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# MHD-Particle-in-cell Method and its Astrophysical Applications

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# Outline

- Motivation and MHD-PIC method
- Application 1: CR acceleration in collisionless shocks
- Application 2: CR streaming instability
- Future directions

# How do CRs interact with a thermal plasma?

- CRs are collisionless and diffuse by scattering off MHD waves/turbulence:

  Galactic CRs’ residence time: ~10s Myr in total.

  Diffusion coefficient: $\kappa \sim R^2/T \sim$ a few x $10^{28}$cm$^2$s$^{-1}$. (e.g., Ginzburg & Syrovatskii 64)

Passive “test particles”

- CRs provide pressure support: $F = -\nabla_\perp P_{\rm CR} = -\frac{\boldsymbol{J}_{\rm CR} \times \boldsymbol{B}}{c}$

- When the bulk CRs drift through background plasma faster than the Alfven speed $v_{D,CR} > v_A$, they will drive *streaming instabilities*.

  (e.g., Kulsrud & Pearce 1969, Bell 2004)

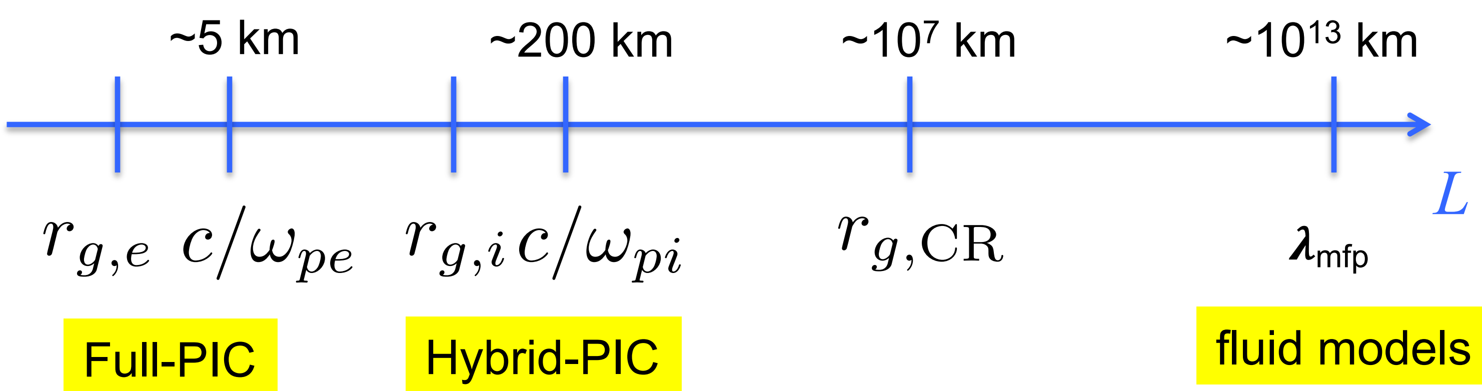
Need kinetic physics: CRs transfer energy/momentum to gas via Alfven waves.

Active CR feedback !

# Simulating CR physics at kinetic level

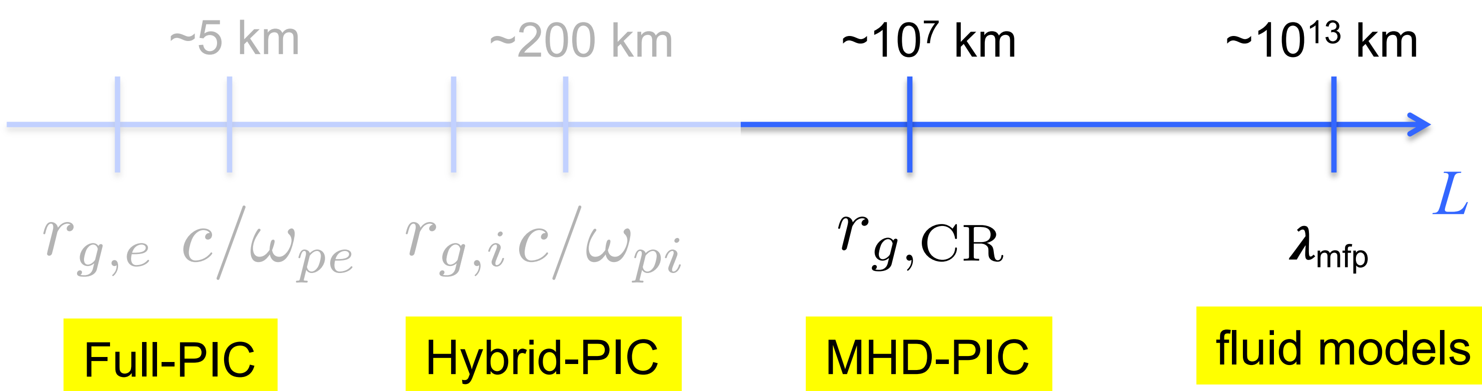
- Minimum requirement: resolve CR gyro-radii.
- Huge scale separation involved, challenging for conventional PIC codes:

  - *Full-PIC*: treat all (background+CR) particles as kinetic particles
  - Hybrid-PIC: all ions (background+CR) are kinetic, electrons as massless fluid



