33}/cm^{2}s

BESIII Detector

MDC

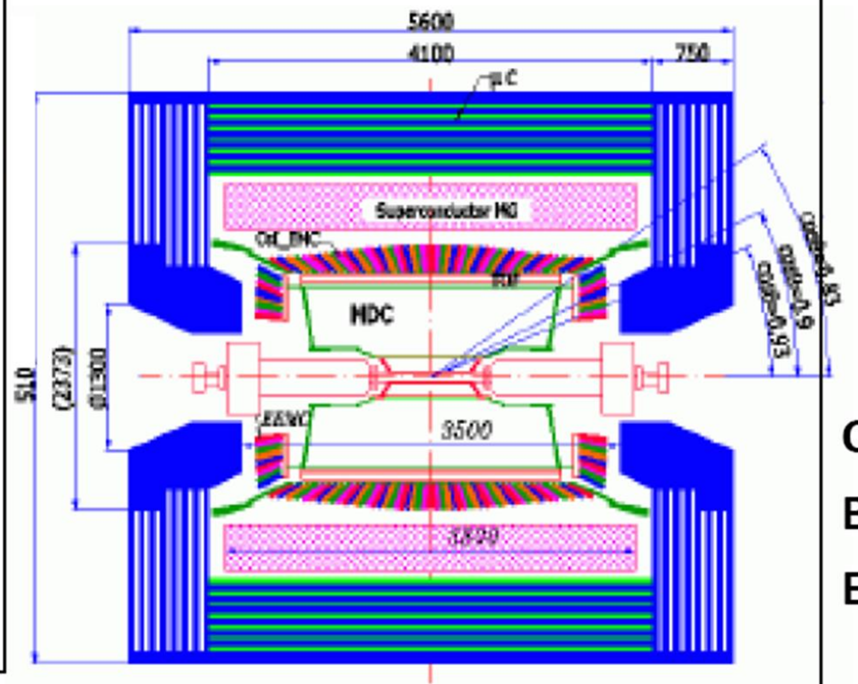


R inner: 63mm ;

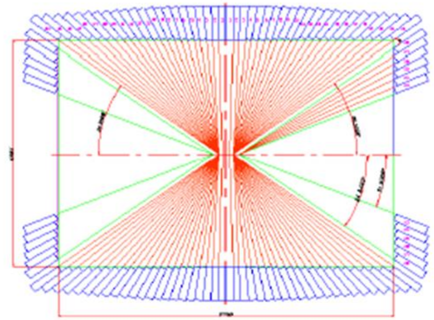
R outer: 810mm

Length: 2582 mm

Layers: 43



CsI(Tl) EMC



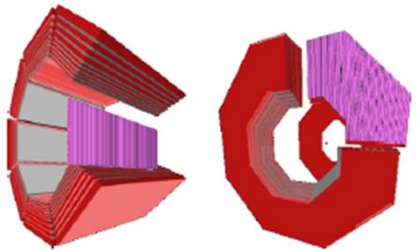
Crystals: 28 cm(15 X_0)

Barrel: $|\cos\theta|<0.83$

Endcap:

$0.85 < |\cos\theta| < 0.93$

RPC MUC



BMUC: 9 layers – 72 modules

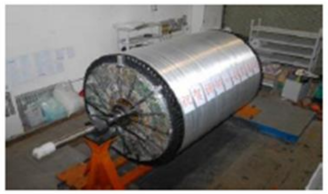
EMUC: 8 layers – 64 modules

TOF

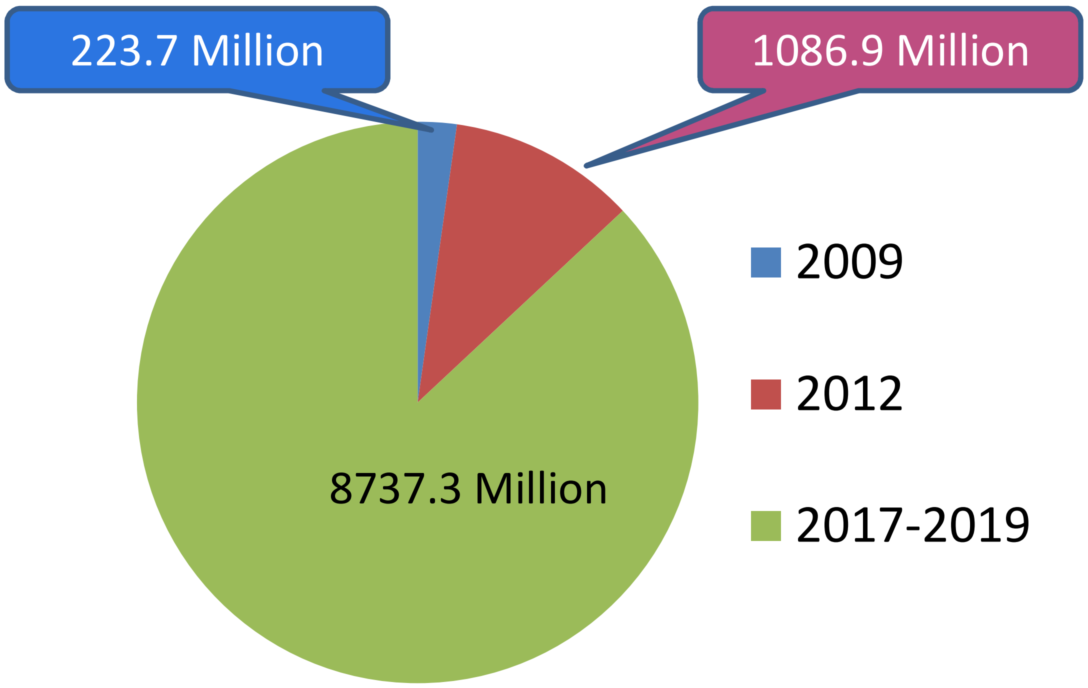
BTOF: two layers

ETOF: 48 scintillators for each

MRPC --- new ETOF



BESIII J/ψ data sets



The analysis based on 2009+2012 J/ψ data:

$\sim$1.31 billion decays

Total 10.047 billion J/ψ decays we have

Role of polarization physics

- Probing of spin degree freedom

  Test the dynamic of SM and low energy hadron interaction

  Existant exp. : RHIC, Jlab, GRAAL, CERN and DESY

  Spin observable, spin-dependent structure function and parton distribution

  Spin crisis at eighties

- BEPCII/BESIII, unpolarized beam, inaccessible polarization of final state by BESIII

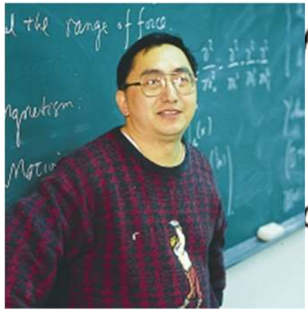
  - Polarized beam for post-BEPCII options, CPV in tau decay, Hyperon weak decay,……
  - Useful tool: transverse polarization of hyperon, spontaneous production at e^+e^- collision
  - $\Lambda \to p\pi^-$ decay plays important role in particle physics

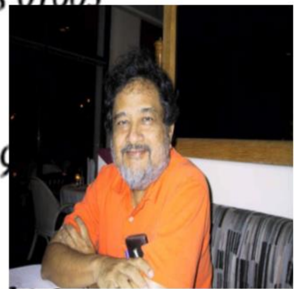
Classic paper

Phys. Rev. D34, 833 (1986)

Hyperon decays and *CP* nonconservation

John F. Donoghue
f Physics and Astronomy, University of Massachusetts, Amherst, Massachusetts 01003



