Understanding the Features in the Cosmic Ray spectra

Paolo Lipari
INFN Roma Sapienza

10th International Workshop on
Air Shower Detection at High Energy

Nanjing, Jan 7, 2020

Some of this presentation based on:

Paolo Lipari and Silvia Vernetto,
"The shape of the cosmic ray proton spectrum"
arXiv:1911.01311 [astro-ph.HE].

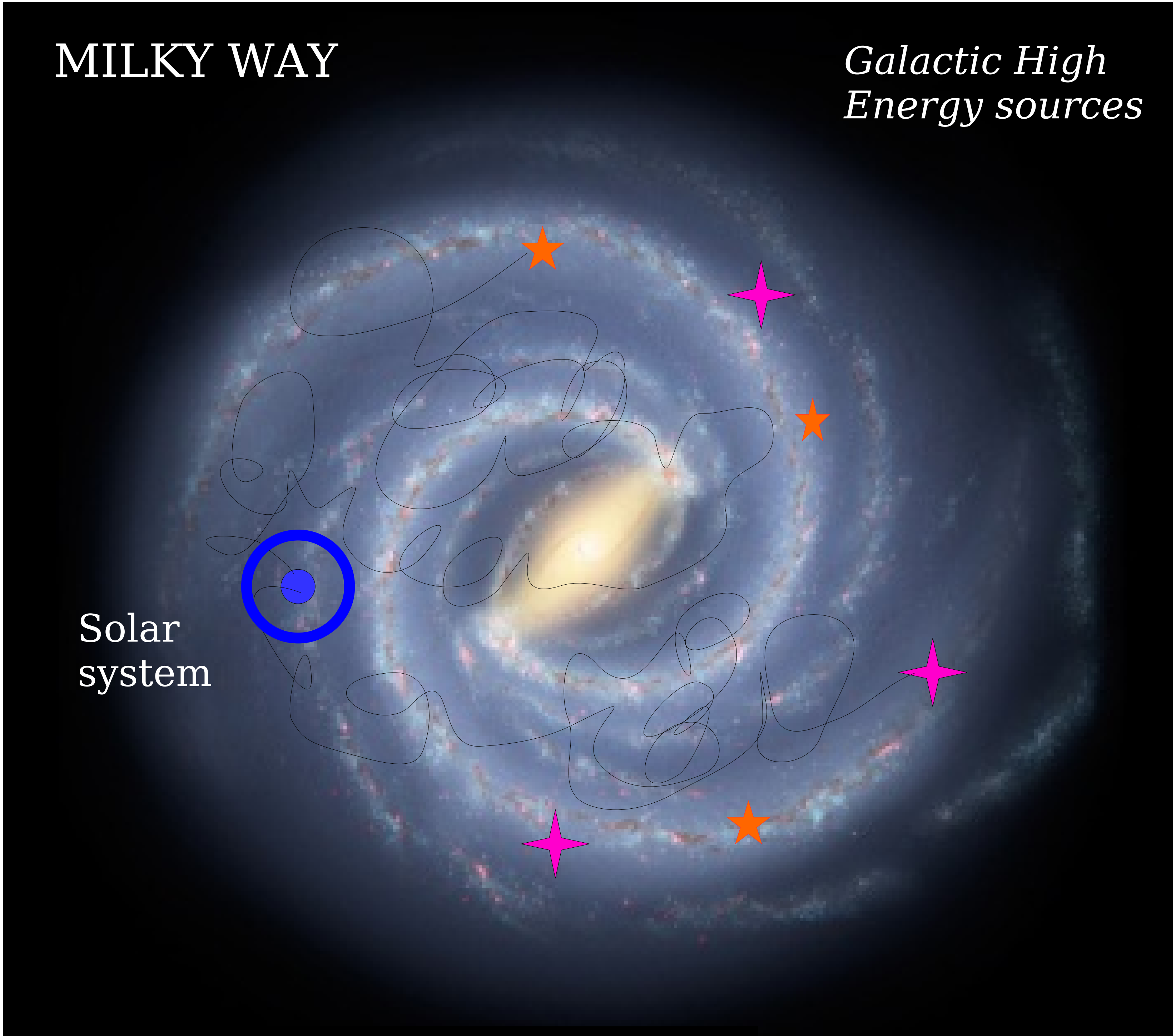
Paolo Lipari
"The The origin of the power–law form
of the extragalactic gamma–ray flux"
arXiv:2001.00982 [astro-ph.HE].

COSMIC RAYS

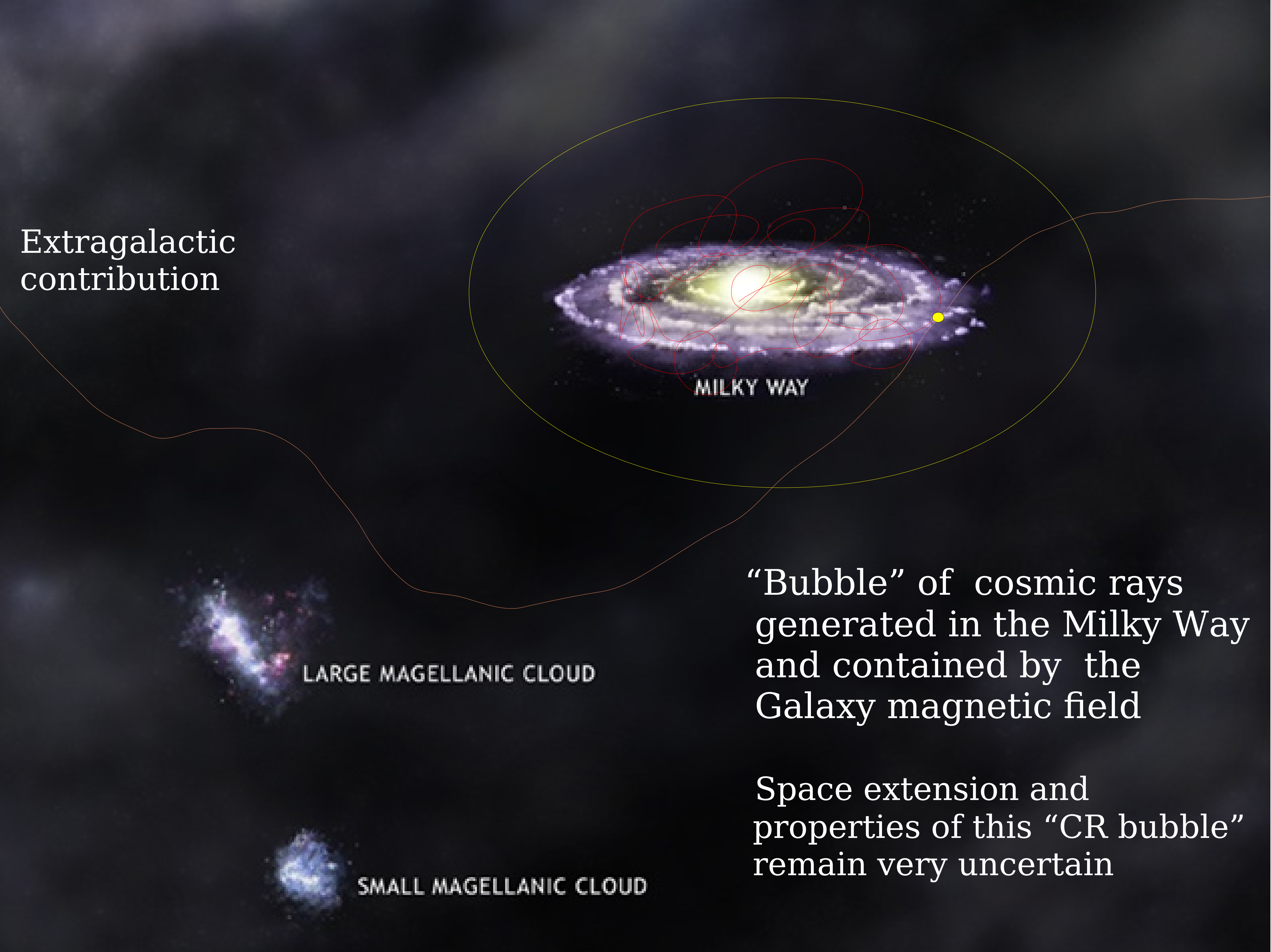
Space and time integrated average of
the emission of particles by *many sources*
in the Galaxy and in the universe,
also shaped by propagation effects.

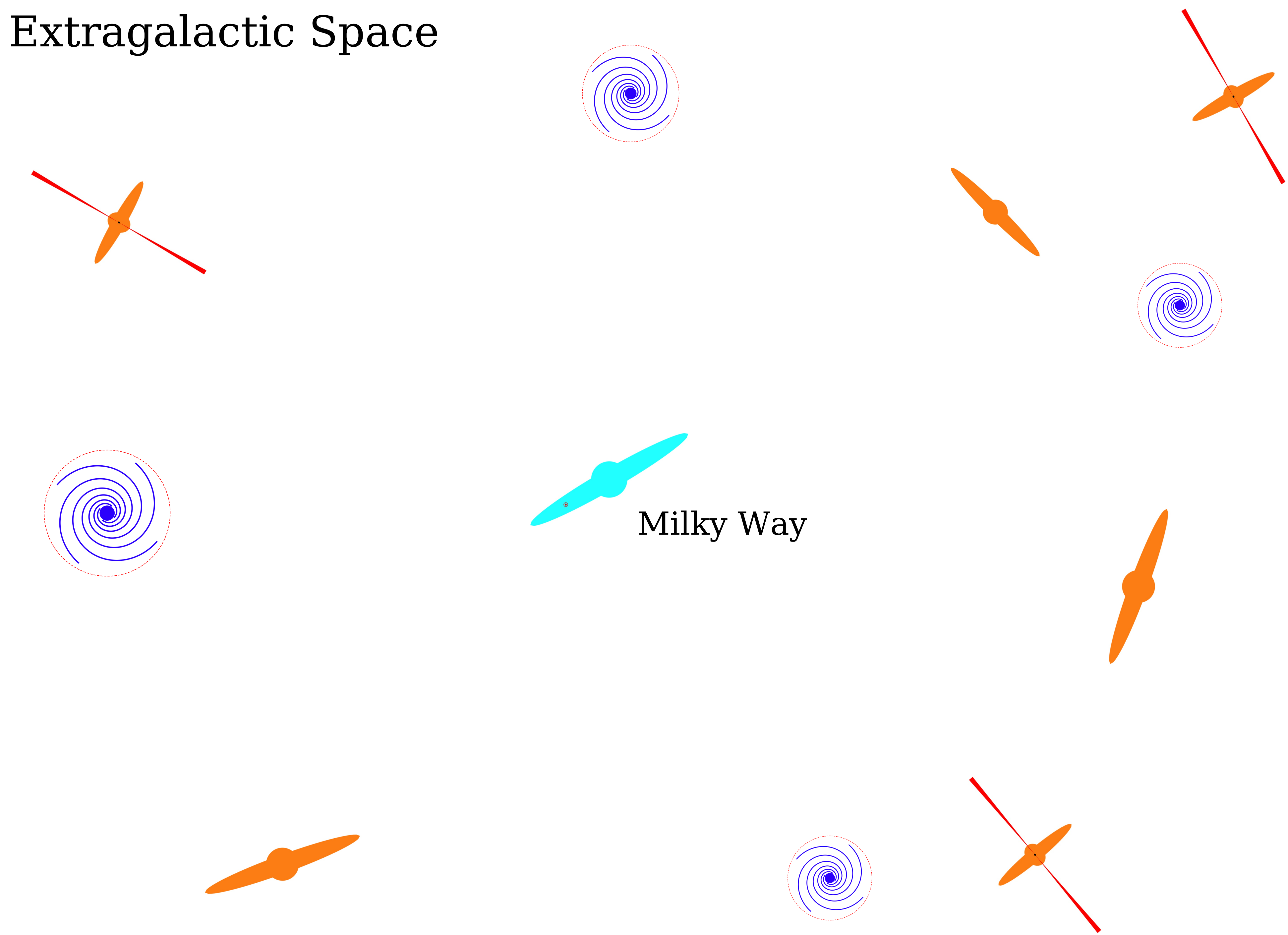
Measurement at
single point (the Earth)
and (effectively) single time (Now)

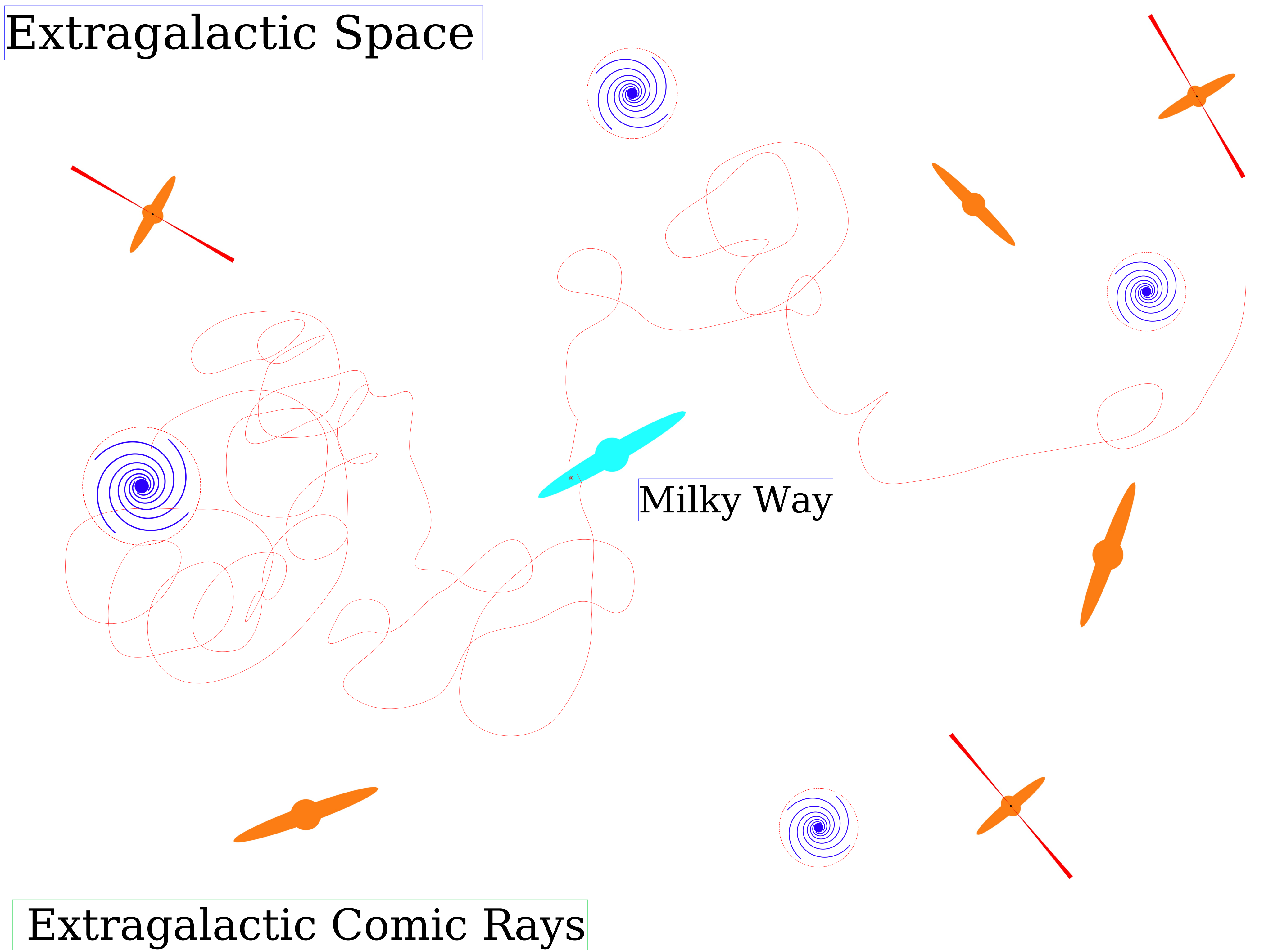
[slow time variations,
geological record carries some information]

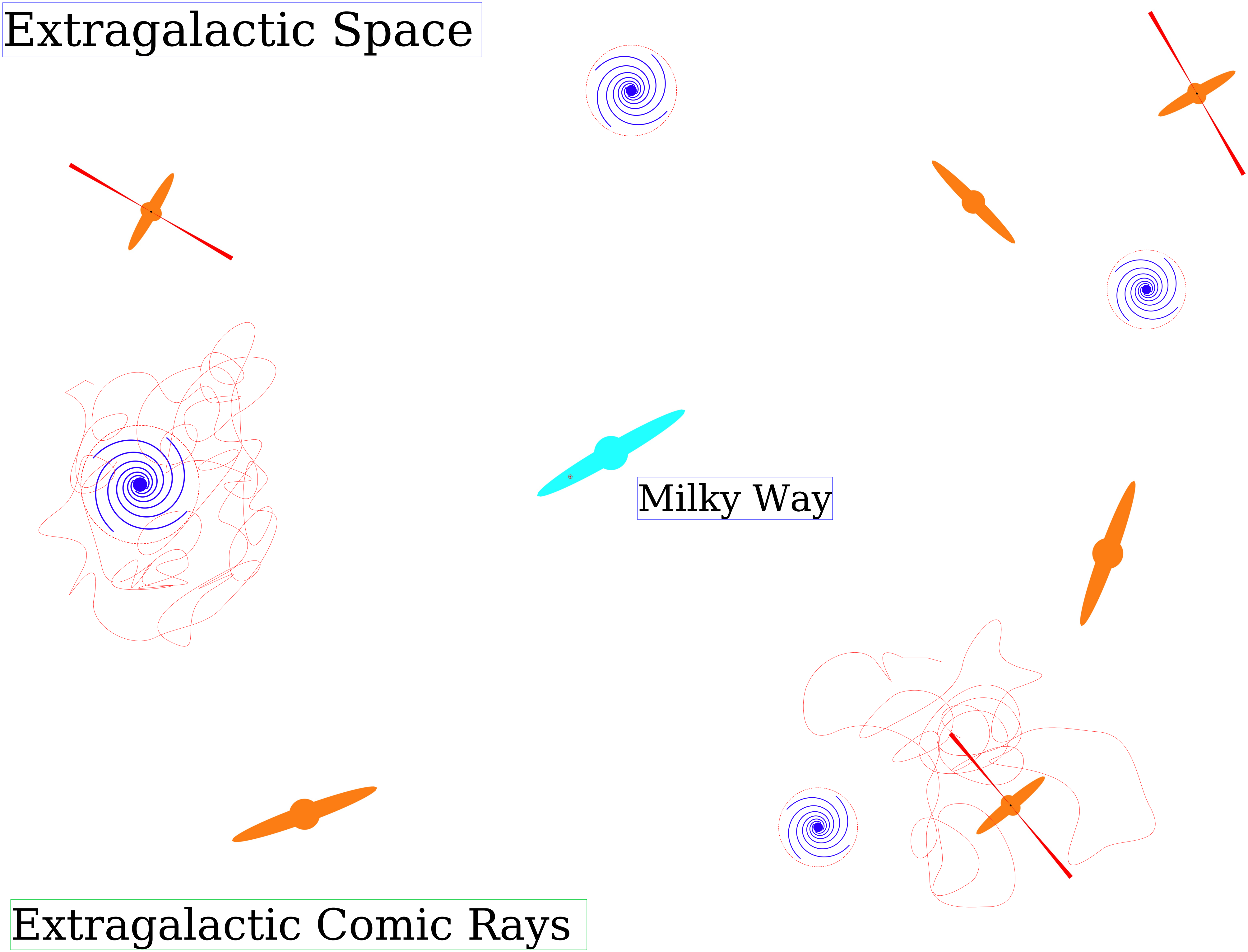


Galactic Cosmic Rays


Extragalactic contribution
MILKY WAY
LARGE MAGELLANIC CLOUD
SMALL MAGELLANIC CLOUD
“Bubble” of cosmic rays generated in the Milky Way and contained by the Galaxy magnetic field
Space extension and properties of this “CR bubble” remain very uncertain


Extragalactic Space
Milky Way


Extragalactic Space
Milky Way
Extragalactic Comic Rays


Extragalactic Space
Milky Way
Extragalactic Comic Rays

Generally accepted ideas about the formation of the

Galactic Cosmic Ray Spectra:

1. The observed CR spectra are shaped by their sources and by propagation effects:

$$\phi(E) \approx \frac{\beta\, c}{4\,\pi} \times Q(E) \times T(E)$$

2. The Sources are discrete, and transients (such as SNR or GRB)

$$Q(E) \approx \frac{1}{\text{time}} \sum_j q_j(E, \vec{x}_j, t_j)$$

3. The (dominant) sources accelerate particles in a broad energy range with a

"universal" source spectral shape

with a unique acceleration mechanism

[probably: Fermi first order acceleration].

The shape of the source spectrum is a power-law

$$q_{\rm source}(E) \propto E^{-\alpha}$$

4. Propagation effects have also a simple ... rigidity.

$$T(E) \approx T_0 \left(\frac{\beta E}{|Z e|} \right)^{-\delta}$$

5. The resulting spectra are then (in a broad energy range) of power law form:

$$\phi(E) \propto E^{-\gamma} \propto E^{-(\alpha+\delta)}$$

With a spectral index
determined by (sources) + (propagation)

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With a spectral index determined by (sources) + (propagation)

Problems for the study of Galactic Cosmic Rays:

- Determine the slopes α δ
- Identify the sources

5. The resulting spectra are then (in a broad energy range) of power law form:

$$\phi(E) \propto E^{-\gamma} \propto E^{-(\alpha+\delta)}$$

With a spectral index determined by (sources) + (propagation)

Recent measurement of the CR spectra have however revealed that they are *not exactly of power-law form*.

