Lepton Flavor Structure & Muonium-antimuonium Conversion

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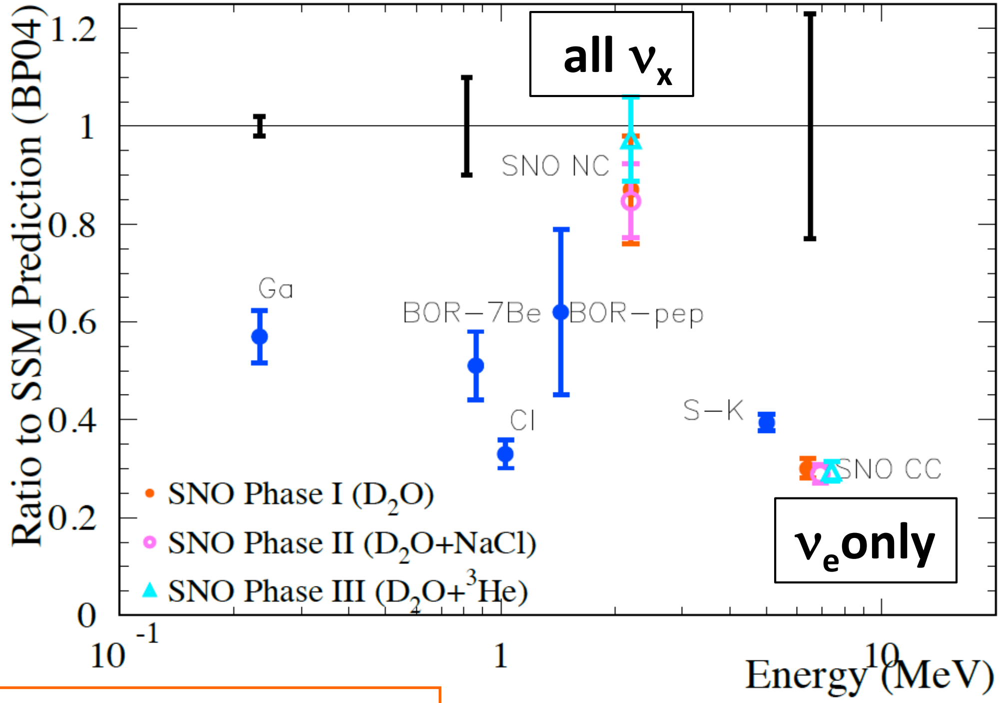
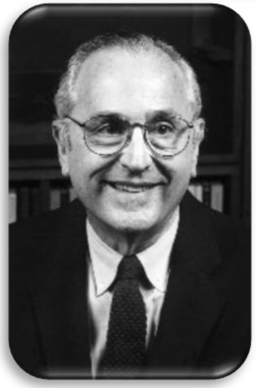
IHEP/UCAS，Beijing

- ◆中微子物理研究现状
- ◆质量模型与实验观测
- ◆轻子味破坏与味结构
- ◆中微子物理未来展望

第三届“高功率强子加速器上的粒子物理前沿研究”研讨会
东莞，2019/12/08

Solar Neutrinos





J. N. Bahcall

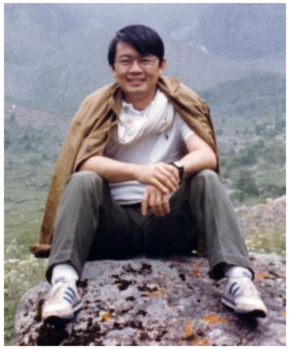
R. Davis Jr.

$$\begin{aligned} \mathrm{CC:} &\quad \nu_e + d \rightarrow p + p + e^- \\ \mathrm{NC:} &\quad \nu_\alpha + d \rightarrow p + n + \nu_\alpha \\ \mathrm{ES:} &\quad \nu_\alpha + e^- \rightarrow \nu_\alpha + e^- \end{aligned}$$

Discovery of solar neutrino oscillations

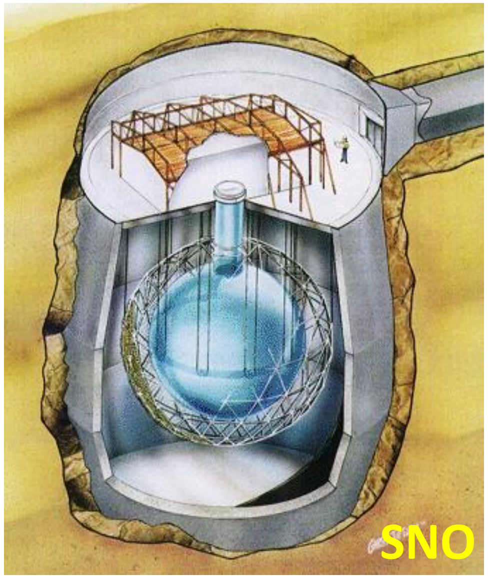
supported by KamLAND

A. B. McDonald

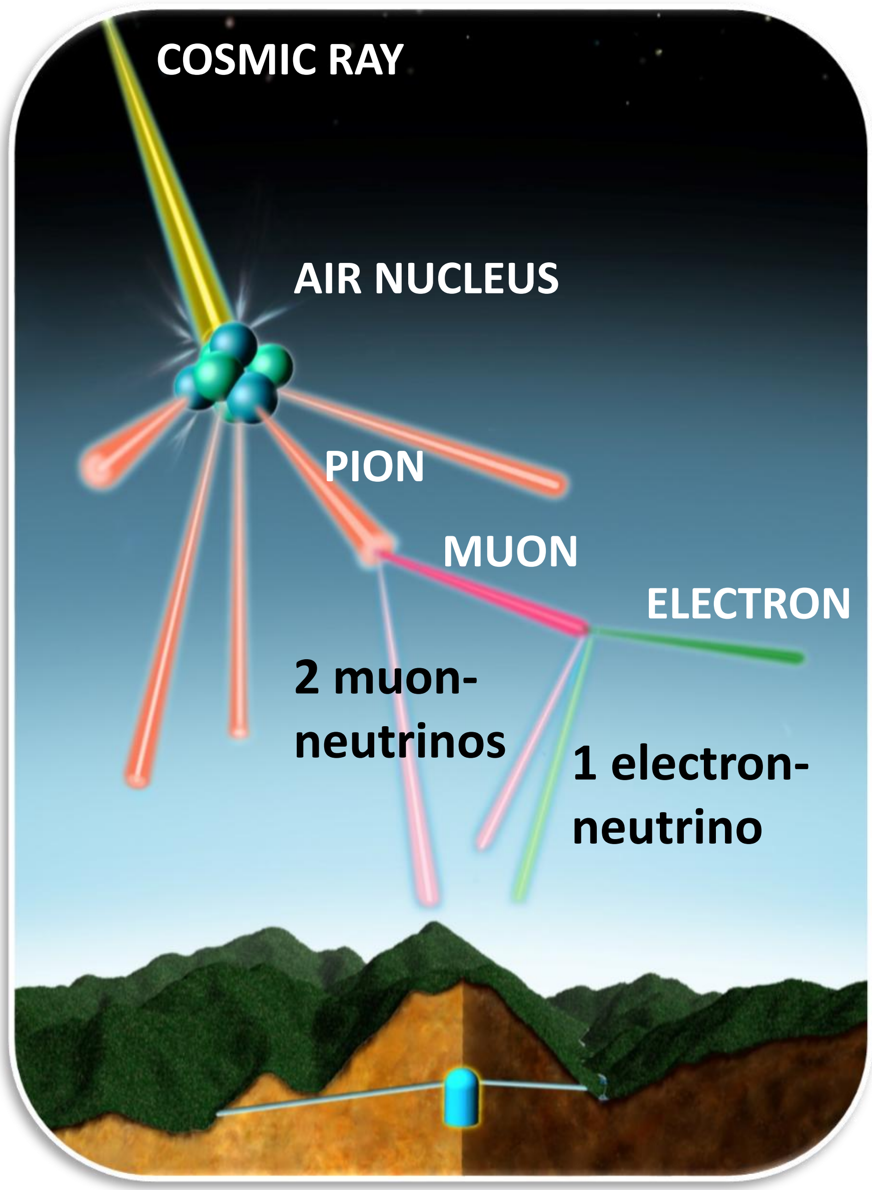


Herbert Chen
陈华森
(1942-1987)

Spokesperson of SNO since 1984

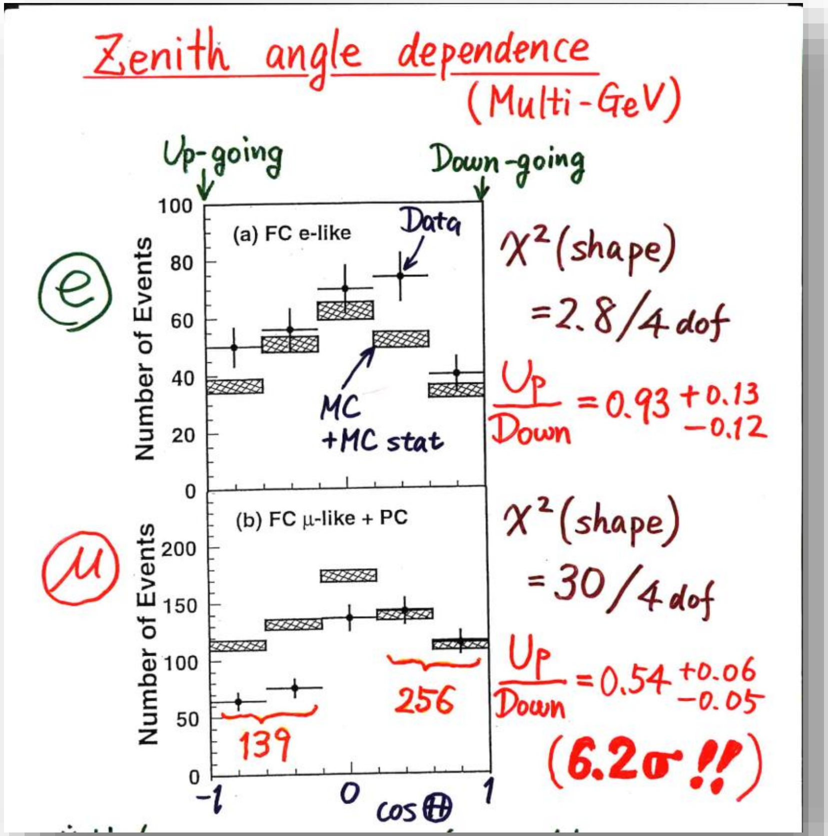


Atmospheric Neutrinos



From Kajita, ICHEP 16

Super-Kamiokande @ Neutrino 98



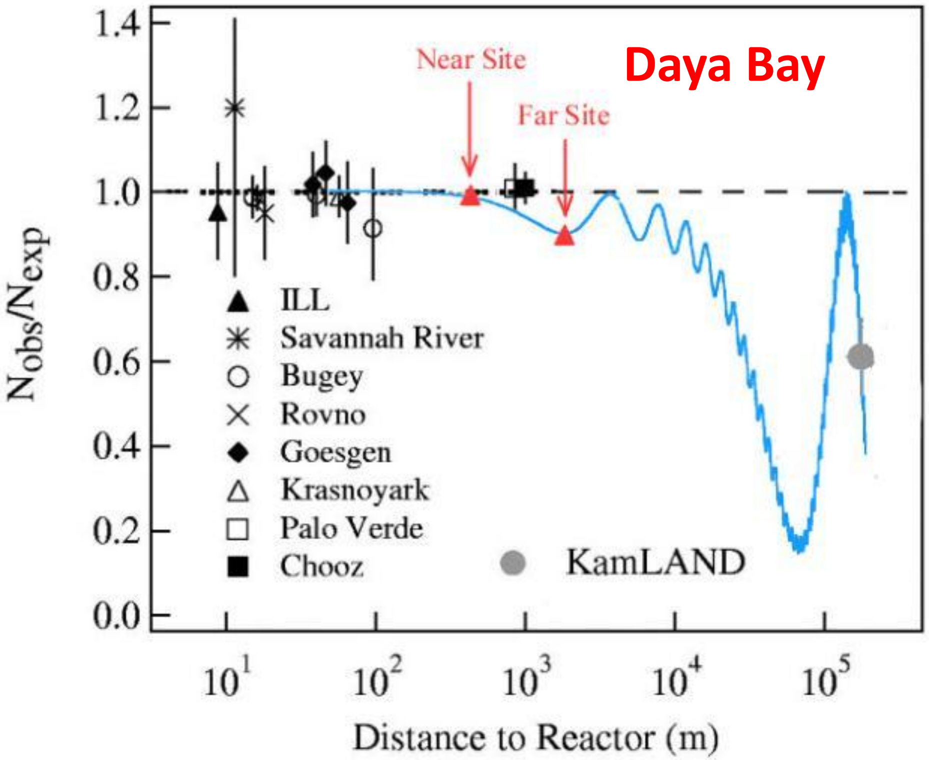
Discovery of atmospheric neutrino oscillations

**supported by
K2K, MINOS, T2K, NOνA**

Yoji Totsuka
(1942-2008)

T. Kajita

Reactor Neutrinos



Dec. 2011

Double Chooz (far detector):

$\sin^2\theta_{13} = 0.022 \pm 0.013$ 1.7σ

Mar. 2012

Daya Bay (near + far detectors):

$\sin^2\theta_{13} = 0.024 \pm 0.004$ 5.2σ

Apr. 2012

RENO (near + far detectors):

$\sin^2\theta_{13} = 0.029 \pm 0.006$ 4.9σ

Discovery of reactor neutrino oscillations

A complete picture of three-flavor neutrino oscillations!