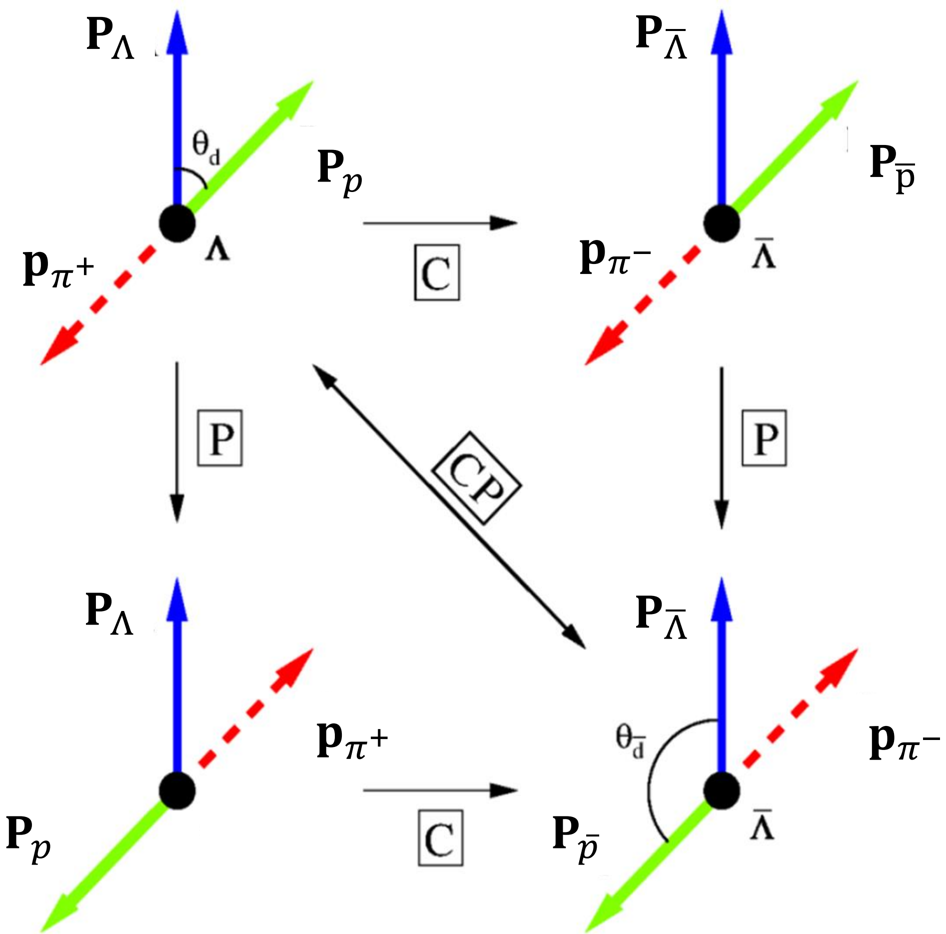


Xiao-Gang He and Sandip Pakvasa
of Physics and Astronomy, University of Hawaii at Manoa, Honolulu, Hawaii 9

(Received 7 March 1986)

We study all modes of hyperon nonleptonic decay and consider the *CP*-odd observables which result. Explicit calculations are provided in the Kobayashi-Maskawa, Weinberg-Higgs, and left-right-symmetric models of *CP* nonconservation.

C- and P- transformation



$$\alpha_\Lambda = \frac{|B_+|^2 - |B_-|^2}{|B_+|^2 + |B_-|^2}, \alpha_{\bar{\Lambda}} = \frac{|\overline{B}_+|^2 - |\overline{B}_-|^2}{|\overline{B}_+|^2 + |\overline{B}_-|^2}$$

CP ***invariance*** **:**

$$\overline{B}_{-\lambda_p} = \eta_\Lambda \eta_p \eta_\pi (-1)^{s_\Lambda - s_p - s_\pi} B_{\lambda_p} = -B_{\lambda_p}$$

If CP invariance:

$$\alpha_\Lambda = -\alpha_{\bar{\Lambda}}$$

CP odd observable:

$$A_\Lambda = \frac{\alpha_\Lambda + \alpha_{\bar{\Lambda}}}{\alpha_\Lambda - \alpha_{\bar{\Lambda}}}$$

$A_\Lambda \sim 10^{-5}$ in CKM mechanism

ancient history

Phys. Rev. Lett. 24, 843 (1970)

ASYMMETRY PARAMETER FOR $\Lambda^0 \to n\pi^0$ †*

S. Olsen, L. Pondrom, R. Handler, and P. Limon‡
University of Wisconsin, Madison, Wisconsin 53706

and

J. A. Smith and O. E. Overseth
University of Michigan, Ann Arbor, Michigan 48104
(Received 18 February 1970)

The asymmetry parameter α_0 for $\Lambda^0 \to n\pi^0$ has been measured relative to α_- by comparing the neutron distribution with the proton distribution from the decay $\Lambda^0 \to p\pi^-$ for polarized Λ^0 hyperons. A sample of 4760 neutron decay events and 8500 proton decay events gave $\alpha_0/\alpha_- = 1.000 \pm 0.068$ in good agreement with the $|\Delta \vec{I}| = \frac{1}{2}$ rule.

Phys. Rev. Lett. 24, 165 (1970)

PROTON POLARIZATION IN $\Sigma^+ \to p\pi^0$ †*

F. Harris and O. E. Overseth
University of Michigan, Ann Arbor, Michigan 48104

and

L. Pondrom and E. Dettmann
University of Wisconsin, Madison, Wisconsin 53706
(Received 10 November 1969)

The polarization of protons from the decay of polarized Σ^+ hyperons has been measured by scattering the protons in a carbon-plate spark chamber. A sample of 1335 useful scatters gave $\alpha_0 = -0.98 \pm 0.05$ and $\varphi_0 = 22° \pm 90°$, where $\tan\varphi_0 = \beta_0/\gamma_0$. Using the data on $\Sigma^+ \to n\pi^+$ and $\Sigma^- \to n\pi^-$ and fitting to the $|\Delta \vec{I}| = \frac{1}{2}$ rule gave $\chi^2 = 0.3$ for 2 degrees of freedom.

Phys. Rev. 184, 1663 (1969)

Final-State Interactions in Nonleptonic Hyperon Decay

O. E. OVERSETH*
The University of Michigan, Ann Arbor, Michigan 48104

AND

S. PAKVASA†
University of Hawaii, Honolulu, Hawaii 96822
(Received 1 April 1969)

We discuss the consequences of including the final-state interactions in the analysis of $\Lambda°$, Σ, and Ξ nonleptonic decays. Emphasis is on the role that the final-state interactions play in tests for T invariance, in tests of the $\Delta I = \frac{1}{2}$ rule (including the resolution of sign ambiguities), and in the determination of the decay amplitudes for these processes.

Previous Measurements

2018 PDG list

α_- FOR $\Lambda \to p\pi^-$

VALUE	EVTS	DOCUMENT ID		TECN	COMMENT
$\mathbf{0.642 \pm 0.013}$	**OUR AVERAGE**				
0.584 ± 0.046	8500	ASTBURY	1975	SPEC	
0.649 ± 0.023	10325	CLELAND	1972	OSPK	
0.67 ± 0.06	3520	DAUBER	1969	HBC	From Ξ decay
0.645 ± 0.017	10130	OVERSETH	1967	OSPK	Λ from $\pi^- p$
0.62 ± 0.07	1156	CRONIN	1963	CNTR	Λ from $\pi^- p$

α_+ FOR $\overline{\Lambda} \to \overline{p}\pi^+$

VALUE	EVTS	DOCUMENT ID		TECN	COMMENT
$\mathbf{-0.71 \pm 0.08}$	**OUR AVERAGE**				
$-0.755 \pm 0.083 \pm 0.063$	$\approx$ 8.7k	ABLIKIM	2010	BES	$J/\psi \to \Lambda\overline{\Lambda}$
-0.63 ± 0.13	770	TIXIER	1988	DM2	$J/\psi \to \Lambda\overline{\Lambda}$

Previous Measurements

2018 PDG list

$(\alpha + \overline{\alpha})/(\alpha - \overline{\alpha})$ in $\Lambda \to p\pi^-$, $\overline{\Lambda} \to \overline{p}\pi^+$

Zero if CP is conserved; α_- and α_+ are the asymmetry parameters for $\Lambda \to p\pi^-$ and $\overline{\Lambda} \to \overline{p}\pi^+$ decay. See also the Ξ^- for a similar test involving the decay chain $\Xi^- \to \Lambda\pi^-$, $\Lambda \to p\pi^-$ and the corresponding antiparticle chain.