The spectral index is *depends on energy* and one observes "features".

Lively debate about the origin of these "features".

CR spectra

$t \approx 1990$

Solar Modulations

$\gamma \approx 2.7$

"Knee" $E \simeq 3$ PeV

$\gamma \approx 3.1$

"Ankle" $E \simeq 6$ EeV

GZK Cutoff ?

CR spectra

$t \approx 2020$

Solar Modulations

"Pamela Hardening"

"10 TeV proton softening" [Dampe (et al.)]

? other features ?

"Knee" $E \simeq 3$ PeV

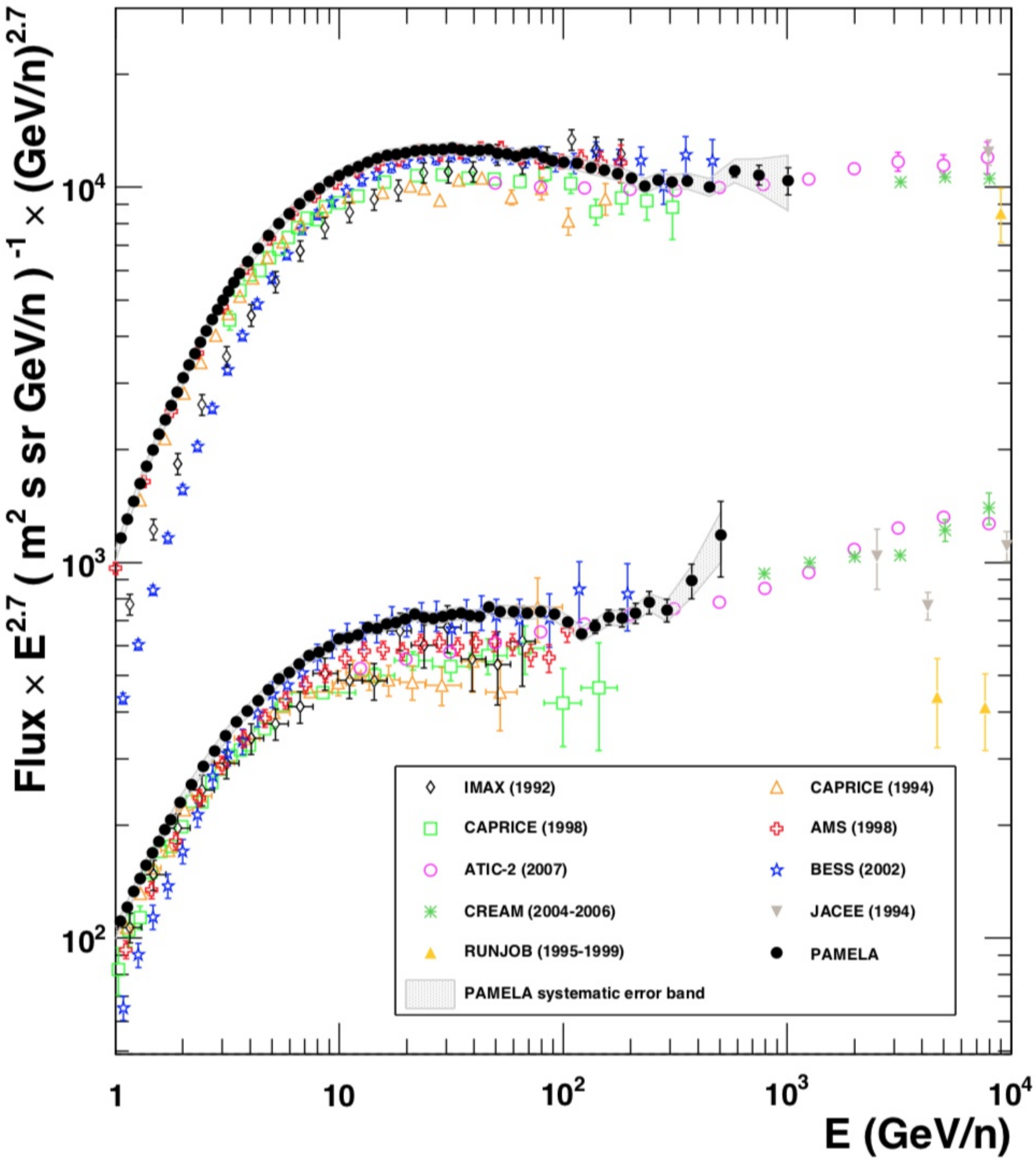
"Low energy ankle"

"2^{nd} Knee"

? other features ?

"Ankle" $E \simeq 6$ EeV

GZK Cutoff ?



PAMELA (2011)

Proton, Helium

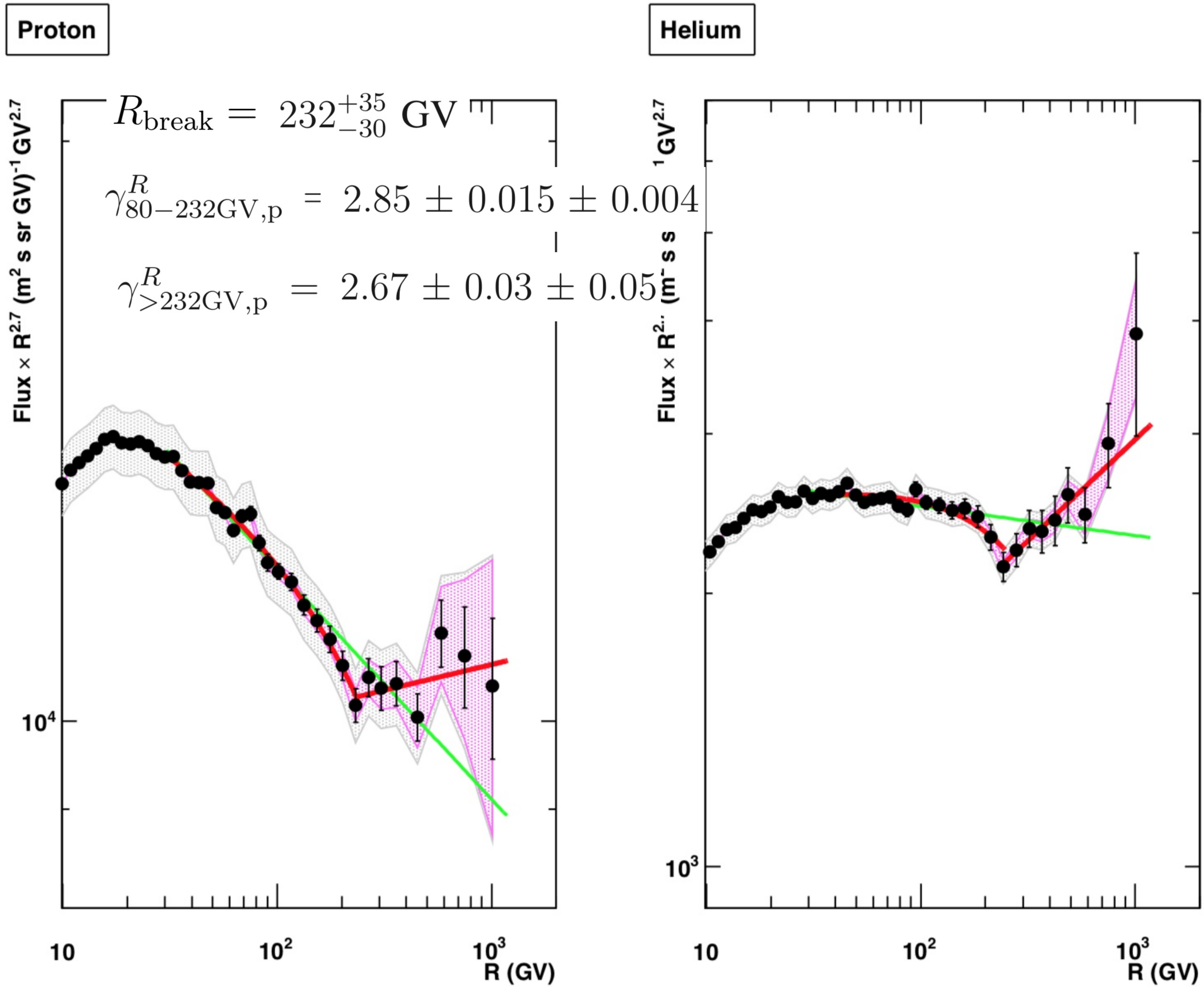
spectra

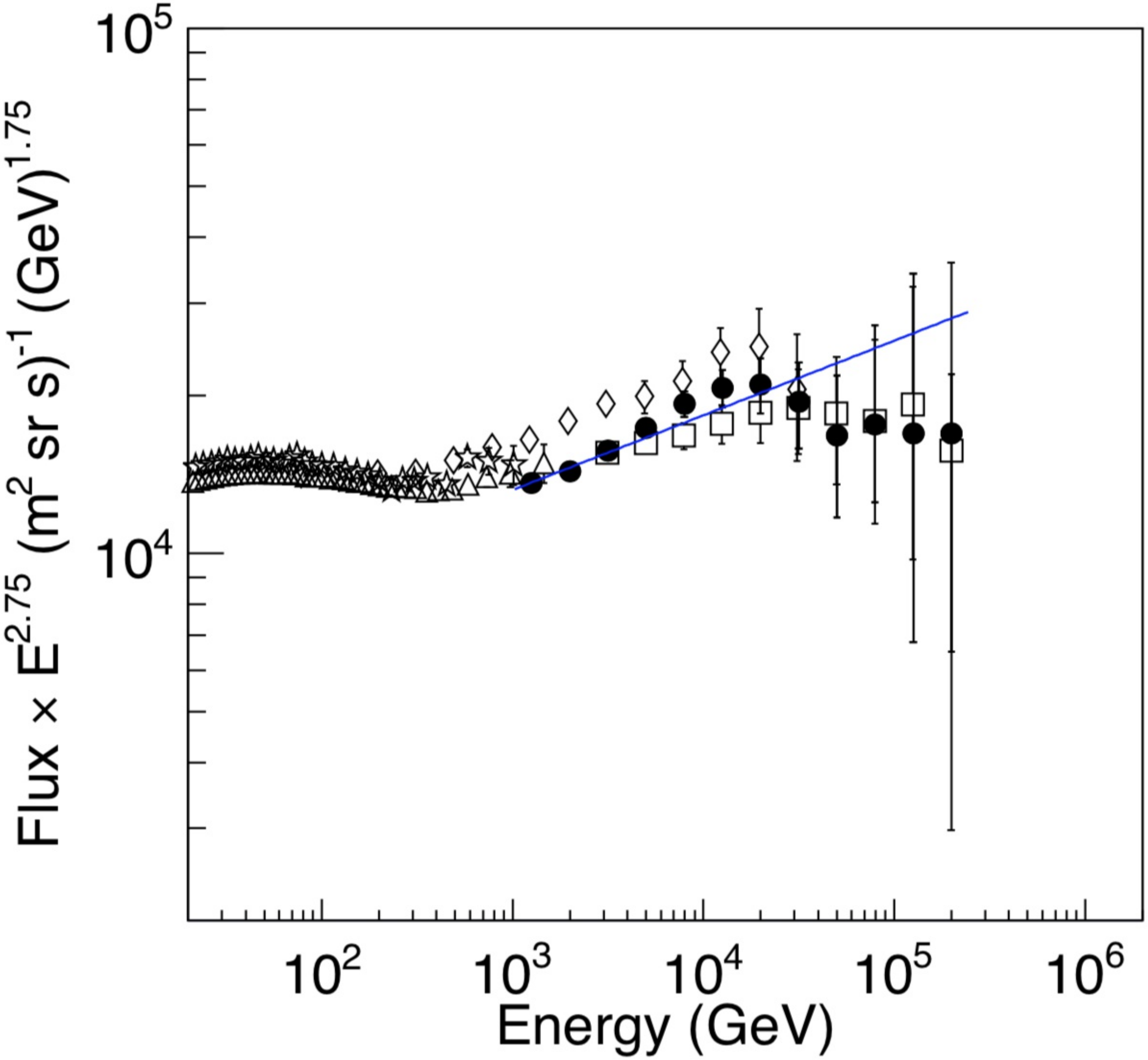
O. Adriani *et al.* [PAMELA Collaboration],
"PAMELA Measurements of Cosmic-ray Proton and Helium Spectra,"
Science **332**, 69 (2011)

$$R_{\rm break} = 232^{+35}_{-30}\ {\rm GV}$$

$$\gamma^{R}_{80-232{\rm GV,p}} = 2.85 \pm 0.015 \pm 0.004$$

$$\gamma^{R}_{>232{\rm GV,p}} = 2.67 \pm 0.03 \pm 0.05$$





Cream III (2010)

Proton Spectrum

“Discrepant Hardening”

$$\alpha_p = 2.61 \pm 0.01$$

H. S. Ahn *et al.*, [CREAM Collaboration]
“Discrepant hardening observed in cosmic-ray elemental spectra,”
Astrophys. J. **714**, L89 (2010)
[arXiv:1004.1123 [astro-ph.HE]].

CREAM data *"hint" of a softening* [E = 10 TeV]

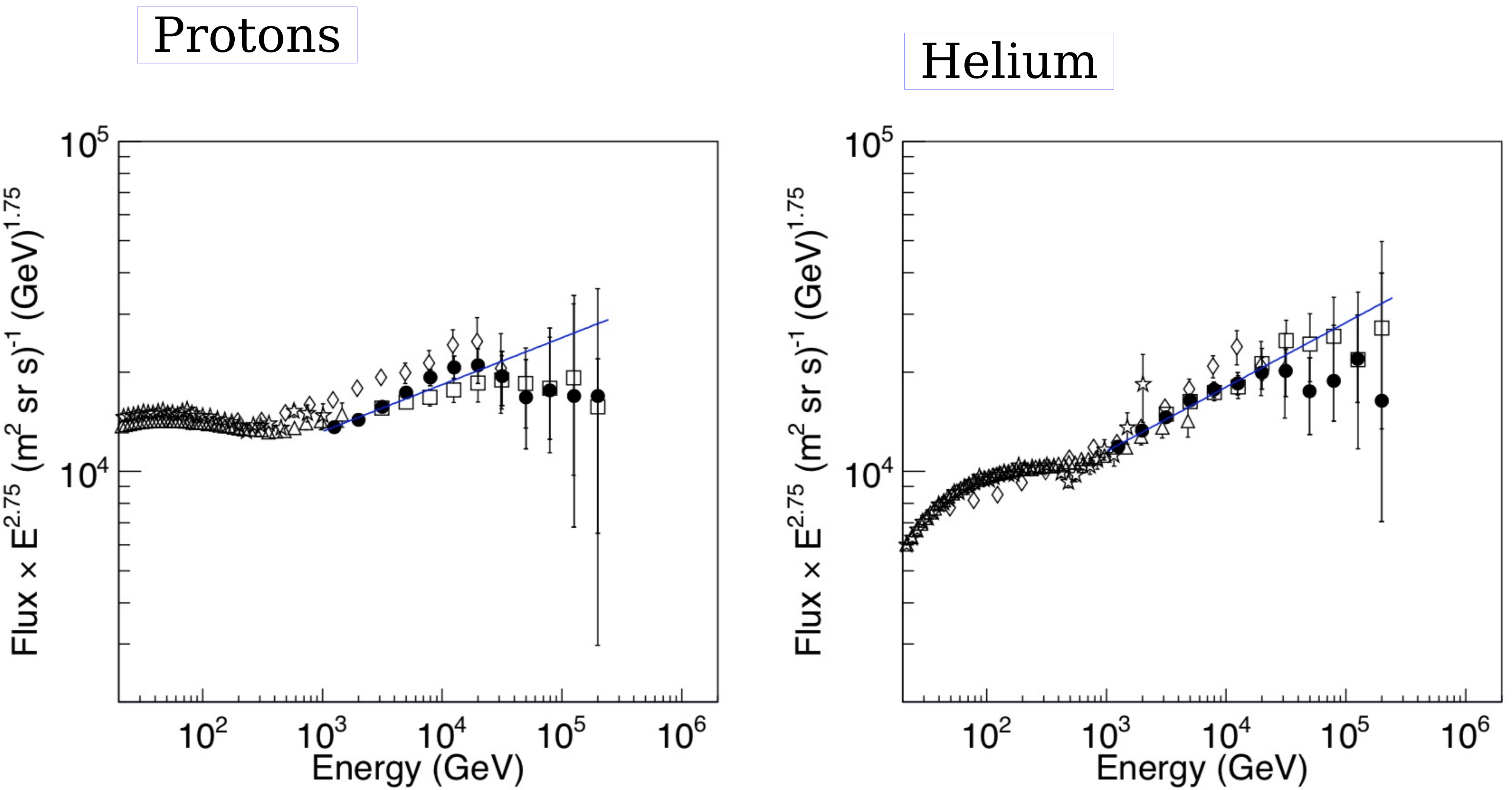
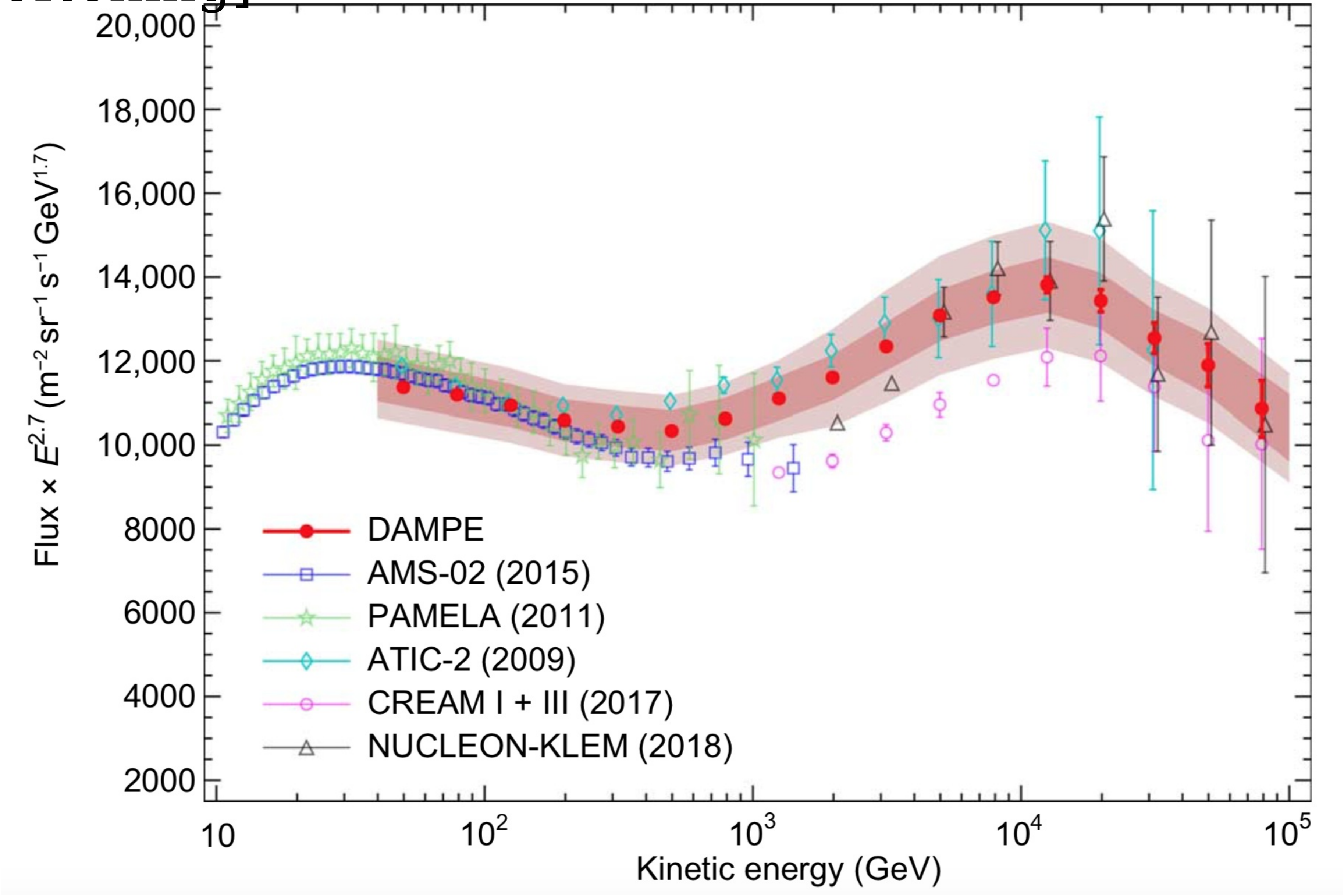
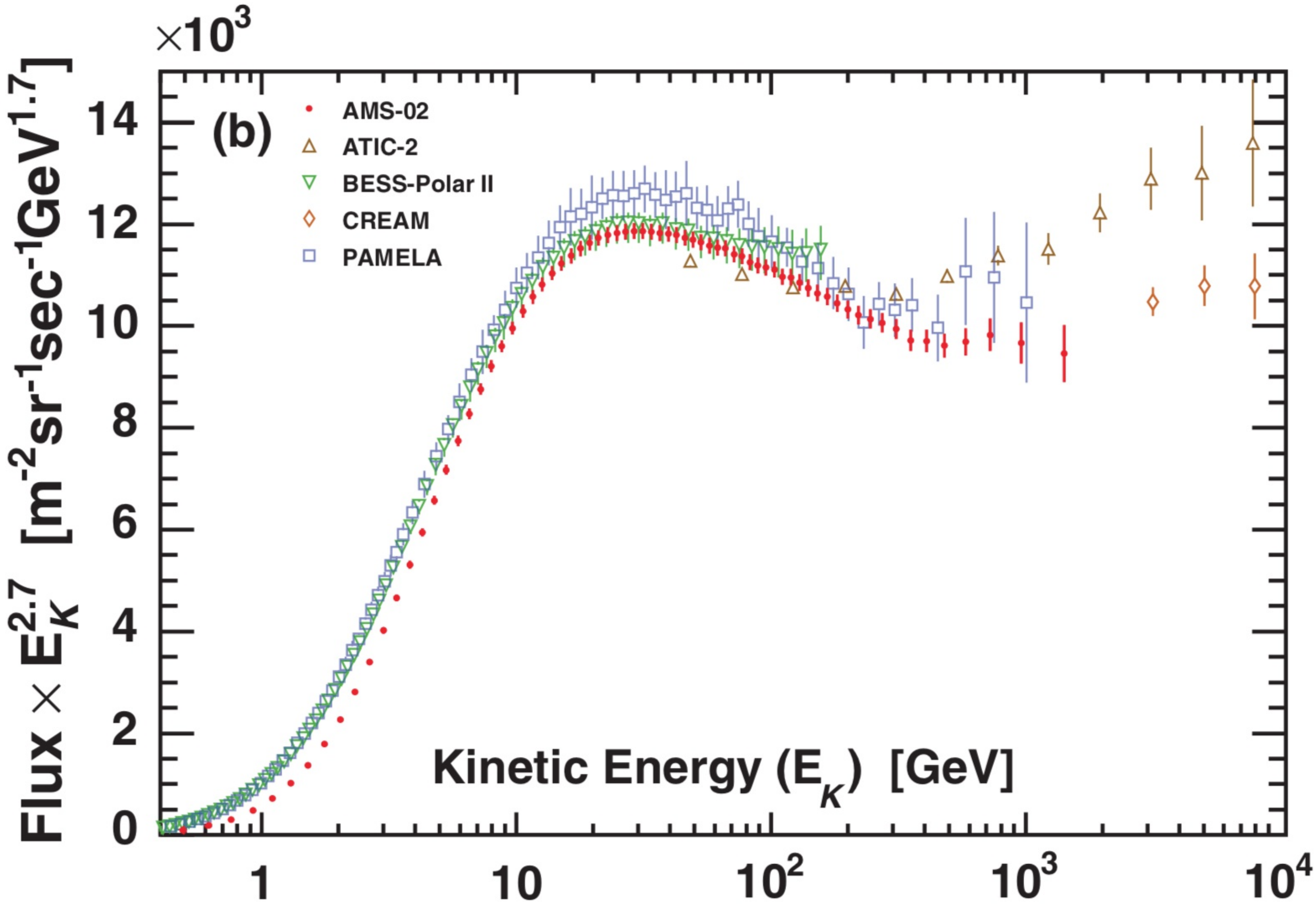


Figure 6. Proton (left) and helium (right) spectra from the combined CREAM-I and CREAM-III data (filled circles). Statistical uncertainties are shown. Selected previous measurements are also shown: AMS-02 (triangles), ATIC-2 (diamonds), and PAMELA (stars).