Lepton Flavor Mixing

Standard Parametrization of the PMNS Matrix

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\boldsymbol{\theta_{23} \sim 45^\circ}$

$\boldsymbol{|\Delta m^2_{32}| \sim 2.5 \times 10^{-3}\ \mathrm{eV}^2}$

Atmospheric,
LBL accelerator

$\boldsymbol{\theta_{13} \sim 9^\circ}$

$\boldsymbol{\delta \sim ?}$

Reactor,
LBL accelerator

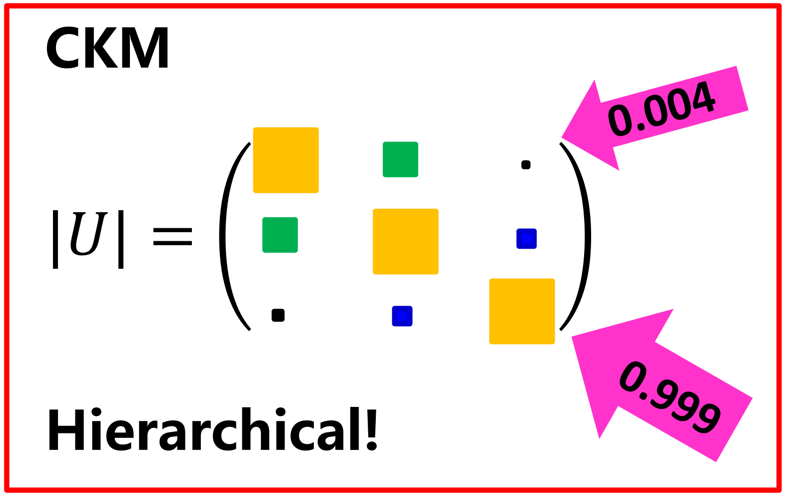
$\boldsymbol{\theta_{12} \sim 34^\circ}$

$\boldsymbol{\Delta m^2_{21} \sim 8 \times 10^{-5}\ \mathrm{eV}^2}$

Solar,
KamLAND

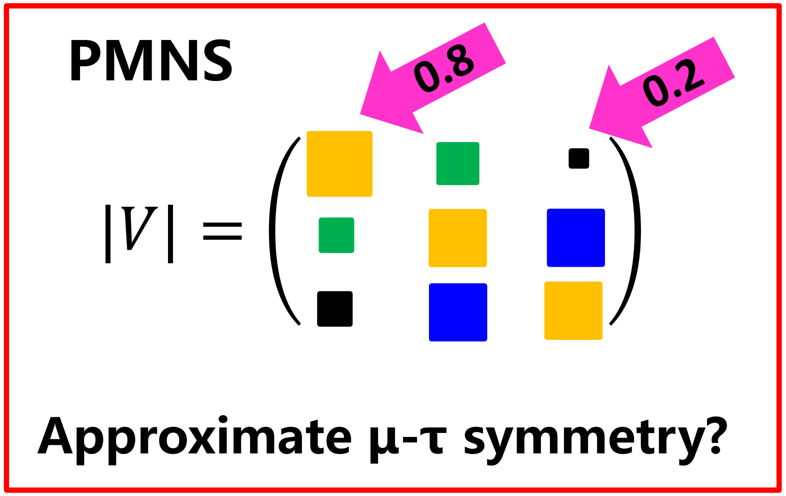
0ν2β, LNV?

Quarks vs. Leptons: A big puzzle of fermion flavor mixings



CKM

Hierarchical!

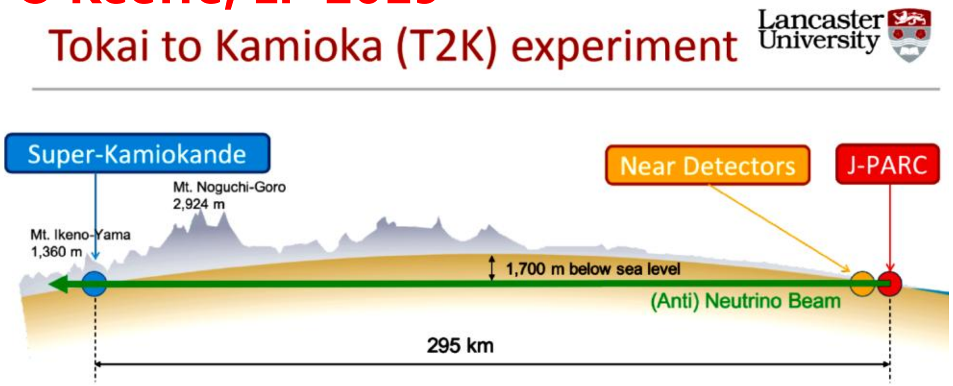


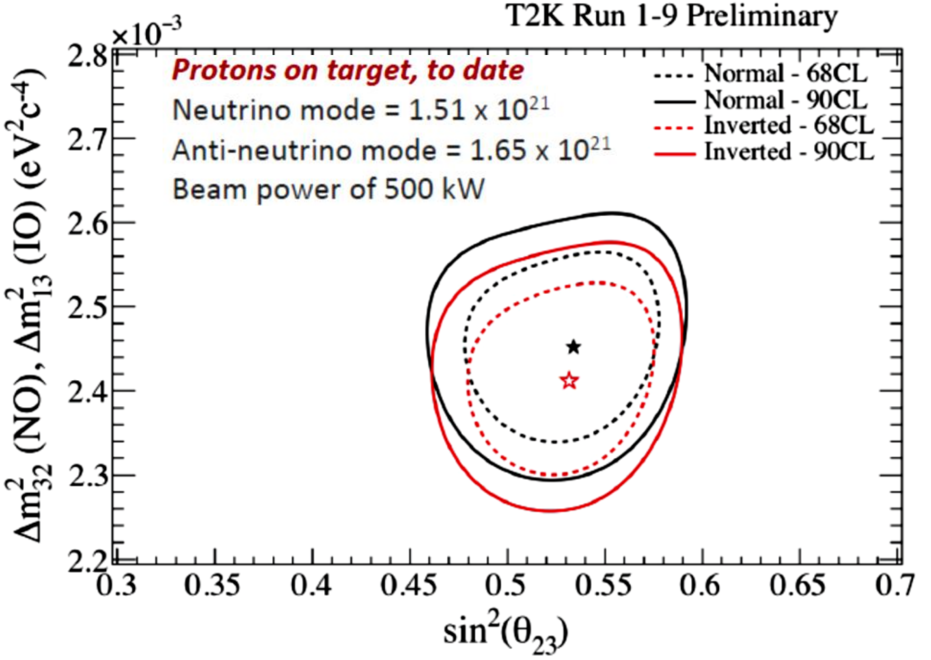
PMNS

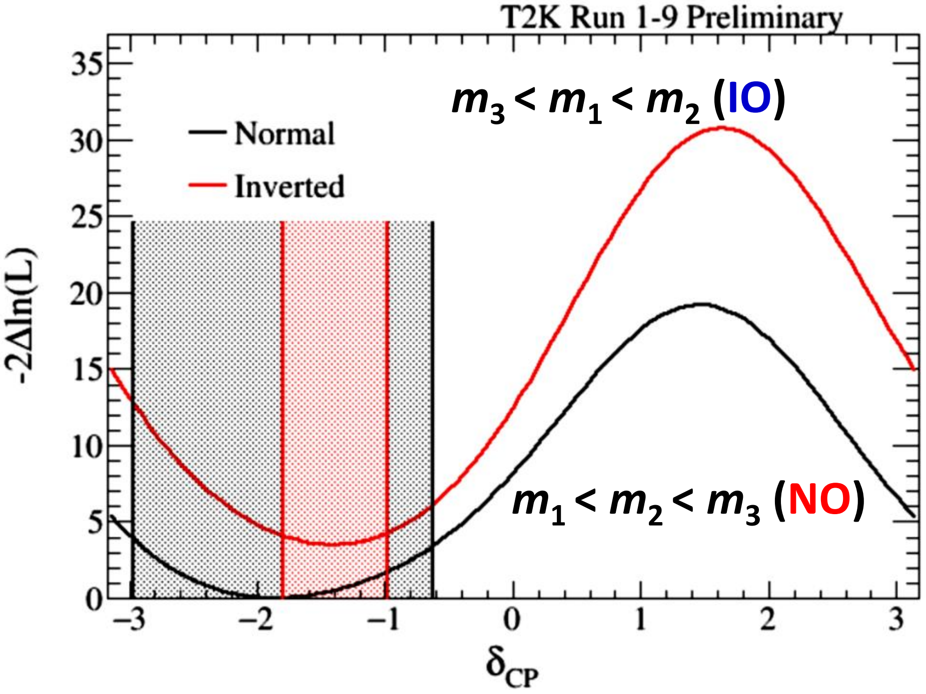
Approximate μ-τ symmetry?

Latest Results from T2K

O'Keeffe, LP 2019







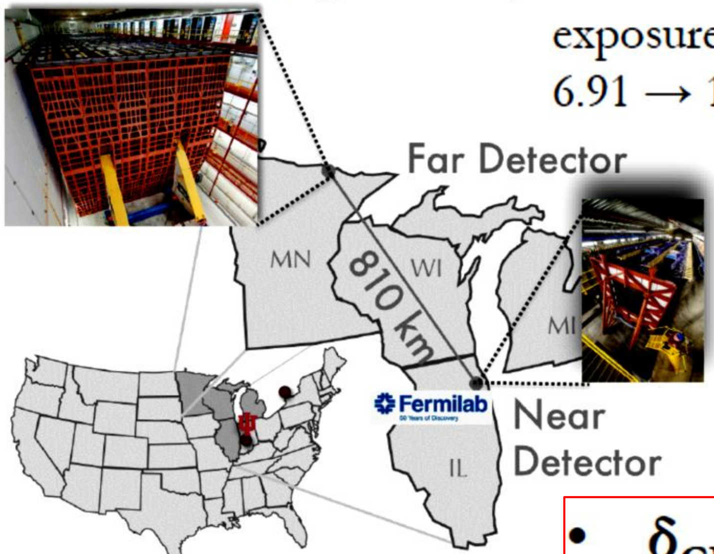
	Normal ordering	Inverted ordering
$\sin^2\theta_{23}$	0.532	0.532
$\lvert\Delta m^2_{32}\rvert \times 10^{-3}$ eV²	2.452	N/A
$\lvert\Delta m^2_{31}\rvert \times 10^{-3}$ eV²	N/A	2.432

T2K favors a mixing angle $\theta_{23} > 45°$, but consistent with maximal mixing

- **Best-fit point: $\delta_{\mathrm{CP}} = -1.885\,\mathrm{rad}/252^{\mathrm{o}}$ (NO) or $\delta_{\mathrm{CP}} = -1.382\,\mathrm{rad}/280^{\mathrm{o}}$ (IO)**
- **The 2σ range: $\delta_{\mathrm{CP}} \in [-2.97, -0.63]$ rad (NO) or $\delta_{\mathrm{CP}} \in [-1.78, -0.98]$ rad (IO)**
- **The CP-conserving values of δ_{CP} (0 and π) are excluded at the 2σ level**

Latest Results from NOvA

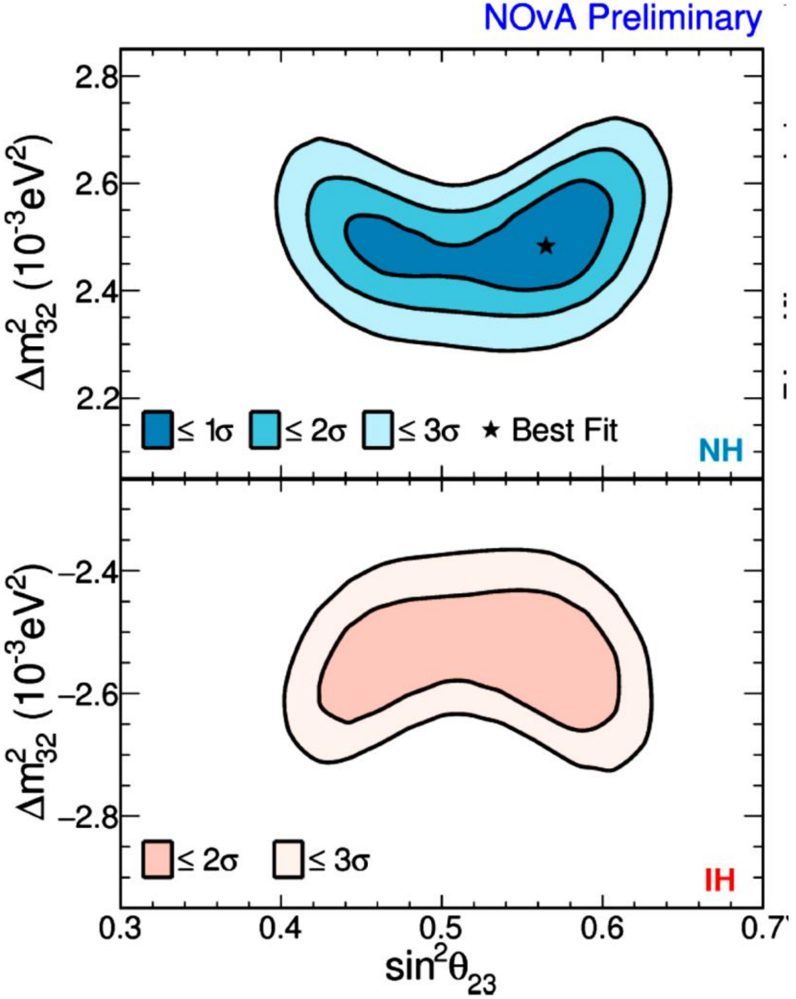
NuMI Off-Axis ν_e Appearance exp

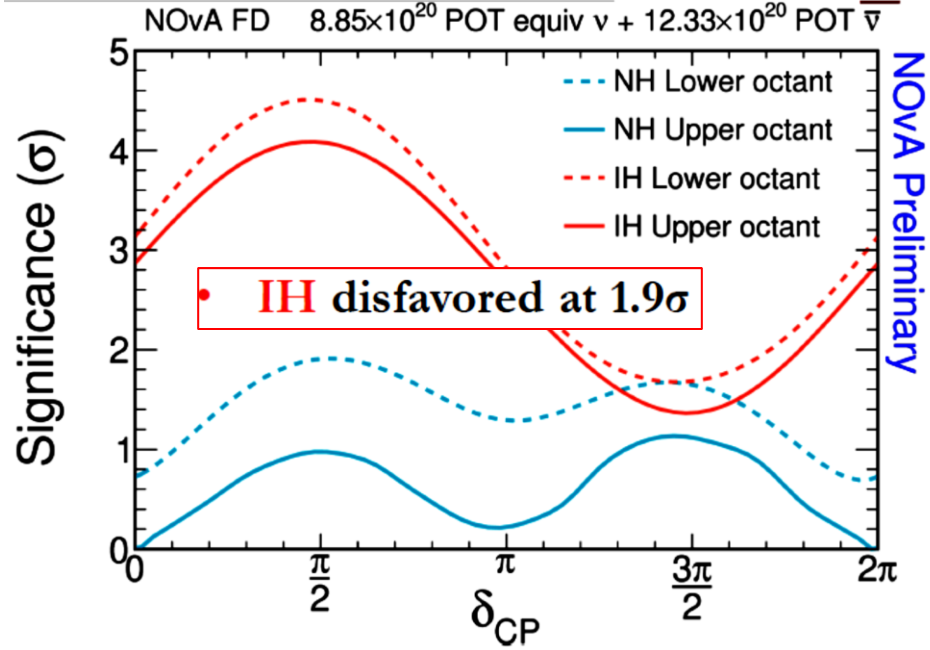


78% increase in exposure (2018 → 2019): $6.91 \rightarrow 12.33 \times 10^{20}$ POT