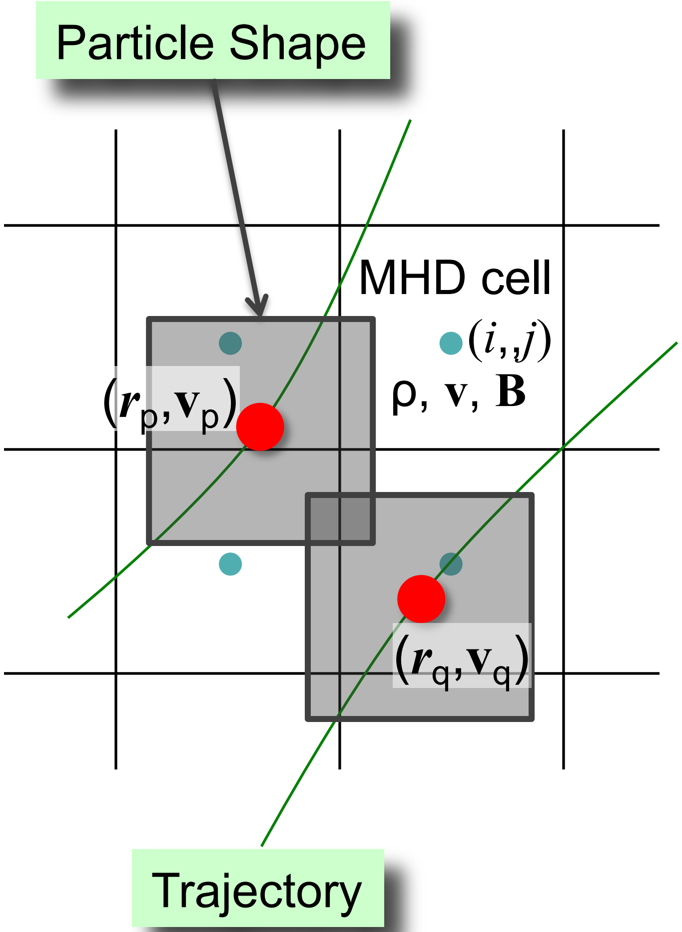
# Simulating CR physics at kinetic level

- Minimum requirement: resolve CR gyro-radii.
- Overcome this scale separation: skipping over the kinetic scales of the background plasma.
  - MHD-PIC: treat background plasma by MHD, while CRs are kinetic



# MHD-PIC approach

- Each computational particle (i.e., super-particle) represents a large collection of real CR particles.
- Each super-particle carries an effective shape, designed to facilitate interpolation from the grid.
- Individual CR particles move under the electro-magnetic field from MHD.
- Total momentum and energy must conserve: particles feedback to MHD cells by depositing changes in momentum and energy locally.



# MHD-PIC: formulation and implementation

Equations for the (relativistic) CR particles:

$$\frac{d(\gamma_j \boldsymbol{u}_j)}{dt} = \frac{q_j}{m_j}\left(\boldsymbol{E} + \frac{\boldsymbol{u}_j}{c} \times \boldsymbol{B}\right)$$

Specify the numerical speed of light $c$ >> any velocities in MHD.

Full equations for the gas:

$$\frac{\partial \rho \boldsymbol{v}}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}\boldsymbol{v} - \boldsymbol{B}\boldsymbol{B} + \mathbf{P}^*) = \text{- Lorentz force on the CRs}$$

$$\frac{\partial E}{\partial t} + \nabla \cdot [(E + P^*)\boldsymbol{v} - \boldsymbol{B}(\boldsymbol{B} \cdot \boldsymbol{v})] = \text{- energy change rate on the CRs}$$

Implemented in the Athena MHD code (Bai, Caprioli, Sironi & Spitkovsky 2015).

(See also van Marle et al. 2018, Mignone et al. 2018)

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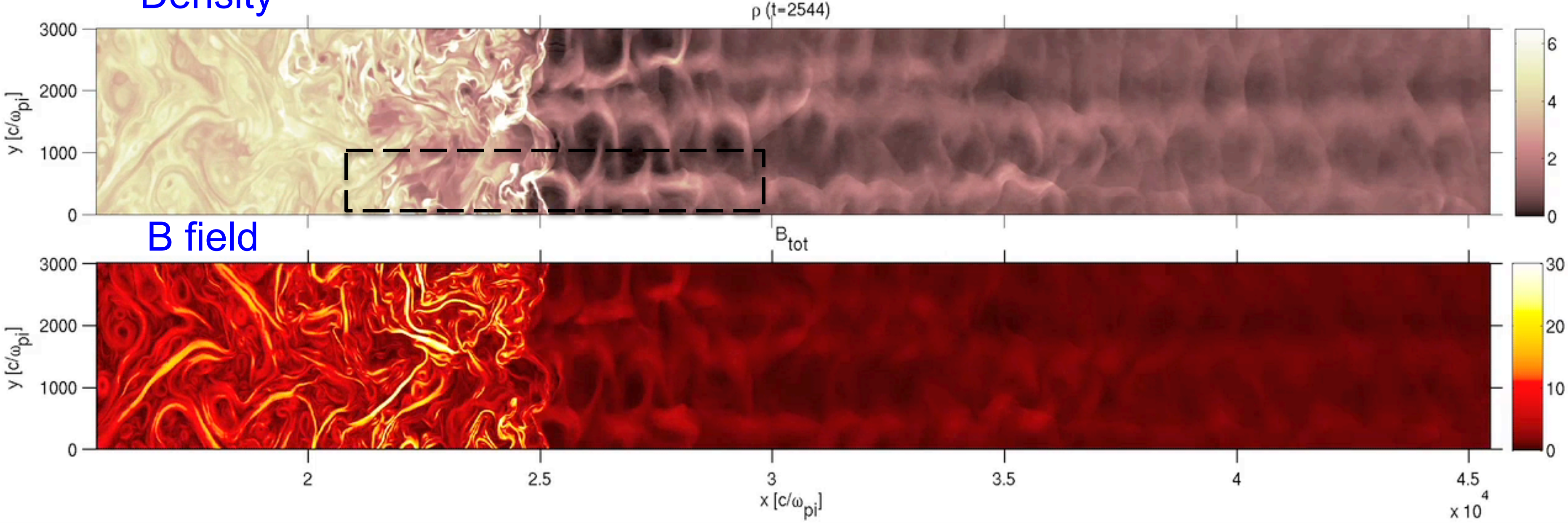
# Non-relativistic shock: MHD-PIC simulation