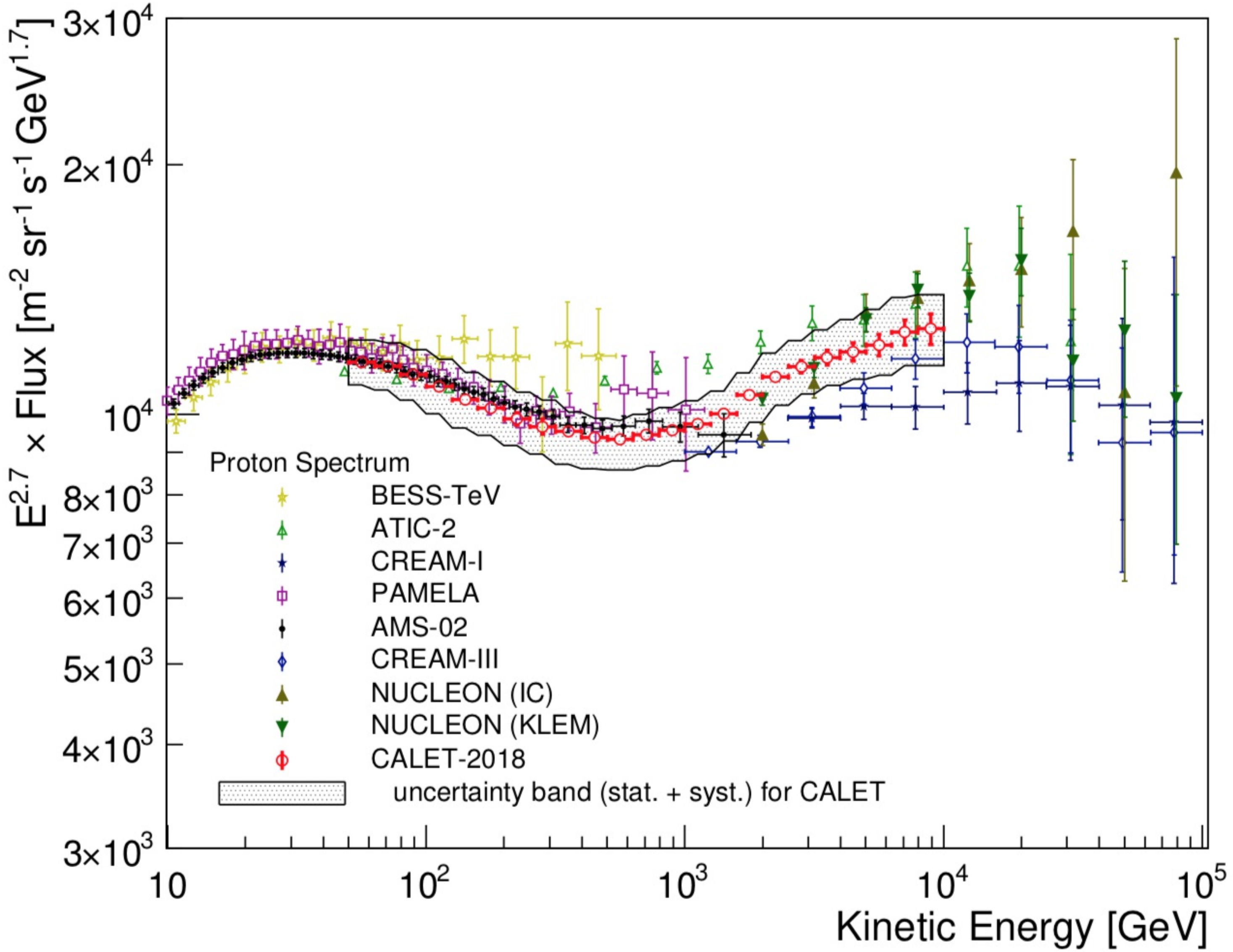
DAMPE (2019)   [Clear detection of a softening]

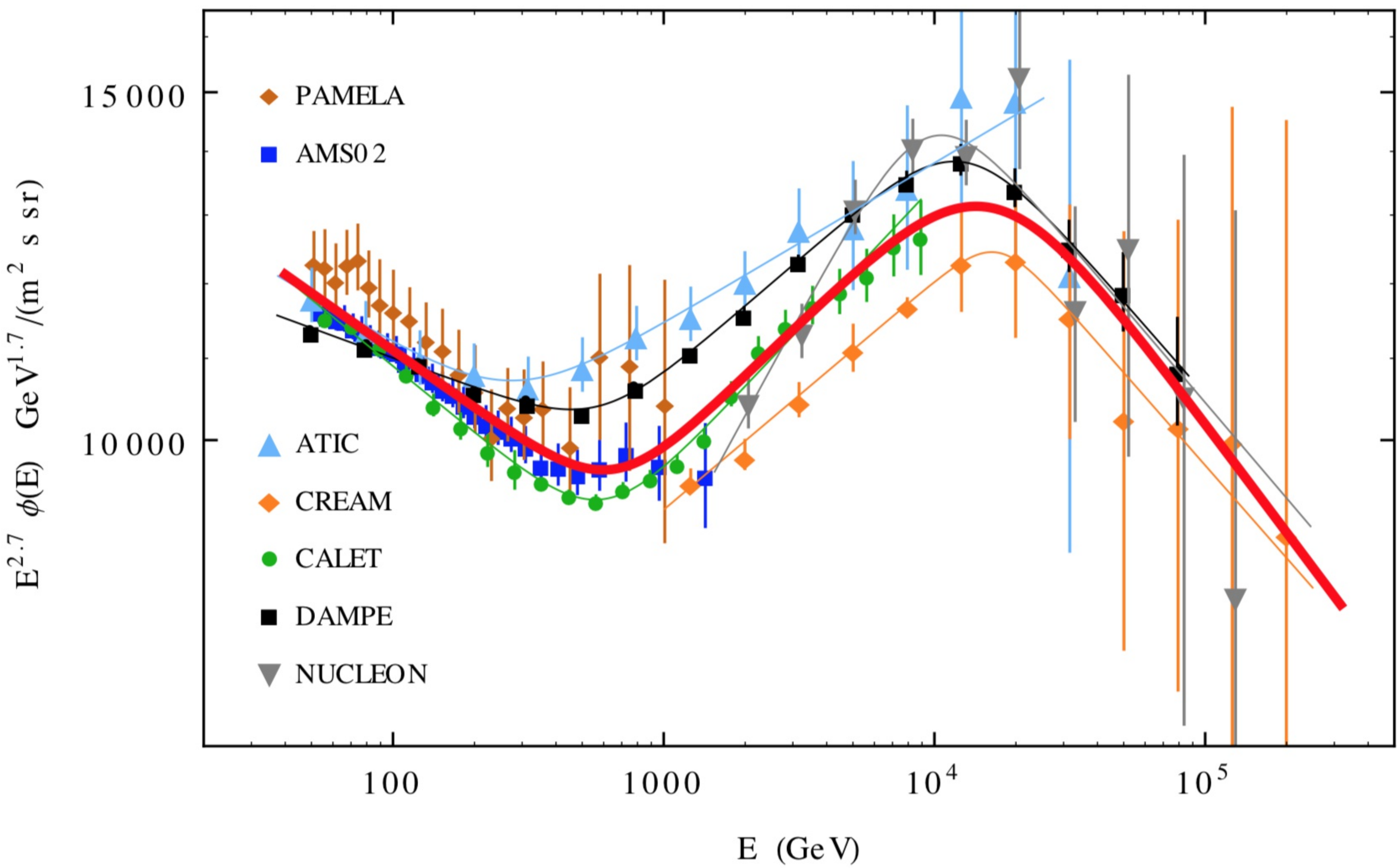


AMS02 (2015) $2.849^{+0.006}_{-0.005}$ $2.716^{+0.037}_{-0.056}$



CALET (2019). Proton flux Description of the Hardening



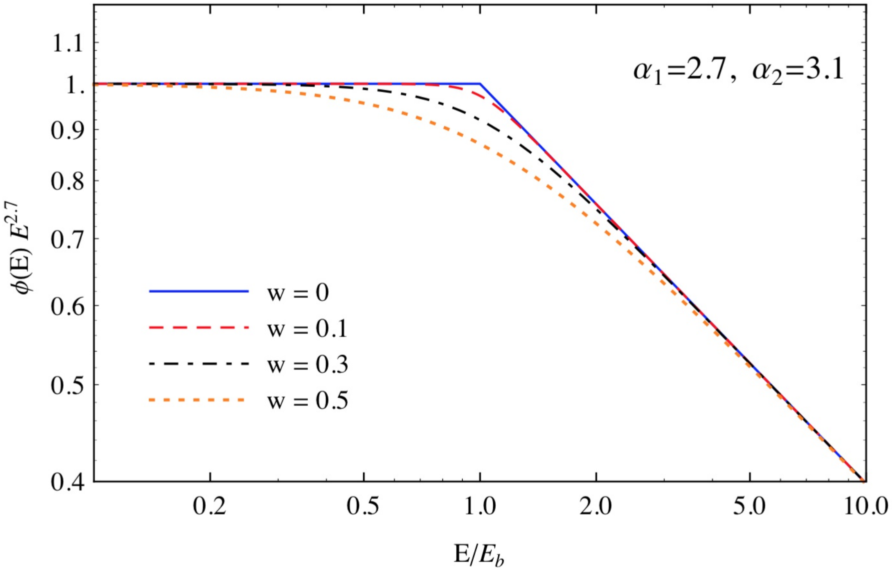


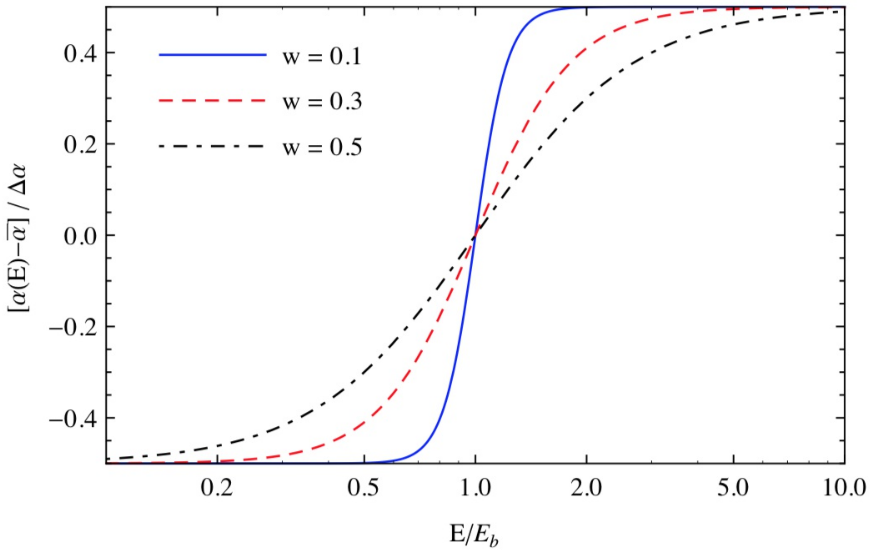
Global fit to to the data [PL + Silvia Vernetto astro-ph/1911.01311]

Functional form for one "spectral feature"

$$\phi(E) = K \left(\frac{E}{E_0}\right)^{-\alpha_1} \left[1 + \left(\frac{E}{E_b}\right)^{1/w}\right]^{-(\alpha_2-\alpha_1)\,w}$$

$$(\Delta \log_{10} E)_{\Delta\alpha/2} = (\log_{10} 9)\, w = 0.954\, w$$





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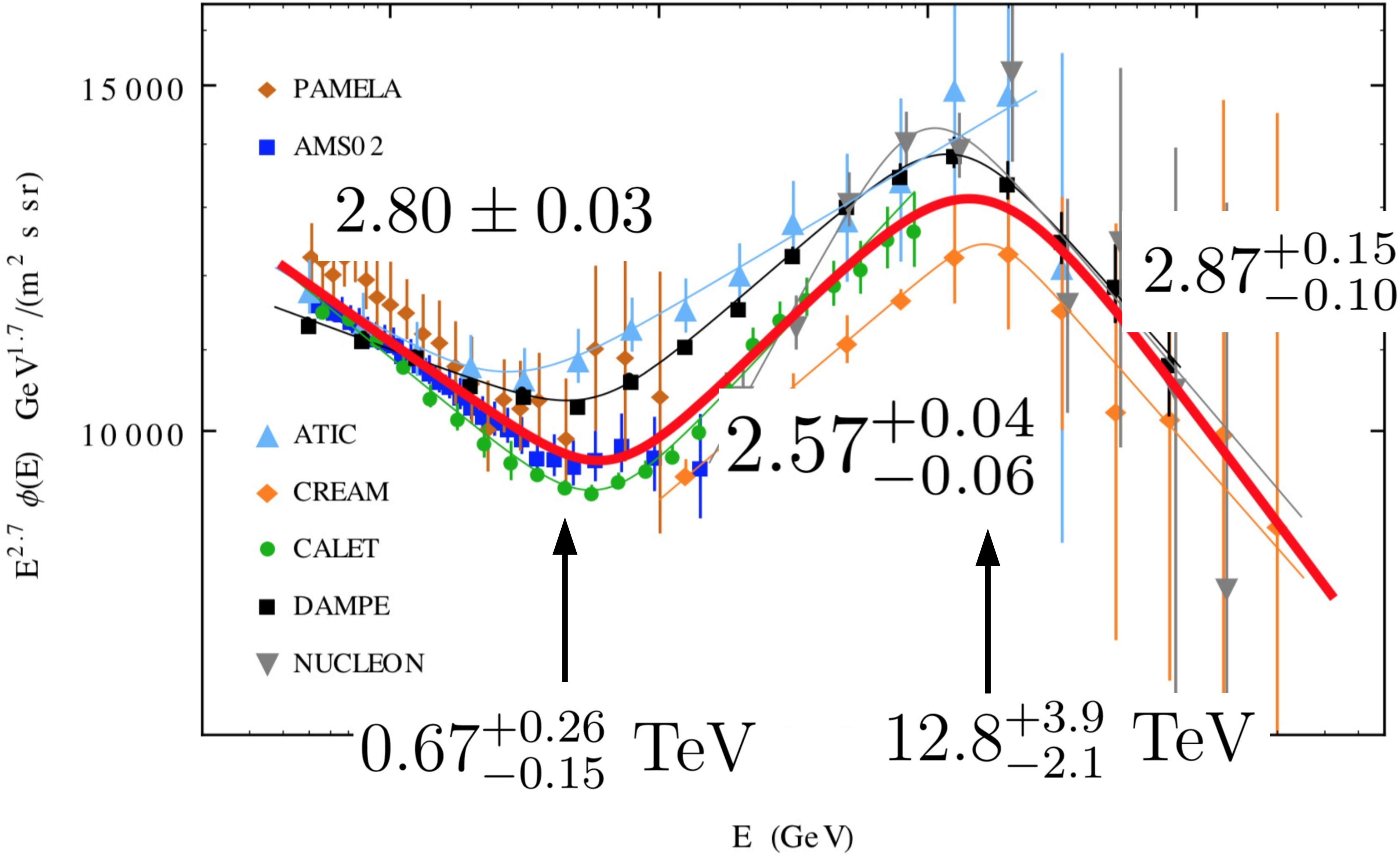
Two spectral features

$$\phi(E) = K \left(\frac{E}{E_0}\right)^{-\alpha_1} \left[1 + \left(\frac{E}{E_b}\right)^{1/w}\right]^{-(\alpha_2-\alpha_1)\,w} \left[1 + \left(\frac{E}{E_b'}\right)^{1/w'}\right]^{-(\alpha_3-\alpha_2)\,w'}$$

	PAMELA	AMS02	ATIC	CREAM	CALET	DAMPE	NUCLEON	All Data
α_1	2.850 ± 0.043	$2.849^{+0.006}_{-0.005}$	$2.79^{+0.20}_{-0.04}$	–	2.81 ± 0.01	2.750 ± 0.005	–	2.80 ± 0.03
α_2	2.67 ± 0.06	$2.716^{+0.037}_{-0.056}$	$2.62^{+0.02}_{-0.05}$	2.58 ± 0.01	2.55 ± 0.01	2.58 ± 0.01	$2.44^{+0.5}_{-0.3}$	$2.57^{+0.04}_{-0.06}$
α_3	–	–	–	$2.84^{+0.07}_{-0.04}$	–	$2.86^{+0.07}_{-0.04}$	$2.86^{+0.5}_{-0.1}$	$2.87^{+0.15}_{-0.10}$
E_b	$0.232^{+0.035}_{-0.030}$	$0.336^{+0.095}_{-0.052}$	$0.275^{+0.140}_{-0.180}$	–	$0.605^{+0.060}_{-0.050}$	$0.574^{+0.044}_{-0.037}$	–	$0.67^{+0.26}_{-0.15}$
w	$0^{+0.25}_{-0}$	$0.04^{+0.21}_{-0.04}$	$0.34^{+0.50}_{-0.34}$	–	0.25 ± 0.05	$0.35^{+0.14}_{-0.31}$	–	0.27 ± 0.19
E'_b	–	–	–	16^{+5}_{-4}	–	$12.8^{+3.9}_{-2.1}$	$9.5^{+8.0}_{-2.0}$	16^{+13}_{-8}
w'	–	–	–	$0^{+0.5}_{-0}$	–	$0.37^{+0.22}_{-0.032}$	$0.24^{+0.7}_{-0.2}$	$0.35^{+0.4}_{-0.3}$
$K_{(0.1\,\mathrm{TeV})}/10^{-2}$	4.60 ± 0.06	4.42 ± 0.02	$4.4^{+0.2}_{-0.4}$	–	4.34 ± 0.01	5.55 ± 0.09	–	4.40 ± 0.02
$K_{(2\,\mathrm{TeV})}/10^{-5}$	–	–	–	1.22 ± 0.01	$0.952^{+0.013}_{-0.015}$	1.15 ± 0.02	1.26 ± 0.04	1.02 ± 0.02
$\chi^2_{\min}$	1.5	3.3	1.5	1.4	17.3	6.4	3.4	46.5
N_{dof}	$16 - 5$	$31 - 5$	$15 - 5$	$12 - 5$	$23 - 5$	$17 - 8$	$11 - 5$	$124 - 8$



Global fit to to the data [PL + Silvia Vernetto astro-ph/1911.01311]

Natural questions:

What is the origin of this spectral structures ?

What are we learning about
the CR sources/propagation ?

Spectra of other nuclear components (He, C, O, ...)
[“Rigidity dependent structures ?]

How should one extrapolate the spectrum
to higher energy ?

Natural questions:

What is the origin of this spectral structures ?