Davies, LP 2019

- $\delta_{CP} = 0.0^{+1.3}_{-0.4}\pi$





- **IH disfavored at 1.9σ**

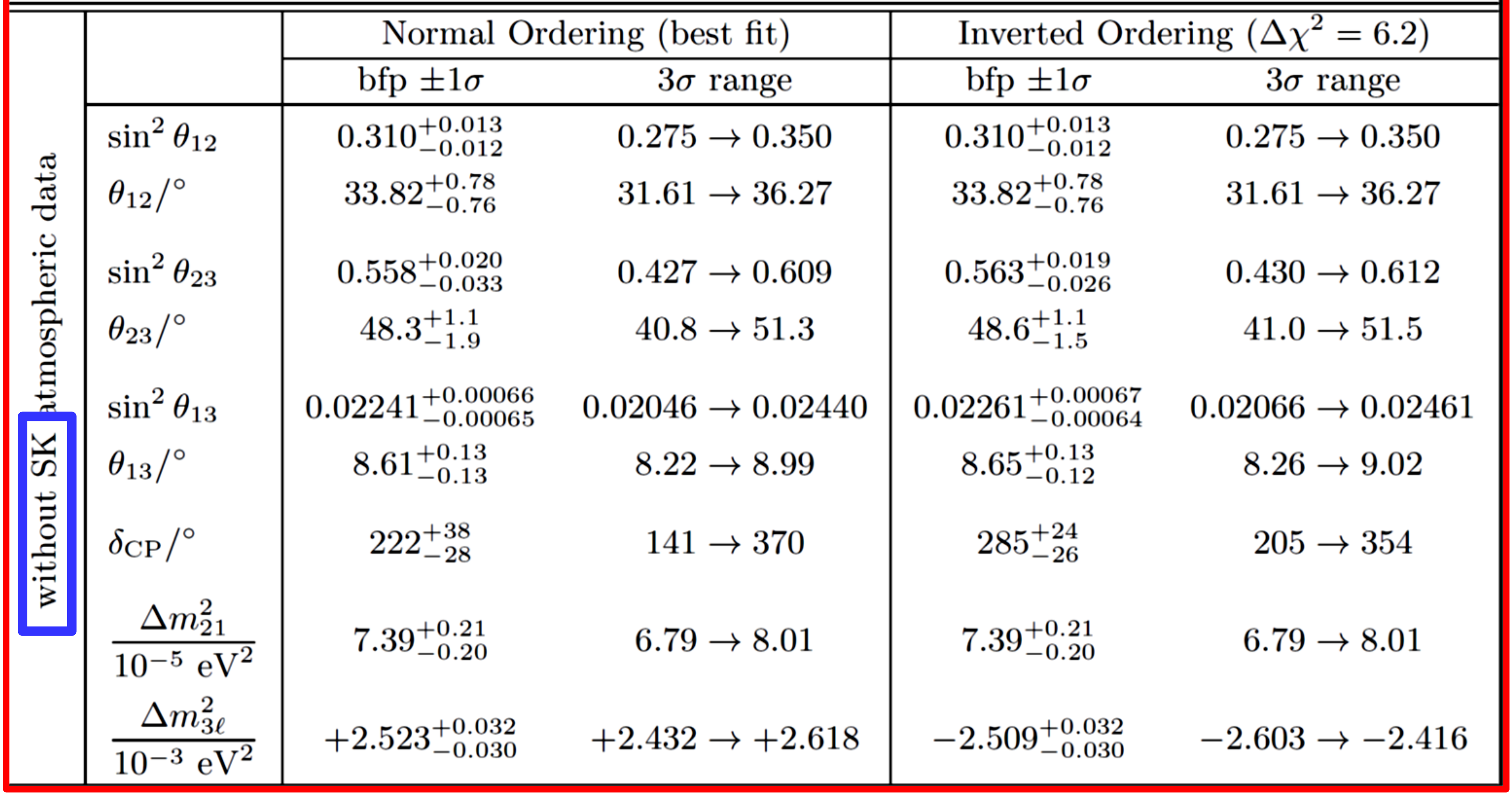
■ **NO and $\theta_{23} > 45^\circ$ now favored**

- $\sin^2\theta_{23} = 0.56^{+0.04}_{-0.03}$
- $\Delta m^2_{32} = +2.48^{+0.11}_{-0.06} \times 10^{-3}\ \mathrm{eV}^2/c^4$ (NH)
- $\sin^2\theta_{23} < 0.5$ (LO) is disfavored at 1.6σ

Global-fit Analysis of Oscillation Data

$m_1 < m_2 < m_3$ (NO) or $m_3 < m_1 < m_2$ (IO)

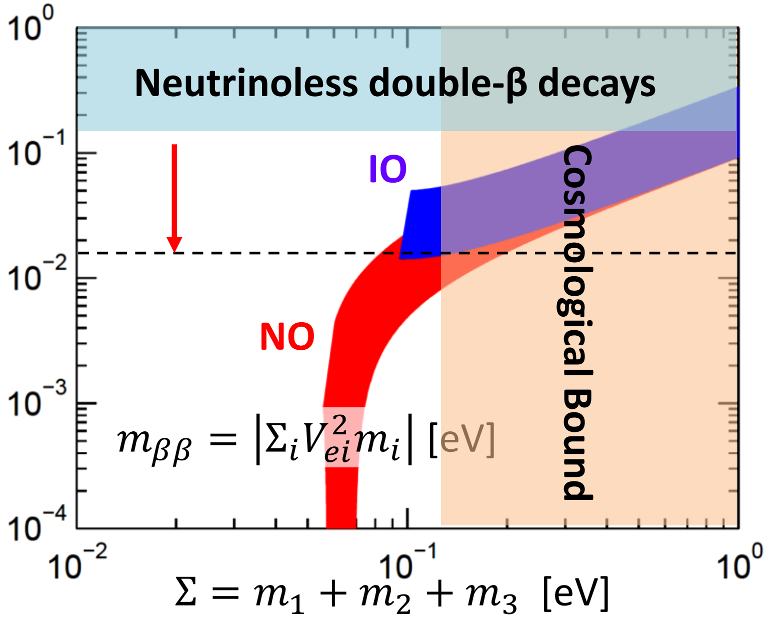
NuFIT 4.1 (2019)

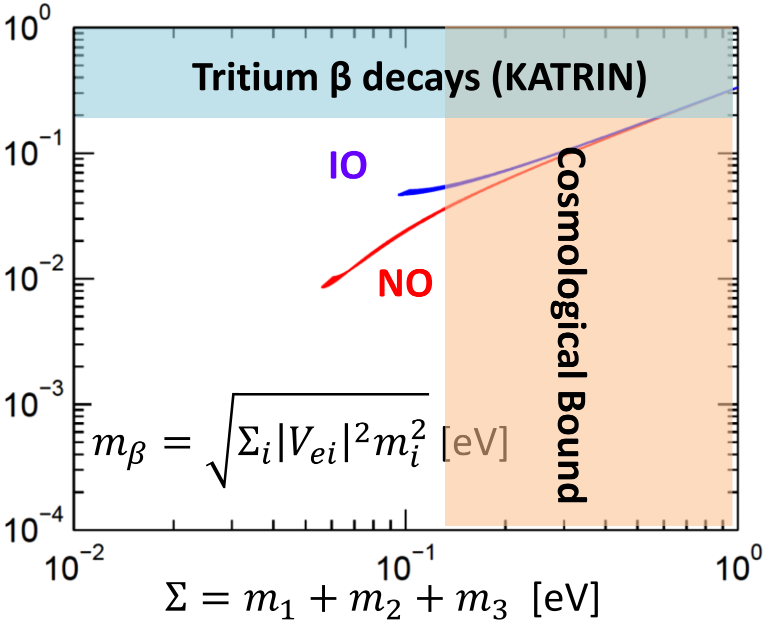


		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.2$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
without SK atmospheric data	$\sin^2\theta_{12}$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$
	$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$
	$\sin^2\theta_{23}$	$0.558^{+0.020}_{-0.033}$	$0.427 \to 0.609$	$0.563^{+0.019}_{-0.026}$	$0.430 \to 0.612$
	$\theta_{23}/^\circ$	$48.3^{+1.1}_{-1.9}$	$40.8 \to 51.3$	$48.6^{+1.1}_{-1.5}$	$41.0 \to 51.5$
	$\sin^2\theta_{13}$	$0.02241^{+0.00066}_{-0.00065}$	$0.02046 \to 0.02440$	$0.02261^{+0.00067}_{-0.00064}$	$0.02066 \to 0.02461$
	$\theta_{13}/^\circ$	$8.61^{+0.13}_{-0.13}$	$8.22 \to 8.99$	$8.65^{+0.13}_{-0.12}$	$8.26 \to 9.02$
	$\delta_{\mathrm{CP}}/^\circ$	222^{+38}_{-28}	$141 \to 370$	285^{+24}_{-26}	$205 \to 354$
	$\dfrac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$
	$\dfrac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.523^{+0.032}_{-0.030}$	$+2.432 \to +2.618$	$-2.509^{+0.032}_{-0.030}$	$-2.603 \to -2.416$

- **Reactor: JUNO, RENO-50**
- **LBL Acc.: T2K, NOνA, LBNF/DUNE**
- **Atm: PINGU, ORCA, Hyper-K, INO**
- **LBL Acc.: LBNF/DUNE**
- **Super-B: ESSνSB, MOMENT**
- **NF & Beta-Beams**

Constraints on neutrino masses

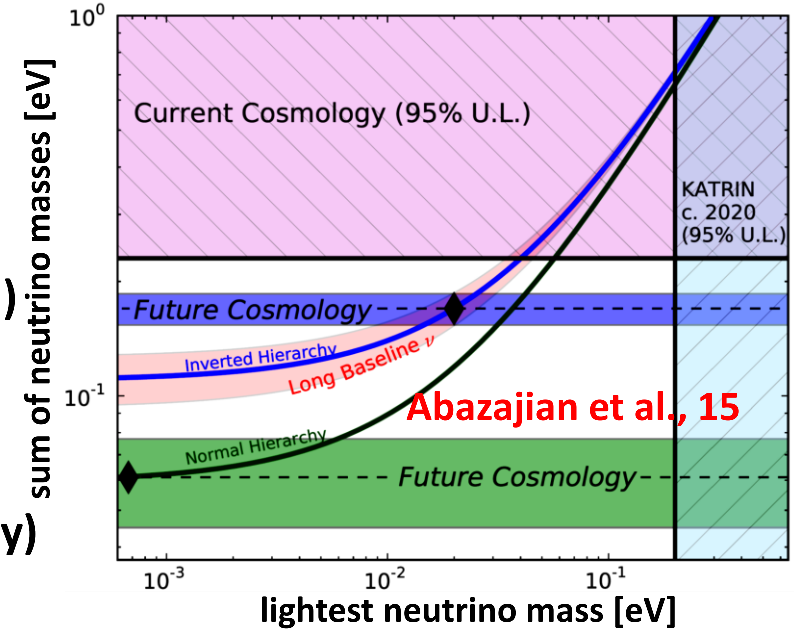




$m_1 < m_2 < m_3$ (NO) or $m_3 < m_1 < m_2$ (IO)

Constraints on absolute neutrino masses

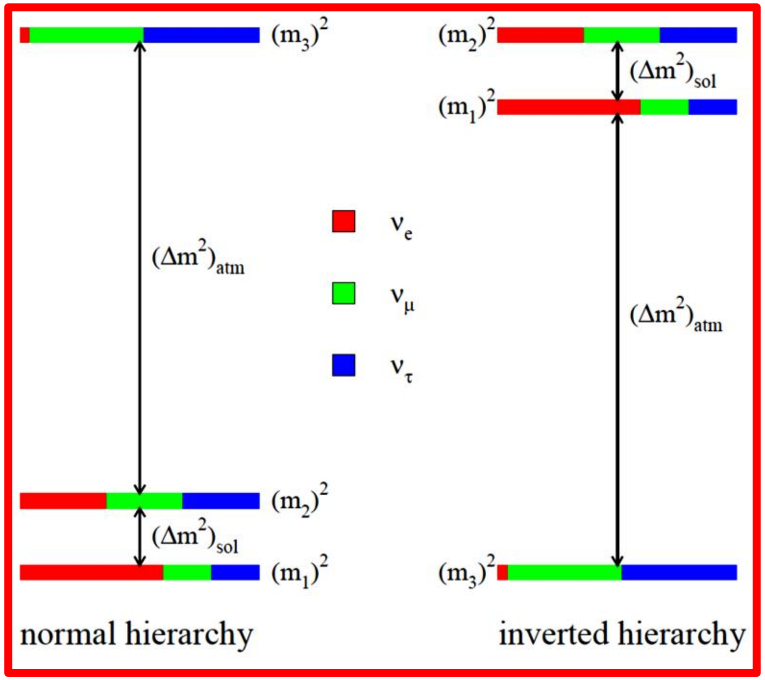
- **Tritium β decays (90% C.L.)**
  $m_\beta < 1.1\ \mathrm{eV}$ **(KATRIN, first result 2019)**
- **Neutrinoless double-β decays (90% C.L.)**
  $m_{\beta\beta} < (0.05\sim0.16)\ \mathrm{eV}$ **(KamLAND-Zen)**
  $(0.17\sim0.49)\ \mathrm{eV}$ **(EXO)**
  $(0.12\sim0.26)\ \mathrm{eV}$ **(GERDA)**
  $(0.11\sim0.50)\ \mathrm{eV}$ **(CUORE)**
- **Cosmological observations (95% probability)**
  $\Sigma < 0.12\ \mathrm{eV}$ **(Planck)**