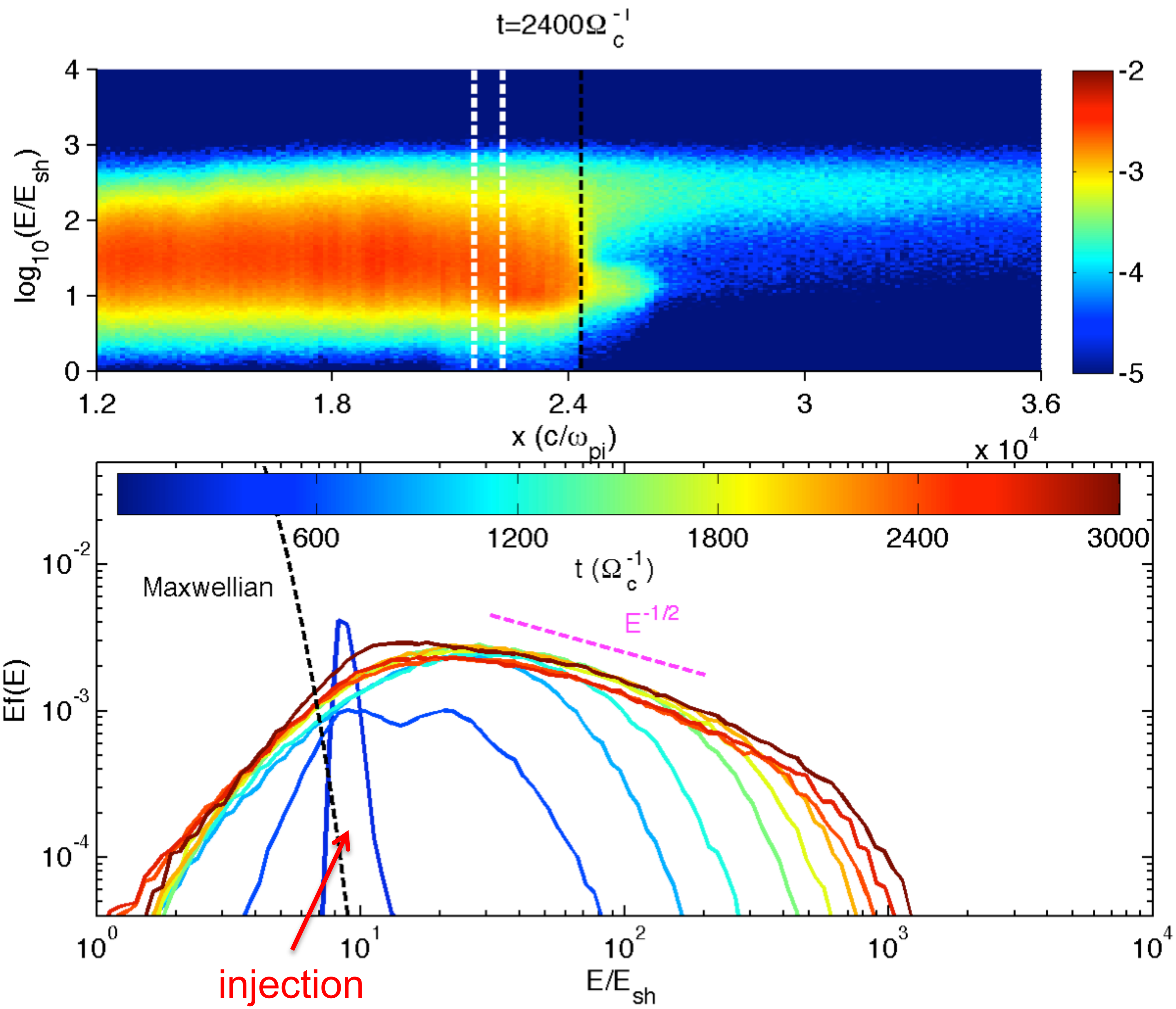

**Fiducial parameters:** $M_A$~30, parallel shock $\theta$=0.

**Resolution:** 12 ion skin depths per cell (*v.s. 0.5 in hybrid-PIC*)

**Particle injection:** artificial (as proof-of-concept) $\eta = 2 \times 10^{-3}$

(to be improved in the near future)