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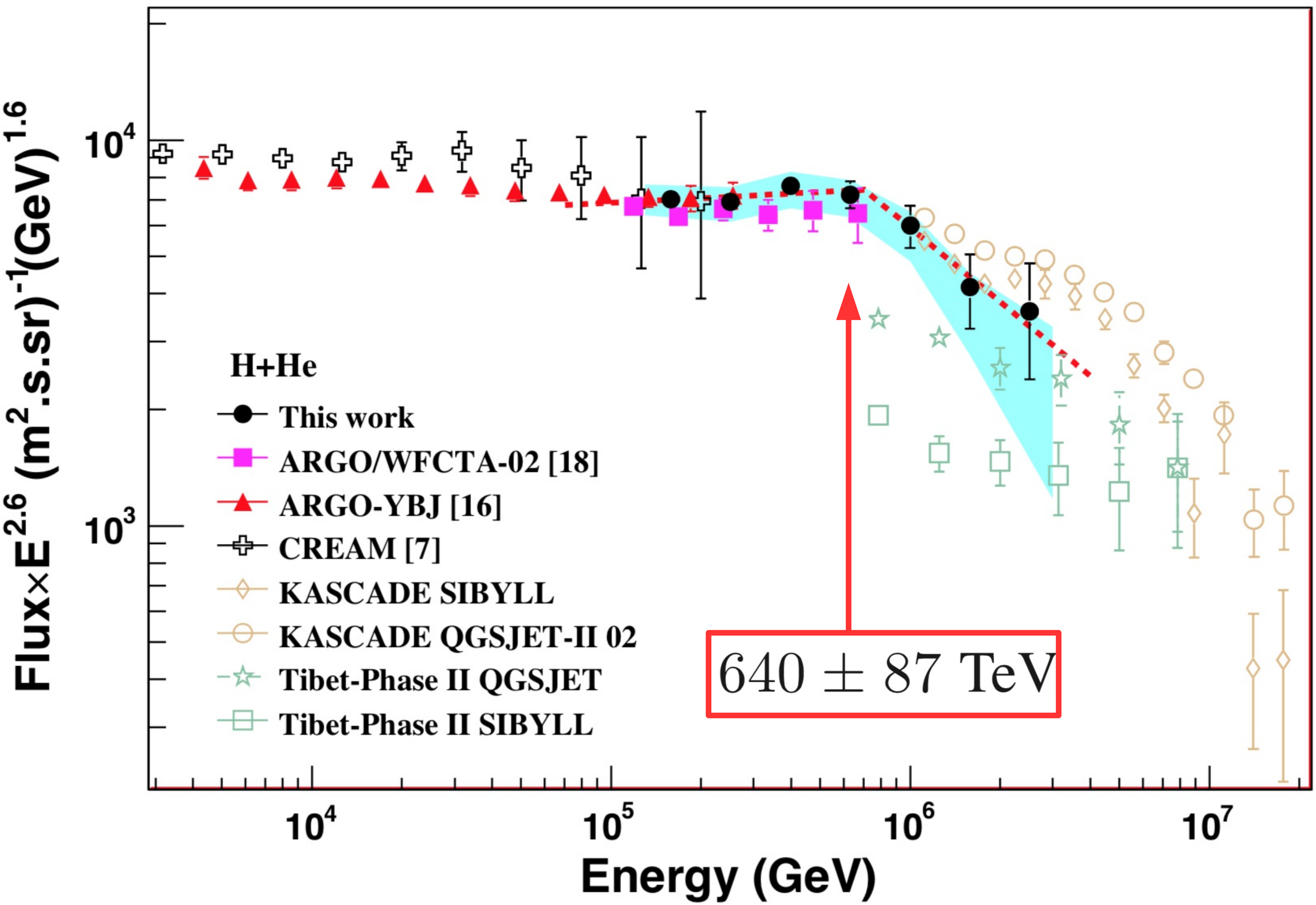
Spectra of other nuclear components (He, C, O, ...)
[“Rigidity dependent structures ?]

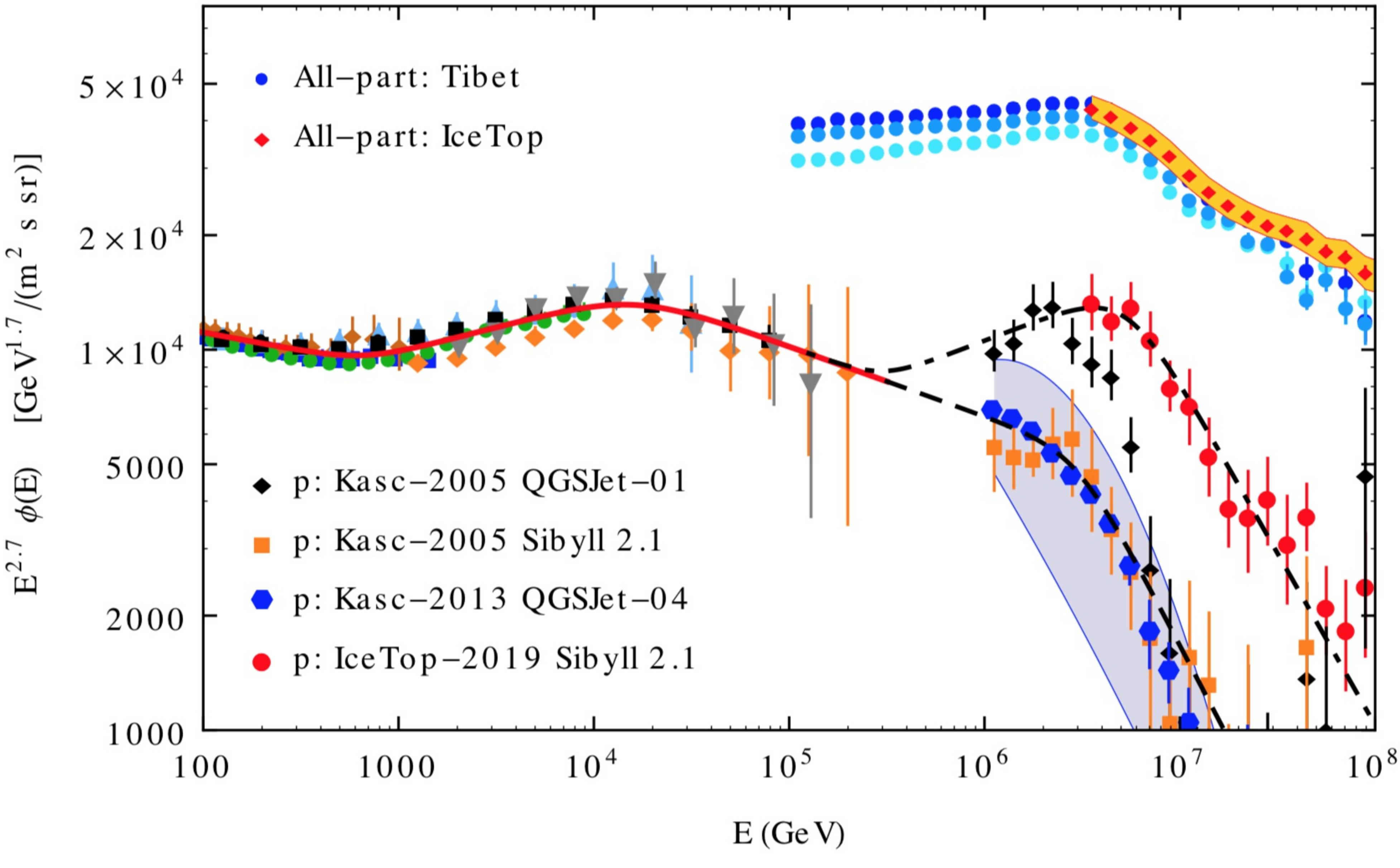
How should one extrapolate the spectrum to higher energy ?

LHAASO can have crucial role for this problem

ARGO-YBJ “light” (proton + helium) flux



Proton Measurements in the Knee region

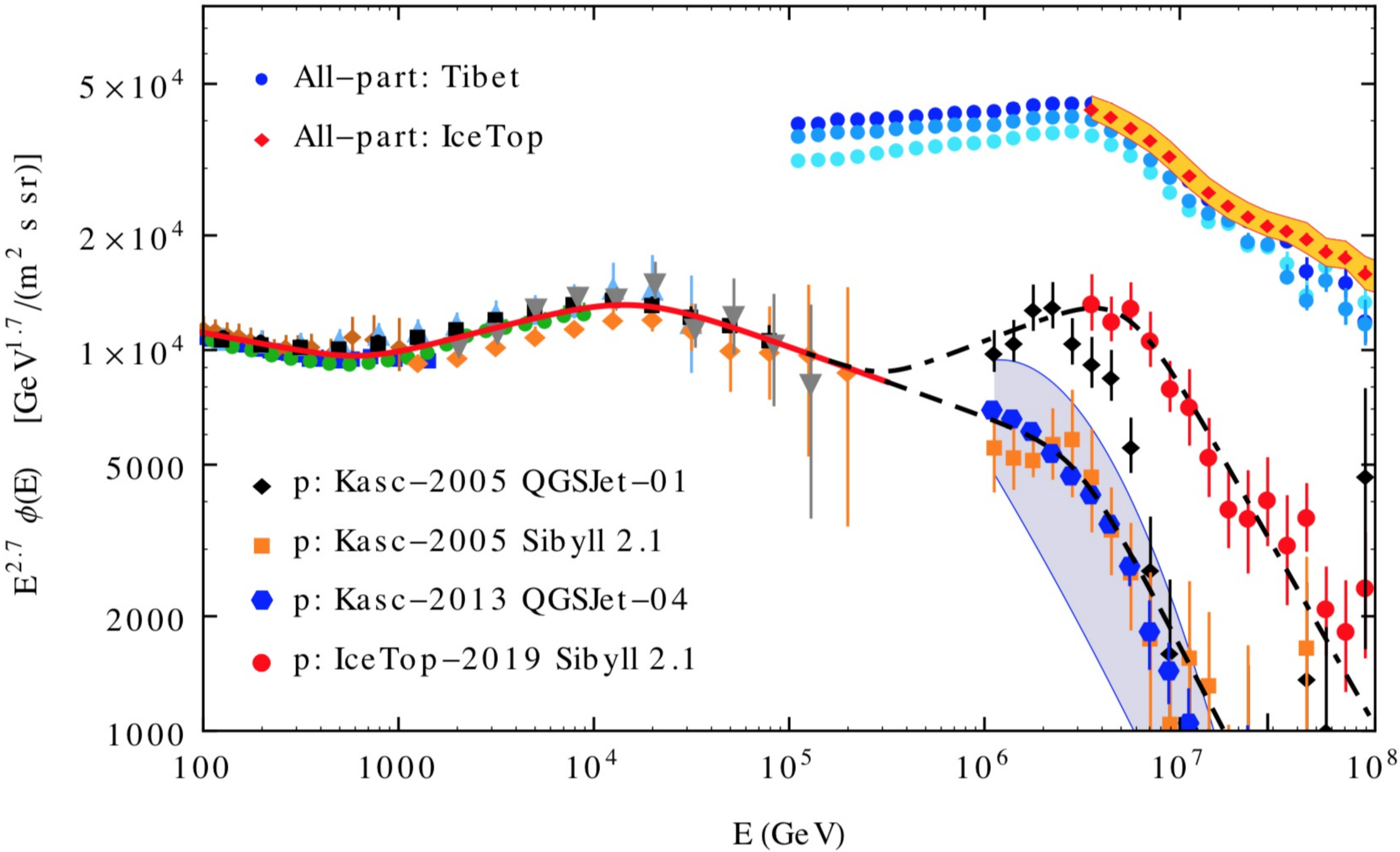


[KASCADE Collaboration]
"KASCADE measurements of energy spectra for elemental groups of cosmic rays: Results and open problems," Astropart. Phys. **24**, 1 (2005) [astro-ph/0505413].

[KASCADE-Grande Collaboration]
"KASCADE-Grande measurements of energy spectra for elemental groups of cosmic rays," Astropart. Phys. **47**, 54 (2013) [arXiv:1306.6283 [astro-ph.HE]].

[IceCube Collaboration],
"Cosmic Ray Spectrum and Composition from PeV to EeV Using 3 Years of Data From IceTop and IceCube," arXiv:1906.04317 [astro-ph.HE].

Proton Measurements in the Knee region



Is there additional "structure" in the proton spectrum ?

Indirect Measurements of CR spectra + composition

- Detection methods + Algorithms

- *Modeling of Hadronic Interactions*

The high quality of the LHAASO observations require (and will stimulate and guide) the development of an understanding of

non-perturbative QCD

the *“Dark side of the Standard Model”*

How can one explain "multi-feature" CR spectra

Literature on the "Pamela Hardening" (1 feature)

$$\phi(E) \propto E^{-\gamma} \propto E^{-(\alpha+\delta)}$$

1. Modify Propagation (critical rigidity) $\delta_1 \quad \delta_2$
2. Two classes of sources $\alpha_1 \quad \alpha_2$

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1. Modify Propagation (critical rigidity) $\delta_1 \quad \delta_2$

2. Two classes of sources $\alpha_1 \quad \alpha_2$

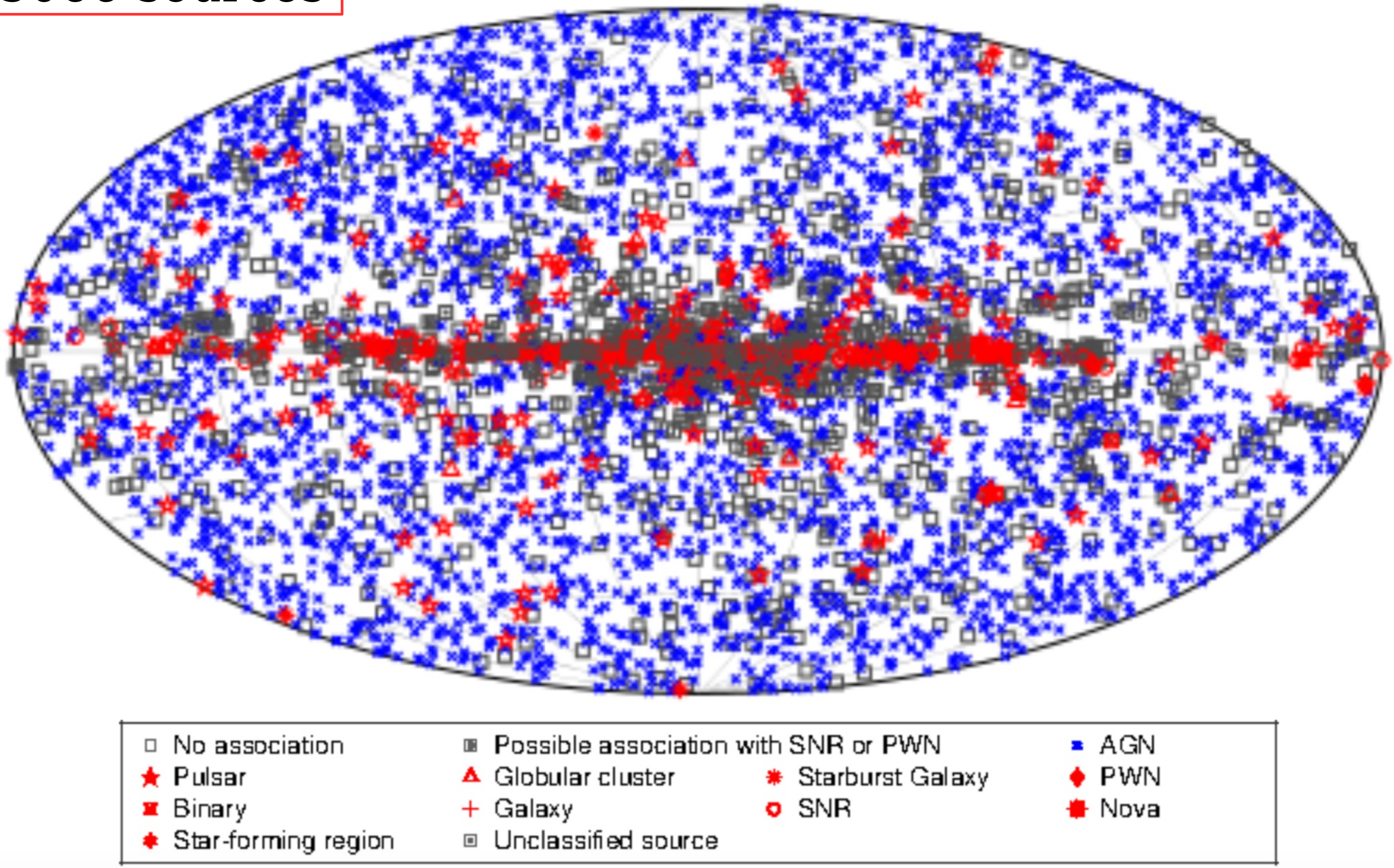
All these models predicted an unbroken spectrum up to the "Knee"
and have to be revised after the discovery of the 10 TeV softening.
Generalizations to 2, 3, features possible but perhaps to *"contrived"*

Well known idea:

Identify the CR sources, and determine (using gamma-ray observations) the spectral shape of freshly accelerated particles inside (or near) these sources

FERMI-LAT Fourth General Catalog (4FGL) '

5066 sources



The 4FGL catalog gives a "best fit" spectrum for all sources. In one of three functional forms:

Power-Law (3543 sources)

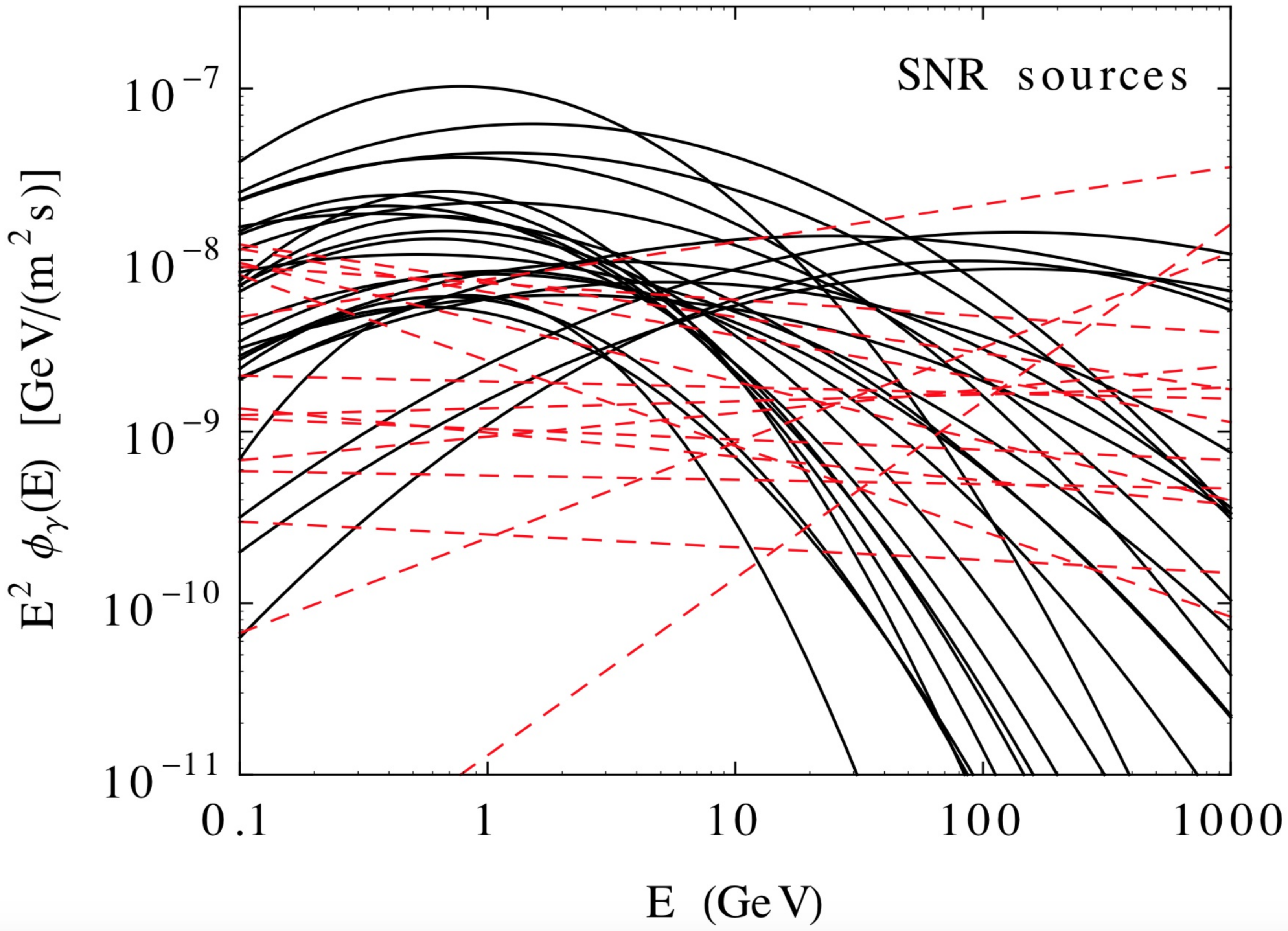
$$\phi_\gamma(E) = \phi_0 \left(\frac{E}{E_0}\right)^{-\alpha}$$