Current Status of Neutrino Physics

Open Questions

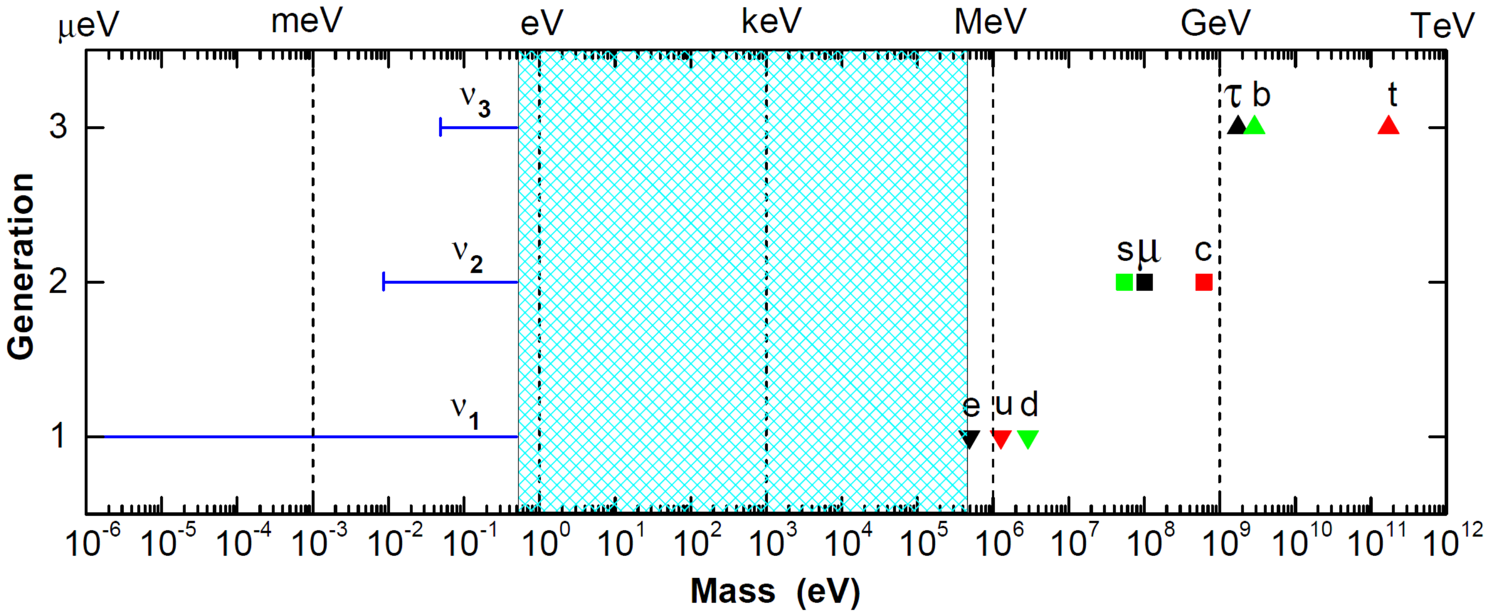
- **Normal or Inverted (sign of Δm^2_{32}?)**
- **Leptonic CP Violation (δ = ?)**
- **Octant of θ_{23} (> or < 45°?)**
- **Absolute Neutrino Masses ($m_{\text{lightest}} = 0$?)**
- **Majorana or Dirac Nature ($\nu = \nu^c$?)**
- **Majorana CP-Violating Phases (how?)**

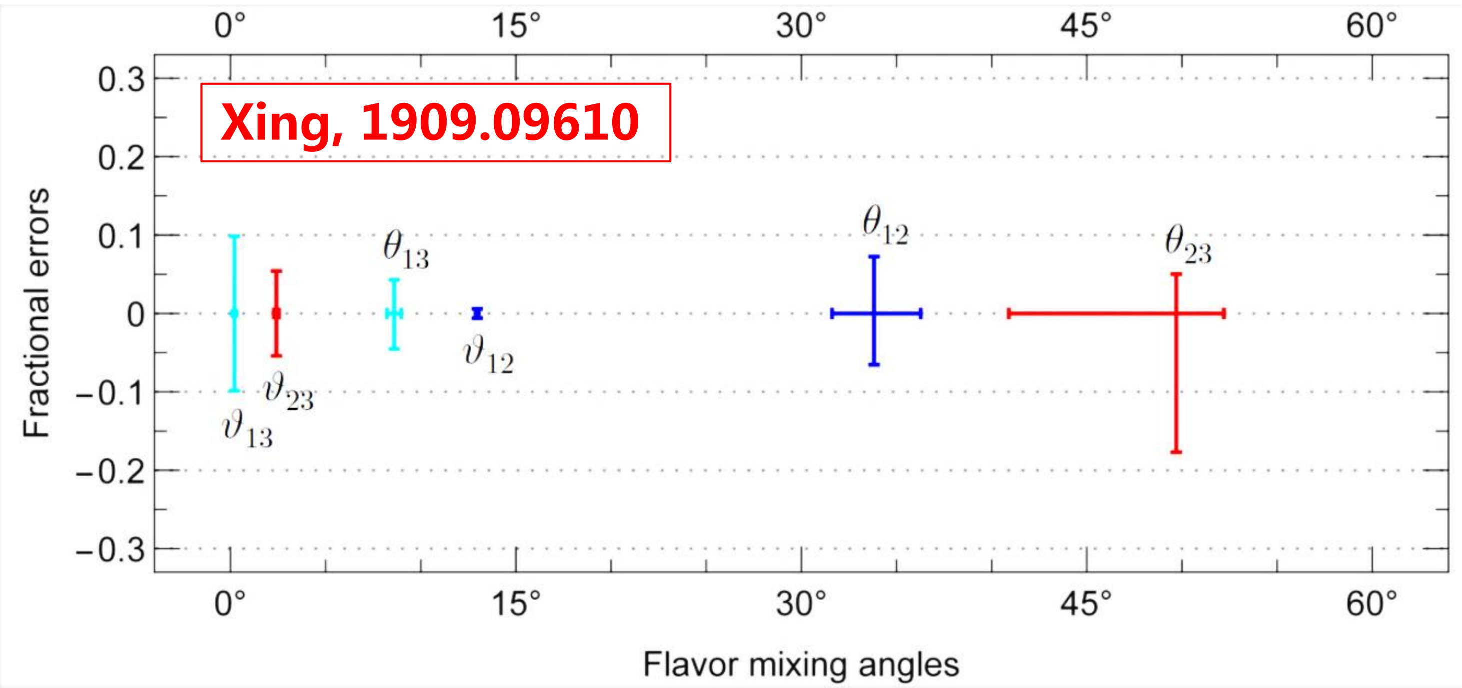


- **Extra Neutrino Species**
- **Exotic Neutrino Interactions**
- **Other LNV & LFV Processes**
- **Leptonic Unitarity Violation**

- **Origin of Neutrino Masses**
- **Flavor Structure (Symmetry?)**
- **Quark-Lepton Connection**
- **Relations to DM, BAU, or NP**

Flavor Puzzles





Xing, 1909.09610

- **Fermion Mass Spectra**
- **Flavor Mixing Pattern**
- **CPV**

Contain most free parameters of the SM

◆ **Origin of Neutrino Masses?**

◆ **Dirac or Majorana?**

Dirac vs. Majorana neutrinos

The simplest way to accommodate tiny neutrino masses

- **Dirac Neutrinos**

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \overline{\nu_{\rm R}}\, {\rm i}\partial\!\!\!/\, \nu_{\rm R} - \left[\overline{\ell_{\rm L}}\, Y_\nu \tilde{H} \nu_{\rm R} + {\rm h.c.}\right]$$

Generate Dirac ν masses in a similar way to that for quarks and charged leptons, after the spontaneous gauge symmetry breaking

$$M_\nu = Y_\nu v$$

$\approx$ **174 GeV**

$\boldsymbol{O(0.1\ {\rm eV})}$

$\boldsymbol{O(10^{-12})}$

Difficulties with Dirac neutrinos

- Tiny Dirac masses worsen fermion mass hierarchy problem (i.e., $m_i/m_t < 10^{-12}$)
- Mandatory lepton number conservation, which is actually accidental in the SM

- **Majorana Neutrinos**

$$-\left[\frac{1}{2}\overline{\nu^{\rm C}_{\rm R}} M_{\rm R} \nu_{\rm R} + {\rm h.c.}\right]$$

Generate tiny Majorana ν masses via the so-called seesaw mechanism

$$M_\nu = v^2 Y_\nu M_{\rm R}^{-1} Y_\nu^{\rm T}$$

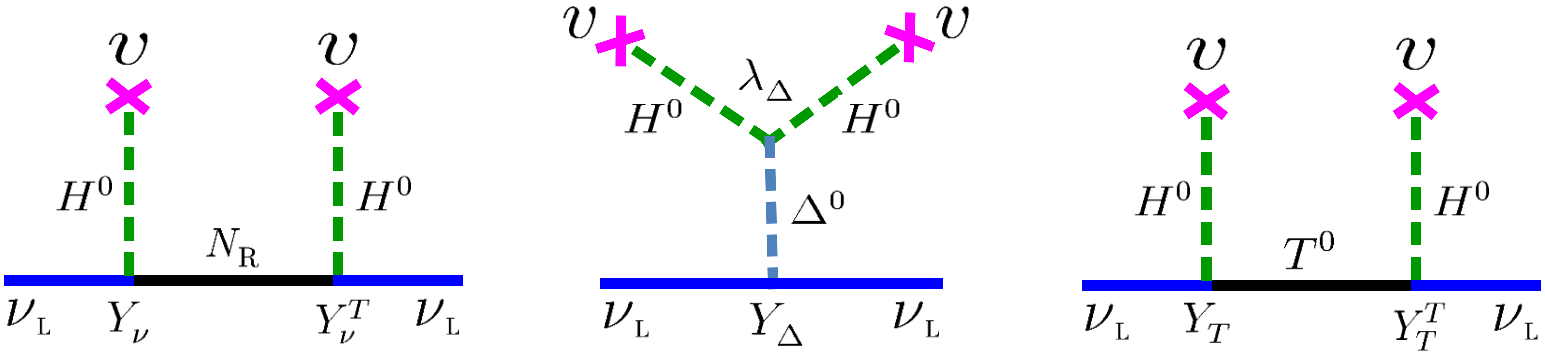
$\boldsymbol{O(0.1\ {\rm eV})}$ $\boldsymbol{O(10^{14}\ {\rm GeV})}$

- Retain the SM symmetries
- GUT or TeV energy scale?

Guide the theorists to build a model for tiny ν masses

Seesaw models for Majorana neutrinos

Majorana neutrinos: a natural way to understand neutrino masses



$$\boxed{M_\nu \approx -v^2 Y_\nu \frac{1}{M_{\rm R}} Y_\nu^T}$$

$$\boxed{M_\nu \approx \lambda_\Delta Y_\Delta \frac{v^2}{M_\Delta}}$$

$$\boxed{M_\nu \approx -v^2 Y_T \frac{1}{M_T} Y_T^T}$$

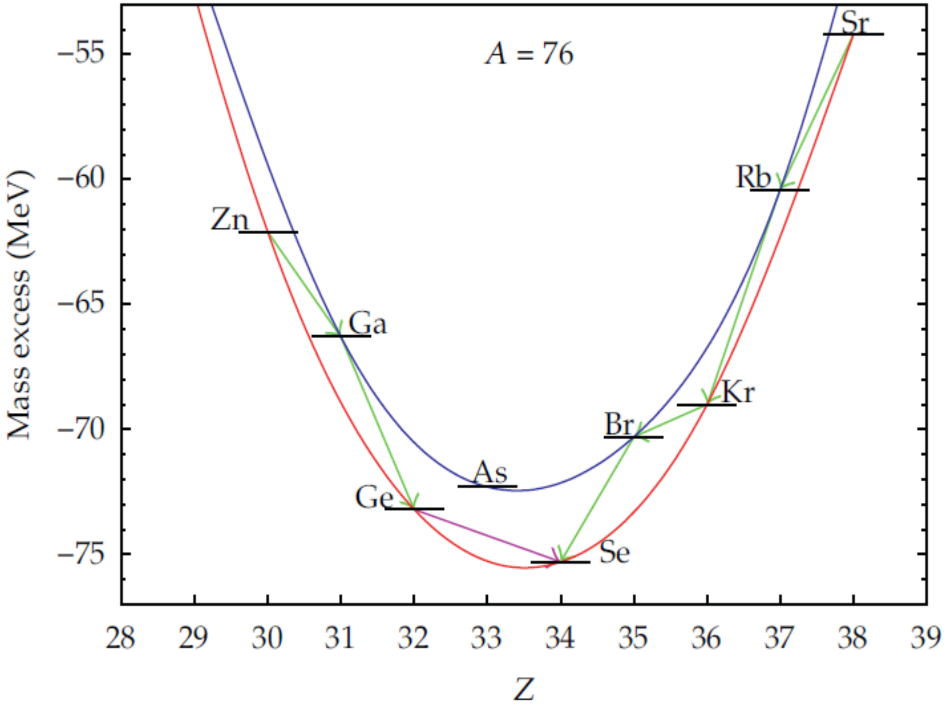
Type-I: SM + 3 right-handed Majorana ν's (Minkowski 77; Yanagida 79; Glashow 79; Gell-Mann, Ramond, Slanski 79; Mohapatra, Senjanovic 79)

Type-II: SM + 1 Higgs triplet (Magg, Wetterich 80; Schechter, Valle 80; Lazarides et al 80; Mohapatra, Senjanovic 80; Gelmini, Roncadelli 80)

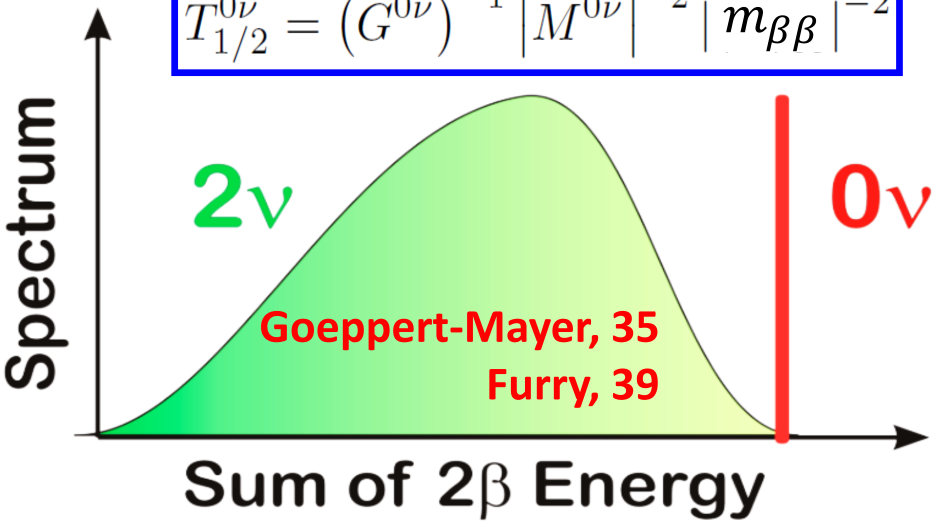
Type-III: SM + 3 triplet fermions (Foot, Lew, He, Joshi 89)