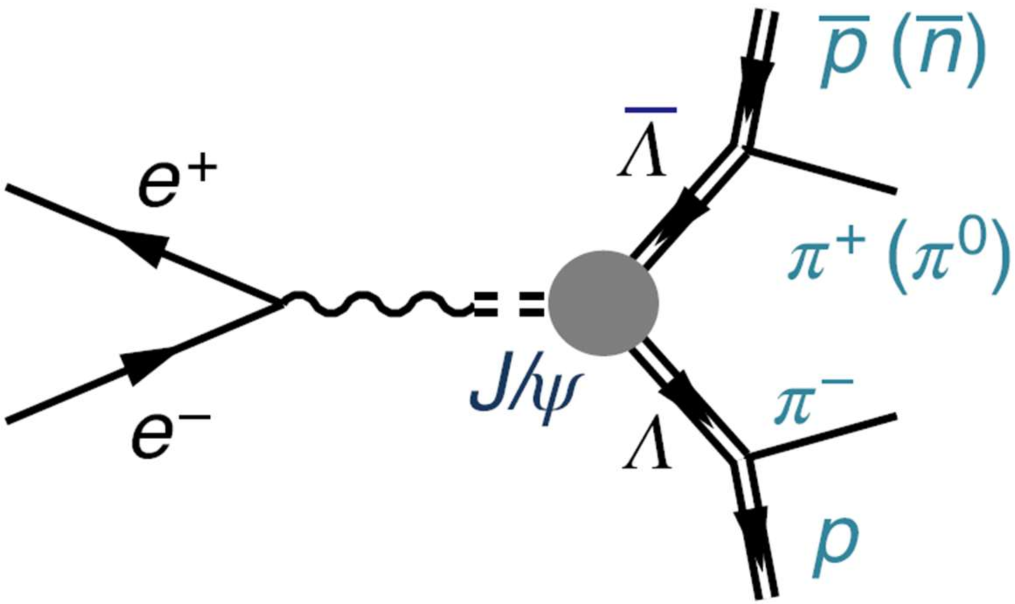
VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*COMMENT*
0.006±0.021 OUR AVERAGE					
$-0.081 \pm 0.055 \pm 0.059$	$\approx$ 8.7k	ABLIKIM	10	BES	$J/\psi \to \Lambda\overline{\Lambda}$
$+0.013 \pm 0.022$	96k	BARNES	96	CNTR	LEAR $\overline{p}p \to \overline{\Lambda}\Lambda$
$+0.01 \pm 0.10$	770	TIXIER	88	DM2	$J/\psi \to \Lambda\overline{\Lambda}$
-0.02 ± 0.14	10k	[1] CHAUVAT	85	CNTR	pp, $\overline{p}p$ ISR
• • • We do not use the following data for averages, fits, limits, etc. • • •					
-0.07 ± 0.09	4063	BARNES	87	CNTR	See BARNES 96

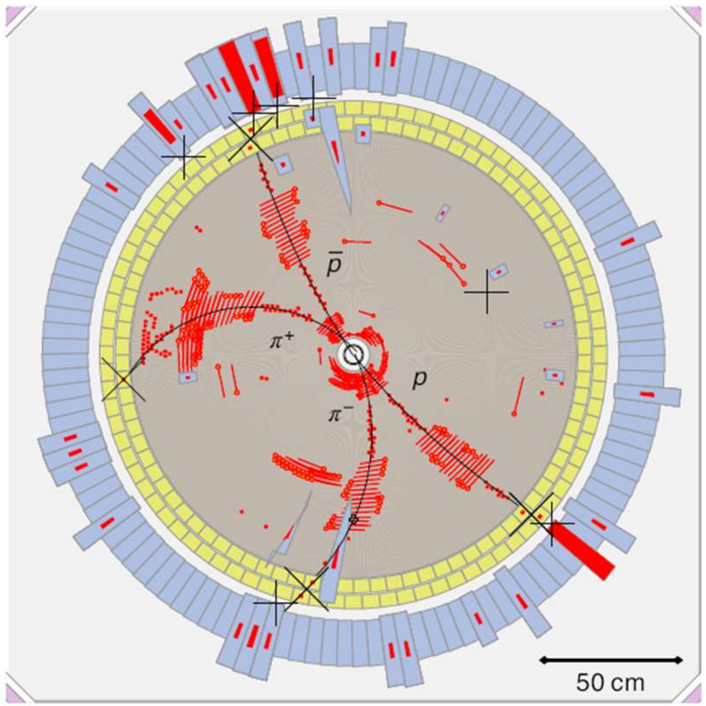
[1] CHAUVAT 85 actually gives $\alpha_+(\overline{\Lambda})/\alpha_-(\Lambda) = -1.04 \pm 0.29$. Assumes polarization is same in $\overline{p}p \to \overline{\Lambda}X$ and $pp \to \Lambda X$. Tests of this assumption, based on C-invariance and fragmentation, are satisfied by the data.

Hyperon decay parameters @ BESIII

$$e^+e^- \to J/\psi \to \Lambda\bar{\Lambda} \to p\bar{p}\pi^+\pi^-$$

Event selection:





- ✓ No PID request for charged tracks
- ✓ $\Lambda/\bar{\Lambda}$ reconstructed with second vertex fit
- ✓ Events reconstructed with kinematic fit

Hyperon decay parameters @ BESIII

$$e^+e^- \rightarrow J/\psi \rightarrow \Lambda\bar{\Lambda} \rightarrow p\bar{p}\pi^+\pi^-$$

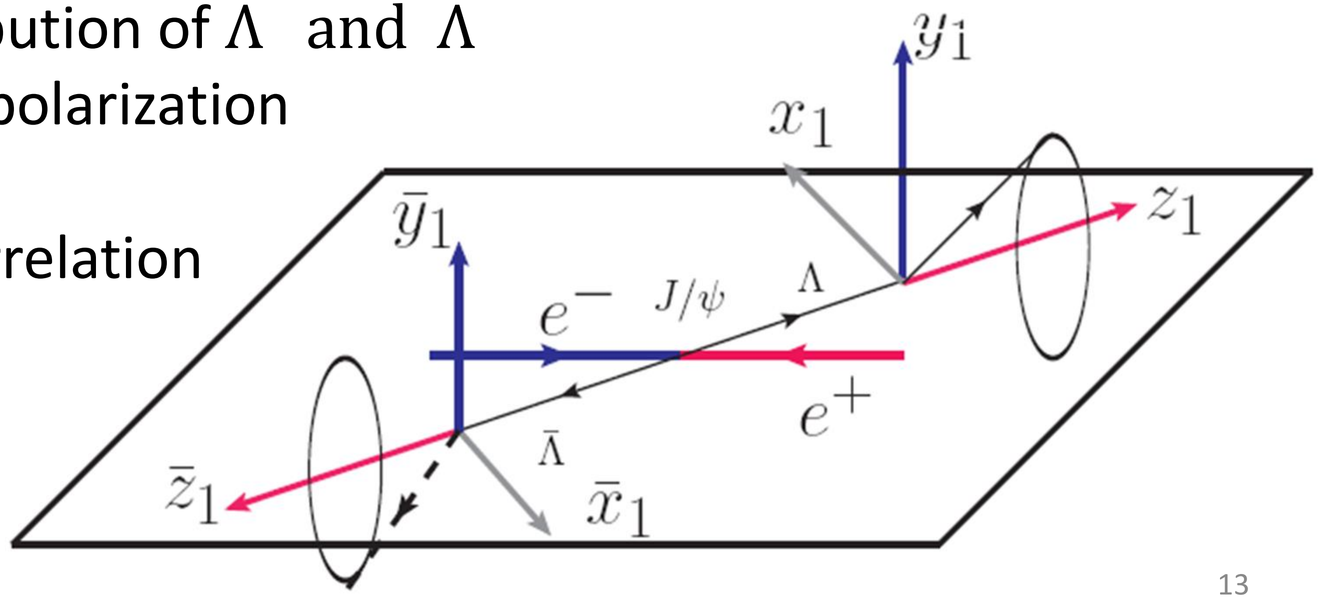
G. Faldt and A. Kupsc
PLB,772, 16 (2017)
EPJA, 52, 141 (2016)
EPJA, 51, 74 (2015).

$$\frac{d\sigma}{d\Omega} \propto T_0 + \sqrt{1-\alpha_{J/\psi}^2}\sin(\Delta)(\alpha_\Lambda T_3 + \alpha_{\bar{\Lambda}} T_4) + \alpha_\Lambda \alpha_{\bar{\Lambda}}[T_1 + \sqrt{1-\alpha_{J/\psi}^2}\cos(\Delta) T_2 + \alpha_{J/\psi} T_5],$$