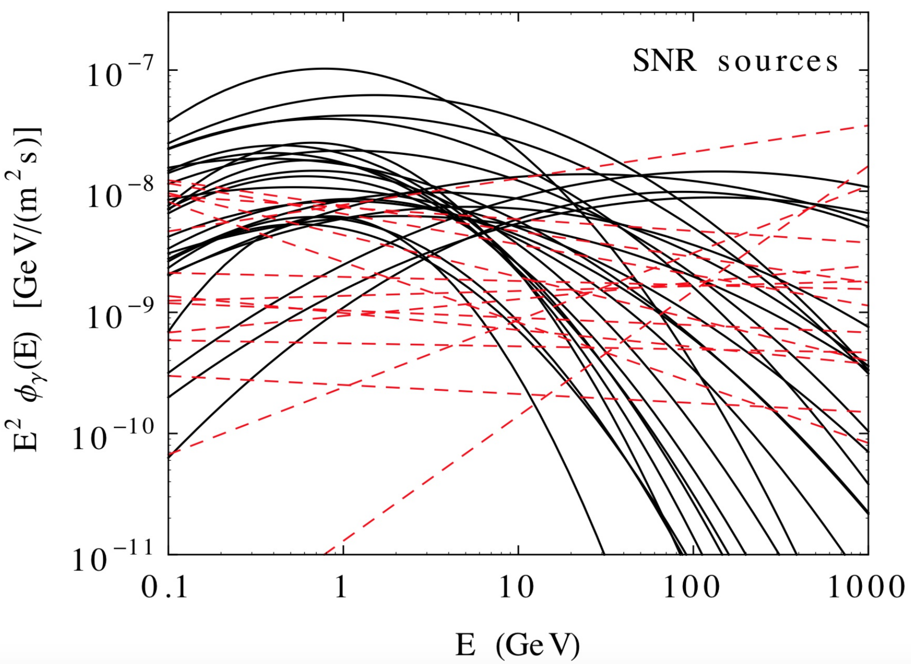
Log-Parabola (1303 sources)

$$\phi(E) = \phi_0 \left(\frac{E}{E_0}\right)^{-(\alpha_0+\beta \ \ln E/E_0)}$$

"Cutoff" (218 Pulsars, LMC, 3C 454.3)

40 Supernova sources in the 4FGL
25 Log-parabola fits [90.1 % of the flux in the 1-100 GeV range]
15 Power-law fits



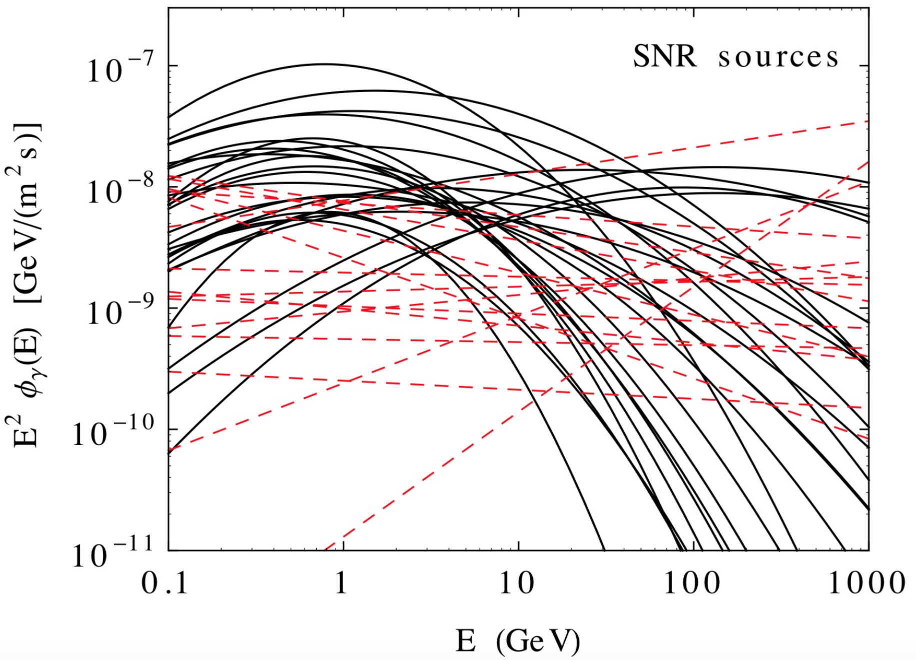


Very broad range of spectral shapes !

Is this consistent with the “standard picture” for Galactic CR acceleration ?

(a) Acceleration in SNR

(b) Power-law spectrum with *unique spectral index*



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Is this consistent with the "standard picture" for Galactic CR acceleration ?

(a) Acceleration in SNR

(b) Power-law spectrum with *unique spectral index*

May be YES.

1. Different ages
2. Different environments

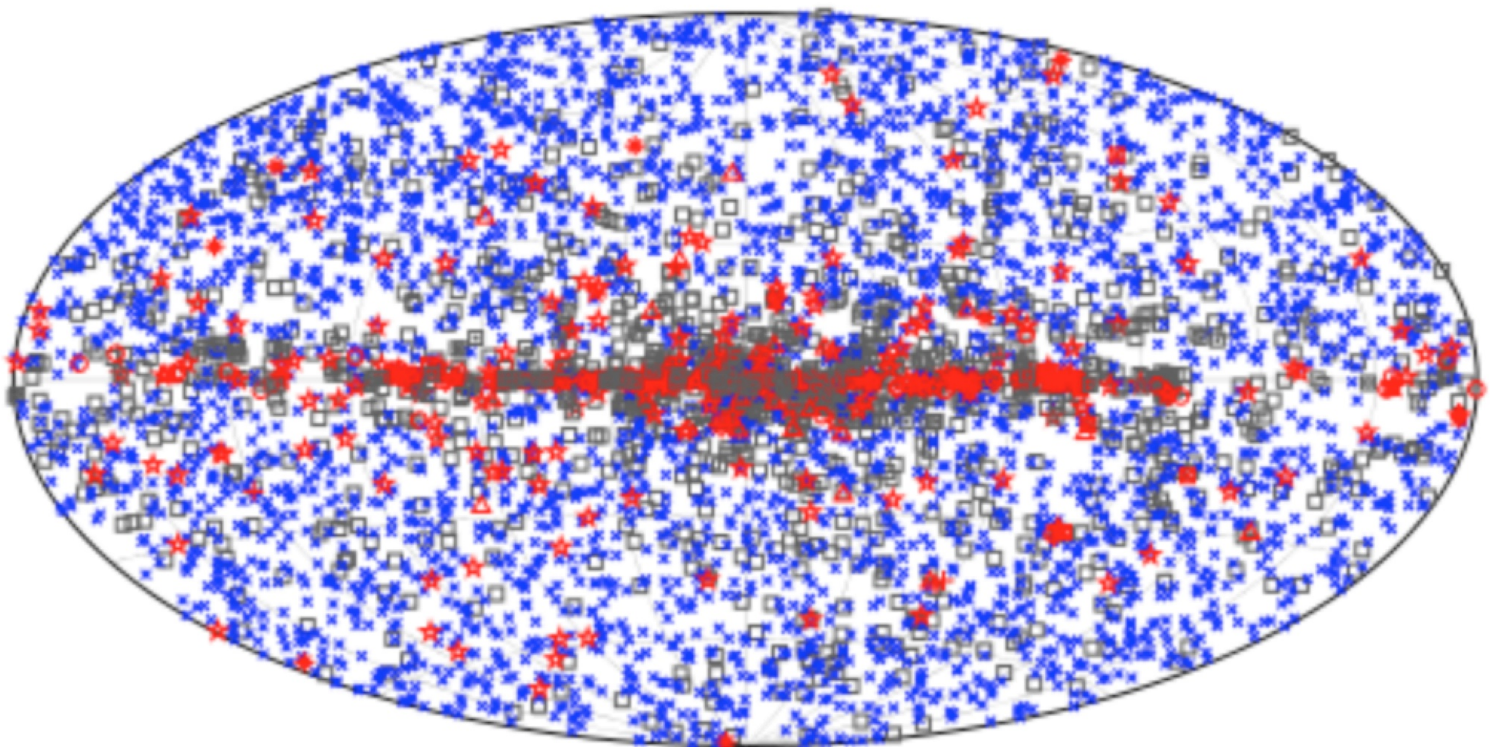
May be NO...

Can the sum of "curved" spectra combine to form a power-law spectrum ?

Can the sum of “curved” spectra
combine to form a power-law spectrum ?

Yes !

It happens for the
the emission of the ensemble of blazars
(that dominate the extragalactic gamma-ray flux).

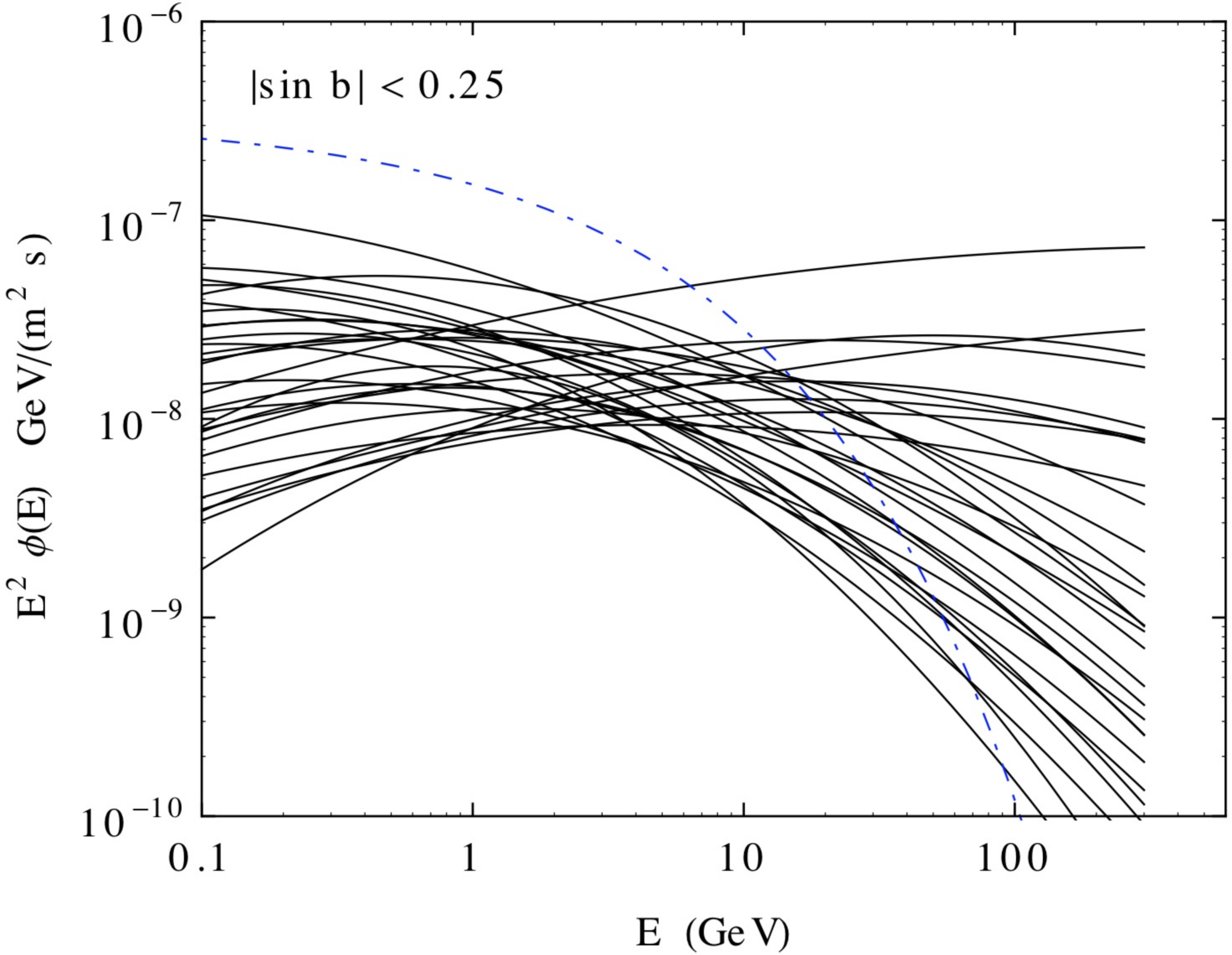


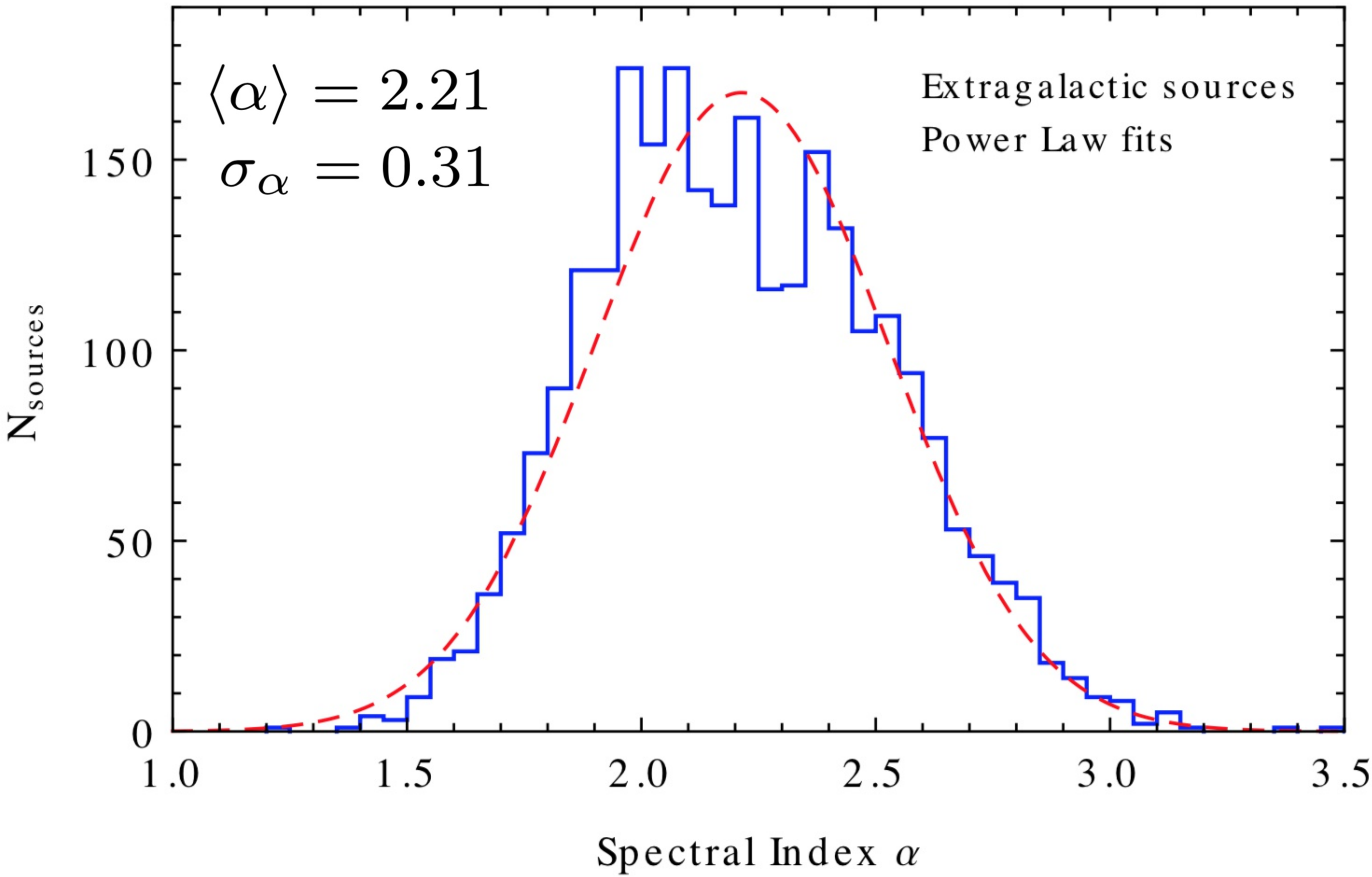
Select sources in the Galactic Polar regions $|\sin b| > 0.25$
Exclude Pulsars:

3223 sources

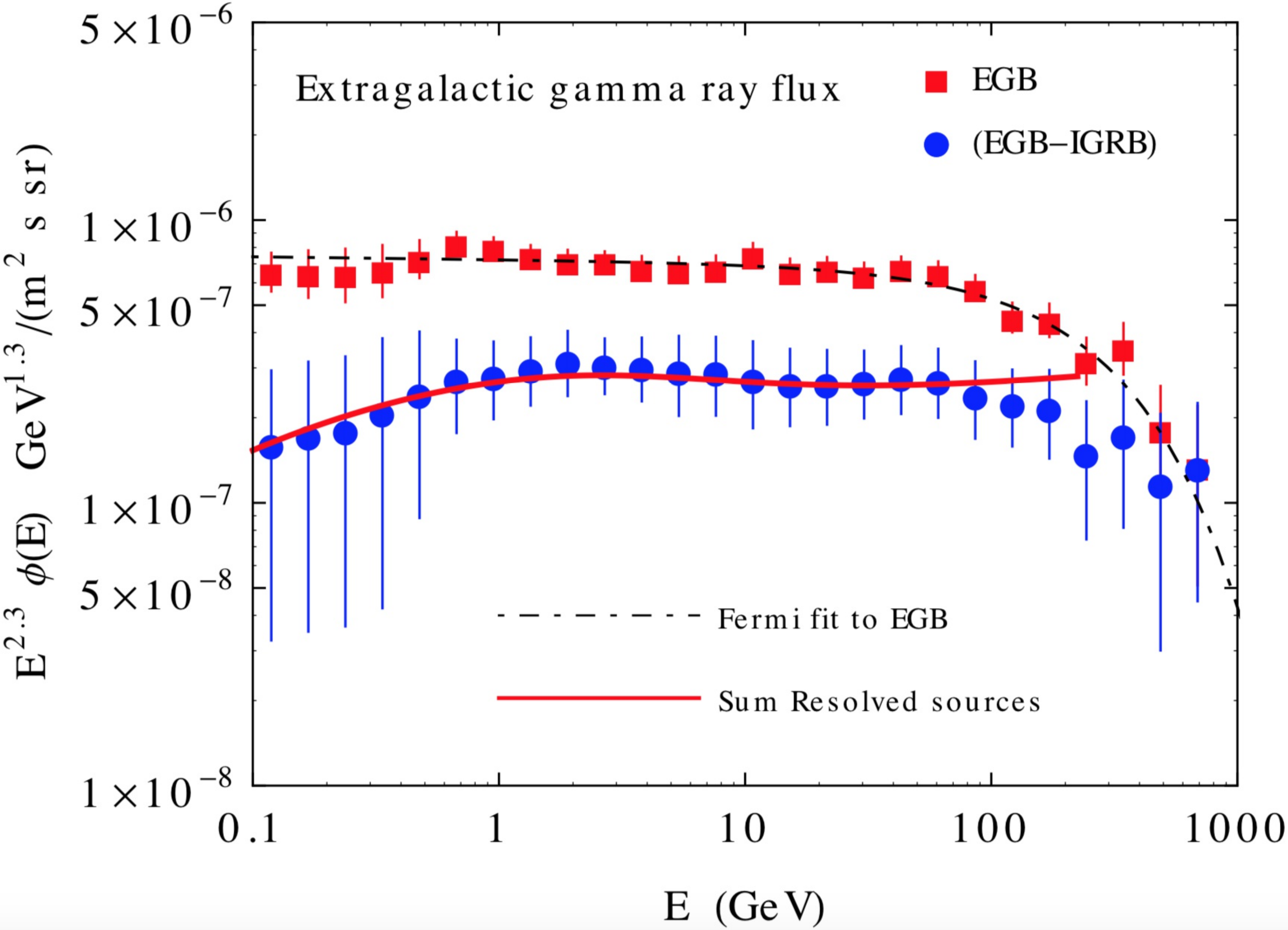
1	"cutoff"	3C 454.3
2629	power-law	
543	log-parabola	(60% of the 1-100 GeV flux)

30 brightest extragalactic sources
[3C454.3 + 29 others all log-parabola]





$\langle\alpha\rangle = 2.21$
$\sigma_\alpha = 0.31$
Extragalactic sources
Power Law fits
N_{sources}
0
50
100
150
1.0
1.5
2.0
2.5
3.0
3.5
Spectral Index α



Extragalactic gamma ray flux
EGB
(EGB−IGRB)
Fermi fit to EGB
Sum Resolved sources
5×10^{-6}
1×10^{-6}
5×10^{-7}
1×10^{-7}
5×10^{-8}
1×10^{-8}
0.1
1
10
100
1000
E (GeV)
$E^{2.3}\ \phi(E)\ \ GeV^{1.3}/(m^2\ s\ sr)$

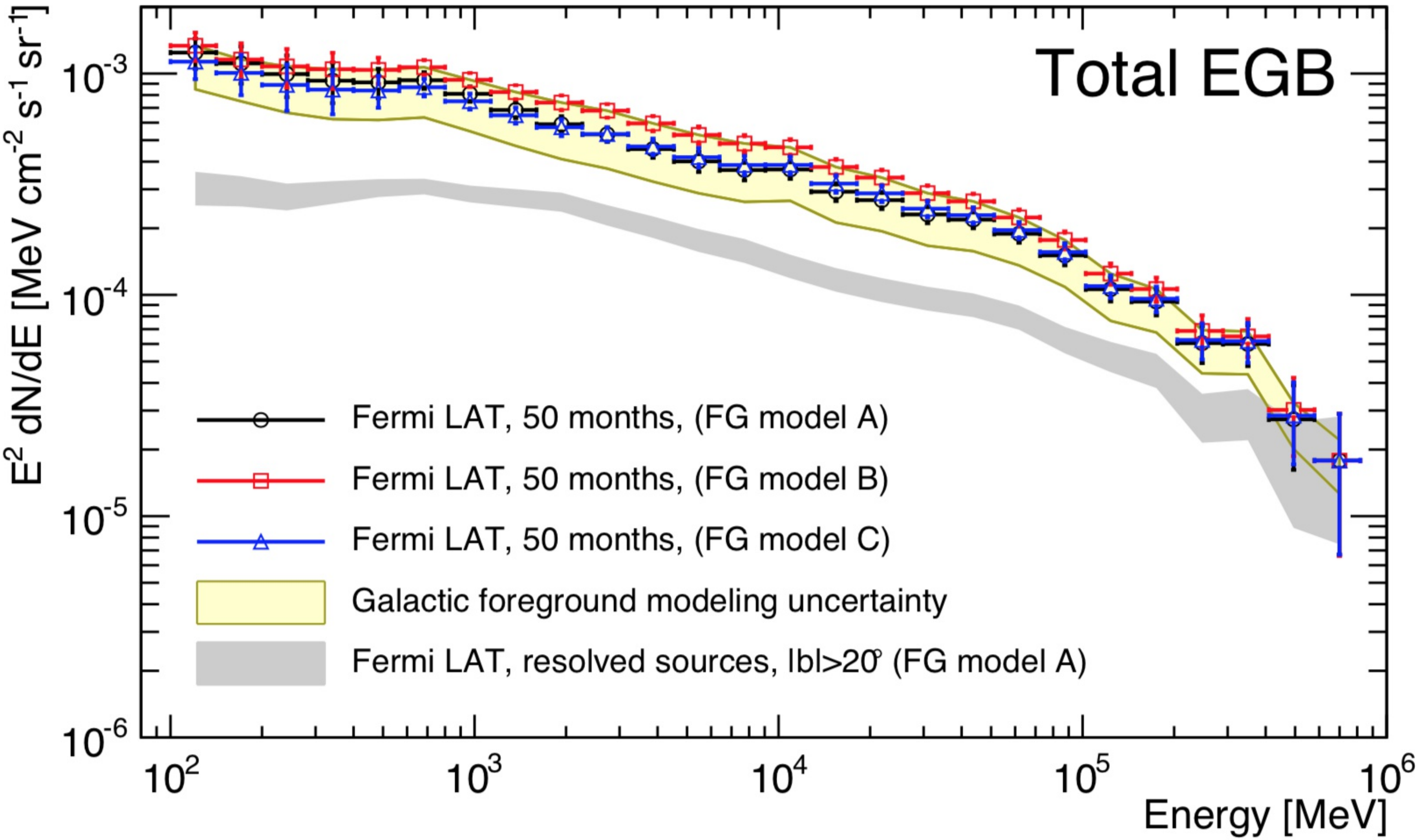


Fig. 8.— Comparison of the total EGB intensities for different foreground models. The total EGB intensity is obtained by summing the IGRB intensity and the cumulative intensity from resolved *Fermi* LAT sources at latitudes $|b| > 20°$ (gray band). See Figure 7 for legend.