# Particle acceleration



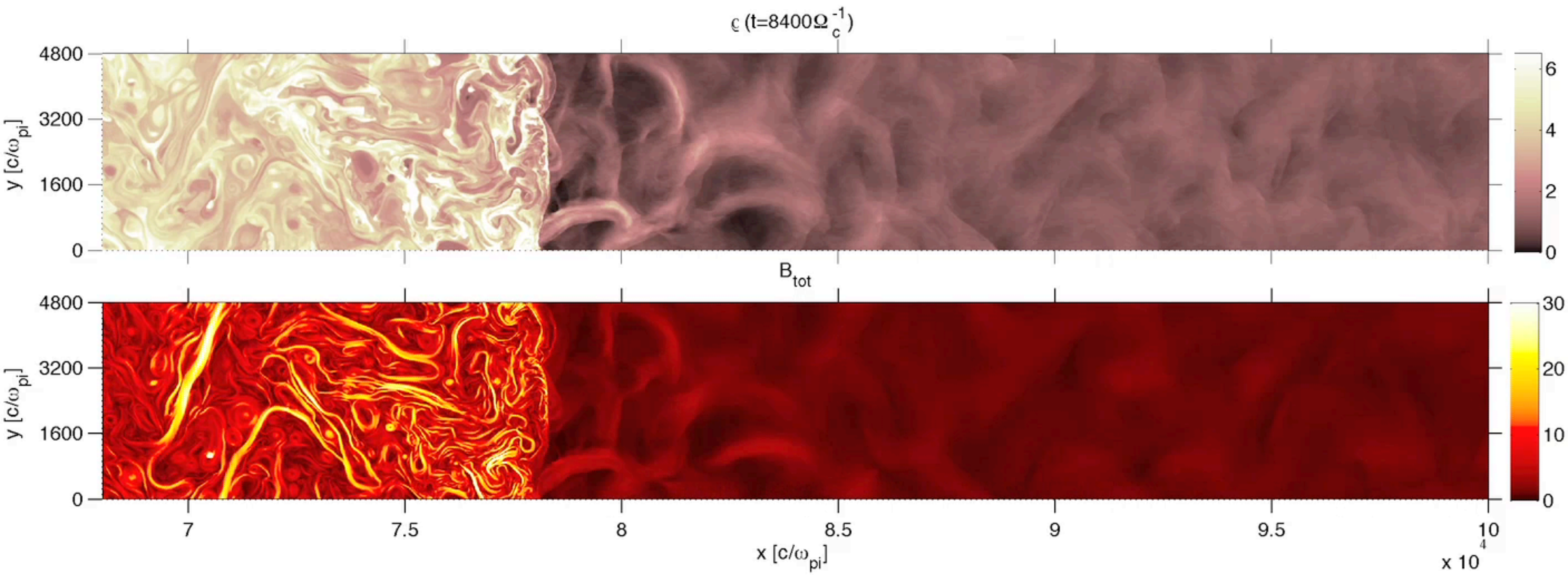

$f(E) \sim E^{-3/2}$ (non-relativistic)

Acceleration efficiency: $\xi \sim 13\%$

Bai+ 2015

# Simulation with relativistic particles

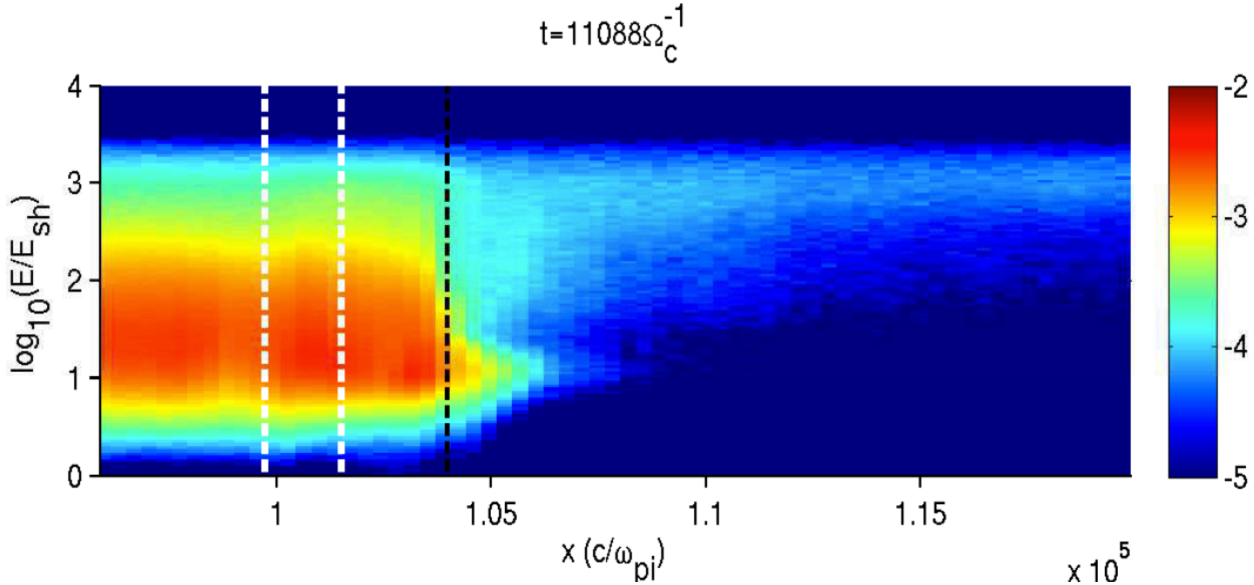
Set numerical speed of light $c$ a factor ~10-20 larger than $v_{sh}$ to follow particle acceleration to relativistic regime.