T_0: angular distribution of Λ and $\bar{\Lambda}$

T_3, T_4, transverse polarization

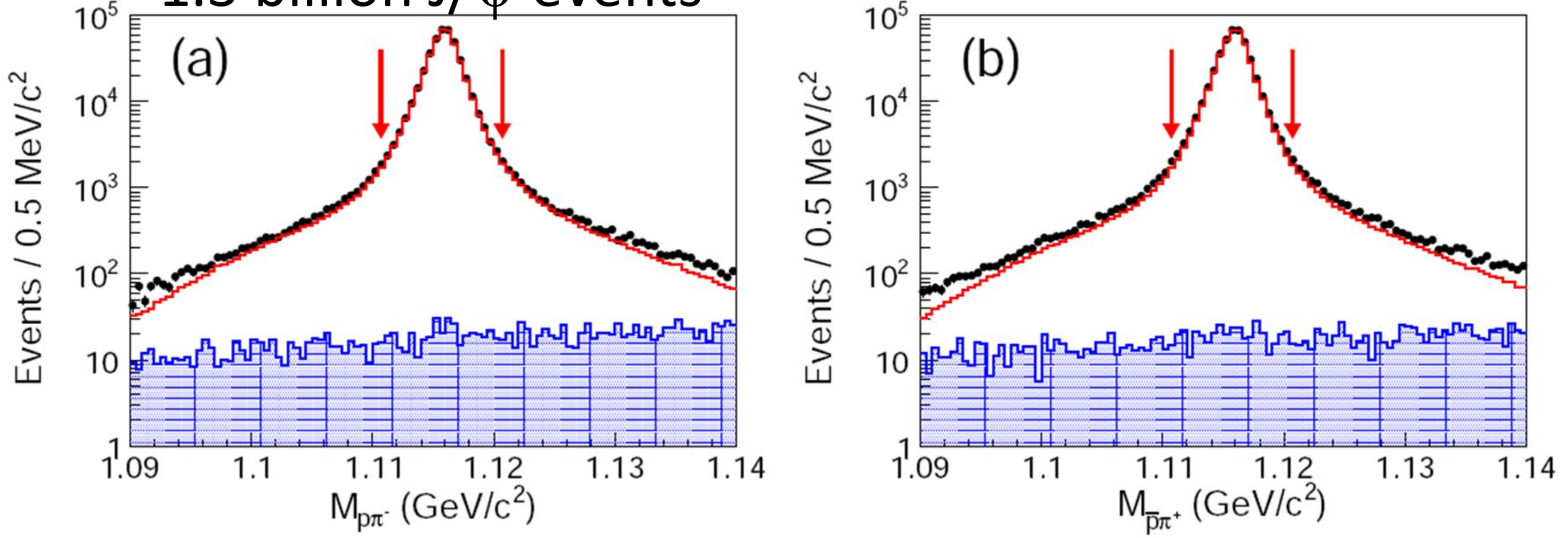
T_1, T_2, T_5: spin correlation



$e^+e^- \to J/\psi \to \Lambda\bar{\Lambda} \to p\bar{p}\pi^+\pi^-,\ p\bar{n}\pi^-\pi^0$

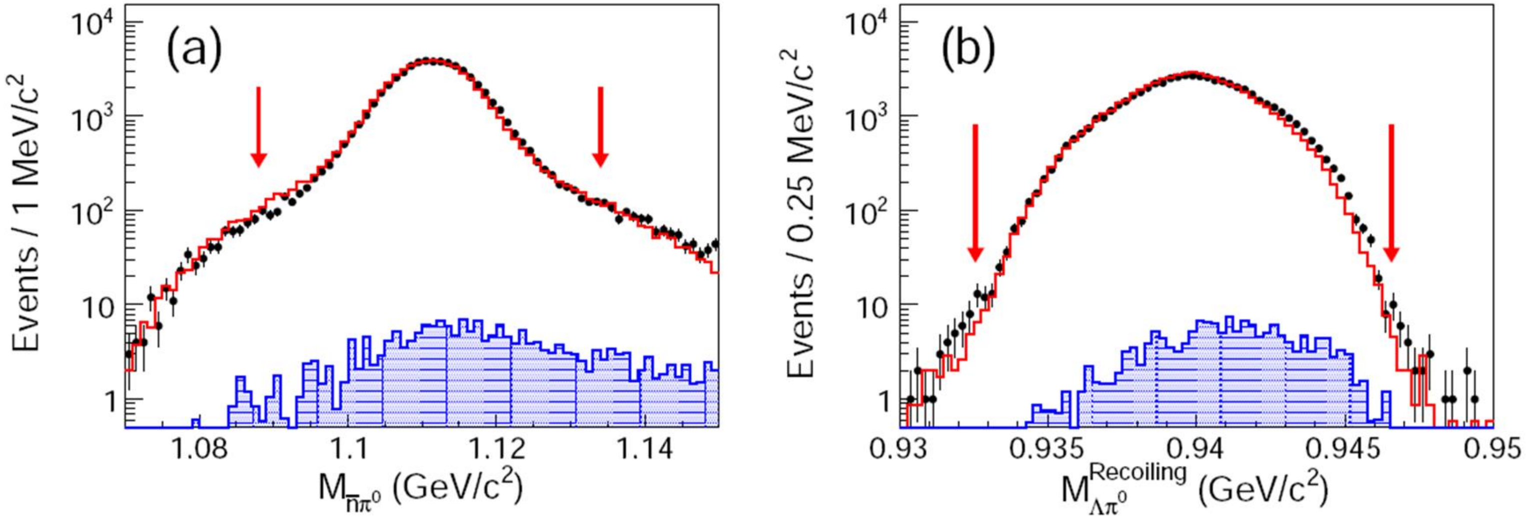
1.3 billion J/ψ events

420,593 events selected

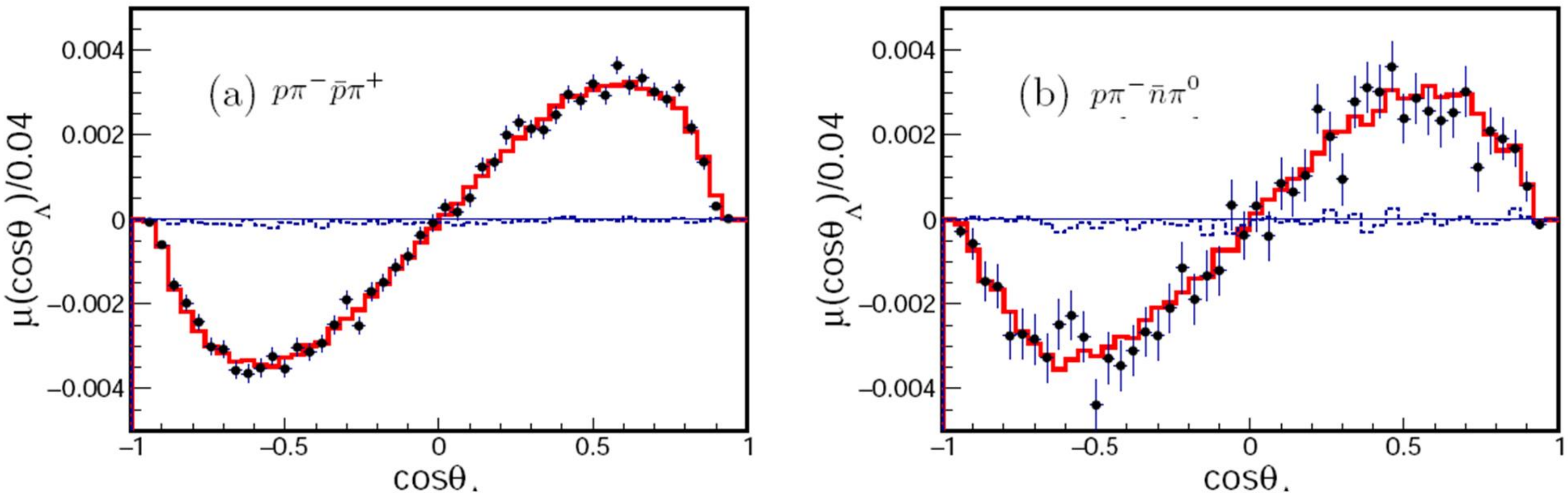


$e^+e^- \to J/\psi \to \Lambda\bar{\Lambda} \to p\bar{n}\pi^0\pi^+$

47,009 events selected



Results of simultaneous fit to two data sets



Parameters	This measurement	Previous results
α_ψ	$0.461 \pm 0.006 \pm 0.007$	0.469 ± 0.027 [19]
α_-	$0.750 \pm 0.009 \pm 0.004$	0.642 ± 0.013 [8]
α_+	$-0.758 \pm 0.010 \pm 0.007$	-0.71 ± 0.08 [8]
$\bar{\alpha}_0$	$-0.692 \pm 0.016 \pm 0.006$	——
$\Delta\Phi$ (rad)	$0.740 \pm 0.010 \pm 0.008$	$\Delta\Phi = 42.4^\circ \pm 0.6^\circ (sta) \pm 0.5^\circ (sys.)$
A_Λ	$\boxed{-0.006 \pm 0.012 \pm 0.007}$	0.006 ± 0.021 [8]
$\bar{\alpha}_0/\alpha_+$	$0.913 \pm 0.028 \pm 0.012$	——