Fermi-LAT fits to the extragalactic flux

$$\phi_\gamma(E) = K_\gamma \; E^{-\alpha} \; e^{-E/E_{\rm cut}}$$

Total Extragalactic Gamma Background (EGB)

$$\alpha = 2.30 \pm 0.02$$

$$E_{\rm cut} = 330 \pm 70 \text{ GeV}$$

Isotropic Gamma Ray Background (IGRB)

$$\alpha = 2.29 \pm 0.02$$

$$E_{\rm cut} = 239 \pm 50 \text{ GeV}$$

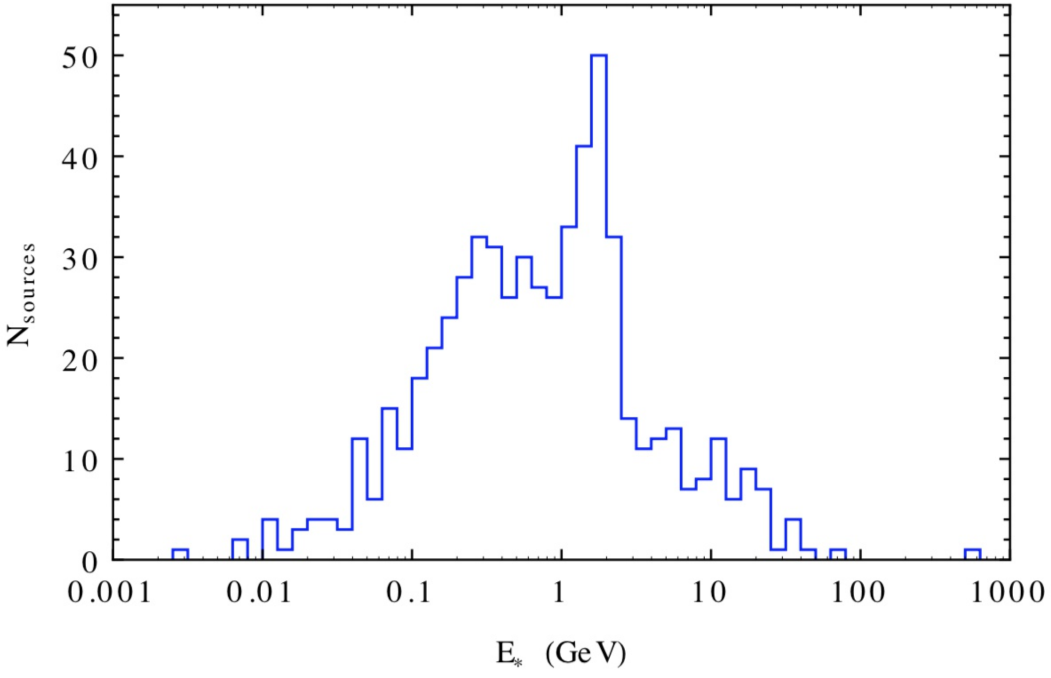
Slope of a Log-Parabola spectrum:

$$\alpha(E) = -\frac{d \log \phi(E)}{d \log E} = \alpha_0 + 2\beta \ \ln \frac{E}{E_0}$$

Takes all values from $-\infty$ to $+\infty$

Characteristic energy E_* $\alpha(E_*) = 2$

[maximum of the SED (spectral index distribution)]



Broad range of values
From 10 MeV
To 100 GeV

It is straightforward to show that:

$$\frac{dQ_\gamma}{dE_*} \propto E_*^{-p} \quad \Longleftrightarrow \quad \phi_\gamma(E) \propto E^{-p}$$

Ensemble of sources
that generate spectra
with characteristic energy E*
depends as a power law on E*

[“hard” sources
are less frequent
that “soft” sources]

$\Longleftrightarrow$

The average spectrum is
a power law
with the same slope

It is straightforward to show that:

$$\frac{dQ_\gamma}{dE_*} \propto E_*^{-p} \iff \phi_\gamma(E) \propto E^{-p}$$

Ensemble of sources
that generate spectra
with characteristic energy E*
depends as a power law on E*

[“hard” sources
are less frequent
that “soft” sources]

$\iff$

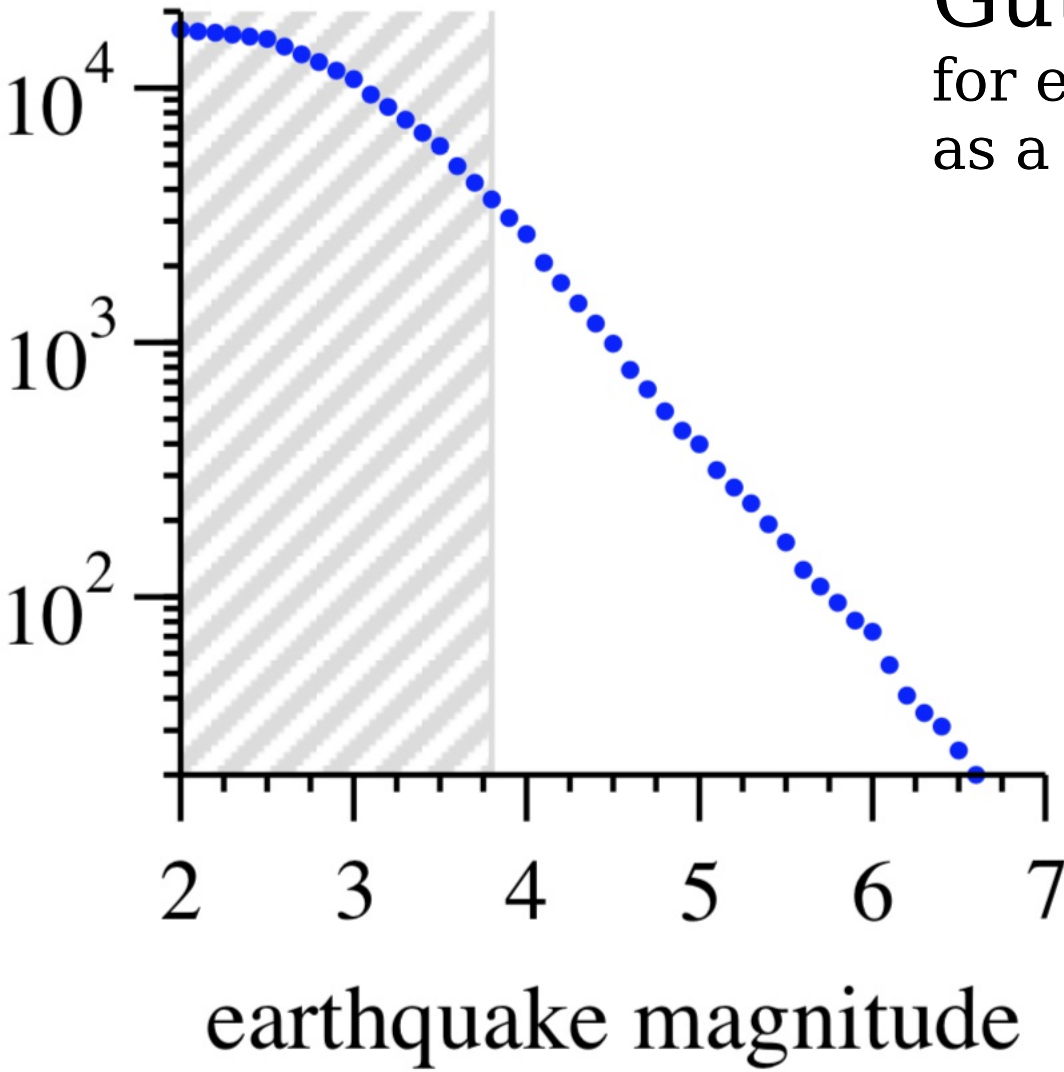
The average spectrum is
a power law
with the same slope

The exponent 2.30 of the extragalactic gamma-ray emission emerges from the “statistical properties” of the ensemble of the blazar flares

Power-law distributions

appear widely in a very broad range of fields:
physics, biology, earth and planetary science
economics and finances, social sciences,
.....

The origin of power-law behavior has been
a topic of debate for more than a century



Gutenberg-Richter law

for earthquake frequency
as a function of magnitude

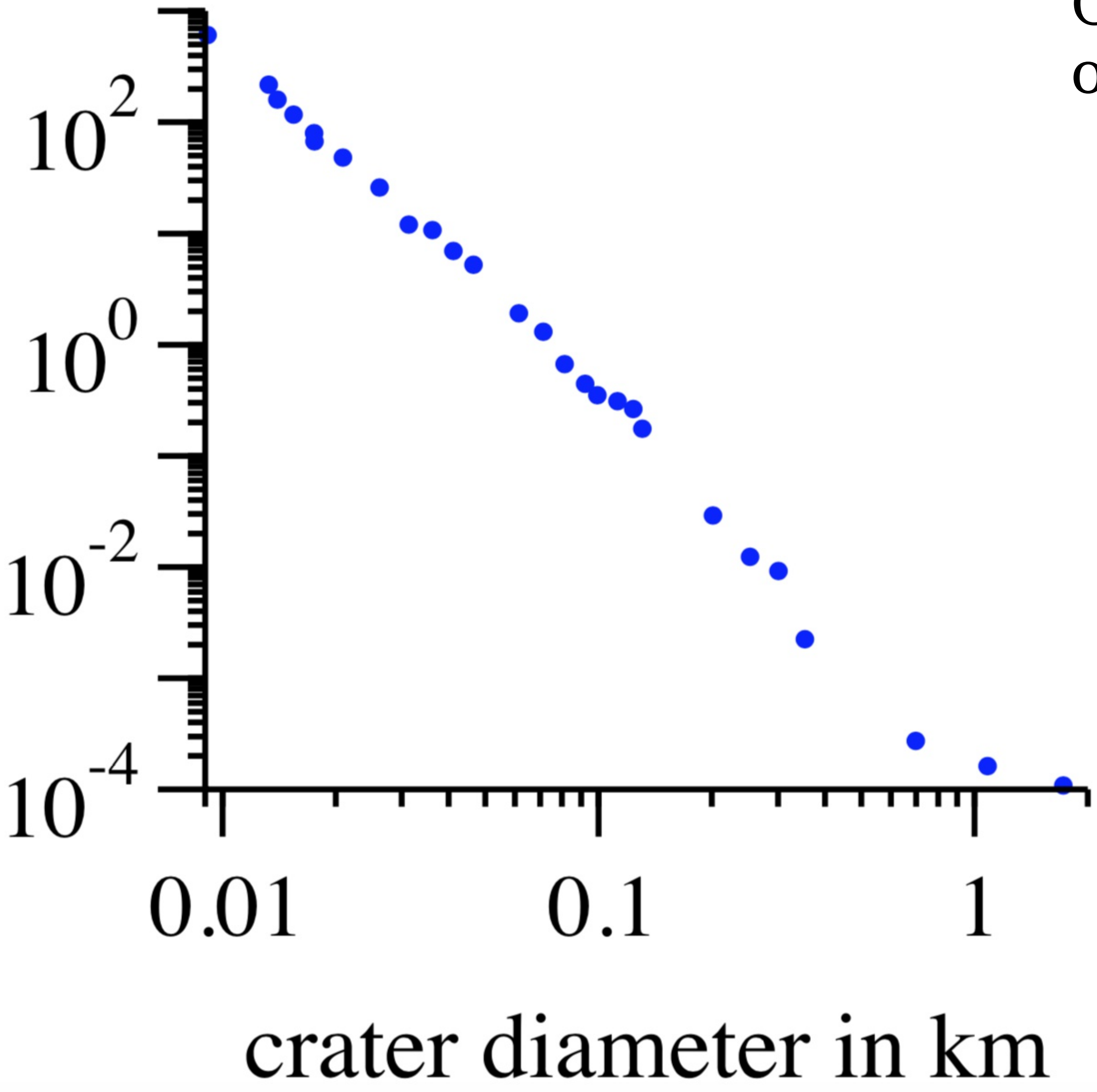
$$\log_{10} N = a - b\, m$$

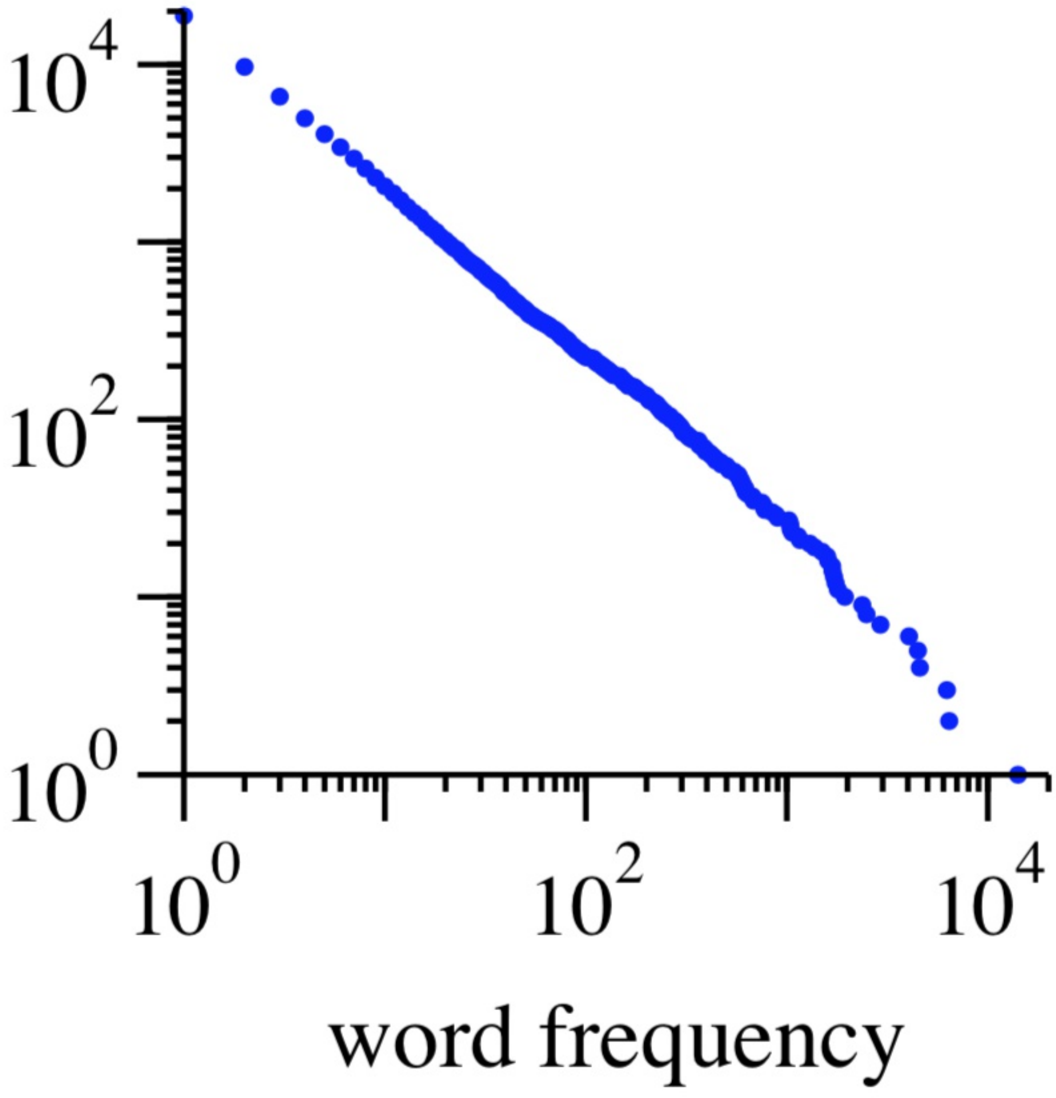
$$dN/d\mathcal{E} \propto \mathcal{E}^{-(\bar{b}+1)}$$

Earthquakes in California
1910-1992

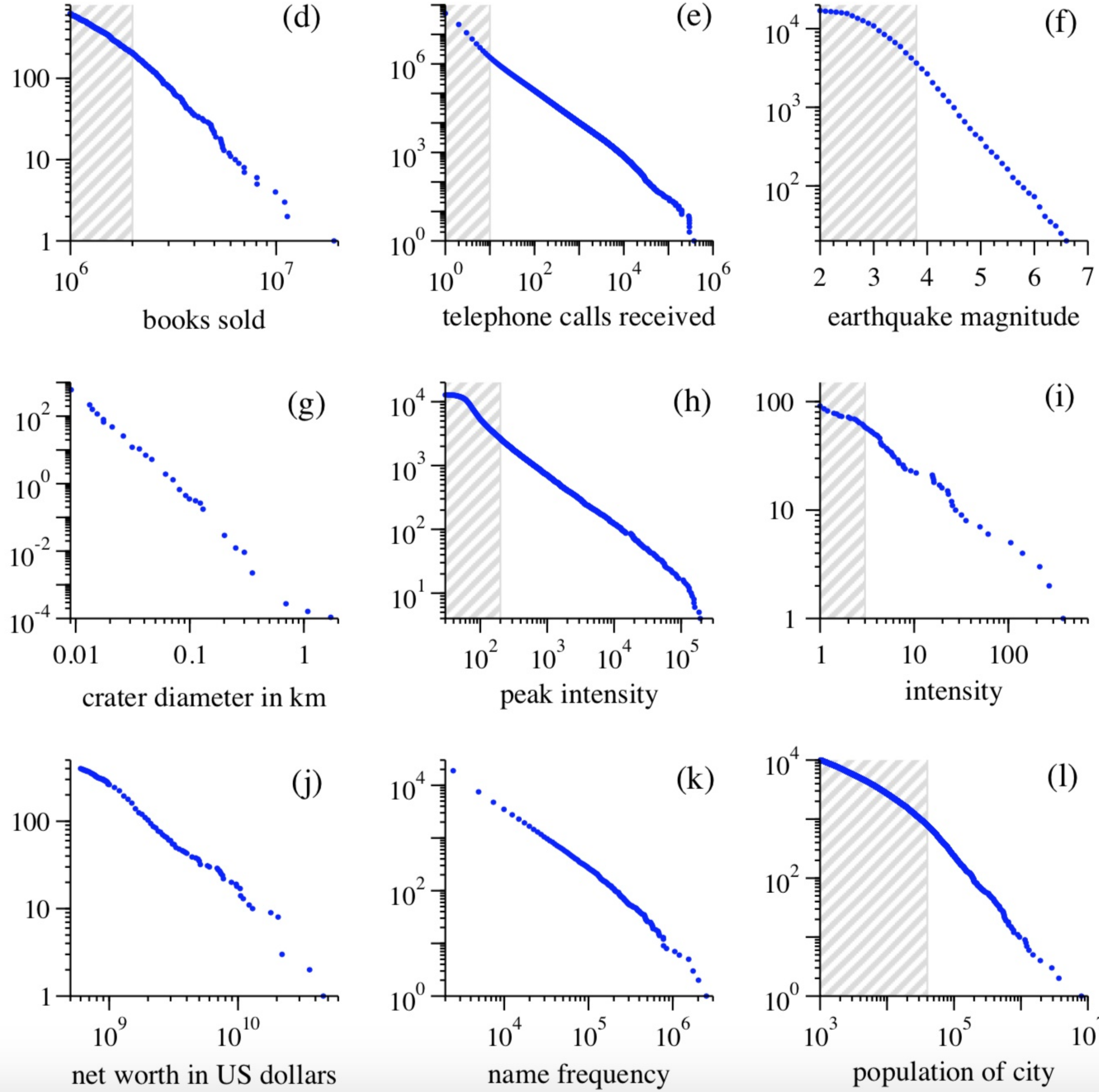
M.E.J. Newman
"Power–laws, Pareto distributions and Zipf's law"
Contemporary Physics **46**, 323 (2005)
arXiv:cond-mat/0412004 [cond-mat.stat-mech]