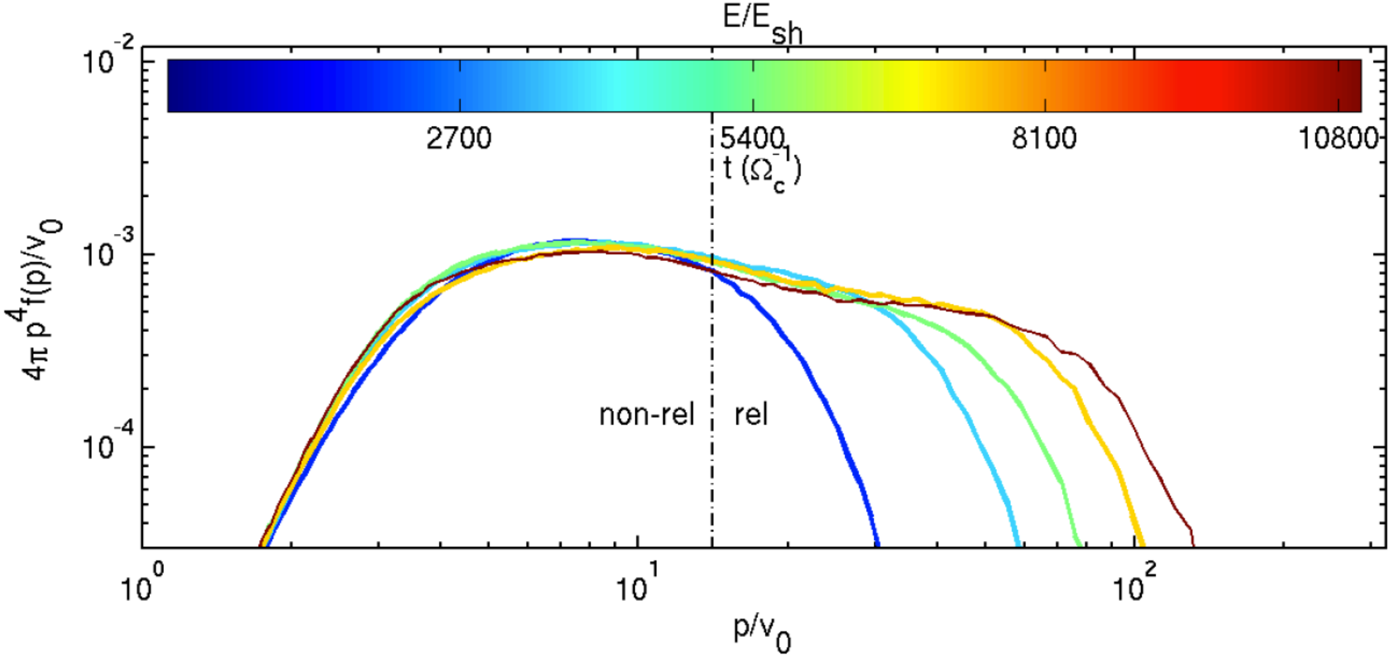



Very large box size (4800 $c/\omega_{pi}$ wide), and very long evolution (~$10^5\Omega_c^{-1}$)

Reduction of shock speed toward later evolution.

# Particle acceleration into relativistic regime

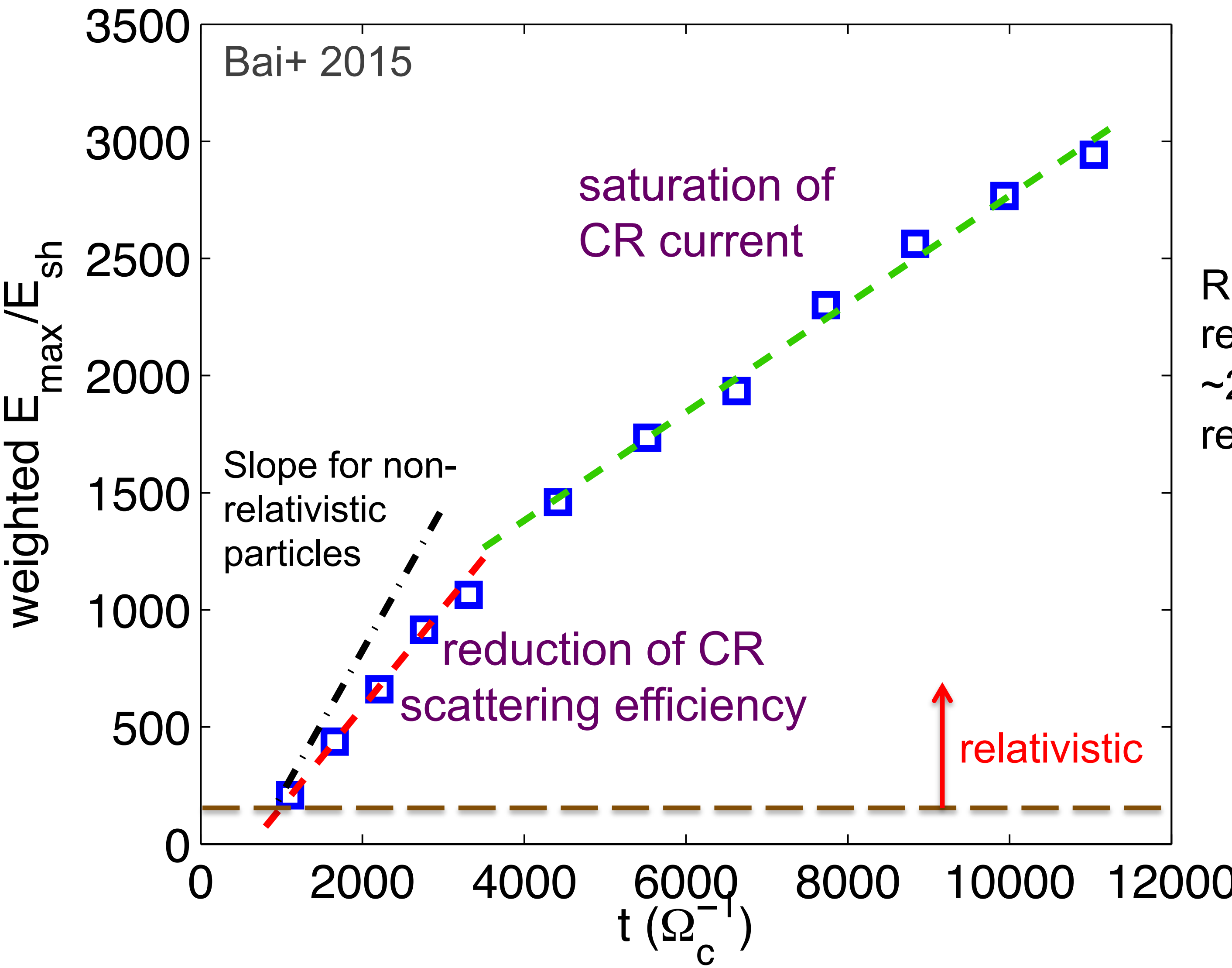



$f(p) \sim p^{-4}$ through the transition + a drop in normalization.

Bai+ 2015

# Evolution of maximum particle energy




Rate of acceleration reduced by a factor ~2 in the relativistic regime

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# Resonant CR streaming instability (CRSI)

When the bulk drift of the CRs exceed the Alfven velocity, they resonantly trigger the CR streaming instability.