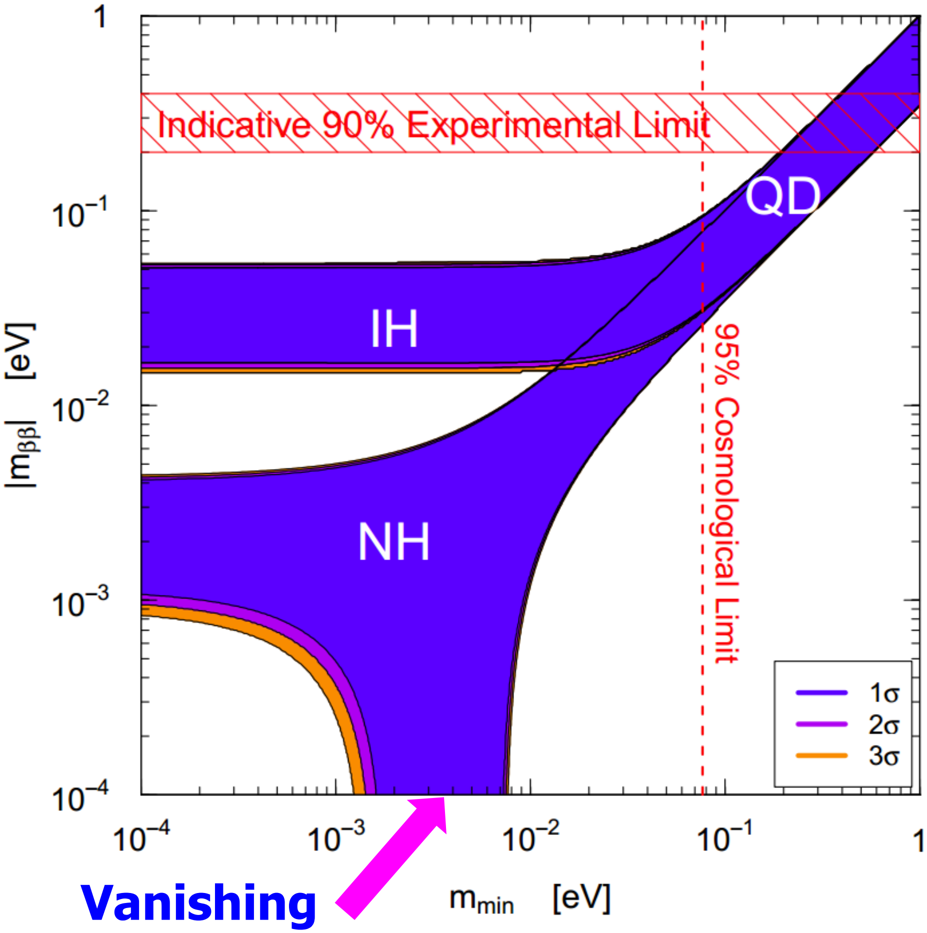
- Can naturally be embedded into the SO(10) GUT (e.g., type-I + type-II seesaw)
- Responsible for both tiny neutrino masses and matter-antimatter asymmetry

Majorana vs. Dirac



$$T^{0\nu}_{1/2} = \left(G^{0\nu}\right)^{-1} \left|M^{0\nu}\right|^{-2} \left|m_{\beta\beta}\right|^{-2}$$

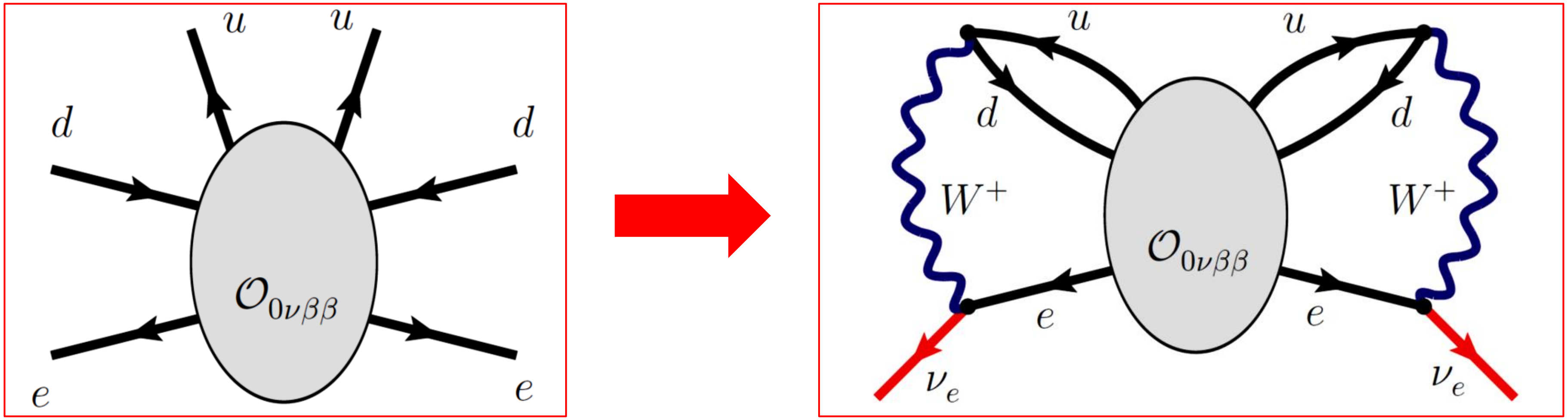




Vanishing 0ν2β mass?

- **Unique feasible way to determine the Majorana nature of neutrinos**
- **Possible to pin down mass ordering**
- **Set constraints on 2 Majorana-type CP-violating phases**

Majorana vs. Dirac



Schechter-Valle Theorem (82): If the 0ν2β decay happens, there must exist an effective Majorana neutrino mass term.

Quantitatively, the 4-loop Majorana mass from the butterfly diagram is EXTREMELY small:

$$\boxed{\delta m_\nu = O(10^{-29}\ \mathrm{eV})}$$

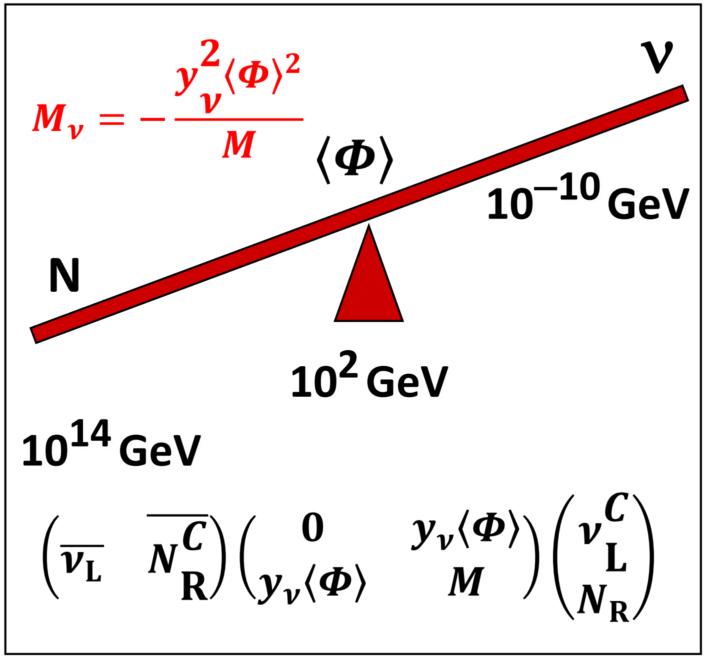
(Duerr, Lindner, Merle, 11; Liu, Zhang, Zhou, 16)

- **Assume 0ν2β decays are governed by short-distance operators**
- **The Schechter-Valle (Black Box) theorem is qualitatively correct, but the induced Majorana masses are too small to be relevant for neutrino oscillations**
- **Other mechanisms are needed to generate neutrino masses**

Test of Seesaw Models

A natural seesaw scale (e.g., type-I)

- **Close to an energy scale of fundamental physics: the GUT scale**



$$M_\nu = -\frac{y_\nu^2\langle\Phi\rangle^2}{M}$$

$$\begin{pmatrix}\overline{\nu_L} & \overline{N_R^C}\end{pmatrix}\begin{pmatrix}0 & y_\nu\langle\Phi\rangle \\ y_\nu\langle\Phi\rangle & M\end{pmatrix}\begin{pmatrix}\nu_L^C \\ N_R\end{pmatrix}$$

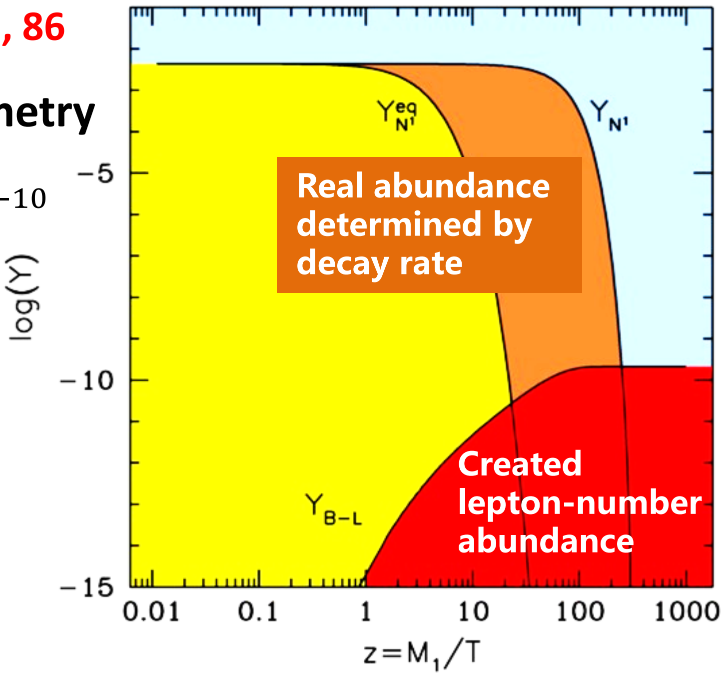
Fukugita, Yanagida, 86

B-number Asymmetry

$$\eta_B = \frac{n_B}{n_\gamma} \simeq 6\times10^{-10}$$

Leptogenesis

- **CP violation**
- **B-L violation**
- **Out-of-equili.**
- **Sphaleron**



Real abundance determined by decay rate

Created lepton-number abundance

Seesaw-induced hierarchy problem **Vissani, 98; Casas et al., 04; Abada et al., 07**

$$\delta M_H^2 = \begin{cases} -\frac{y_i^2}{8\pi^2}\left(\Lambda^2 + M_i^2 \ln\frac{M_i^2}{\Lambda^2}\right) & \text{(Type I)} \\ \frac{3}{16\pi^2}\left[\lambda_3\left(\Lambda^2 + M_\Delta^2\ln\frac{M_\Delta^2}{\Lambda^2}\right) + 4\lambda_\Delta^2 M_\Delta^2 \ln\frac{M_\Delta^2}{\Lambda^2}\right] & \text{(Type II)} \\ -\frac{3y_i^2}{8\pi^2}\left(\Lambda^2 + M_i^2\ln\frac{M_i^2}{\Lambda^2}\right) & \text{(Type III)} \end{cases}$$

In type-I seesaw models:

$$M_i \lesssim 10^7\ \text{GeV}\left(\frac{0.2\ \text{eV}}{m_i}\right)^{1/3}$$

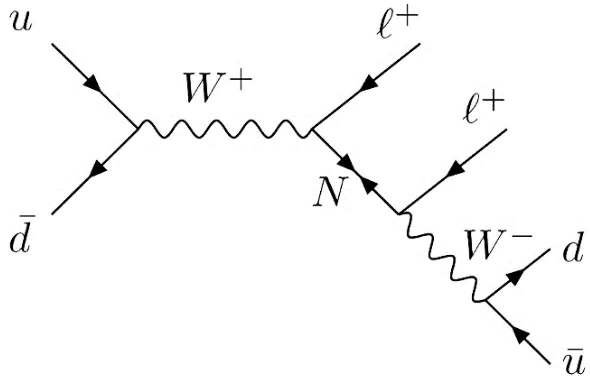
for $\delta M_H^2 \sim 0.1\ \text{TeV}^2$

Test of Seesaw Models

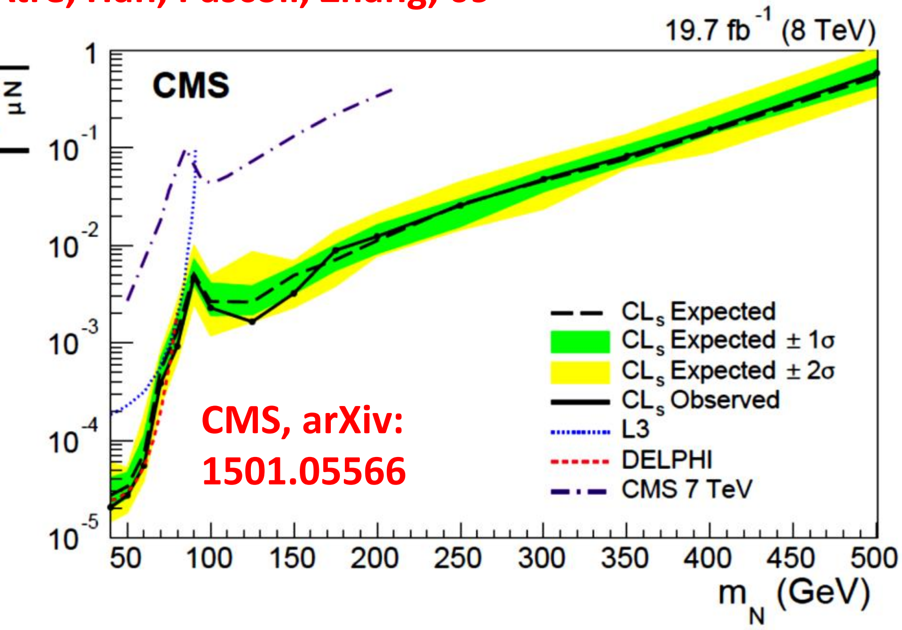
Seesaw models at the EW or TeV scales

- **motivated by the naturalness and testability problems of conventional seesaws**