Systematic uncertainties

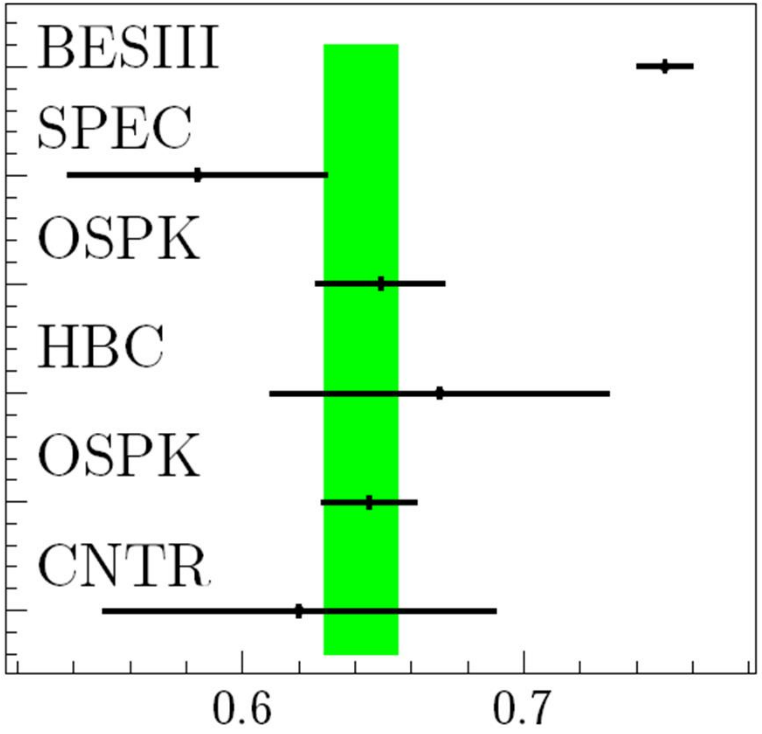
- Challenge to estimate detector asymmetry in tracking, and kinematic fit
- Systematic uncertainties well controlled

Relative systematic uncertainties (%)

Source	α_ψ	α_-	α_+	$\bar{\alpha}_0$	$\Delta\Phi$
Tracking, π^0, $\bar{n}$	1.5	0.1	0.3	0.6	1.1
Kinematic fit	0.2	0.1	0.8	0.6	0.0
Fit method	0.0	0.5	0.4	0.4	0.1
Total	1.5	0.5	0.9	0.8	1.1

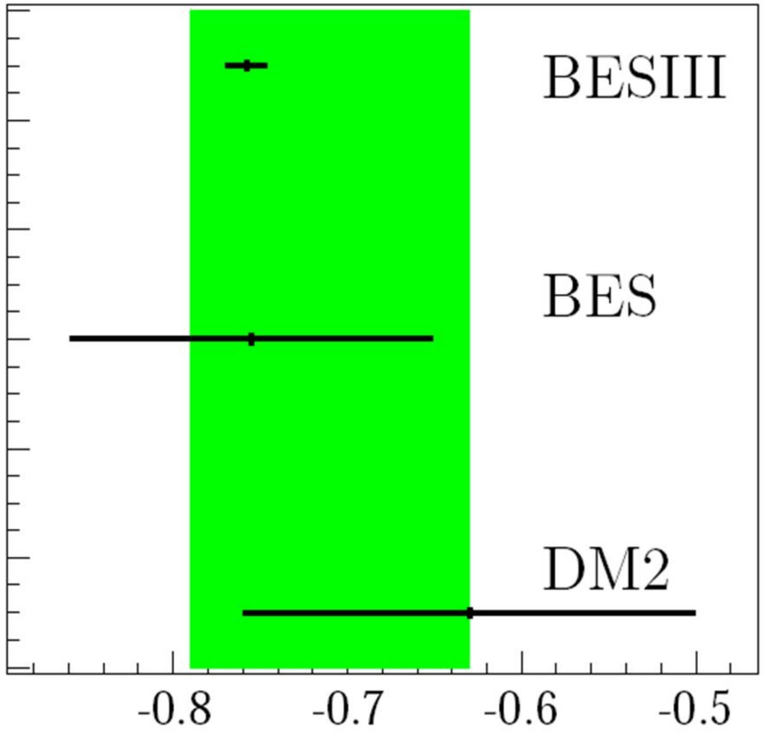
- Justify results with $J/\psi \rightarrow \Xi^-\bar{\Xi}^+ \rightarrow \Lambda\bar{\Lambda}\pi^+\pi^- \rightarrow p\bar{p}2(\pi^+\pi^-)$

BESIII: 0.750 ± 0.010



(a) α_- for $\Lambda \to p\pi^-$

BESIII: -0.758 ± 0.012



(b) α_+ for $\bar{\Lambda} \to \bar{p}\pi^+$

BESIII, arXiv: 1808.08917,

[Nature Phys. 15 (2019) 631-634]

Quick response from PDG2019

α_- FOR $\Lambda \to p\pi^-$

2019 PDG list

VALUE	EVTS	DOCUMENT ID		TECN	COMMENT
$0.750 \pm 0.009 \pm 0.004$	420k	ABLIKIM	2018AG	BES3	J/ψ to $\Lambda\overline{\Lambda}$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
0.584 ± 0.046	8500	ASTBURY	1975	SPEC	
0.649 ± 0.023	10325	CLELAND	1972	OSPK	
0.67 ± 0.06	3520	DAUBER	1969	HBC	From Ξ decay
0.645 ± 0.017	10130	OVERSETH	1967	OSPK	Λ from $\pi^- p$
0.62 ± 0.07	1156	CRONIN	1963	CNTR	Λ from $\pi^- p$

α_+ FOR $\overline{\Lambda} \to \overline{p}\pi^+$

VALUE	EVTS	DOCUMENT ID		TECN	COMMENT
$-0.758 \pm 0.010 \pm 0.007$	420k	ABLIKIM	2018AG	BES3	J/ψ to $\Lambda\overline{\Lambda}$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
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-0.63 ± 0.13	770	TIXIER	1988	DM2	$J/\psi \to \Lambda\overline{\Lambda}$

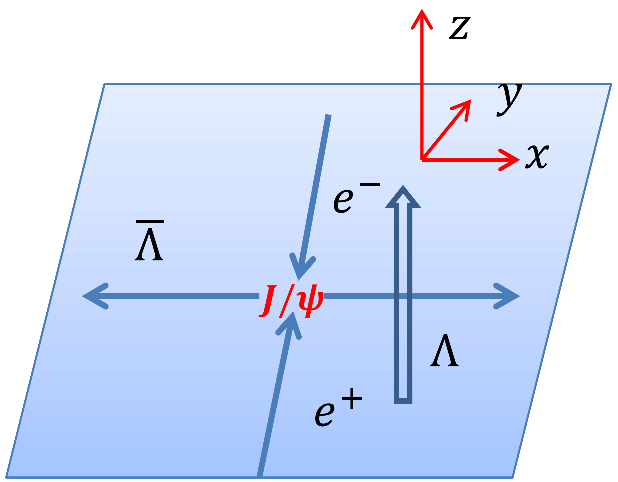
Where does the TP come from?