Characteristic growth rate:

$$\Gamma(k) \approx \Omega_c \frac{N_{\rm CR}(p > p_{\rm res}(k))}{n_i} \frac{v_D - v_A}{v_A}$$

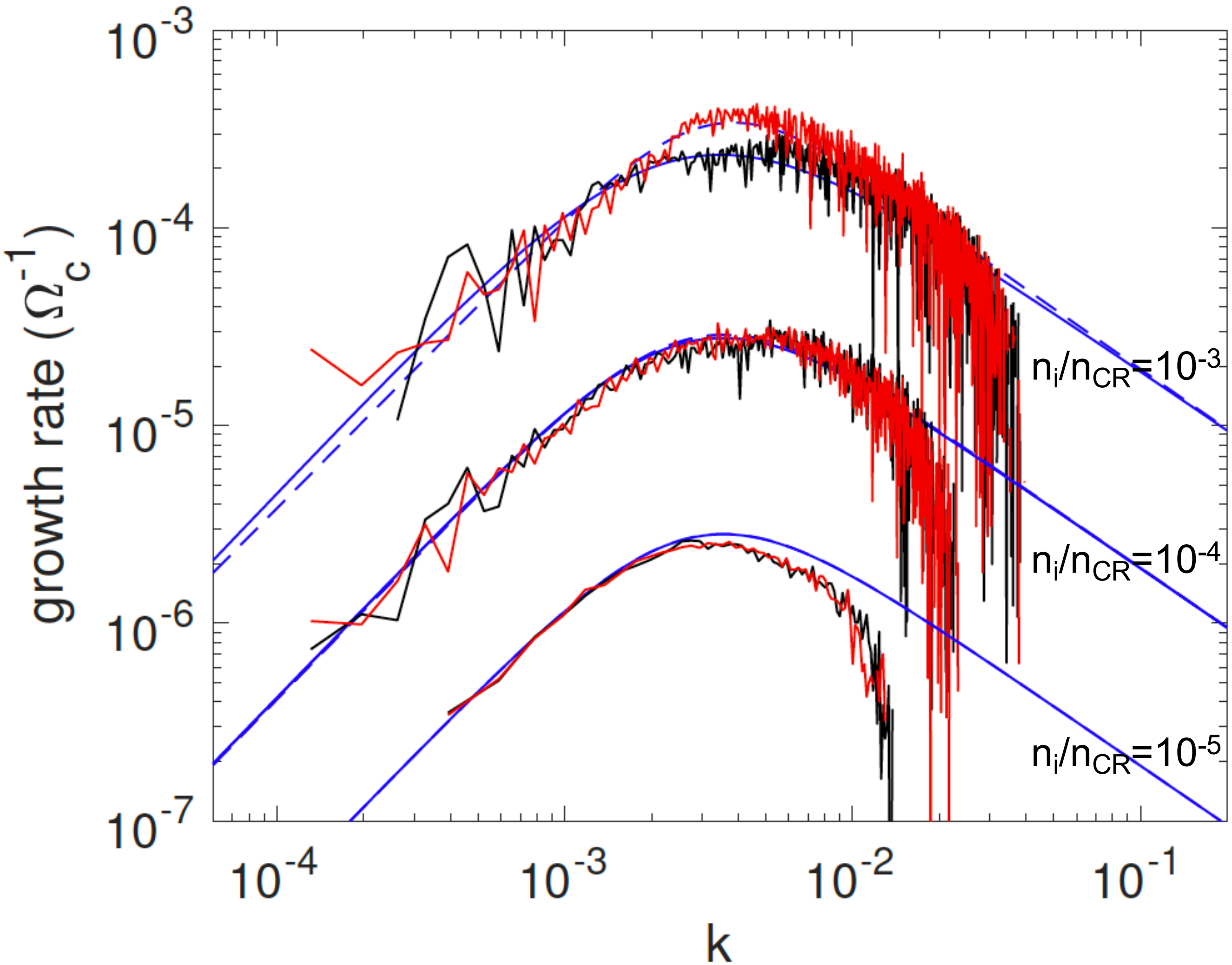
Driven primarily by low-energy CRs (the dominant population by number)

(e.g., Kulsrud & Pearce 69, Skilling 75)

Important feedback mechanism to galaxy formation and evolution:

- CR self-confinement
- CR-drive galactic outflows

# Matching analytical dispersion relation

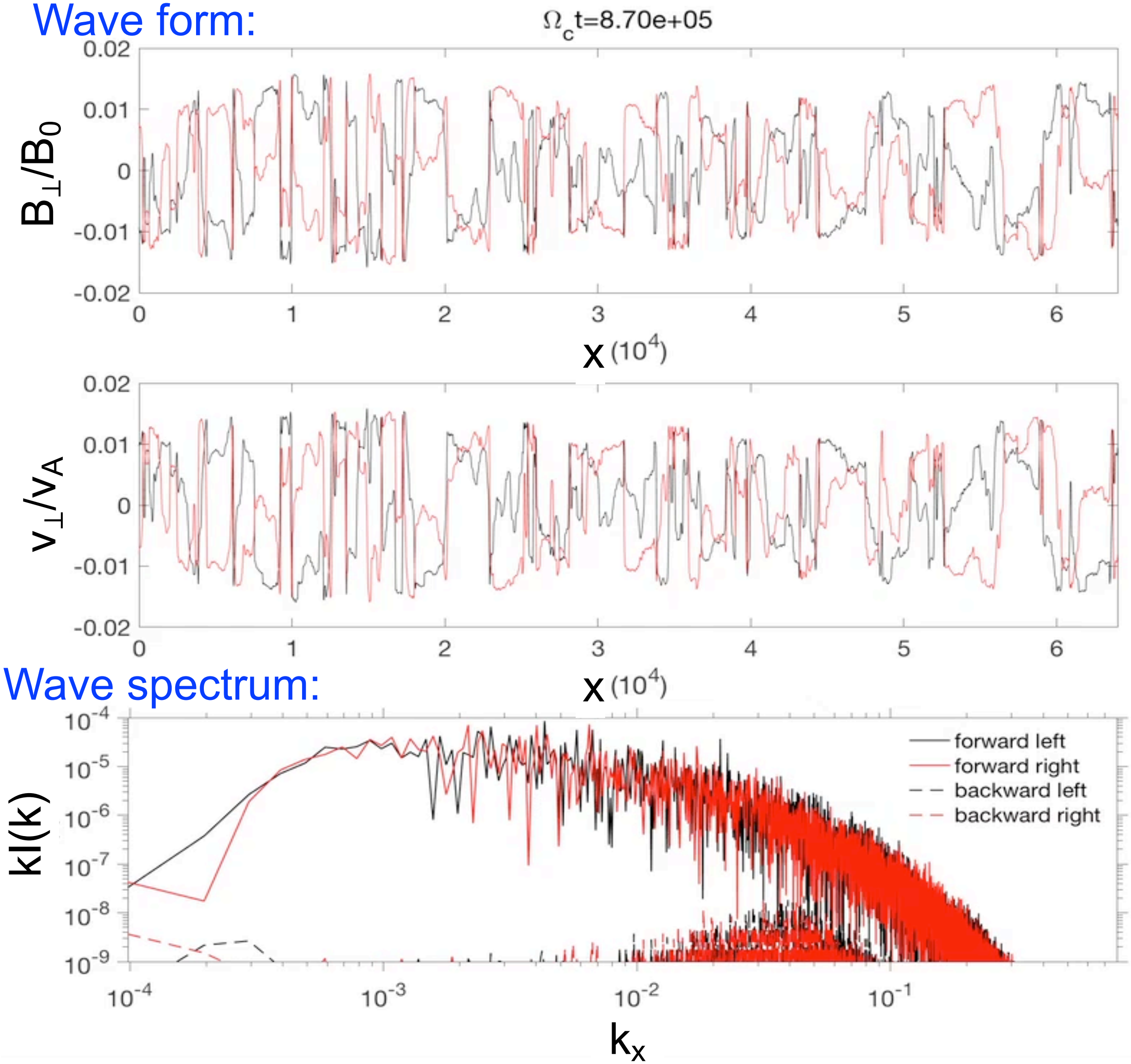



Accurately reproduce the linear growth rate over broad spectrum.

Bai et al. 2019

# 1D simulation: growth and saturation

Wave form:




Simulation performed in the rest frame of the CRs.

Periodic BC.

Gas travels to the left at $v_D$.

Fiducial parameters:

$v_D=2v_A$

$N_{CR}/n_0=10^{-4}$

$p_0/m=300v_A$

2048 ppc, Lx~50 most unstable wavelength.

Bai et al. 2019

# Towards saturation: quasi-linear diffusion

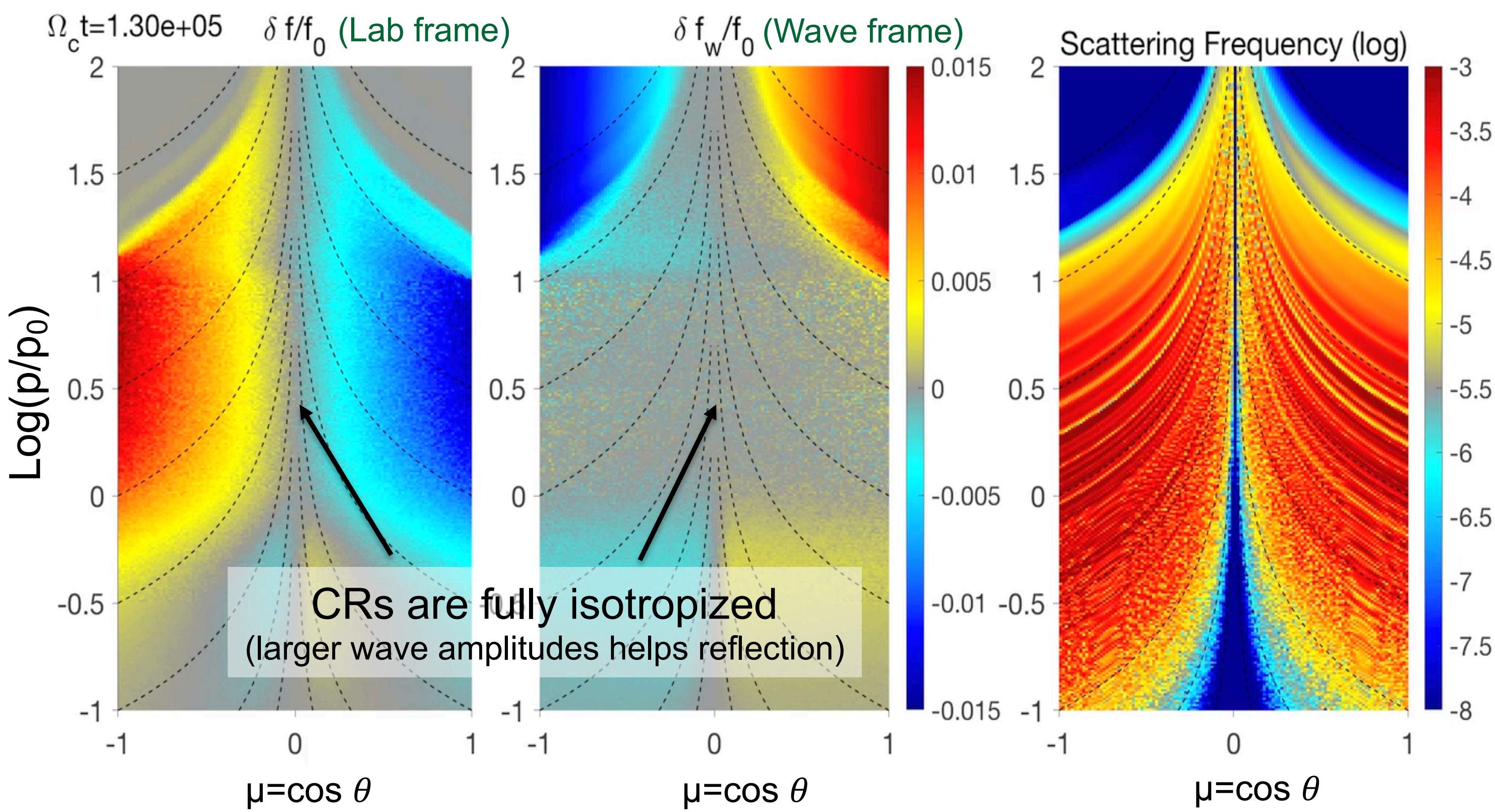
$$\frac{\partial f_w}{\partial t} = \frac{\partial}{\partial \mu_w}\left[\frac{1-\mu_w^2}{2}\nu(\mu_w)\frac{\partial f_w}{\partial \mu_w}\right] + \text{reflection}$$