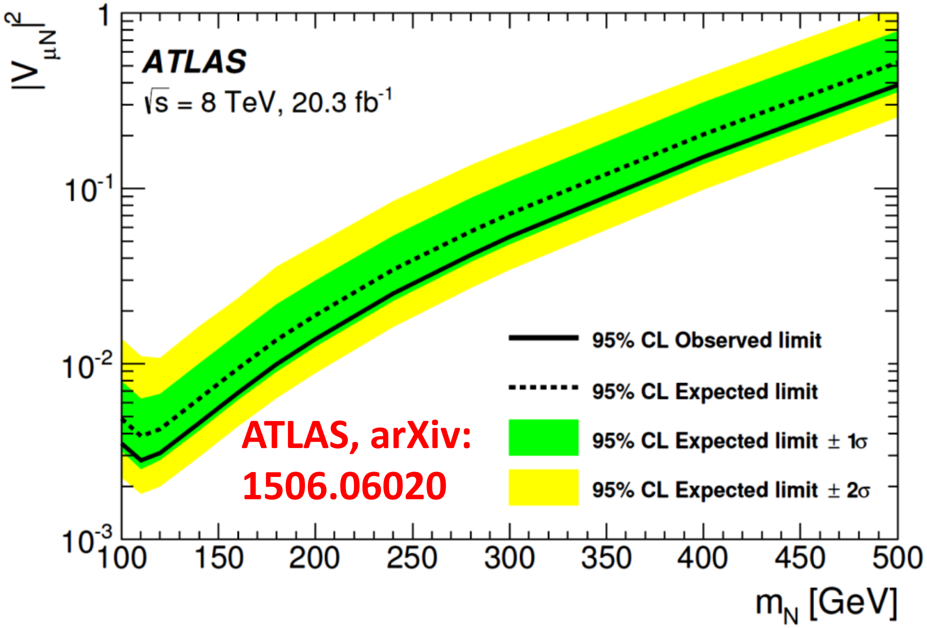
Han, Zhang, 06; Atre, Han, Pascoli, Zhang, 09



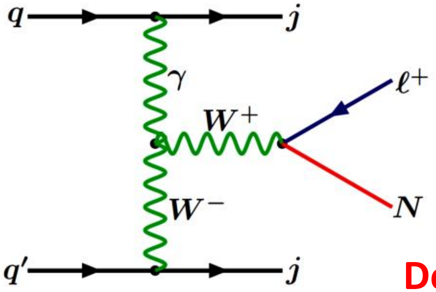
Signals:
same-sign
dileptons



CMS, arXiv: 1501.05566



ATLAS, arXiv: 1506.06020



For M_N > 600 GeV, t-channel γ-mediated production dominates over Drell-Yan process

Dev et al., 14; Alva et al., 16

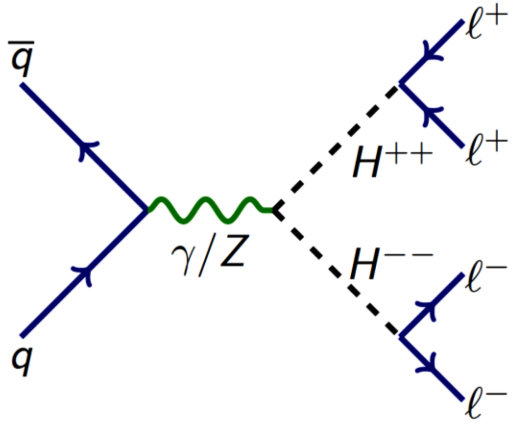
Type-II: 1207.2666 (CMS), 1412.0237 (ATLAS)

Type-III: 1506.01291 (CMS), 1506.01839 (ATLAS)

Test of Seesaw Models

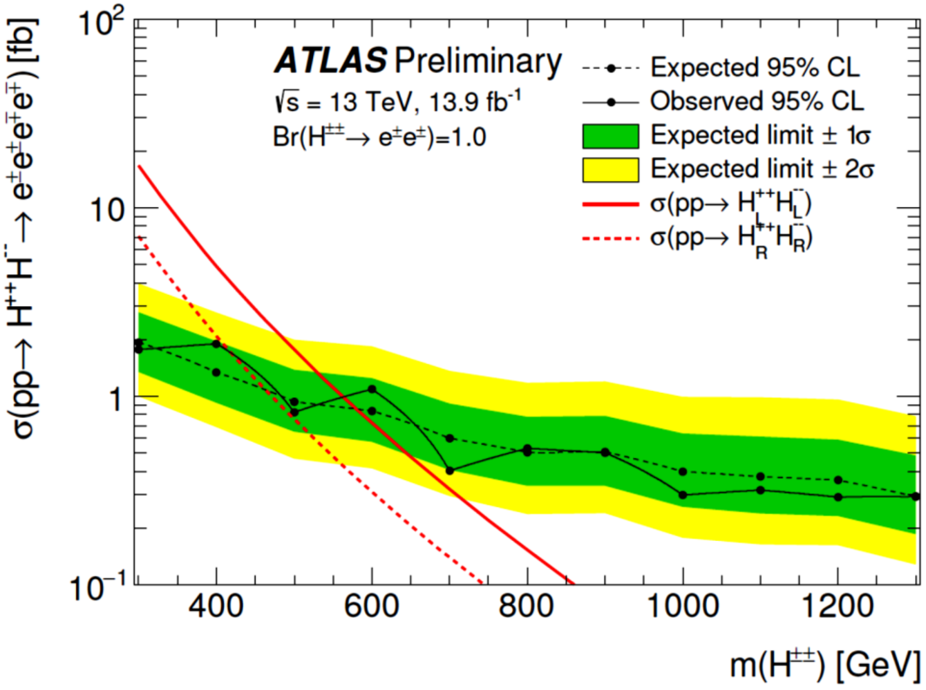
Searches for doubly-charged Higgs bosons

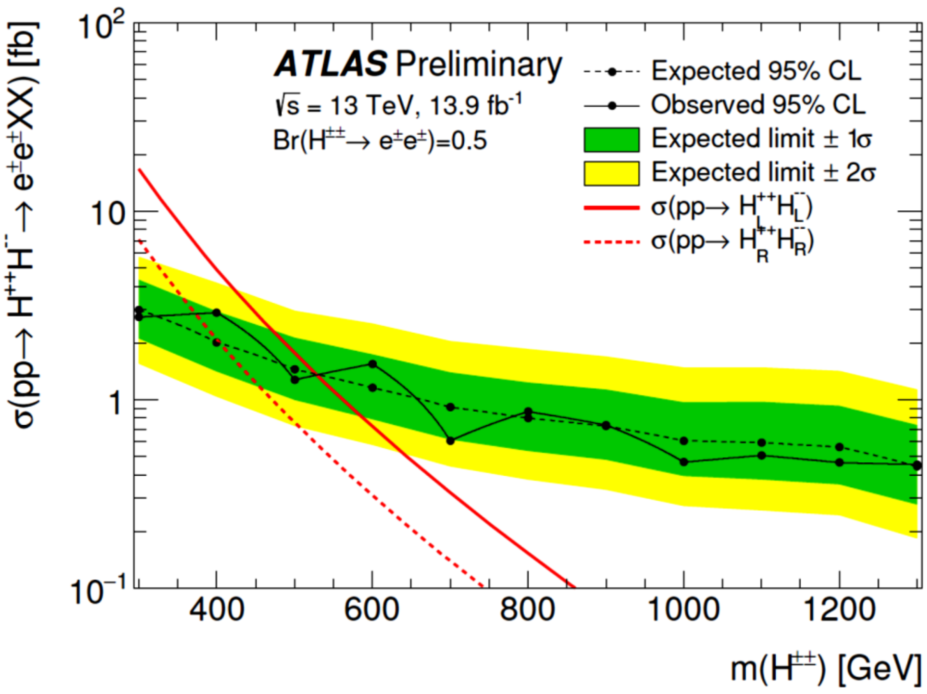
Chun et al., 03; Han et al., 05; Raidal et al., 07; Perez et al., 08; Chao et al., 08; Z.L. Han, R. Ding, Y. Liao, 12, 15



- Depending on the triplet vev, the dominant decay channel is either leptons or W's
- Couplings directly related to neutrino masses and flavor mixing parameters
- Current constants on masses depend on branching ratios of doubly-charged Higgs decays

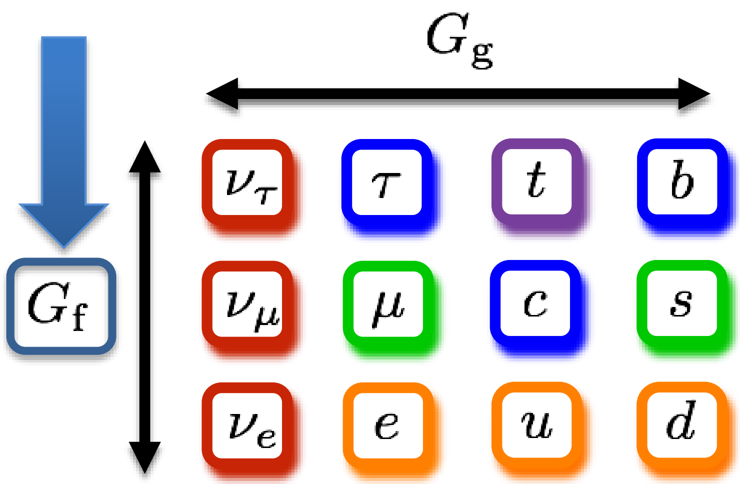
ATLAS-CONF-2016-051





Lepton Flavor Structure: Symmetries

Flavor Symmetry

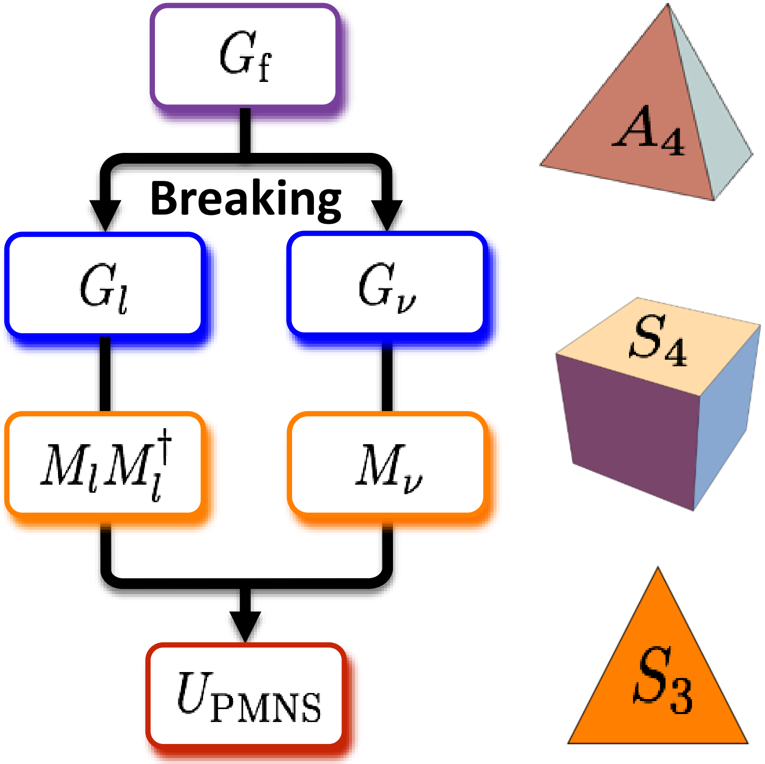


Tri-bimaximal neutrino mixing matrix

Harrison, Pekins, Scott, 02; Xing, 02; He, Zee, 03

$$V_0 = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Paradigm of flavor symmetries



PMNS matrix is (partially) determined by the structure of symmetry groups

See, Ishimori et al., 10; Altarelli, Feruglio, 10; King et al., 14, King, 18; Xing, 19, for recent reviews

Lepton Flavor Structure: Symmetries

Allowed ranges of PMNS matrix elements (@ 3σ)

$$\begin{pmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ |U_{\mu1}| & |U_{\mu2}| & |U_{\mu3}| \\ |U_{\tau1}| & |U_{\tau2}| & |U_{\tau3}| \end{pmatrix} = \begin{pmatrix} 0.801 \to 0.845 & 0.514 \to 0.580 & 0.137 \to 0.158 \\ 0.225 \to 0.517 & 0.441 \to 0.699 & 0.614 \to 0.793 \\ 0.246 \to 0.529 & 0.464 \to 0.713 & 0.590 \to 0.776 \end{pmatrix}$$

In the standard parametrization:

Xing, Zhou, 08, 14; Xing, Luo, 14; Y.L. Zhou, 15
Xing, Zhao, Rept. Prog. Phys., 16, for a review

μ-τ symmetry $|U_{\mu i}| = |U_{\tau i}|$:

(1) $\theta_{23} = 45^o$ & $\theta_{13} = 0$ (excluded)

(2) $\theta_{23} = 45^o$ & $\delta = 90^o$ or 270^o (allowed)

Partial μ-τ symmetry $|U_{\mu 1}| = |U_{\tau 1}|$:

$\theta_{23} \neq 45^o$ & $\delta \approx 270^o$ (favored by NOνA)

μ-τ reflection symmetry

Harrison, Scott, 02, 04; Grimus, Lavoura, 04

$$M_\nu = \begin{pmatrix} A & B & B^* \\ B & C & D \\ B^* & D & C^* \end{pmatrix}$$

Invariant under: $\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \to \begin{pmatrix} \nu_e^c \\ \nu_\tau^c \\ \nu_\mu^c \end{pmatrix}$

Predictions: $\theta_{23} = 45^\circ$, $\delta = 90^\circ$ or 270°, but θ_{12} and θ_{13} are left arbitrary

Lepton Flavor Structure: Texture Zeros

Two-zero Textures of M_ν

Frampton, Glashow, Marfatia, 02;
Xing, 02; Fritzsch, Xing, Zhou, 11

A₁ $\begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}$

A₂ $\begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix}$

B₁ $\begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}$

B₂ $\begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}$

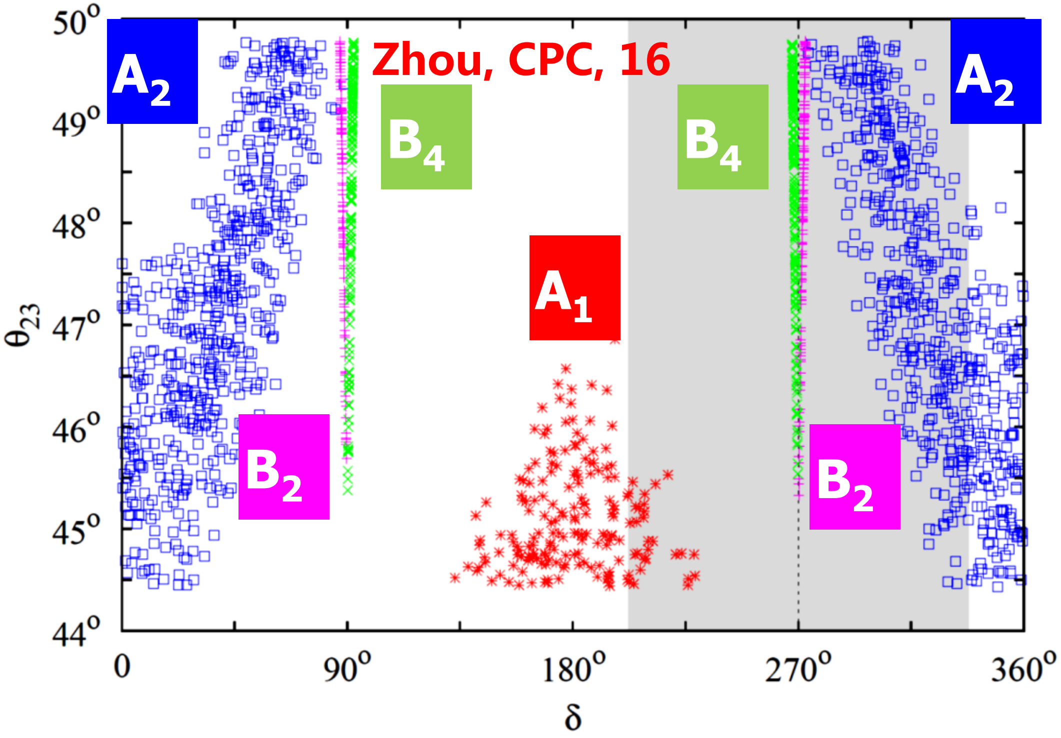
B₃ $\begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}$