- From the e^+/e^- beam ?
  - ✘ No, BEPC beams unpolarized
- From the e^+/e^- natural polarization when circulating in the BEPCII storage ring ?
  - ✘ Sokolov-Ternov effects: 4.3 hs @ψ' peak, but beam lifetime ~ 2.0 hs
- From the J/ψ spin transfer ?
  - ✓ Yes, it does from the J/ψ tensor polarization

J/ψ polarization: $\mathcal{P}_z = 0,\ T_{zz} = \frac{1}{\sqrt{6}}$

Λ transverse polarization:

$$\mathcal{P}_y = \sqrt{6}\frac{T_{zz}\sin\theta\cos\theta\sin\Delta\sqrt{1-\alpha_\psi^2}}{1+\alpha_\psi\left[\frac{1}{3}+\frac{1}{\sqrt{6}}T_{zz}(1+3\cos 2\theta)\right]}$$



$\mathcal{P}_y$ manifest if $\sin\Delta \neq 0$

CPV observables in Ξ^- decay

decay rate difference

$$\frac{\Gamma_{\bar{\Lambda}\pi^+} - \Gamma_{\Lambda\pi^-}}{\Gamma} \equiv 0$$

← $\Lambda\pi$ final states are purely Ispin=1, only $\Delta I=1/2$ transitions allowed, no $\Delta I=3/2$ transition possible

decay asymmetry difference

$$\alpha_{\mp} = \pm\frac{2\,\mathrm{Re}(S^*P)}{|S|^2+|P|^2} = \pm\frac{2|S||P|\cos(\Delta_S \pm \phi_{CP})}{|S|^2+|P|^2}$$

$$\frac{\alpha_- + \alpha_+}{\alpha_- - \alpha_+} = \frac{\sin\Delta_S \sin\phi_{CP}}{\cos\Delta_S \cos\phi_{CP}} = \tan\Delta_S \tan\phi_{CP}$$

← in this case, the strong phase ($\Delta_s = \delta_S - \delta_P$) is measureable (see below)

final-state polarization difference

$$\beta_{\mp} = \pm\frac{2\,\mathrm{Im}(S^*P)}{|S|^2+|P|^2} = \pm\frac{2|S||P|\sin(\Delta_S \pm \phi_{CP})}{|S|^2+|P|^2}$$

$$\frac{\beta_- + \beta_+}{\alpha_- - \alpha_+} = \frac{\cos\Delta_S \sin\phi_{CP}}{\cos\Delta_S \cos\phi_{CP}} = \tan\phi_{CP}$$

← Strong phase cancels out

$$\frac{\beta_- - \beta_+}{\alpha_- - \alpha_+} = \frac{\sin\Delta_S \cos\phi_{CP}}{\cos\Delta_S \cos\phi_{CP}} = \tan\Delta_S$$

← measures the strong phase

big advantage for Ξ over Λ

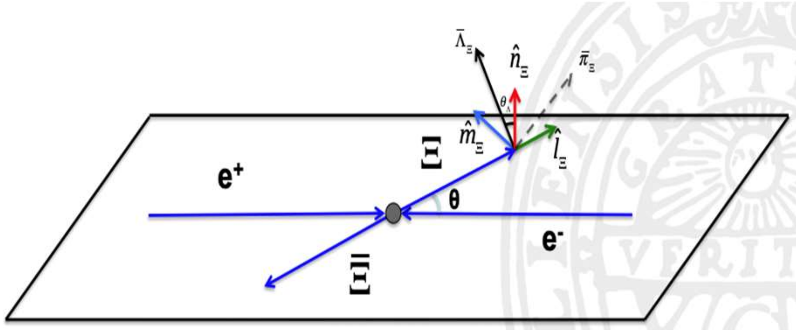
CPV with $J/\psi \to \Xi^-(\Lambda\pi^-)\overline{\Xi}^+(\Lambda\pi^+)$+c.c.

缺点:

complicated topology: 9-dimensions

$\theta_\Xi, \theta_\Lambda, \phi_\Lambda, \theta_{\overline{\Lambda}}, \phi_{\overline{\Lambda}}, \theta_p, \phi_p, \theta_{\bar{p}}, \phi_{\bar{p}}$

72 terms, 8 parameters to determine



Low rate compared to $\Lambda\overline{\Lambda}$

1.3B J/ψ: 420K $\Lambda(p\pi^-)\overline{\Lambda}(\bar{p}\pi^+)$evts

61K $\Xi(\Lambda\pi^-)$ $\Xi(\Lambda\pi^-)$

优点:

- **$\Lambda\overline{\Lambda}$ polarizations are measureable via their parity-violating $p\pi^-$ ($p\pi^+$) decays;**
- **β_- and β_0 parameters can be determined.**
- **Preliminary results indicate that the Ξs are even more polarized than the Λs.**

Σ^+?

α_0 FOR $\Sigma^+ \to p\pi^0$

VALUE	EVTS		DOCUMENT ID	
$-0.980^{+0.017}_{-0.015}$ OUR FIT				
$-0.980^{+0.017}_{-0.013}$ OUR AVERAGE				
$-0.945^{+0.055}_{-0.042}$	1259	15	LIPMAN	73
-0.940 ± 0.045	16k		BELLAMY	72
$-0.98^{+0.05}_{-0.02}$	1335	16	HARRIS	70
-0.999 ± 0.022	32k		BANGERTER	69

50 year-old measurements,

probably wrong for the same reason the Λ measurements were wrong

Σ^+ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$p\pi^0$	(51.57 ± 0.30) %
Γ_2	$n\pi^+$	(48.31 ± 0.30) %

$\alpha_0\approx1$ ➔ S-wave ≈ P-wave

interference is maximum

well suited for $\alpha_0+\bar{\alpha}_0/\alpha_0-\bar{\alpha}_0$

No measurements of $\bar{\alpha}_0$ or $\bar{\alpha}_-$

$\Gamma(p\pi^0) \approx \Gamma(n\pi^+)$ to ~10% ➔ $T_{3/2} \approx$ 5% $T_{1/2}$

$\Delta\Gamma$ will be suppressed

$\Gamma(\Sigma^+ \to n\ell^+\nu)/\Gamma(\Sigma^- \to n\ell^-\bar{\nu})$

Test of $\Delta S = \Delta Q$ rule.

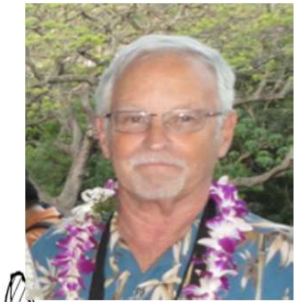
VALUE	EVTS	DOCL
<0.043 OUR LIMIT	Our 90% CL limit,	

PDG 2018 $\Delta S=\Delta Q$ limit is not severe,

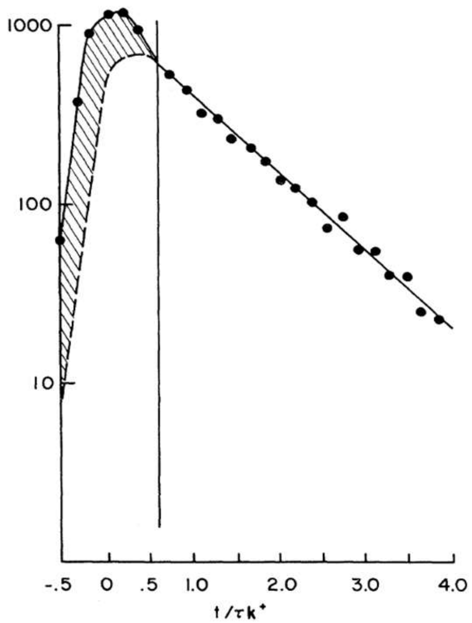
BESIII can probably improve on this by a large factor

Harris et al., α_0 parameter in $\Sigma^+ \rightarrow p\pi^0$ decays

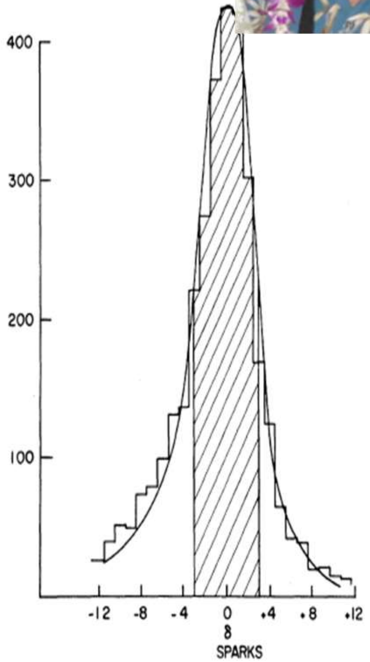
Phys. Rev. Lett. 24, 165 (1970)