Cumulative distributions of Moon craters





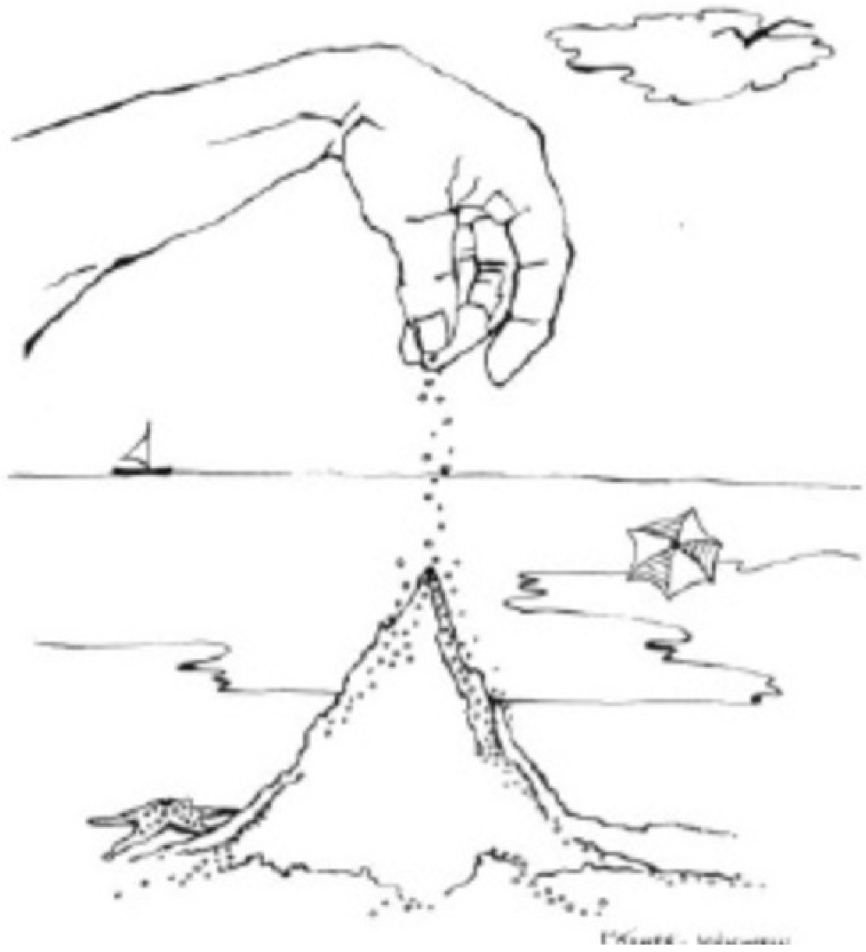
Frequency
of unique words
in Moby Dick
(Heman Melville)



(d)
100
10
1
10^6
10^7
books sold
(e)
10^6
10^3
10^0
10^0
10^2
10^4
10^6
telephone calls received
(f)
10^4
10^3
10^2
2
3
4
5
6
7
earthquake magnitude
(g)
10^2
10^0
10^{-2}
10^{-4}
0.01
0.1
1
crater diameter in km
(h)
10^4
10^3
10^2
10^1
10^2
10^3
10^4
10^5
peak intensity
(i)
100
10
1
1
10
100
intensity
(j)
100
10
1
10^9
10^{10}
net worth in US dollars
(k)
10^4
10^2
10^0
10^4
10^5
10^6
name frequency
(l)
10^4
10^2
10^0
10^3
10^5
10^7
population of city

quantity	n	$\langle x \rangle$	σ	$x_{\max}$	$\hat{x}_{\min}$	$\hat{\alpha}$	n_{tail}	p
count of word use	18 855	11.14	148.33	14 086	7 ± 2	1.95(2)	2958 ± 987	**0.49**
protein interaction degree	1846	2.34	3.05	56	5 ± 2	3.1(3)	204 ± 263	**0.31**
metabolic degree	1641	5.68	17.81	468	4 ± 1	2.8(1)	748 ± 136	0.00
Internet degree	22 688	5.63	37.83	2583	21 ± 9	2.12(9)	770 ± 1124	**0.29**
telephone calls received	51 360 423	3.88	179.09	375 746	120 ± 49	2.09(1)	$102\,592 \pm 210\,147$	**0.63**
intensity of wars	115	15.70	49.97	382	2.1 ± 3.5	1.7(2)	70 ± 14	**0.20**
terrorist attack severity	9101	4.35	31.58	2749	12 ± 4	2.4(2)	547 ± 1663	**0.68**
HTTP size (kilobytes)	226 386	7.36	57.94	10 971	36.25 ± 22.74	2.48(5)	6794 ± 2232	0.00
species per genus	509	5.59	6.94	56	4 ± 2	2.4(2)	233 ± 138	**0.10**
bird species sightings	591	3384.36	10 952.34	138 705	6679 ± 2463	2.1(2)	66 ± 41	**0.55**
blackouts ($\times 10^3$)	211	253.87	610.31	7500	230 ± 90	2.3(3)	59 ± 35	**0.62**
sales of books ($\times 10^3$)	633	1986.67	1396.60	19 077	2400 ± 430	3.7(3)	139 ± 115	**0.66**
population of cities ($\times 10^3$)	19 447	9.00	77.83	8 009	52.46 ± 11.88	2.37(8)	580 ± 177	**0.76**
email address books size	4581	12.45	21.49	333	57 ± 21	3.5(6)	196 ± 449	**0.16**
forest fire size (acres)	203 785	0.90	20.99	4121	6324 ± 3487	2.2(3)	521 ± 6801	0.05
solar flare intensity	12 773	689.41	6520.59	231 300	323 ± 89	1.79(2)	1711 ± 384	**1.00**
quake intensity ($\times 10^3$)	19 302	24.54	563.83	63 096	0.794 ± 80.198	1.64(4)	$11\,697 \pm 2159$	0.00
religious followers ($\times 10^6$)	103	27.36	136.64	1050	3.85 ± 1.60	1.8(1)	39 ± 26	**0.42**
freq. of surnames ($\times 10^3$)	2753	50.59	113.99	2502	111.92 ± 40.67	2.5(2)	239 ± 215	**0.20**
net worth (mil. USD)	400	2388.69	4 167.35	46 000	900 ± 364	2.3(1)	302 ± 77	0.00
citations to papers	415 229	16.17	44.02	8904	160 ± 35	3.16(6)	3455 ± 1859	**0.20**
papers authored	401 445	7.21	16.52	1416	133 ± 13	4.3(1)	988 ± 377	**0.90**
hits to web sites	119 724	9.83	392.52	129 641	2 ± 13	1.81(8)	$50\,981 \pm 16\,898$	0.00
links to web sites	241 428 853	9.15	106 871.65	1 199 466	3684 ± 151	2.336(9)	$28\,986 \pm 1560$	0.00

Concept of "Self Organized Criticality"

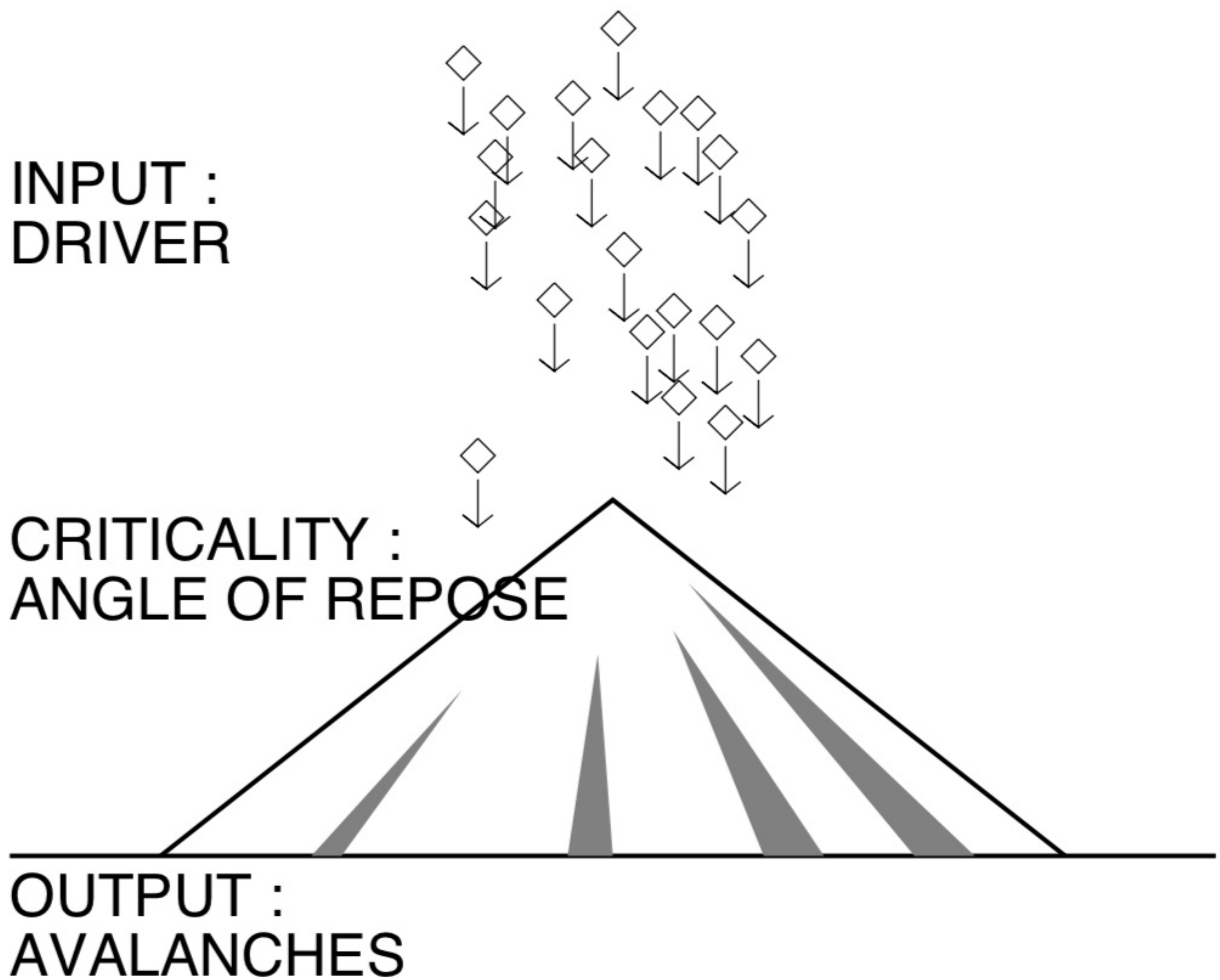


"Sand Pile"
model

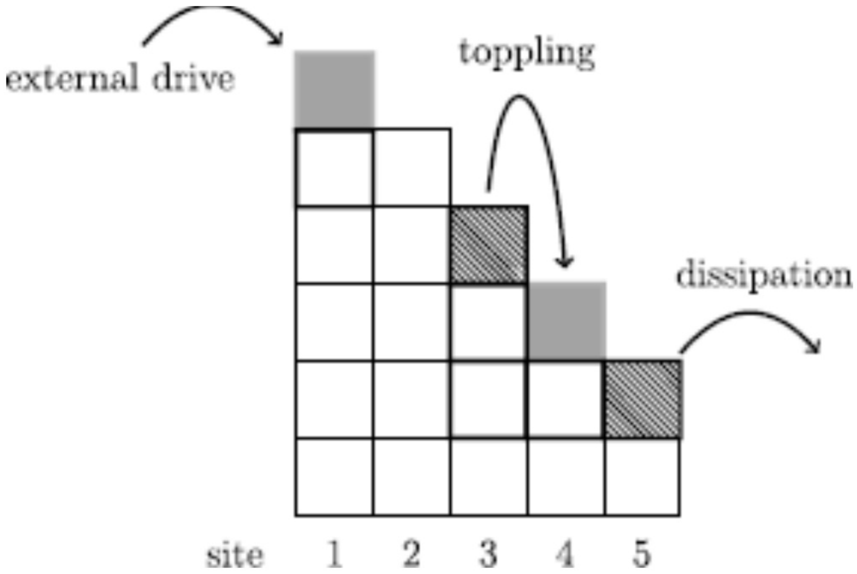
P. Bak, C. Tang and K. Wiesenfeld,
"Self-organized criticality: An Explanation of 1/f noise"
Phys. Rev. Lett. **59**, 381 (1987).

P. Bak, C. Tang and K. Wiesenfeld,
"Self-organized criticality"
Phys. Rev. A **38**, 364 (1988).

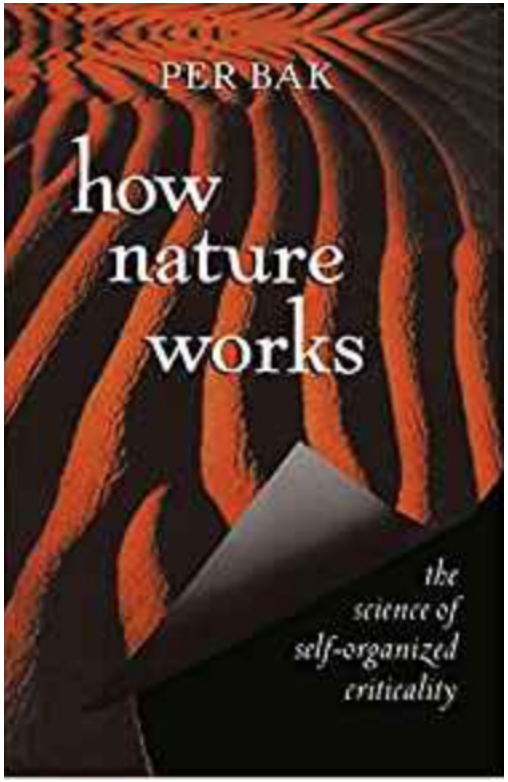
Sandpile SOC Paradigm



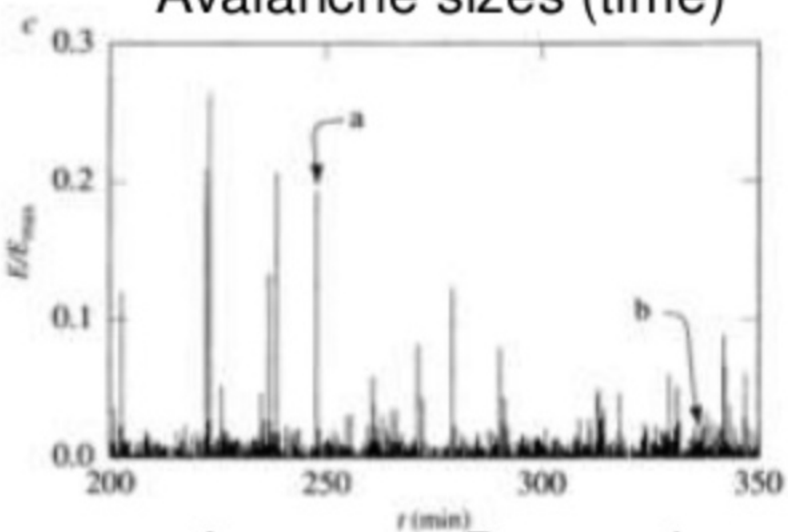
Cellular Automata
Model of a “sand pile"

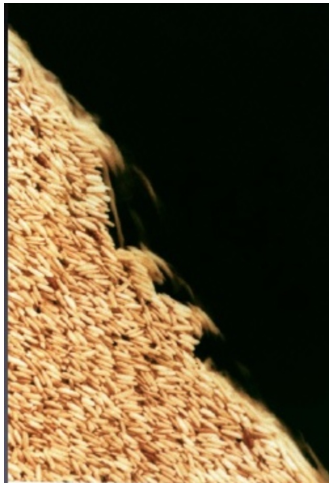


Per Bak (1948-2002)

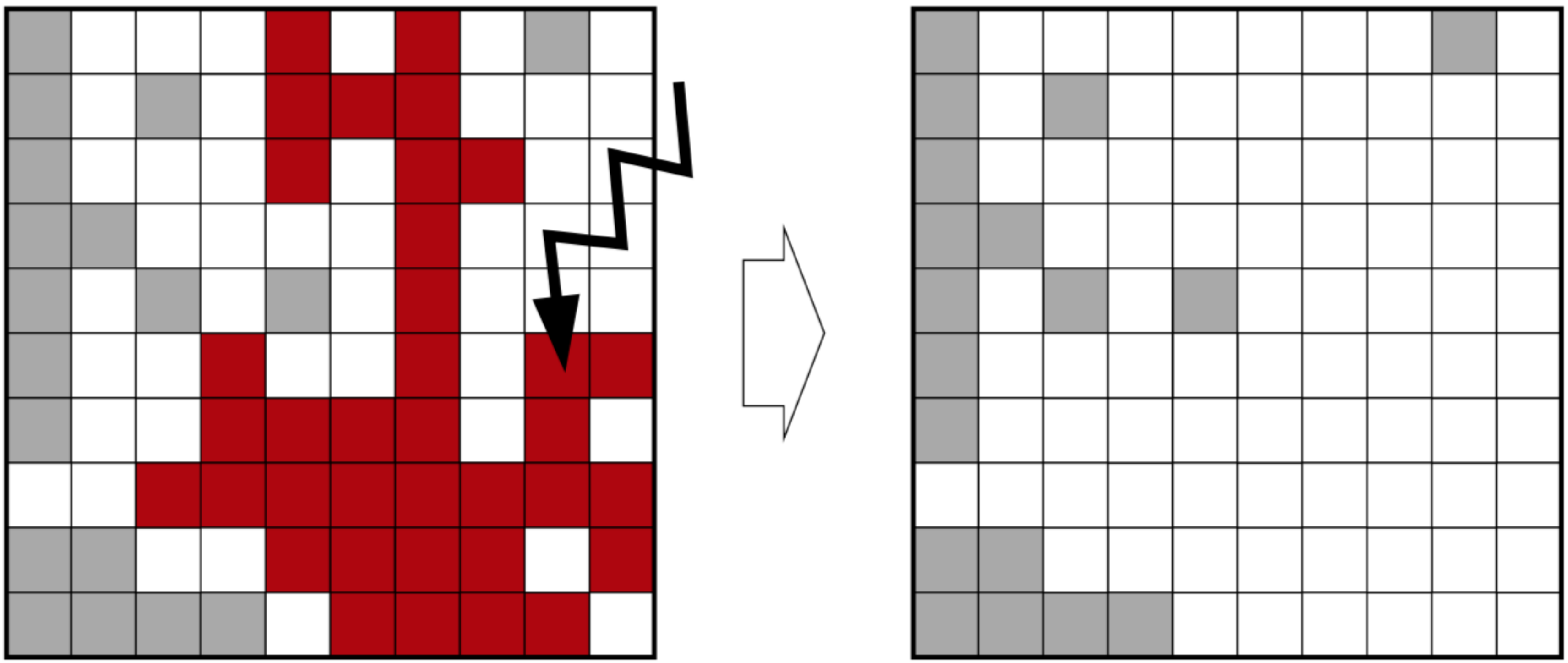


Avalanche sizes (time)





“Forest fire model”



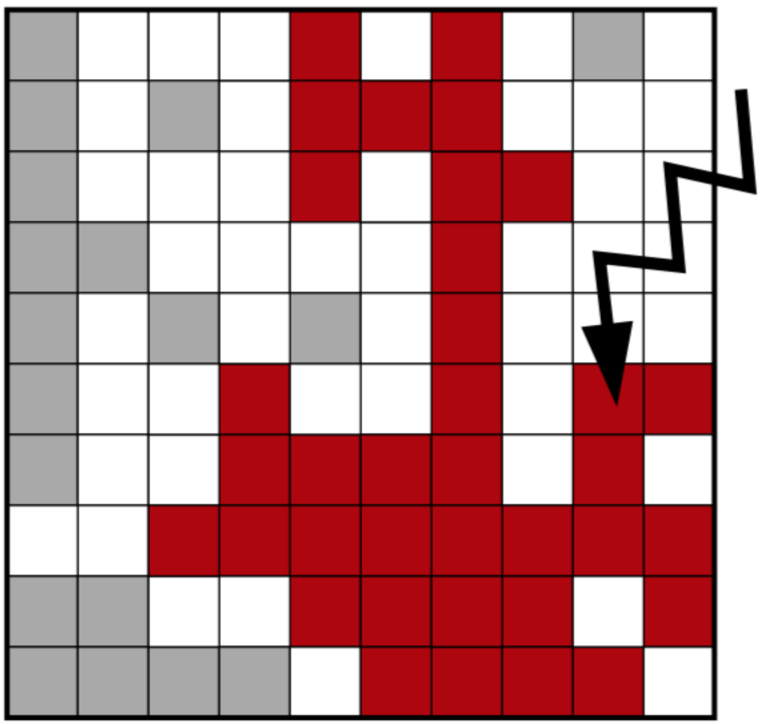
Lattice represents a “landscape”.