B₄ $\begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & 0 \end{pmatrix}$

C $\begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}$

Consistent with nonzero θ_{13} & can be realized by A_4 symmetry

(a) NH; (b) $\theta_{23} > 45^\circ$; (c) $\delta \sim 270^\circ$

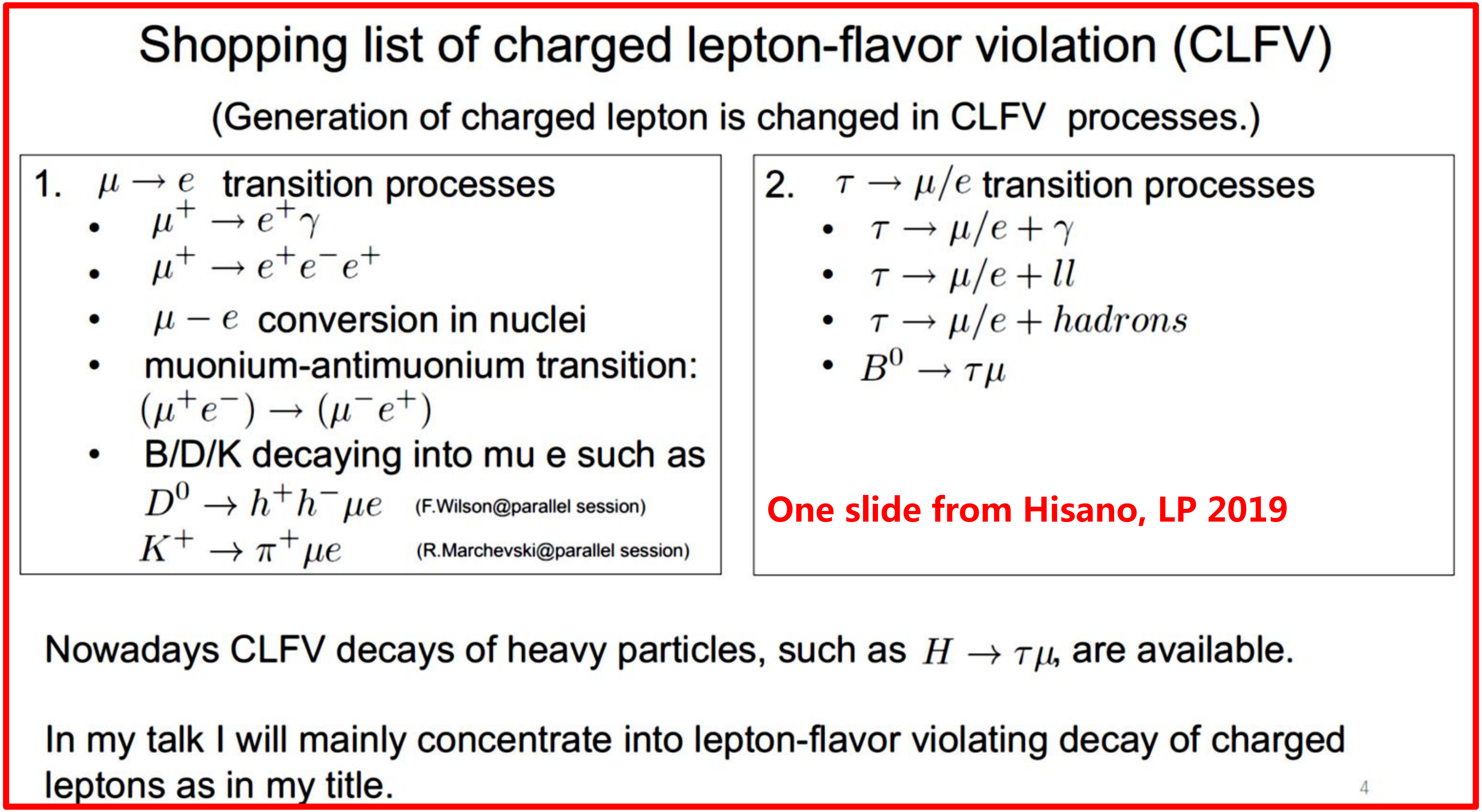


Model building in the type-I+II seesaw model

$l_{\alpha L}$	$E_{\alpha L}$	N_R	Φ_i	φ, ϕ	Δ
1,1′,1″	3	1	3	1,1′	1

$$M_\nu = u \begin{pmatrix} 0 & 0 & a_\Delta \\ 0 & b_\Delta & 0 \\ a_\Delta & 0 & 0 \end{pmatrix} - \frac{v^2}{M} \begin{pmatrix} a_\nu^2 & 0 & 0 \\ 0 & 0 & b_\nu c_\nu \\ 0 & b_\nu c_\nu & 0 \end{pmatrix}$$

Lepton Flavor Structure: CLFV



Shopping list of charged lepton-flavor violation (CLFV)

(Generation of charged lepton is changed in CLFV processes.)

1. $\mu \rightarrow e$ transition processes
   - $\mu^+ \rightarrow e^+\gamma$
   - $\mu^+ \rightarrow e^+e^-e^+$
   - $\mu - e$ conversion in nuclei
   - muonium-antimuonium transition: $(\mu^+e^-) \rightarrow (\mu^-e^+)$
   - B/D/K decaying into mu e such as
     $D^0 \rightarrow h^+h^-\mu e$ (F.Wilson@parallel session)
     $K^+ \rightarrow \pi^+\mu e$ (R.Marchevski@parallel session)

2. $\tau \rightarrow \mu/e$ transition processes
   - $\tau \rightarrow \mu/e + \gamma$
   - $\tau \rightarrow \mu/e + ll$
   - $\tau \rightarrow \mu/e + hadrons$
   - $B^0 \rightarrow \tau\mu$

Nowadays CLFV decays of heavy particles, such as $H \rightarrow \tau\mu$, are available.

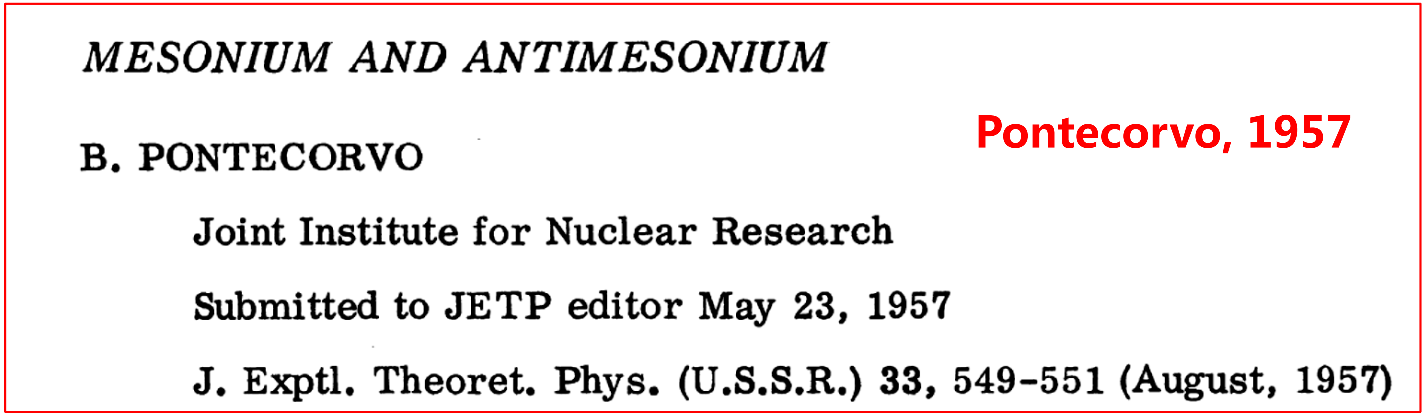
In my talk I will mainly concentrate into lepton-flavor violating decay of charged leptons as in my title.

4

One slide from Hisano, LP 2019

- **CLFV as natural as neutrino oscillations in neutrino mass models**
- **Necessary to probe lepton flavor structure/Complementary info.**
- **Pay more attention to muonium-antimuonium conversion**

History of Theoretical Studies



MESONIUM AND ANTIMESONIUM

B. PONTECORVO

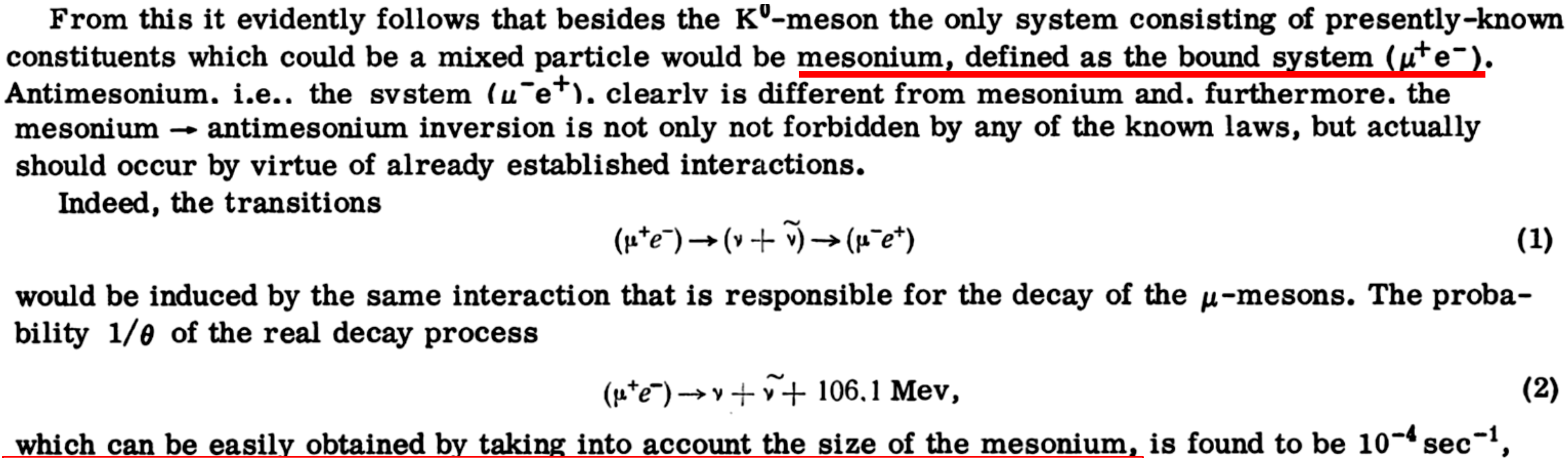
Joint Institute for Nuclear Research

Submitted to JETP editor May 23, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 549-551 (August, 1957)

Pontecorvo, 1957

Бруно Понтекорво



From this it evidently follows that besides the K^0-meson the only system consisting of presently-known constituents which could be a mixed particle would be mesonium, defined as the bound system (μ^+e^-). Antimesonium. i.e.. the svstem (μ^-e^+). clearlv is different from mesonium and. furthermore. the mesonium → antimesonium inversion is not only not forbidden by any of the known laws, but actually should occur by virtue of already established interactions.

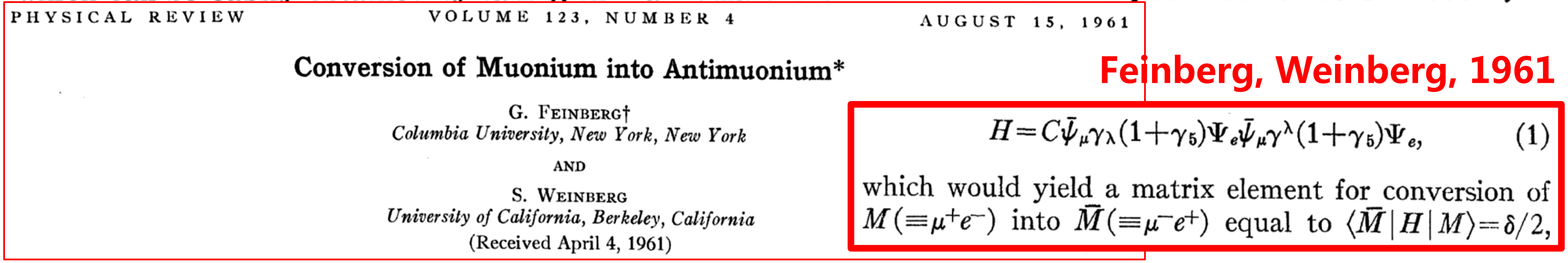
Indeed, the transitions

$$(\mu^+e^-)\rightarrow(\nu+\tilde{\nu})\rightarrow(\mu^-e^+) \tag{1}$$

would be induced by the same interaction that is responsible for the decay of the μ-mesons. The probability $1/\theta$ of the real decay process

$$(\mu^+e^-)\rightarrow\nu+\tilde{\nu}+106.1\ \text{Mev}, \tag{2}$$

which can be easily obtained by taking into account the size of the mesonium, is found to be $10^{-4}\ \text{sec}^{-1}$,



PHYSICAL REVIEW VOLUME 123, NUMBER 4 AUGUST 15, 1961

Conversion of Muonium into Antimuonium*

G. Feinberg†
Columbia University, New York, New York

AND

S. Weinberg
University of California, Berkeley, California

(Received April 4, 1961)