Scattering frequency:

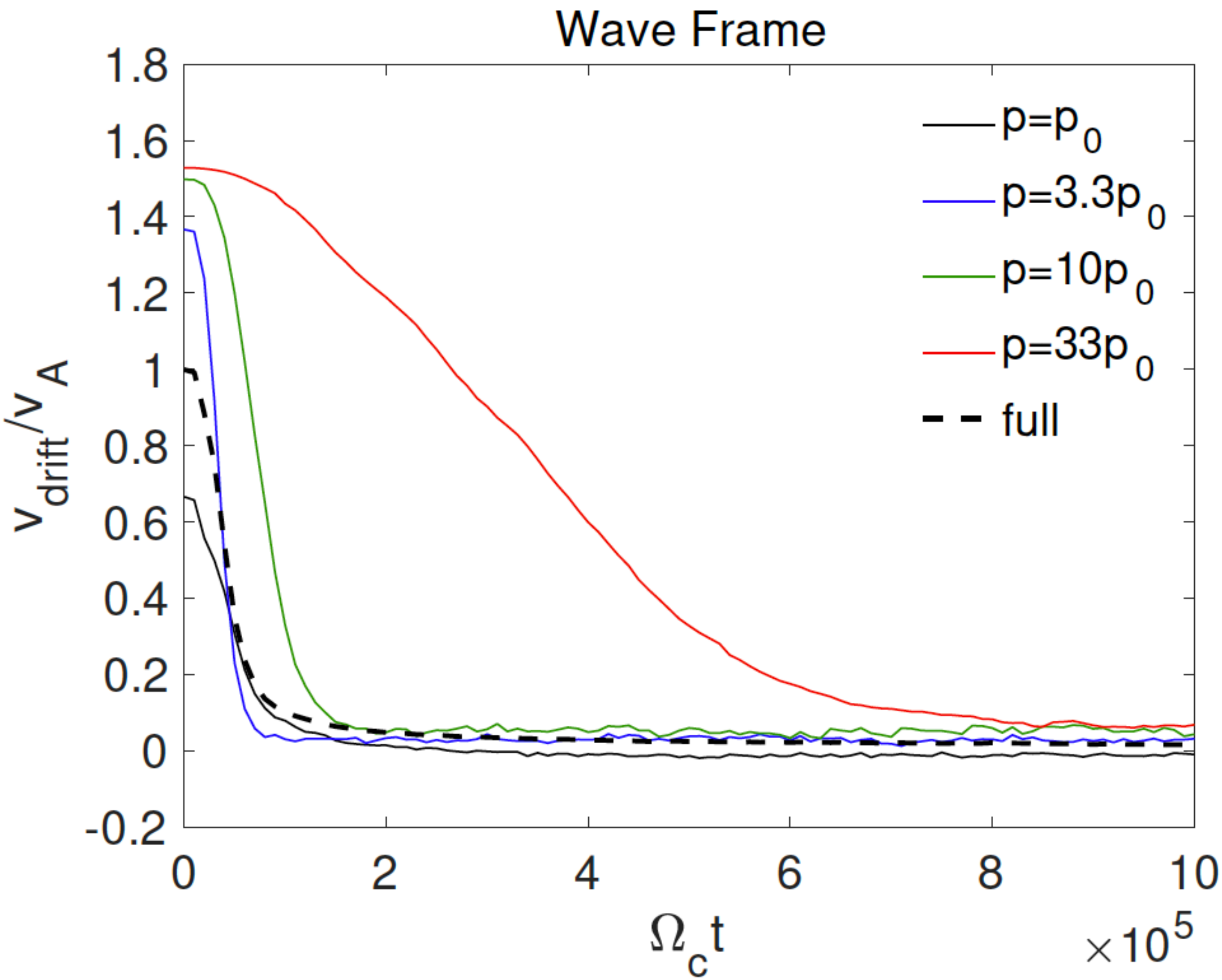
$$\nu(\mu_w) = \pi \Omega k_{\text{res}} I(k_{\text{res}})$$

wave intensity

Parameters: $v_D = 2v_A$; $N_{CR}/n_0 = 10^{-3}$



# Reduction of CR drift speeds




$v_D=2v_A$

$N_{CR}/n_0=10^{-3}$

Bai et al. 2019

# Future directions

- Incorporate wave damping physics
- Incorporate multi-phase ISM
- Other gyro-resonant instabilities
- CR source problem
- CR escape
- Towards 2D/3D

# Summary

- Motivation and development of MHD-PIC method
  - To study kinetic aspect of CRs interacting with background plasmas
  - PIC for CRs, MHD for background plasmas, valid on scales > ion skin depth, implemented to Athena MHD code (fully conservative).
- MHD-PIC simulations of CR acceleration
  - Reproduce hybrid-PIC results using much larger box at much reduced cost, and can follow CR acceleration to relativistic regime.
  - Need (artificial) injection prescription.
- MHD-PIC simulations of resonant CR streaming instability
  - Overcome the challenges: developed $\delta f$ method to reduce noise
  - First numerical study: confirmation of linear growth rates, and can follow CR quasi-linear evolution, overcome 90deg problem.
- Future: more realistic microphysics, CR escape…