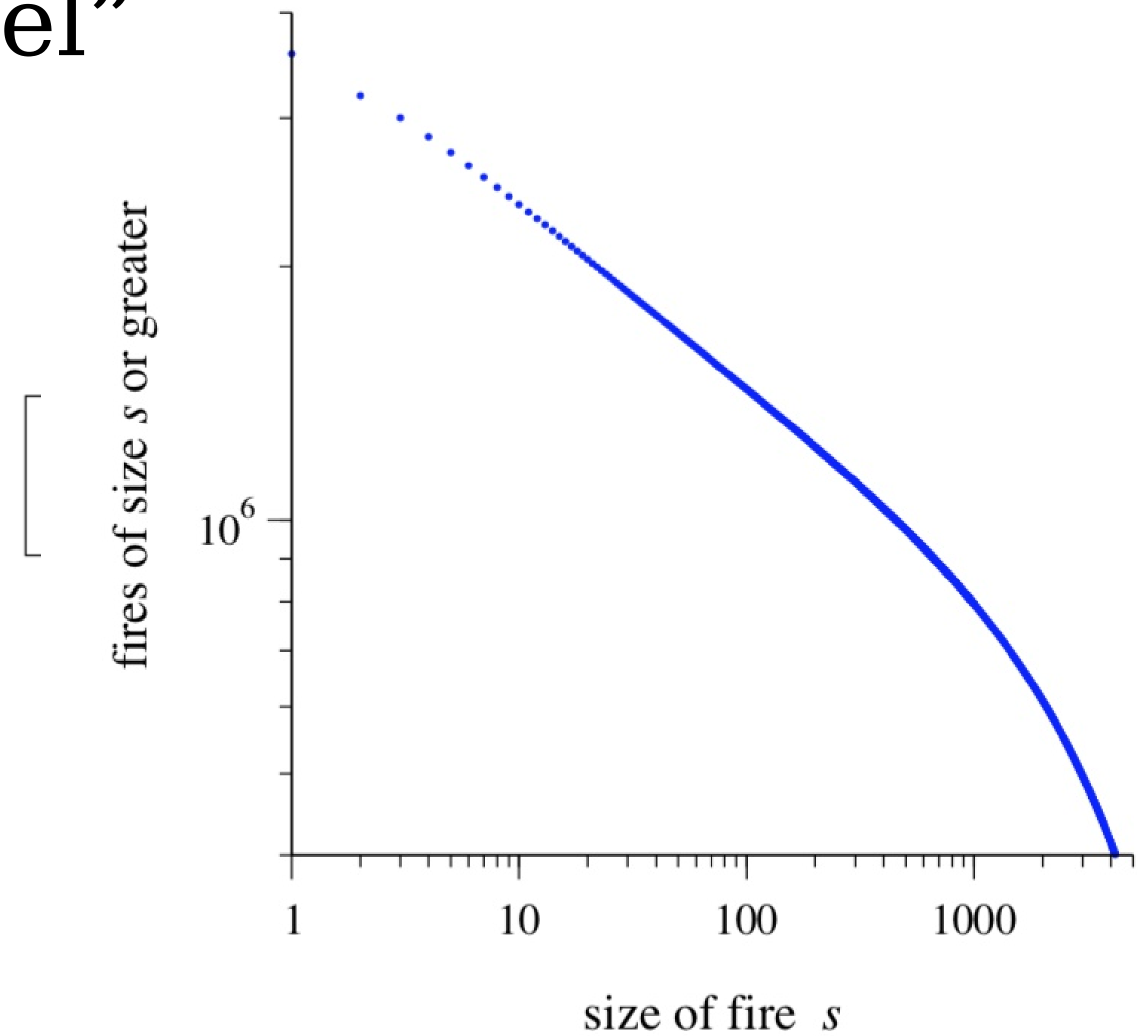
One tree can grow in each square.

Trees grow randomly in an empty square.

Fires (“lightnings”) start randomly in one filled square and the fire propagates to all adjacent filled points, burning the entire connected cluster.

“Forest fire model”

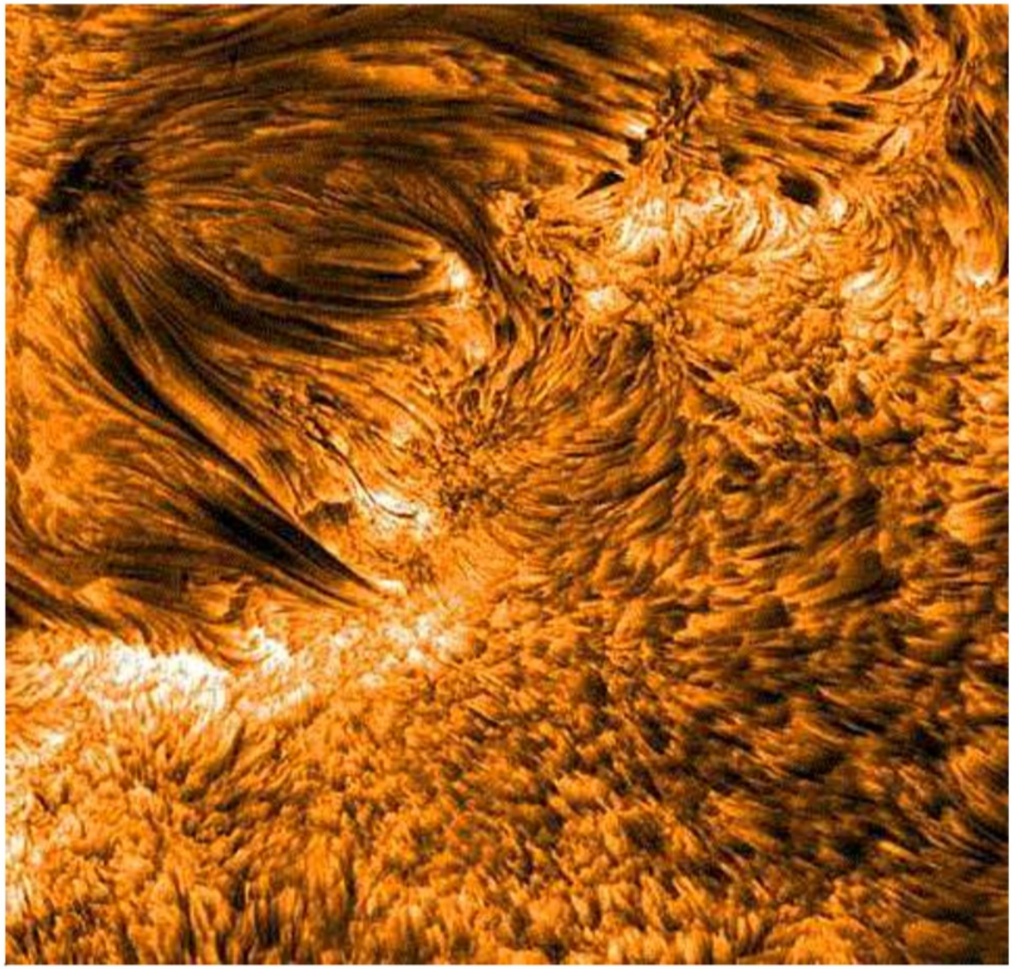




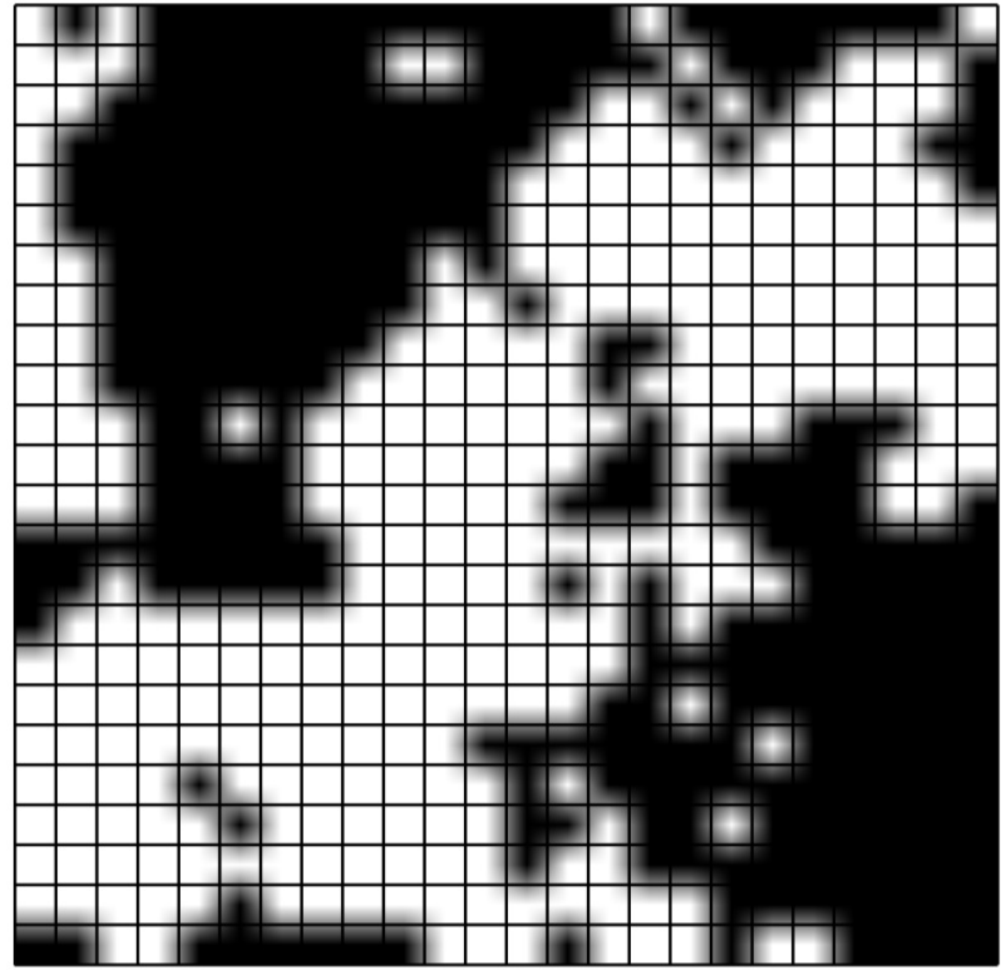
B. Drossel and F. Schwabl,
“Self–organized critical forest–fire model”,
Phys. Rev. Lett. **69**, 1629 (1992)

Cumulative distribution
of size of “fires” in a
numerical simulation
for a (5000 x 5000) lattice

Real-World Microscopic Structure



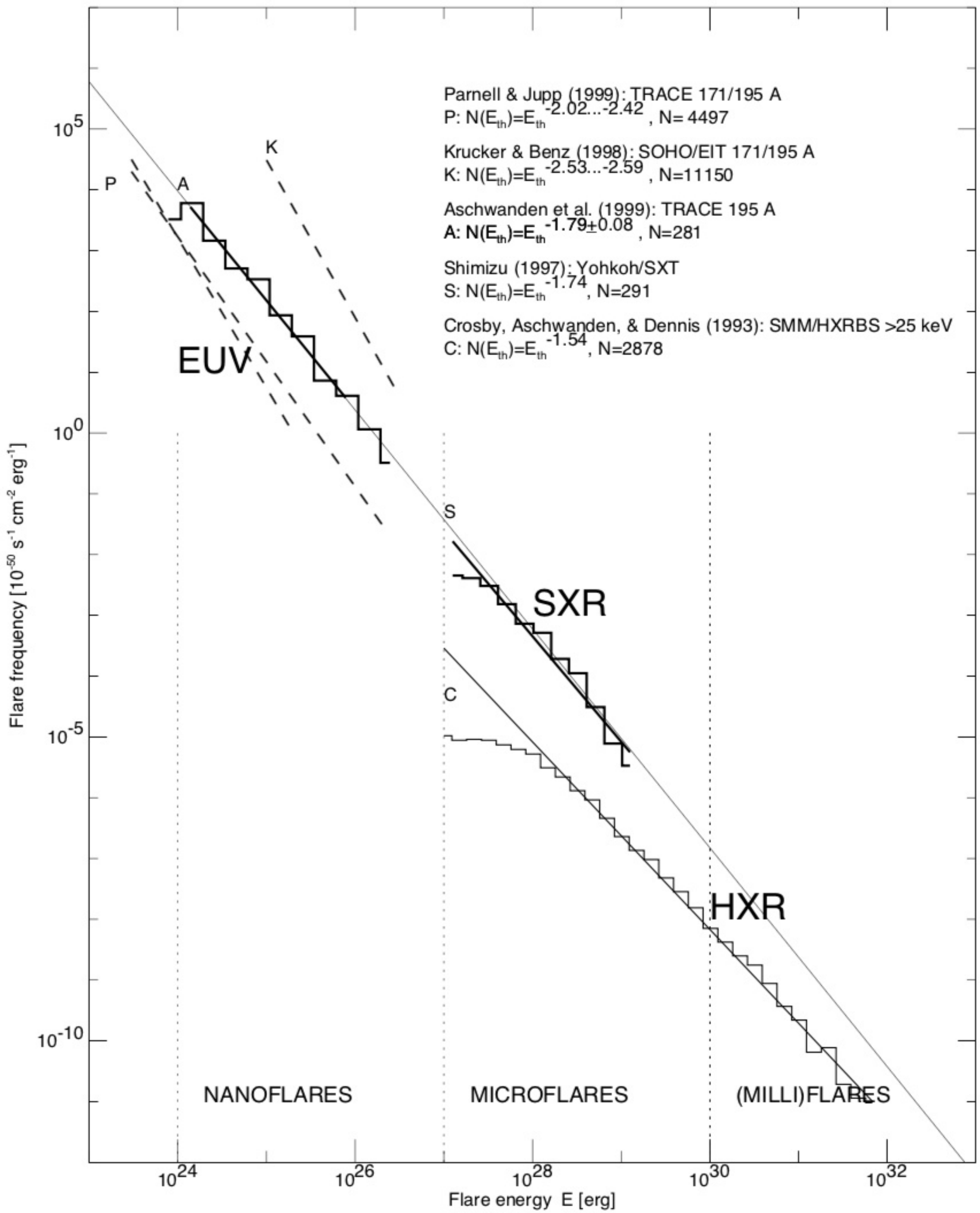
Numeric Lattice Simulation



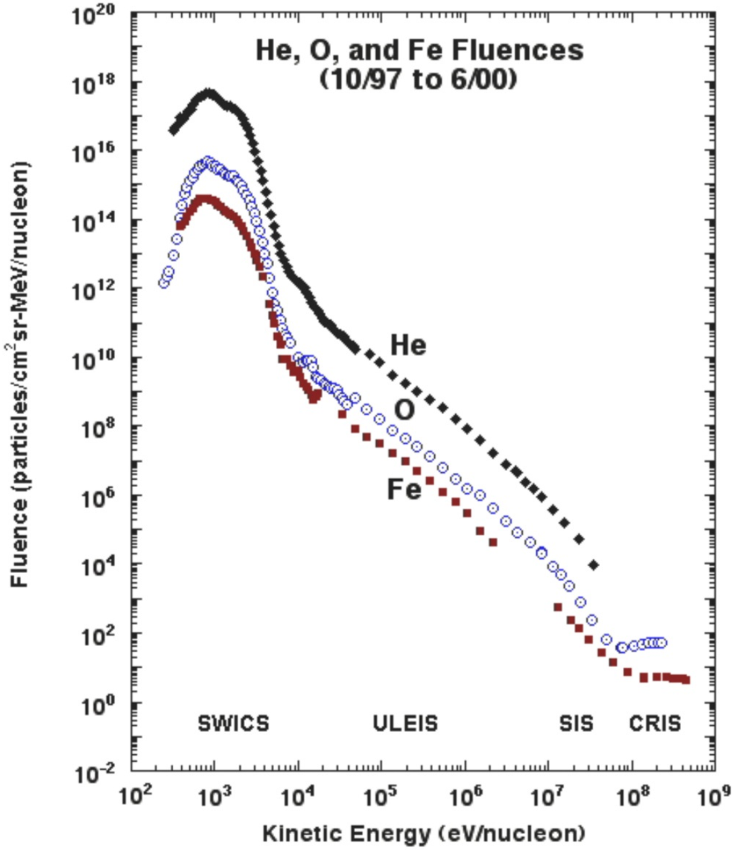
[lattice size: 1000 km]

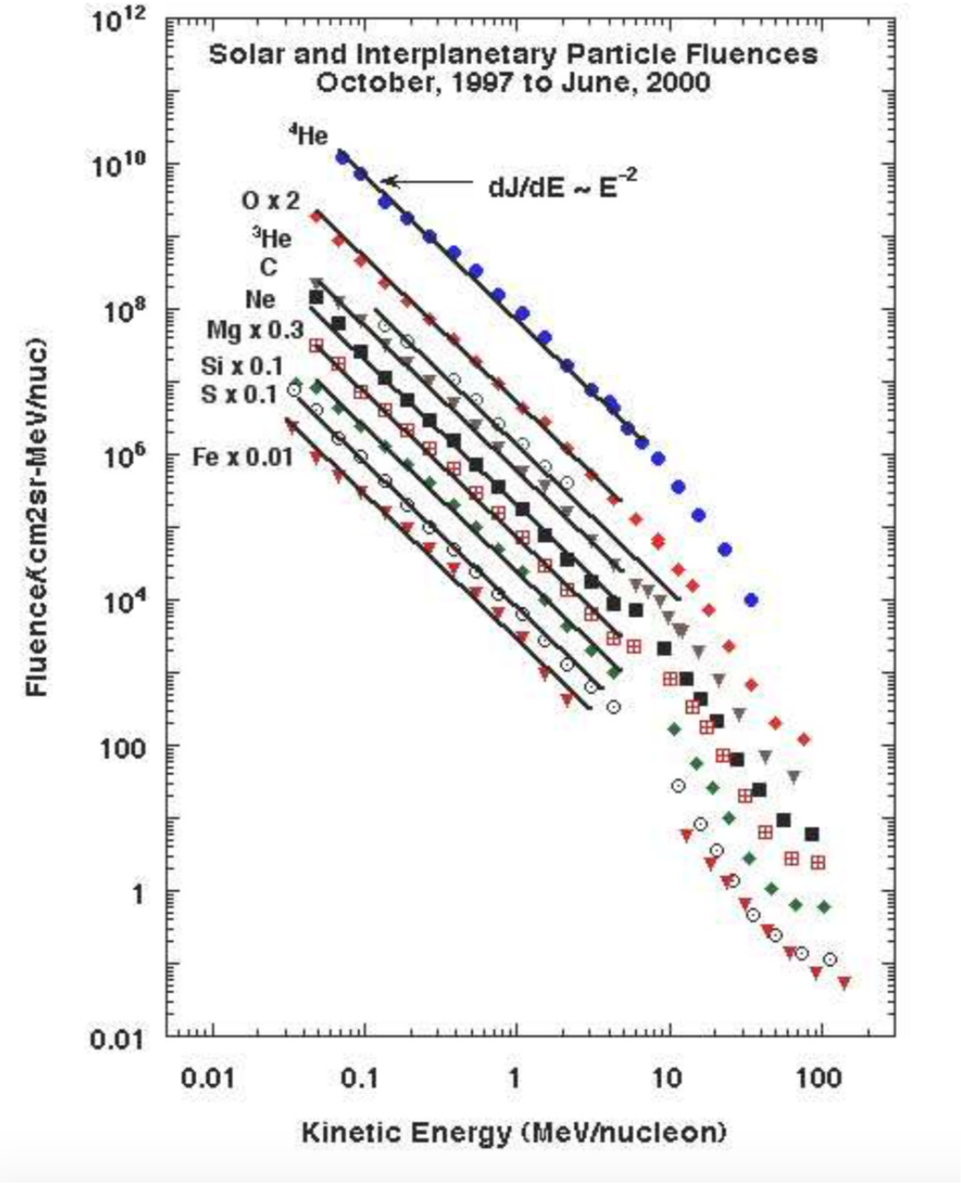
Edward T. Lu & Russell J. Hamilton
"Avalanches and the Distribution of Solar Flares"
Ap J. **380** 89, (1991)

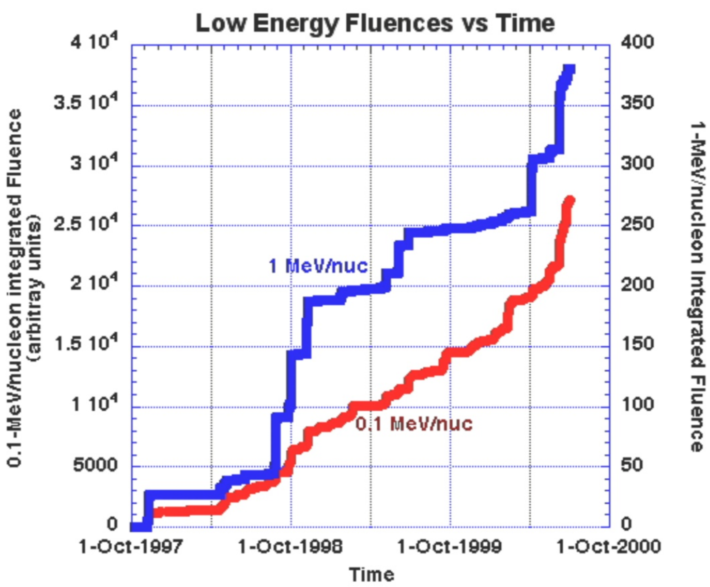
M. J. Aschwanden *et al.*,
"25 Years of Self-Organized Criticality: Solar and Astrophysics,"
Space Sci. Rev. **198**, no. 1-4, 47 (2016)
[arXiv:1403.6528 [astro-ph.IM]].



Distribution of energy of solar flares







R.A. Mewaldt *et al.*
"Long-Term Fluences of Energetic Particles in the Heliosphere"
27th ICRC Hamburg, (2001).

Final Comments:

[*] A detailed study of the (component separated) CR spectra in the entire energy range [10 TeV – 10 PeV] is a challenging and very important task.

[*] Modeling of hadronic interactions is crucial for the success of the LHAASO program on Cosmic Ray spectra.

[*] "Curved" Log-parabola spectra account for most of the gamma-ray spectra observed by Fermi. An in depth study is necessary.

[*] There is evidence that the statistical properties of blazar flaring is a "Critical Phenomenon" [analogy with solar flares should be studied in more detail]

[*] The concepts could perhaps also be relevant for Galactic Cosmic Rays (with a source spectrum Formed by components of different shape, that Combine to a sum that is only approximately a power-law]