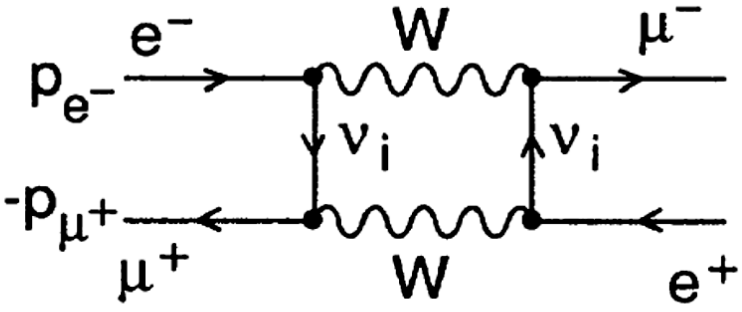
Feinberg, Weinberg, 1961

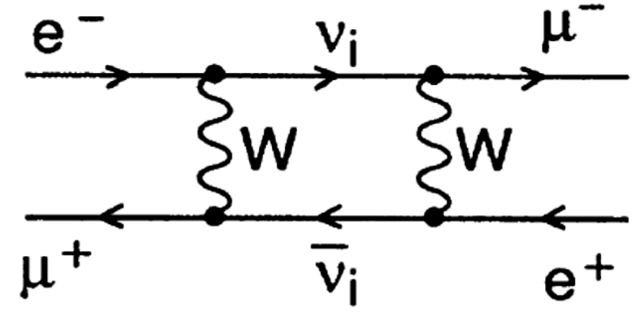
$$H=C\bar{\Psi}_\mu\gamma_\lambda(1+\gamma_5)\Psi_e\bar{\Psi}_\mu\gamma^\lambda(1+\gamma_5)\Psi_e, \tag{1}$$

which would yield a matrix element for conversion of $M(\equiv\mu^+e^-)$ into $\bar{M}(\equiv\mu^-e^+)$ equal to $\langle\bar{M}|H|M\rangle=\delta/2$,

History of Theoretical Studies

Extend the SM by three right-handed neutrinos: Dirac neutrino model





Effective Hamiltonian for M-$\bar{M}$ conversion

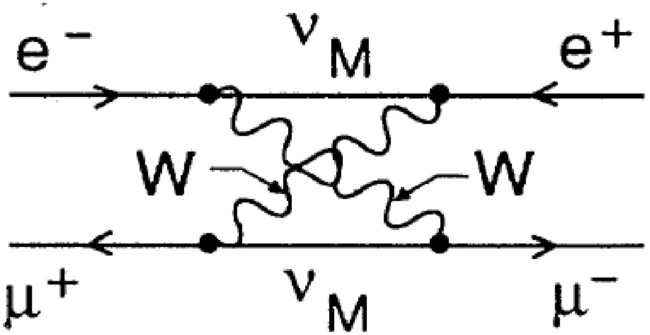
$$\mathcal{H}_{\text{eff}}=\frac{G_A}{\sqrt{2}}\bar{\psi}_\mu\gamma^\alpha(1+\gamma_5)\psi_e\bar{\psi}_\mu\gamma_\alpha(1+\gamma_5)\psi_e$$
$$+\frac{G_B}{\sqrt{2}}\bar{\psi}_\mu(1-\gamma_5)\psi_e\bar{\psi}_\mu(1-\gamma_5)\psi_e+\text{H.c.}$$

Theoretical predictions

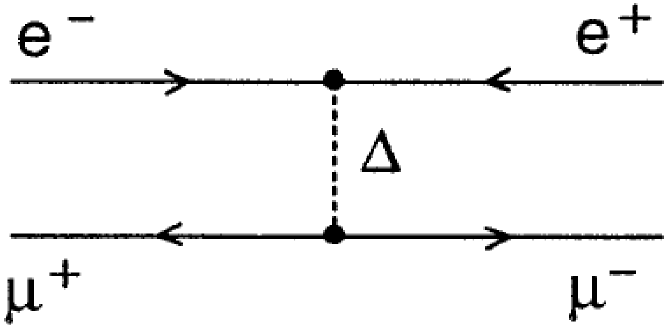
$$\boxed{G_A, G_B<\frac{G_F^2\sqrt{2}}{8\pi^2}m_\mu^2=7\times10^{-9}G_F}$$

Swartz, PRD, 1989 & refs therein

Massive Majorana neutrinos



Doubly-charged Higgs boson



Type-II Seesaw Models

Extend the SM by introducing a scalar triplet with Y = -2

$$\mathcal{L}_{\rm m} = -\frac{1}{2}\overline{\ell_{\alpha \rm L}}\left(Y_\Delta\right)_{\alpha\beta} i\sigma_2 \Delta \ell^{\rm C}_{\beta \rm L} + \text{h.c.} \qquad \Delta \equiv \sqrt{2}\begin{pmatrix}\Delta^-/\sqrt{2} & -\Delta^0\\ \Delta^{--} & -\Delta^-/\sqrt{2}\end{pmatrix}$$

The simplest scalar potential

$$V(H,\Delta) = -\mu^2 H^\dagger H + \lambda\left(H^\dagger H\right)^2 + \frac{1}{2}M_\Delta^2 \,\text{Tr}\left(\Delta^\dagger \Delta\right) - \left[\lambda_\Delta M_\Delta H^T i\sigma_2 \Delta H + \text{h.c.}\right]$$

After the spontaneous gauge symmetry breaking

$$v_\Delta = \lambda_\Delta \frac{v^2}{M_\Delta} \qquad \boxed{M_\nu = Y_\Delta v_\Delta = V\widehat{M}_\nu V^{\rm T}} \quad \Rightarrow \quad Y_\Delta = \frac{V\widehat{M}_\nu V^T}{v_\Delta}$$

$\Uparrow$

Completely reconstructed from neutrino oscillation data

The model receives constraints from precision electroweak data, g-2, LFV and direct searches from LHC (depending on M_Δ & v_Δ)

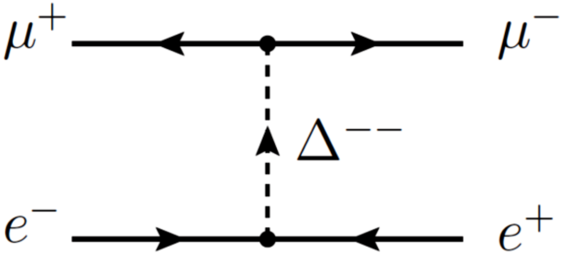
$v_\Delta < 10^{-4}$ GeV: $\Delta^{\pm\pm} \to l^\pm l^\pm$ $\qquad$ $v_\Delta > 10^{-4}$ GeV: $\Delta^{\pm\pm} \to W^\pm W^\pm$

Type-II Seesaw Models

Precision electroweak data (e.g., ρ parameter): $v_\Delta < 1\ \mathrm{GeV}$

LFV ($\mu \to e\gamma / \mu \to eee / \mu N \to eN$): $M_\Delta v_\Delta > 100\ \mathrm{GeV} \cdot \mathrm{eV}$

Calculate the probability of M-$\bar{M}$ conversion **Li, Schmidt, 1907.06963**



$$H_{M\bar{M}} = \frac{(Y_\Delta)_{ee}(Y_\Delta)^*_{\mu\mu}}{2M_\Delta^2}[\bar{\mu}\gamma^\sigma P_{\mathrm{L}} e] \cdot [\bar{\mu}\gamma_\sigma P_{\mathrm{L}} e]$$

Without Magnetic field

$$\boxed{P_{M\bar{M}} = \sum_{i=1}^{4} \left|\langle \lambda_i^{\bar{M}} | H_{M\bar{M}} | \lambda_i^{M} \rangle\right|^2 / 2\gamma^2}$$

Muon decay Rate:

$$\gamma = G_F^2 m_\mu^5 / 192\pi^3$$

The constraint on the probability **Willmann *et al.*, PRL, 1999**

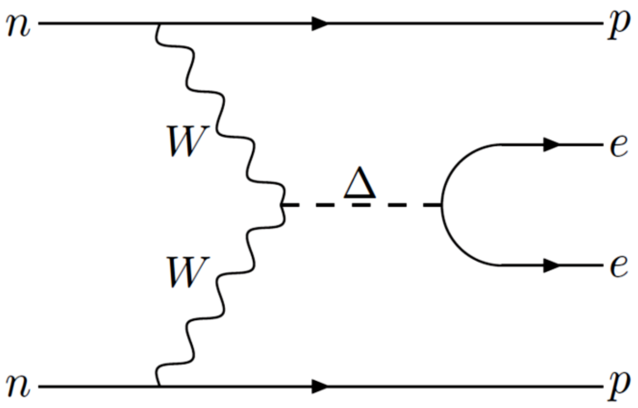
$$P_{M\bar{M}}(B = 0.1\ T) = 0.36\ P_{M\bar{M}} < 8.3 \times 10^{-11} \Rightarrow \boxed{\frac{|(Y_\Delta)_{ee}||(Y_\Delta)_{\mu\mu}|}{(M_\Delta^2/\mathrm{GeV}^2)} < 2.0 \times 10^{-7}}$$

Type-II Seesaw Models

Connection with neutrino mass matrix

$$\frac{|(Y_\Delta)_{ee}||(Y_\Delta)_{\mu\mu}|}{(M_\Delta^2/\mathrm{GeV}^2)} = \frac{|(M_\nu)_{ee}||(M_\nu)_{\mu\mu}|}{(M_\Delta^2/\mathrm{GeV}^2)\cdot v_\Delta^2} < 2.0\times 10^{-7}$$

0νββ decays dominated by light neutrinos for $M_\Delta > 100\ \mathrm{GeV}$



$$|(M_\nu)_{ee}| < 0.1\ \mathrm{eV}$$

Take $M_\Delta v_\Delta = 800\ \mathrm{GeV}\cdot\mathrm{eV}$ to evade the constraints

Come back to viable neutrino mass matrices

$$\mathbf{A_2}\begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix} \qquad \mathbf{B_2}\begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}$$

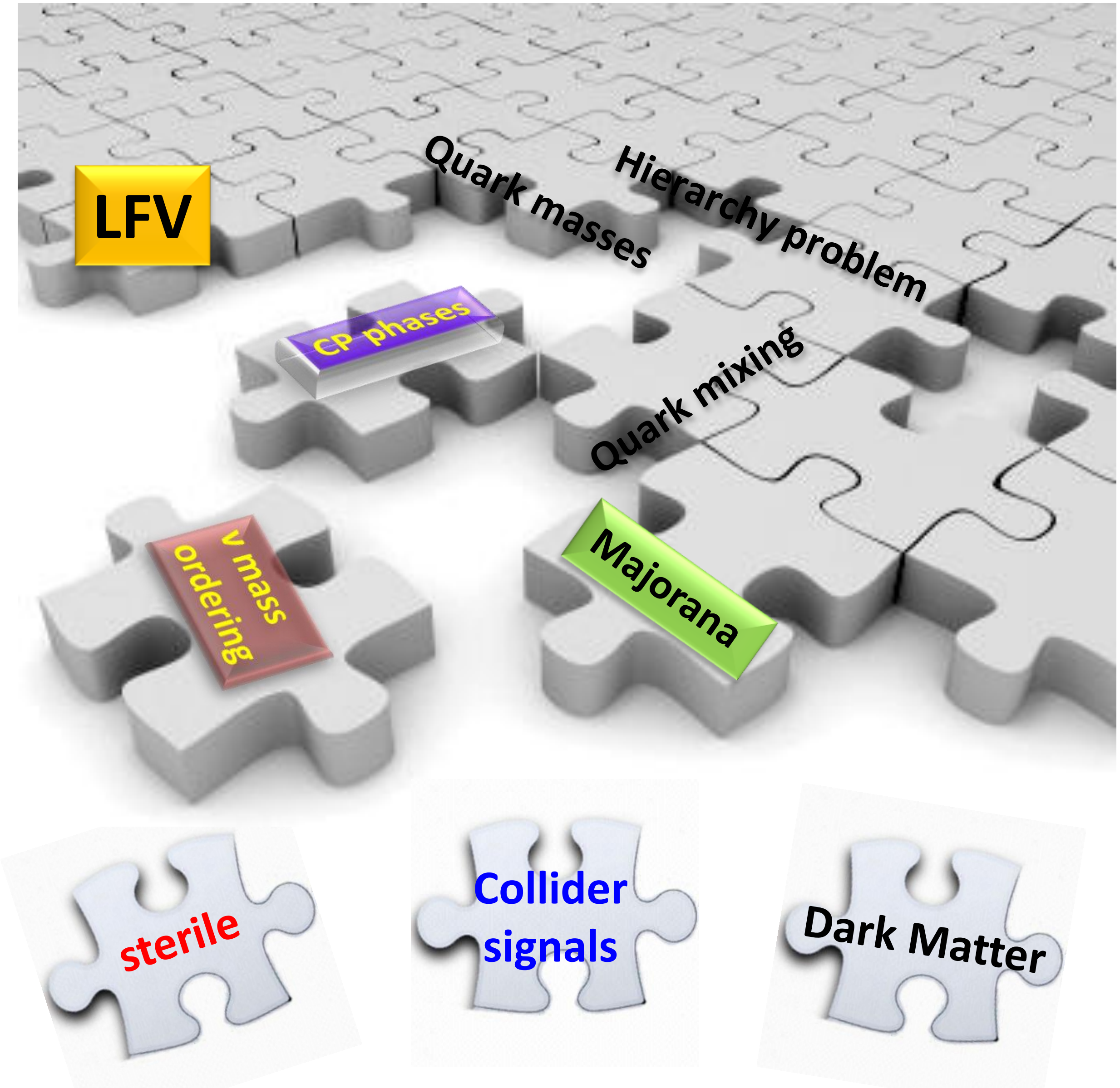
$$(Y_\Delta)_{ee}(Y_\Delta)_{\mu\mu} = 0 \qquad (Y_\Delta)_{ee}(Y_\Delta)_{\mu\mu} \neq 0$$

For two-zero texture B_2

$$\frac{|(M_\nu)_{ee}||(M_\nu)_{\mu\mu}|}{(M_\Delta^2/\mathrm{GeV}^2)\cdot v_\Delta^2} \approx 1.6\times 10^{-8}$$

Discovery may be around the corner (one order of magnitude below)!

Outlook



- Neutrino mass ordering and lepton CP violation will be measured in the oscillation experiments

- Possible to pin down the absolute neutrino mass and the Majorana nature of massive neutrinos

- LFV and LNV processes may shed light on the lepton flavor structure

- Future large hadron and lepton colliders will help us explore the origin of neutrino masses

A long way to go, but be optimistic that the future of neutrino physics is bright!