Fred Harris PhD Thesis

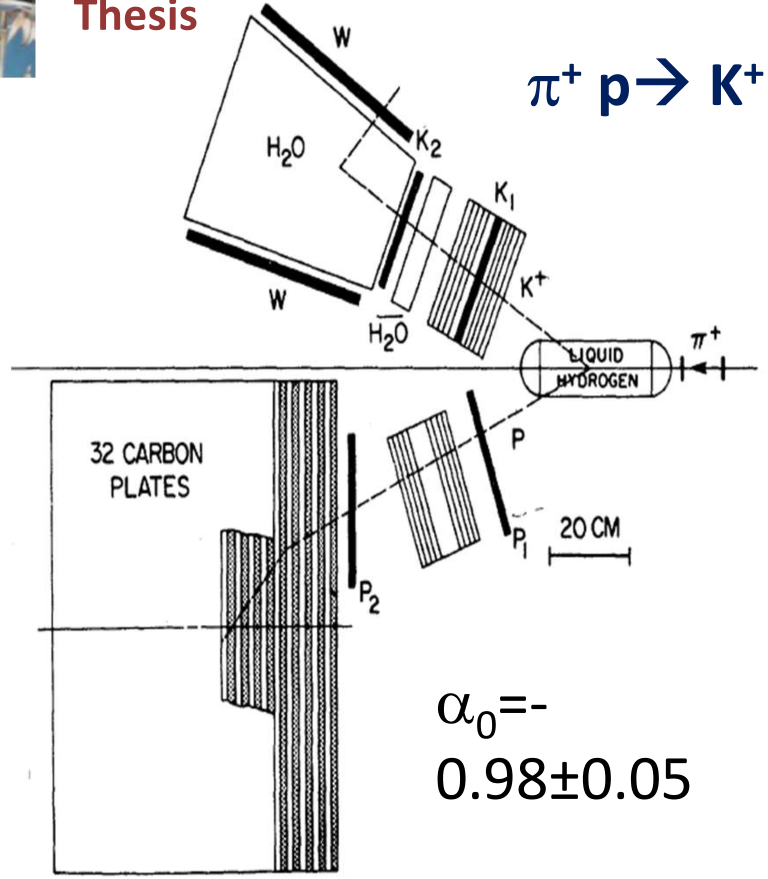


exit-entry time in the H_2O tank



observed-expected range of the proton

$\pi^+ p \rightarrow K^+ \Sigma^+$



α_0=-0.98±0.05

Fred used thin (1 cm-thick) carbon plates. Thus, according to Tajima-san's hypothesis, his analyzing powers were more reliable

Σ^-?

40~50 year-old measurements,

probably wrong for the same reason the Λ measurements were wrong

α_- FOR $\Sigma^- \rightarrow n\pi^-$

VALUE	EVTS	DOCUMENT ID	
-0.068 ± 0.008 OUR AVERAGE			
-0.062 ± 0.024	28k	HANSL	78
-0.067 ± 0.011	60k	BOGERT	70
-0.071 ± 0.012	51k	BANGERTER	69

$\alpha_0 \approx 0$ ➔ 1 partial wave dominates

interference is small not well suited for $\alpha_- + \alpha_+ / \alpha_- - \alpha_+$ measurements

Σ^- DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$n\pi^-$	(99.848 ± 0.005) %

no measurements of α_+

single dominant decay mode

no suitable for $\Delta\Gamma$ measurements

Ω^-?

α FOR $\Omega^- \to \Lambda K^-$

Some early results have been omitted.

VALUE	EVTS	DOCUMENT ID	
0.0180±0.0024 OUR AVERAGE			
+0.0207±0.0051±0.0081	960k	[7] CHEN	05
+0.0178±0.0019±0.0016	4.5M	[7] LU	05A

α FOR $\Omega^- \to \Xi^0 \pi^-$

VALUE	EVTS	DOCUMENT ID	
+0.09±0.14	1630	BOURQUIN	84

α FOR $\Omega^- \to \Xi^- \pi^0$

VALUE	EVTS	DOCUMENT ID	
+0.05±0.21	614	BOURQUIN	84

Ω^- DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	ΛK^-	(67.8±0.7) %
Γ_2	$\Xi^0 \pi^-$	(23.6±0.7) %
Γ_3	$\Xi^- \pi^0$	(8.6±0.4) %

$\Gamma(\Xi^0\pi^-)$ =(2.74 + 0.15) x $\Gamma(\Xi^-\pi^0)$

ΔI=1/2 rule expectation: ≈ 2 : ← $T_{3/2}$ ≈ 0.15 $T_{1/2}$

$\Delta\Gamma$ will be enhanced (compared to Λs)

α≈0 ➔ 1 partial wave dominates all modes

interference is small, not well suited

for α + α/α□α measurements

BESIII should check all these measurements

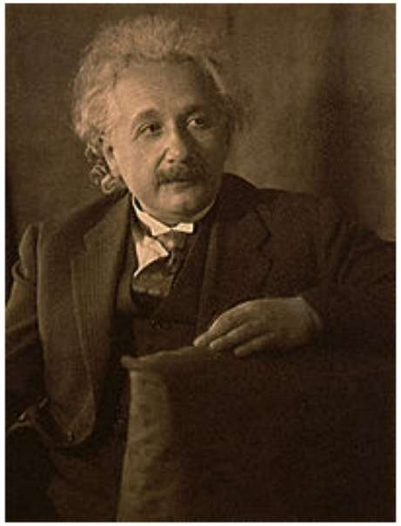
this $\Delta_s=\delta_{3/2}-\delta_{1/2}$

$$\frac{Bf(\Omega^-\to\Xi^-\pi^0)}{Bf(\Omega^-\to\Xi^0\pi^-)}-\frac{Bf(\bar{\Omega}^+\to\bar{\Xi}^+\pi^0)}{Bf(\bar{\Omega}^+\to\bar{\Xi}^0\pi^=)}=\underbrace{\boxed{2\left(1+\sqrt{2}\right)\left(T_{3/2}/T_{1/2}\right)\sin\Delta_s\sin\phi_{CP}}}$$

sensitivity is reduced but only by a factor of ~0.7

EPR paradox test using charmonium decays

Einstein-Podolsky-Rosen (EPR) paradox

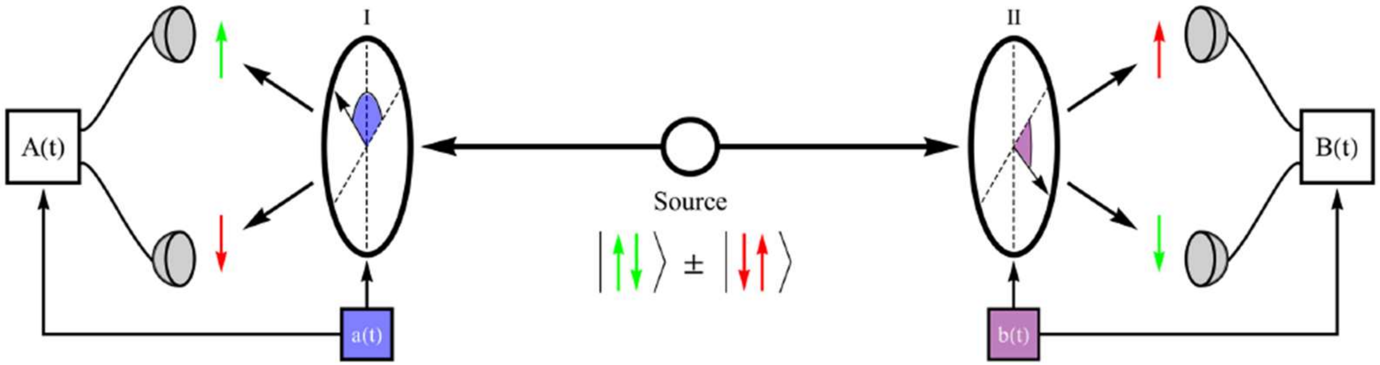

Albert Einstein

- EPR paradox[1] (1935) initiated an intense discussion on the philosophical foundations of quantum mechanics.
- Physics quantity corresponding to "element of reality" can be predicted without any uncertainty.
- A given example: two particle system with sum of momentum $(p_1 + p_2)$ and position difference $(x_1 - x_2)$
- They concluded: the premises of "element of reality" is wrong or Q.M. is incomplete.

Ref.[1] . Einstein, A; B Podolsky and N. Rosen, Phys. Rev. 47, 10 (1935), "Can Quantum-Mechanical description of Physical Reality be considered Complete?"

Local hidden variable theory (LHVT)

Bohm
Spin
setup



- Local hidden variable theory:

  A spin singlet formed by two spin-1/2 particles

  $|\psi_0\rangle = \frac{1}{\sqrt{2}}(\,|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)$. In Q.M.

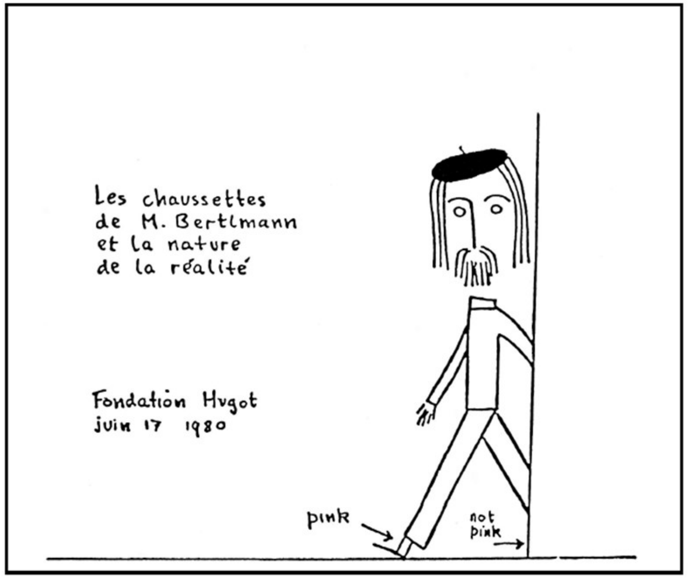
  $$A:\ \ \vec{\sigma}_1 \cdot \vec{a} \qquad\qquad B: \vec{\sigma}_2 \cdot \vec{a}$$

- In LHVT, $A(\vec{a},\vec{\lambda}) = \mathrm{sign}\,(\vec{a}\cdot\vec{\lambda}), B(\vec{b},\vec{\lambda}) = \mathrm{sign}\,(\vec{a}\cdot\vec{\lambda})$, probability describing correlation between them

  $$P(\vec{a},\vec{b}) = \int d\lambda\rho(\lambda)A(\vec{a},\vec{\lambda})B(\vec{b},\vec{\lambda})$$

BELL inequality(BI)

Cartoon of
Berltmann's socks
by Bell (1980)



- BELL inequality

(J.S. Bell, Physics, 1, 195(1964)):

$$1 + P(\vec{b}, \vec{c}) \geq |P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c})|$$

- BI developed many variants, and tested in optics and high energy physics.

Test BI in charmonium decays

- $\eta_c \to \Lambda\bar{\Lambda} \to p\bar{p}\pi^+\pi^-$
  - Q.M.: Λ: $\mathrm{M_A} = (4\pi)^{-1/2}\left(S + P\vec{\sigma}_A \cdot \vec{A}\right)$,

    $\bar{\Lambda}$: $\mathrm{M_B} = (4\pi)^{-1/2}\left(S + P\vec{\sigma}_B \cdot \vec{B}\right)$;

$$I\left(\vec{A}, \vec{B}\right) = \left(\frac{|S|^2 + |P|^2}{4\pi}\right)^2 (1 + \alpha_\Lambda^2 \vec{A} \cdot \vec{B}\ )$$

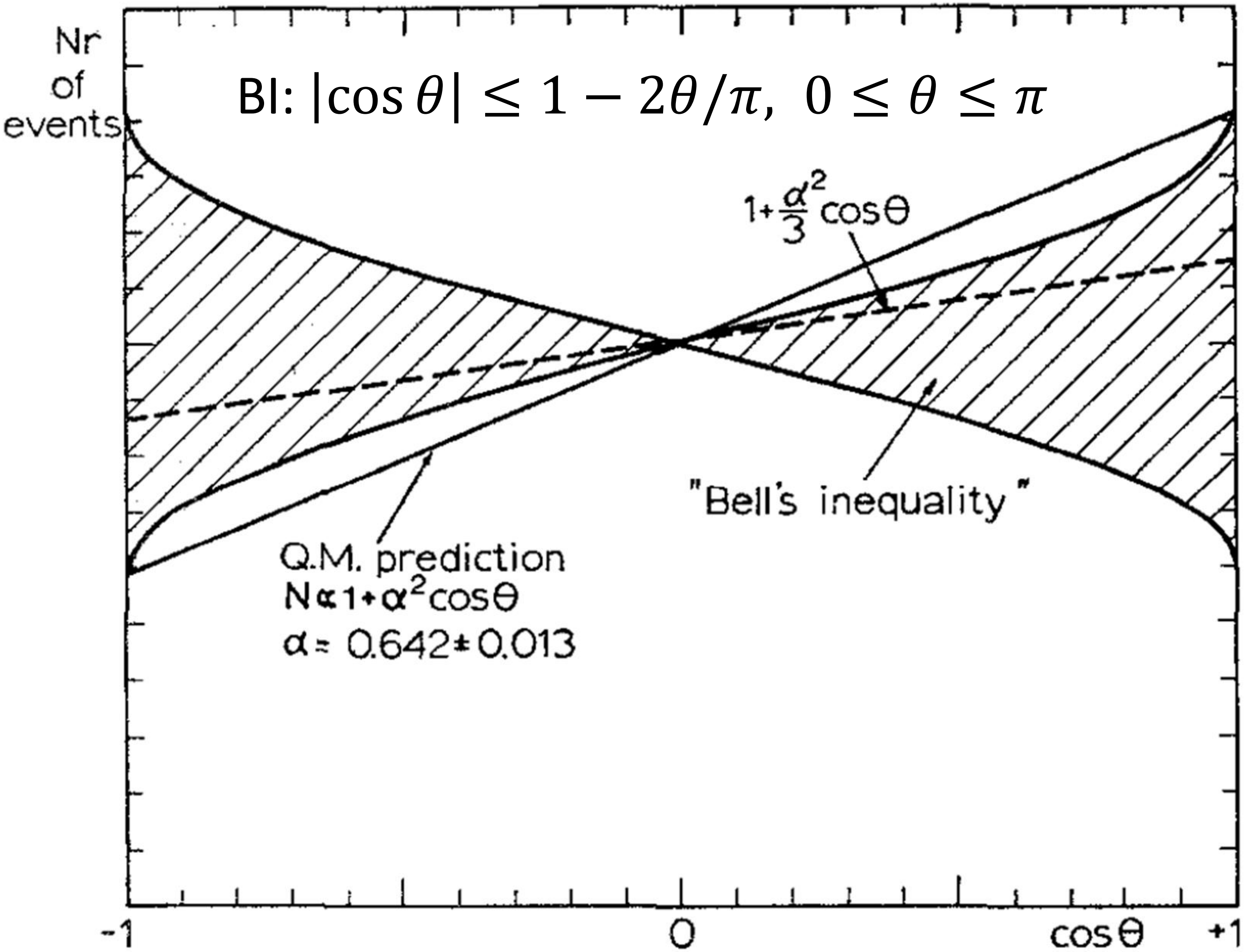
- In LHVT

$$I\left(\vec{A}, \vec{B}\right) = \left(\frac{|S|^2 + |P|^2}{4\pi}\right)^2 \frac{1}{4\pi} \int (1 + \alpha_\Lambda X \cdot A)(1 + \alpha_\Lambda X . B) d\Omega$$

$$= \left(\frac{|S|^2 + |P|^2}{4\pi}\right)^2 (1 + \frac{1}{3} {\alpha_\Lambda}^2 \vec{A} \cdot \vec{B}\ )$$

Test BI in $\eta_c \to \Lambda\bar{\Lambda}$ decays

N.A. Tornqvist, Foundation of Physics, 11, 171(1981)



Test BI in $J/\psi \to \Lambda\bar{\Lambda}$ decays

优点： 10billion J/ψ events provides about 3.3 million exclusive events, and accuracy for α_Λ and $\alpha_{\bar{\Lambda}}$ was measured up to a few percentage .

缺点： D-wave between the $\Lambda\,\bar{\Lambda}$ introduce a factor to suppress the slope of the proton angular distribution

Summary

- Let us summarize what we have learned ……